Introduction to Time and Frequency Metrology concepts

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May 23rd, 2022

The International System of Units (SI)

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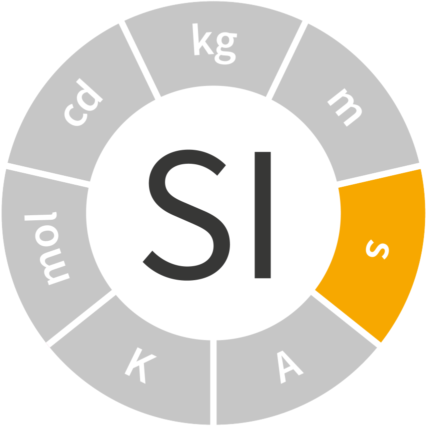


T & F in the International System (SI)

Second (s): Definition

The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{\mathrm{Cs}}$, the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.

Segundo (s): Definición (No oficial!)

El segundo, cuyo símbolo es s, es la unidad de tiempo del SI. Se lo define estableciendo el valor numérico fijo de la frecuencia del cesio $\Delta\nu_{\mathrm{Cs}}$, la frecuencia de la transición entre niveles hiperfinos del estado fundamental no perturbado del átomo de cesio-133, igual a 9 192 631 770 cuando es expresada en unidades de Hz, que es igual a s^{-1}

This definition implies the exact relation $\Delta\nu_{\mathrm{Cs}} = 9\,192\,631\,770\,\mathrm{Hz}$. Inverting this relation gives an expression for the unit second in terms of the defining constant $\Delta\nu_{\mathrm{Cs}}$:

$$1Hz = \frac{\Delta\nu_{\mathrm{Cs}}}{9192631770}$$

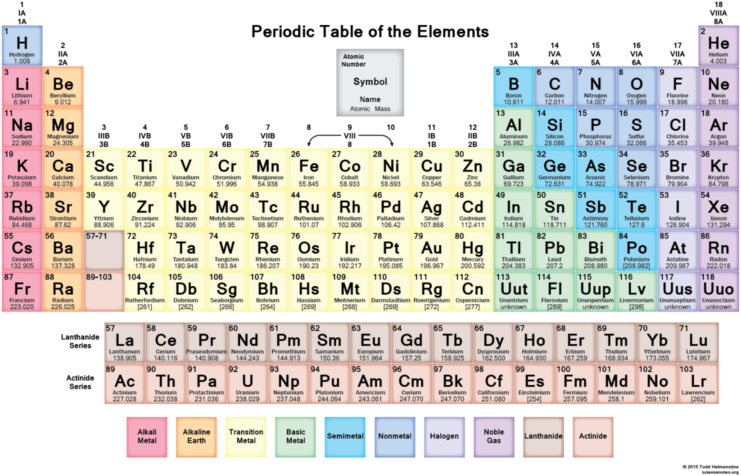
or

$$1s = \frac{9192631770}{\Delta\nu_{\mathrm{Cs}}}$$

The effect of this definition is that the second is equal to the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the unperturbed ground state of the ^{133}Cs atom.

`https://www.bipm.org/en/si-base-units/second`

The International System of Units (SI)



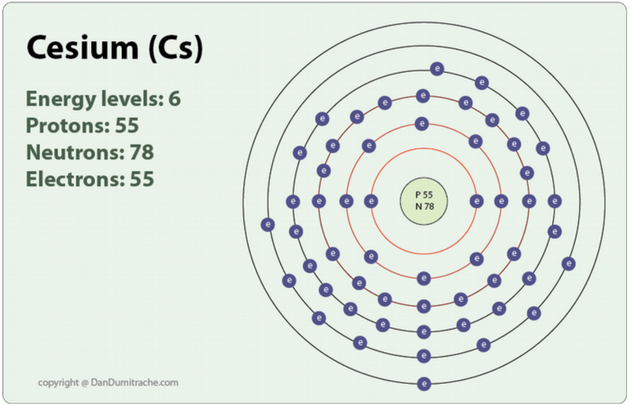
Periodic Table of the Elements

Atomic Number
Symbol
Name
Atomic Mass

1 IA 1A	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8	9 VIII 8	10	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A
1 H Hydrogen 1.008																	2 He Helium 4.003
3 Li Lithium 6.941	4 Be Beryllium 9.012											5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
11 Na Sodium 22.990	12 Mg Magnesium 24.305											13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.867	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.631	33 As Arsenic 74.922	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 84.798
37 Rb Rubidium 84.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.711	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.294
55 Cs Cesium 132.905	56 Ba Barium 137.328	57-71	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.085	79 Au Gold 196.967	80 Hg Mercury 200.592	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [208.982]	85 At Astatine 209.987	86 Rn Radon 222.018
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [268]	110 Ds Darmstadtium [269]	111 Rg Roentgenium [272]	112 Cn Copernicium [277]	113 Uut Ununtrium unknown	114 Fl Flerovium [289]	115 Uup Ununpentium unknown	116 Lv Livermorium [298]	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown

Lanthanide Series	57 La Lanthanum 138.905	58 Ce Cerium 140.116	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.243	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.500	67 Ho Holmium 164.930	68 Er Erbium 167.259	69 Tm Thulium 168.934	70 Yb Ytterbium 173.055	71 Lu Lutetium 174.967
Actinide Series	89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium [254]	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium [262]

The International System of Units (SI)



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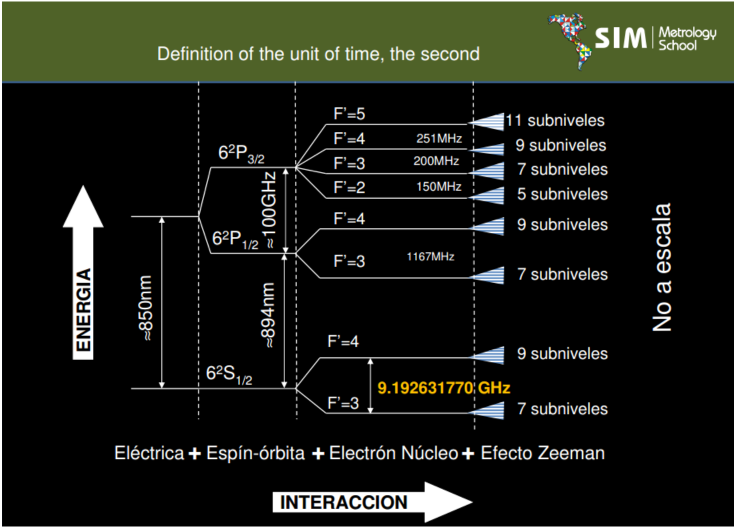

Definition of the unit of time, the second

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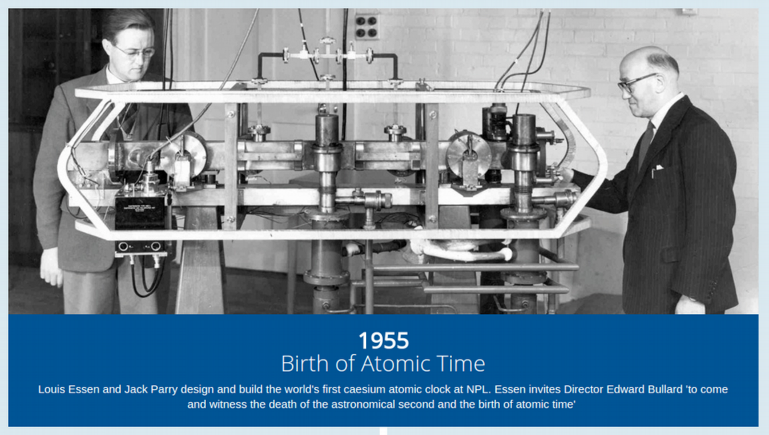
The International System of Units (SI)



1955

Birth of Atomic Time

Louis Essen and Jack Parry design and build the world's first caesium atomic clock at NPL. Essen invites Director Edward Bullard 'to come and witness the death of the astronomical second and the birth of atomic time'

The International System of Units (SI)

Time and frequency measurements play a fundamental role in basic reseach.

- Measurement of the fundamental constants (c, α, R) and their possibly time variation
- Test the validity of the special and general theory of relativity
- Very high accuracy spectroscopy
- Astronomy, Radio Astronomy and Astrophysics

The International System of Units (SI)

Time and frequency metrology is also crucial in telecommunication networks, navegation systems among other important technological applications

- Communication (5G)
- Satellite Navigation (GPS, GLONASS, GALILEO)
- Time Synchronization for World Financial Markets (High-Frequency Trading)

The International System of Units (SI)

Evolution of the accuracy of atomic clocks

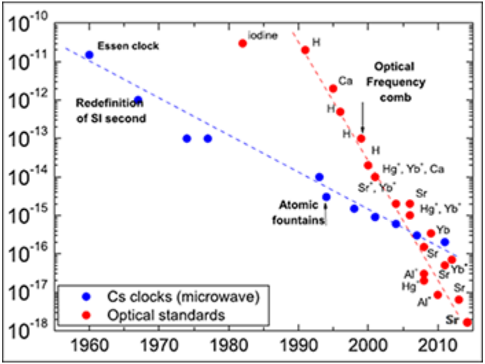


Figure : Evolution of the accuracy of atomic clocks

http://www.bipm.org/en/news/full-stories/2017-07-definition-second.html

T & F in the International System (SI)

Two units of measurement in the International System (SI) apply to time and frequency metrology

Second (s)

- The base unit for the quantity *time* is the second
- One of the 7 base SI units
- The symbol for second is s

Hertz (Hz)

- The derived unit for the quantity *frequency* is the hertz
- Defined as events per second
- One of 22 SI units derived from base units
- The symbol for hertz is Hz

Three basic types of time and frequency information

Date and Time-of-Day

The notation used to describe when an event occurred. *I was born on September 3, 1980 at 0800 UTC*

Time Interval

The duration between two events *The record in the 100 metres for men is 9.58 seconds.*

Frequency

The rate of a repetitive event *This device has to be calibrated once a year*

T & F in the International System (SI)

The units of time of day are defined as multiples of the SI second

- 1 minute = 60 seconds
- 1 hour = 60 minutes or 3600 seconds
- The hour and the minute are Non-SI units accepted for use with the SI
- The symbols are h and min. Remember: ~~hs~~ nor ~~mins~~

Hour and minutes are based on the sexagesimal (base 60) system that is around 4000 years old. Days are based on the duodecimal (base 12) system that is at least 3500 years old.

`https://www.bipm.org/en/publications/si-brochure/table6.html`

T & F in the International System (SI)

The units of time interval are defined as fractional parts of the SI second

- $\text{millisecond} = 1 \times 10^{-3}\,\mathrm{s}$
- $\text{microsecond} = 1 \times 10^{-6}\,\mathrm{s}$
- $\text{nanosecond} = 1 \times 10^{-9}\,\mathrm{s}$
- $\text{picosecond} = 1 \times 10^{-12}\,\mathrm{s}$

The sub-second units are all relatively new (within the last few hundred years) and all use the decimal (base 10) system.

T & F in the International System (SI)

The units of frequency are expressed in hertz, or in multiples of the hertz

- hertz (Hz) = one event or cycle per second
- kilohertz (kHz)$= 1 \times 10^3\,\mathrm{Hz}$
- megahertz (MHz)$= 1 \times 10^6\,\mathrm{Hz}$
- gigahertz (GHz) $= 1 \times 10^9\,\mathrm{Hz}$

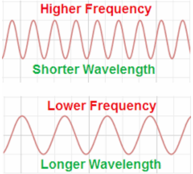
The relationship between frequency and time interval

We can measure frequency to get time interval, or we can measure time interval to get frequency. This is because frequency is the reciprocal of time interval:

$$f = \frac{1}{T} \tag{1}$$

Where T is the period of the signal in seconds and f is the frequency in hertz.

Wavelength

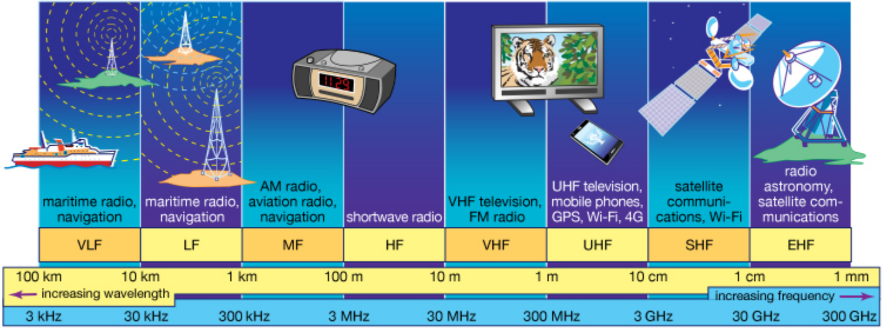


The wavelength is the length of one complete wave cycle, expressed in units of length

$$\lambda = \frac{c}{f} \tag{2}$$

- Where c is the speed of light: a constant of $299\,727\,738\,\mathrm{m\,s^{-1}}$
- To get λ in meters, it is common to use $\lambda = \frac{300}{f}$, where f is in MHz.
- Wavelength is mosly used in waves propagating in free space

Frequency bands



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Clocks and Oscillators

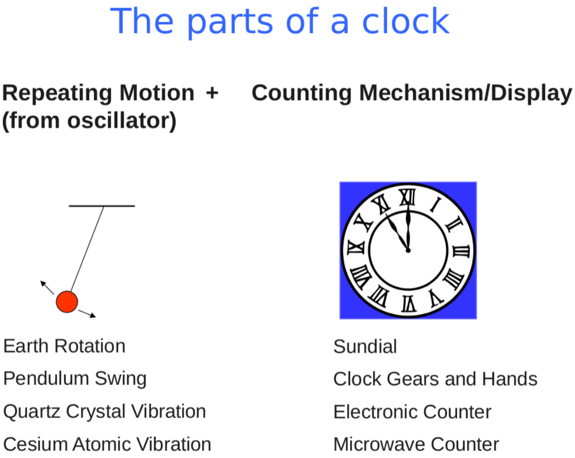
A clock counts cycles of a frequency and records units of time interval, such as seconds, minutes, hours, and days. A clock consists of an oscillator, a counter, and a display.

Oscillators are the heart of all clocks. They produce a periodic event that repeats at a nearly constant rate. This rate is called the resonance frequency. The best clocks contain the best oscillators.

Synchronization and Syntonization

- **Synchronization** is the process of setting two or more clocks to the same time.
- **Syntonization** is the process of setting two or more oscillators to the same frequency.

Inside a clock



The parts of a clock

Repeating Motion + (from oscillator)	**Counting Mechanism/Display**
Earth Rotation	Sundial
Pendulum Swing	Clock Gears and Hands
Quartz Crystal Vibration	Electronic Counter
Cesium Atomic Vibration	Microwave Counter

Inside a clock

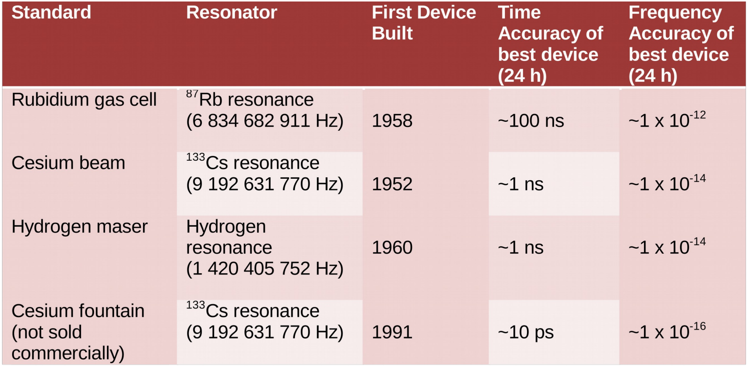
The words “clock” and “oscillator” are often used incorrectly by metrologists

To most people, a clock is a device that displays the time of day. It answers perhaps the world’s most common question: What time is it now?

Technically, an oscillator is the reference or “time base” for the ticks of a clock.

However, metrologists often refer to oscillators as clocks. Thus, you will probably hear the term “clock” in this meeting when we are referring to devices that produce frequency, but that do not always keep time or have a display.

Atomic clocks



Standard	Resonator	First Device Built	Time Accuracy of best device (24 h)	Frequency Accuracy of best device (24 h)
Rubidium gas cell	^{87}Rb resonance (6 834 682 911 Hz)	1958	~100 ns	$\sim 1 \times 10^{-12}$
Cesium beam	^{133}Cs resonance (9 192 631 770 Hz)	1952	~1 ns	$\sim 1 \times 10^{-14}$
Hydrogen maser	Hydrogen resonance (1 420 405 752 Hz)	1960	~1 ns	$\sim 1 \times 10^{-14}$
Cesium fountain (not sold commercially)	^{133}Cs resonance (9 192 631 770 Hz)	1991	~10 ps	$\sim 1 \times 10^{-16}$

Rubidium (f $\sim$ 6,8 GHz) is used as secondary representation of the second.
HTTP://WWW.BIPM.ORG/EN/PUBLICATIONS/MISES-EN-PRATIQUE/STANDARD-FREQUENCIES.HTML

Stability and accuracy

Stability

- In T & F, it indicates how well an oscillator can produce the same frequency over a given period of time. Stability doesn't indicate whether the time or frequency is "right" or "wrong", but only whether it stays the same.
- It is quantified by the Allan Variance

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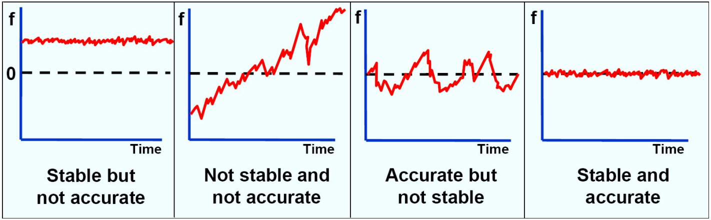
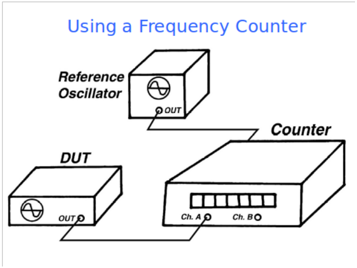
Accuracy

- Indicates how well an oscillator has been set on time or frequency
- Is normally expressed as a dimensionless number (unitless): $\frac{\Delta f}{f}$

Stability and accuracy



Accuracy



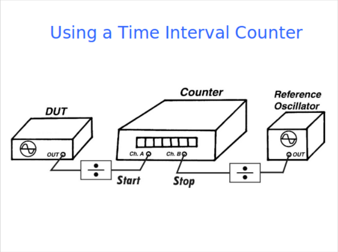
Accuracy evaluation

$$\frac{\Delta f}{f} = \frac{f_{measured} - f_{nominal}}{f_{nominal}}$$

- $f_{measured}$ is the reading of the counter
- $f_{nominal}$ is the nominal frequency of the oscillator under test (10 MHz, for example)

Accuracy in time domain

The same can be done if you measure time difference:



Using a Time Interval Counter

Accuracy evaluation in time domain

$$-\frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}$$

The quantity Δt is the phase change expressed in time units, estimated by the difference of two readings from a time interval counter or oscilloscope. T is the duration of the measurement, also expressed in time units.

Accuracy in time domain

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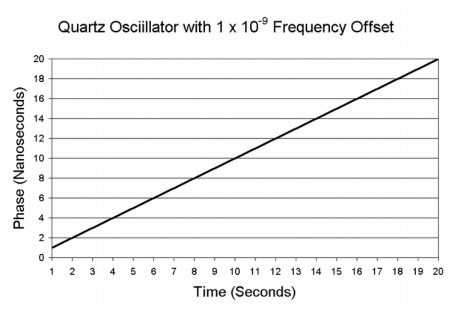
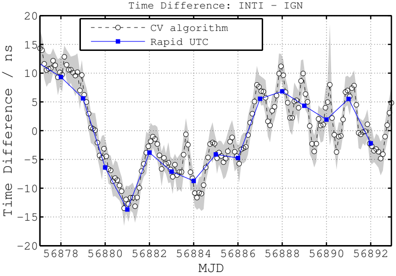


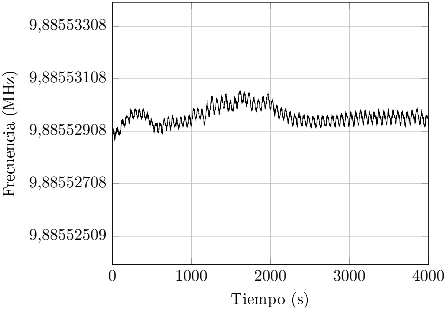
Accuracy evaluation in time domain

$$-\frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}$$

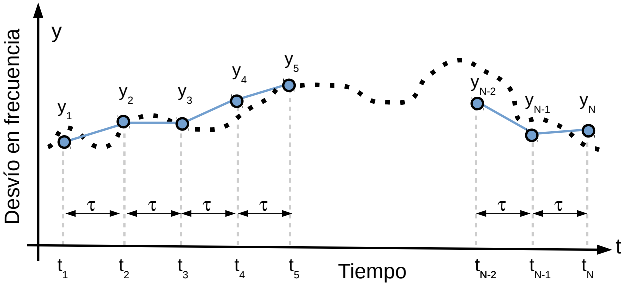
If you compute the slope of the Phase difference, you obtain $\frac{\Delta t}{T}$ in $\frac{ns}{s}$

Standard deviation cannot be used in T&F...





Allan Variance



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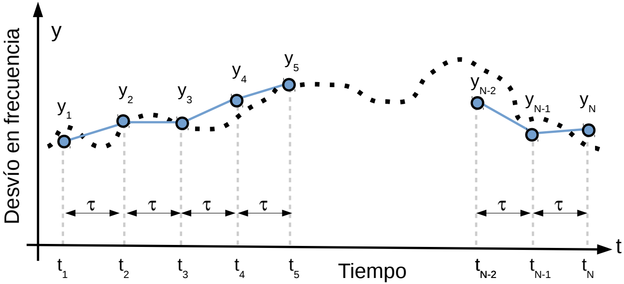
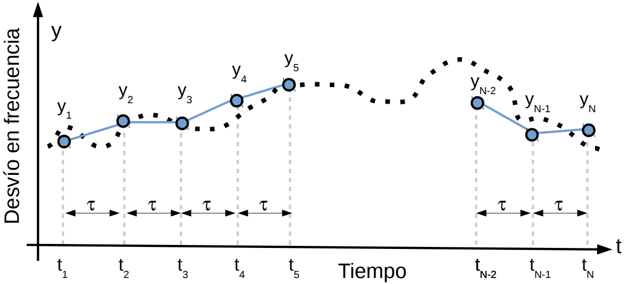


Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2}\langle(\Delta y)^2\rangle$

Allan Variance

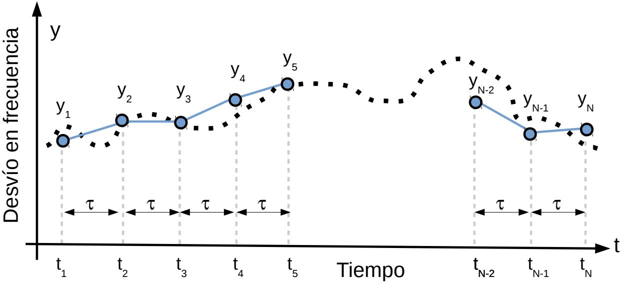


Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2}\langle(\Delta y)^2\rangle$

Estimator:

$$\hat{\sigma}_y^2(\tau) = \frac{1}{2N}\sum_{i=1}^{N}(\bar{y}_2 - \bar{y}_1)^2$$

Allan Variance



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$$\hat{\sigma}_y^2(\tau) = \frac{1}{2N}\sum_{i=1}^{N}(\bar{y}_2 - \bar{y}_1)^2$$

- $\bar{y}_2 - \bar{y}_1 \Rightarrow$ Normal Distribution
- $(\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_1^2$ Distribution
- $\sum_{i=1}^{N}(\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_N^2$ Distribution

Stability

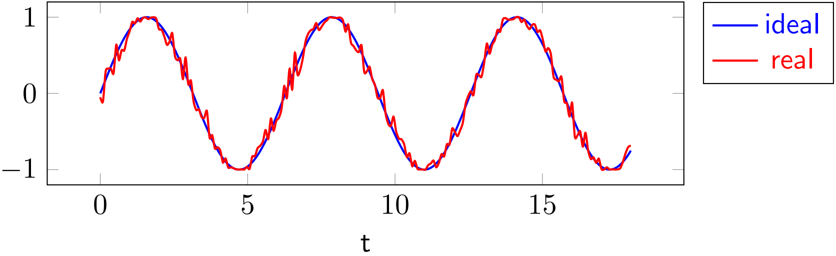
Ideal oscillator

$$V_o \cos(2\pi\nu_o t + \varphi)$$

Stability

Ideal oscillator

$$V_o \cos(2\pi\nu_o t + \varphi)$$



Stability

Ideal oscillator

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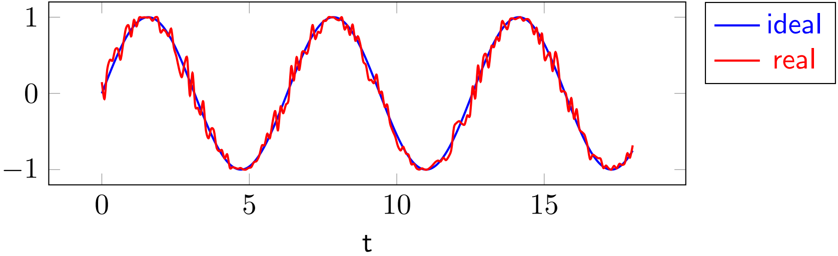

Real Oscillator model

$$[V_o + \varepsilon(t)] \cos(2\pi\nu_o t + \varphi(t));$$

$\varphi(t)$: phase noise

Stability

$$\boxed{V_o \cos(2\pi\nu_o t + \underline{\varphi(t)})}$$

Instant frequency: $\nu(t)$

$$\nu(t) = \nu_o + \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

Frequency deviation: $y(t)$

$$y(t) = \frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2\pi\nu_0}\frac{d\varphi(t)}{dt}$$

Stability

$$\boxed{V_o \cos(2\pi\nu_o t + \underline{\varphi(t)})}$$

Instant frequency: $\nu(t)$

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Frequency deviation: $y(t)$

$$y(t) = \frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2\pi\nu_0}\frac{d\varphi(t)}{dt}$$

$$\boldsymbol{x(t) \equiv \frac{\varphi(t)}{2\pi\nu_0} \Rightarrow y(t) = \frac{dx}{dt}}$$

Time dependence of Allan variance

From PSD to $\sigma_y(\tau)$

$$\sigma_y^2(\tau) = \int_0^\infty S_y(f)\left(2\frac{\sin^4(\pi\tau f)}{(\pi\tau f)^2}\right)df$$

Time dependence of Allan variance

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Power law noise model

$$S_y(f) = h_{-2} f^{-2} + h_{-1} f^{-1} + \underbrace{h_0 f^0}_{\text{White noise}} + h_1 f^1 + h_2 f^2$$

Time dependence of Allan variance

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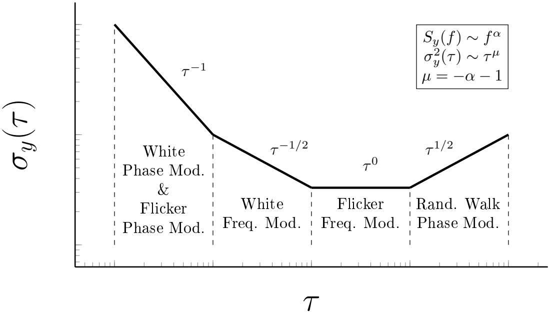
Slope relationships

$$S_y(f) \sim f^\alpha$$
$$\sigma_y^2(\tau) \sim \tau^{-\alpha-1}$$

Time dependence of Allan variance



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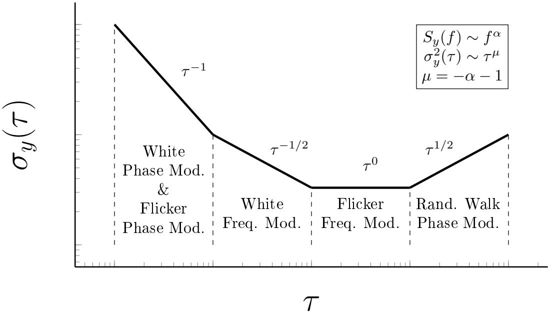
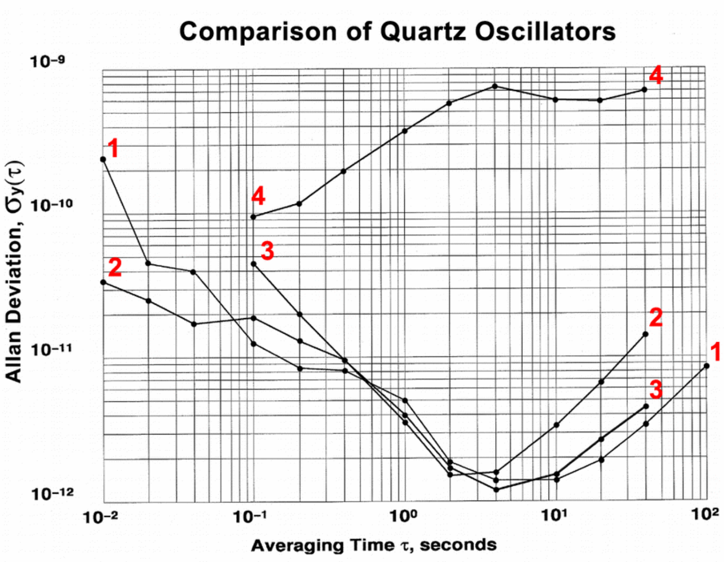


$S_y(f)$	$\sigma_y(\tau)$	**Noise Type**	**Origin**
$h_{-2}f^{-2}$	$\tau^{1/2}$	Random Walk F. Mod.	Ambient
$h_{-1}f^{-1}$	τ^{0}	Flicker Freq. Mod.	Resonator
h_0	$\tau^{-1/2}$	White Freq. Mod.	Thermal noise
$h_1 f^1$	τ^{-1}	Flicker Phase Mod.	Electric noise

Time dependence of Allan variance



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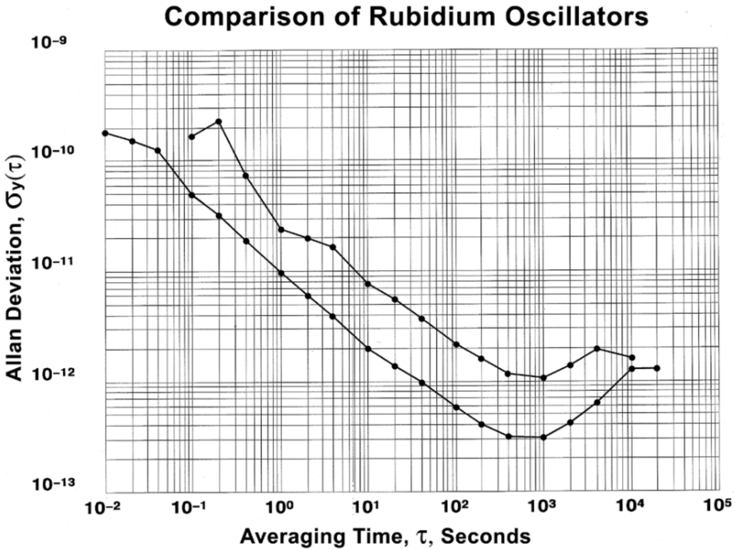

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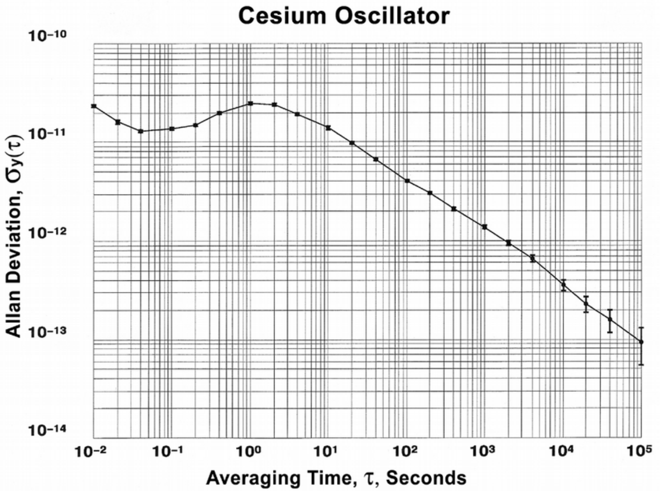
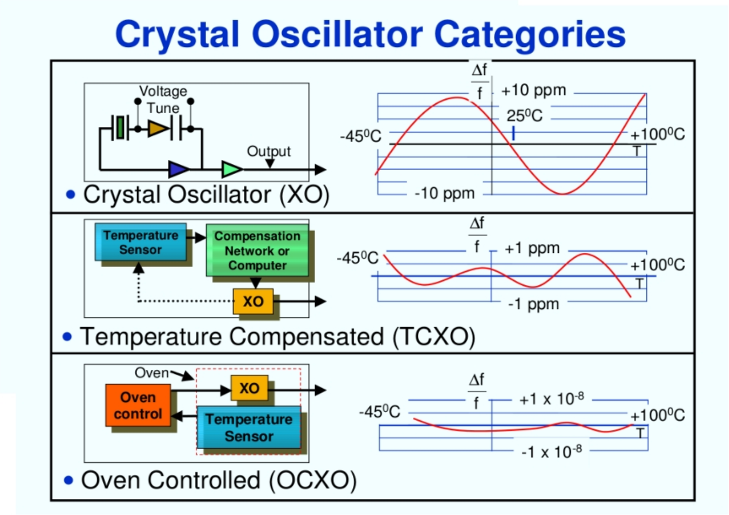


Tools for Allan variance calculation

- `http://www.stable32.com/`
- `http://www.alavar.org/`
- `https://www.mathworks.com/help/nav/ref/allanvar.html`
- `AMTyF ask for it to tiempo@cenam.mx`

Types of quartz oscillators



Crystal Oscillator Categories

- Crystal Oscillator (XO)
- Temperature Compensated (TCXO)
- Oven Controlled (OCXO)

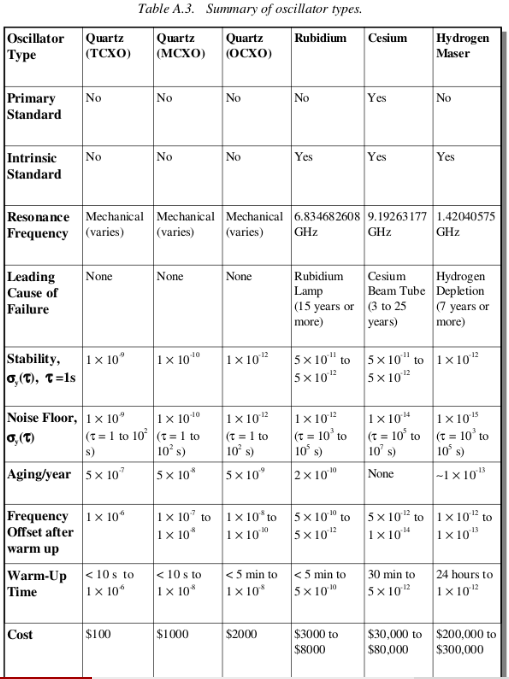


Table A.3. Summary of oscillator types.

Oscillator Type	**Quartz (TCXO)**	**Quartz (MCXO)**	**Quartz (OCXO)**	**Rubidium**	**Cesium**	**Hydrogen Maser**
Primary Standard	No	No	No	No	Yes	No
Intrinsic Standard	No	No	No	Yes	Yes	Yes
Resonance Frequency	Mechanical (varies)	Mechanical (varies)	Mechanical (varies)	6.834682608 GHz	9.19263177 GHz	1.42040575 GHz
Leading Cause of Failure	None	None	None	Rubidium Lamp (15 years or more)	Cesium Beam Tube (3 to 25 years)	Hydrogen Depletion (7 years or more)
Stability, $\sigma_y(\tau)$, τ=1s	1×10^{-9}	1×10^{-10}	1×10^{-12}	5×10^{-11} to 5×10^{-12}	5×10^{-11} to 5×10^{-12}	1×10^{-12}
Noise Floor, $\sigma_y(\tau)$	1×10^{-9} ($\tau = 1$ to 10^2 s)	1×10^{-10} ($\tau = 1$ to 10^2 s)	1×10^{-12} ($\tau = 1$ to 10^2 s)	1×10^{-12} ($\tau = 10^3$ to 10^5 s)	1×10^{-14} ($\tau = 10^5$ to 10^7 s)	1×10^{-15} ($\tau = 10^3$ to 10^5 s)
Aging/year	5×10^{-7}	5×10^{-8}	5×10^{-9}	2×10^{-10}	None	$\sim 1 \times 10^{-13}$
Frequency Offset after warm up	1×10^{-6}	1×10^{-7} to 1×10^{-8}	1×10^{-8} to 1×10^{-10}	5×10^{-10} to 5×10^{-12}	5×10^{-12} to 1×10^{-14}	1×10^{-12} to 1×10^{-13}
Warm-Up Time	< 10 s to 1×10^{-6}	< 10 s to 1×10^{-8}	< 5 min to 1×10^{-8}	< 5 min to 5×10^{-10}	30 min to 5×10^{-12}	24 hours to 1×10^{-12}
Cost	\$100	\$1000	\$2000	\$3000 to \$8000	\$30,000 to \$80,000	\$200,000 to \$300,000

References

1. Lombardi, M. A. (2008). Selecting a Primary Frequency Standard for a Calibration Laboratory. Cal Lab Magazine: The International Journal of Metrology, 33-39.
2. Riley, W. J. (2008). Handbook of Frequency Stability Analysis.–National Institute of Standards and Technology (NIST), US Department of Commerce. NIST Special Publication, 1065.
3. Lombardi, M. A., Novick, A. N., Neville-Neil, G., & Cooke, B. (2016). Accurate, traceable, and verifiable time synchronization for world financial markets. Journal of research of the NIST, 121, 436-463.

The End

" Never measure anything but frequency "

Arthur Schawlow, Nobel Prize in Physics 1981 "for his contribution to the development of laser spectroscopy"