

Introduction to Time and Frequency Metrology concepts

Diego Luna

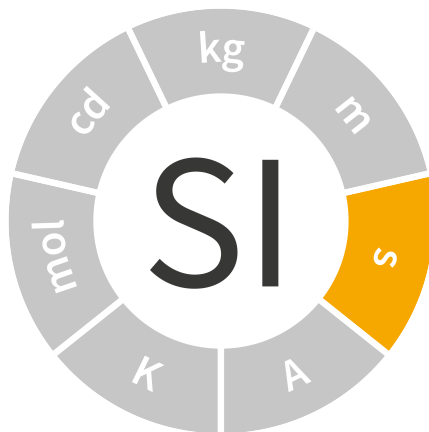


October 23, 2017

The International System of Units (SI)



The International System of Units (SI)



T & F in the International System (SI)

Second (s): Definition

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Segundo (s): Definición (No oficial!)

El segundo es la duración de 9 192 631 770 períodos de la radiación correspondiente a la transición entre dos niveles hiperfinos del estado fundamental del átomo de cesio 133

The International System of Units (SI)

Periodic Table of the Elements																																			
1 IA 1A		2 IIA 2A												13 IIIA 3A		14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A															
1 H Hydrogen 1.008																																			
3 Li Lithium 6.941		4 Be Beryllium 9.012												5 B Boron 10.811		6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180															
11 Na Sodium 22.990		12 Mg Magnesium 24.305		3 IIIB 3B		4 IVB 4B		5 VB 5B		6 VIB 6B		7 VIIB 7B		8 VIII 8		9 VIII 9		10 VIII 10		11 IB 1B		12 IIB 2B		13 Al Aluminum 26.982		14 Si Silicon 28.086		15 P Phosphorus 30.974		16 S Sulfur 32.065		17 Cl Chlorine 35.453		18 Ar Argon 39.948	
19 K Potassium 39.098		20 Ca Calcium 40.078		21 Sc Scandium 44.956		22 Ti Titanium 47.867		23 V Vanadium 50.942		24 Cr Chromium 51.996		25 Mn Manganese 54.938		26 Fe Iron 55.845		27 Co Cobalt 58.933		28 Ni Nickel 58.693		29 Cu Copper 63.546		30 Zn Zinc 65.38		31 Ga Gallium 69.723		32 Ge Germanium 72.631		33 As Arsenic 74.922		34 Se Selenium 78.971		35 Br Bromine 79.904		36 Kr Krypton 84.796	
37 Rb Rubidium 84.468		38 Sr Strontium 87.62		39 Y Yttrium 88.906		40 Zr Zirconium 91.224		41 Nb Niobium 92.906		42 Mo Molybdenum 95.95		43 Tc Technetium 98.907		44 Ru Ruthenium 101.07		45 Rh Rhodium 102.906		46 Pd Palladium 106.42		47 Ag Silver 107.868		48 Cd Cadmium 112.411		49 In Indium 114.818		50 Sn Tin 118.710		51 Sb Antimony 121.757		52 Te Tellurium 127.6		53 I Iodine 126.904		54 Xe Xenon 131.294	
55 Cs Cesium 132.905		56 Ba Barium 137.327		57-71 Lanthanide Series		72 Hf Hafnium 178.49		73 Ta Tantalum 180.948		74 W Tungsten 183.84		75 Re Rhenium 186.207		76 Os Osmium 190.23		77 Ir Iridium 192.22		78 Pt Platinum 195.085		79 Au Gold 196.967		80 Hg Mercury 200.592		81 Tl Thallium 204.383		82 Pb Lead 207.2		83 Bi Bismuth 208.980		84 Po Polonium [209]		85 At Astatine [210]		86 Rn Radon [222]	
87 Fr Francium [223]		88 Ra Radium [226]		89-103 Actinide Series		104 Rf Rutherfordium [261]		105 Db Dubnium [262]		106 Sg Seaborgium [266]		107 Bh Bohrium [264]		108 Hs Hassium [265]		109 Mt Meitnerium [268]		110 Ds Darmstadtium [271]		111 Rg Roentgenium [272]		112 Cn Copernicium [285]		113 Uut Ununtrium [288]		114 Fl Flerovium [289]		115 Uup Ununpentium [291]		116 Lv Livermorium [293]		117 Uus Ununseptium [294]		118 Uuo Ununoctium [294]	
Lanthanide Series				57 La Lanthanum 138.905		58 Ce Cerium 140.116		59 Pr Praseodymium 140.908		60 Nd Neodymium 144.243		61 Pm Promethium 144.913		62 Sm Samarium 150.36		63 Eu Europium 151.964		64 Gd Gadolinium 157.25		65 Tb Terbium 158.925		66 Dy Dysprosium 162.500		67 Ho Holmium 164.930		68 Er Erbium 167.259		69 Tm Thulium 168.934		70 Yb Ytterbium 173.055		71 Lu Lutetium 174.967			
Actinide Series				89 Ac Actinium 227.028		90 Th Thorium 232.038		91 Pa Protactinium 231.036		92 U Uranium 238.029		93 Np Neptunium 237.048		94 Pu Plutonium 244.064		95 Am Americium 243.061		96 Cm Curium 247.070		97 Bk Berkelium 247.070		98 Cf Californium 251.080		99 Es Einsteinium [254]		100 Fm Fermium [257]		101 Md Mendelevium [258]		102 No Nobelium [259]		103 Lr Lawrencium [262]			
Alkali Metal		Alkaline Earth		Transition Metal		Basic Metal		Semimetal		Nonmetal		Halogen		Noble Gas		Lanthanide		Actinide																	

The International System of Units (SI)

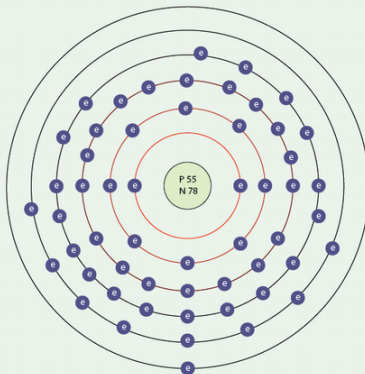
Cesium (Cs)

Energy levels: 6

Protons: 55

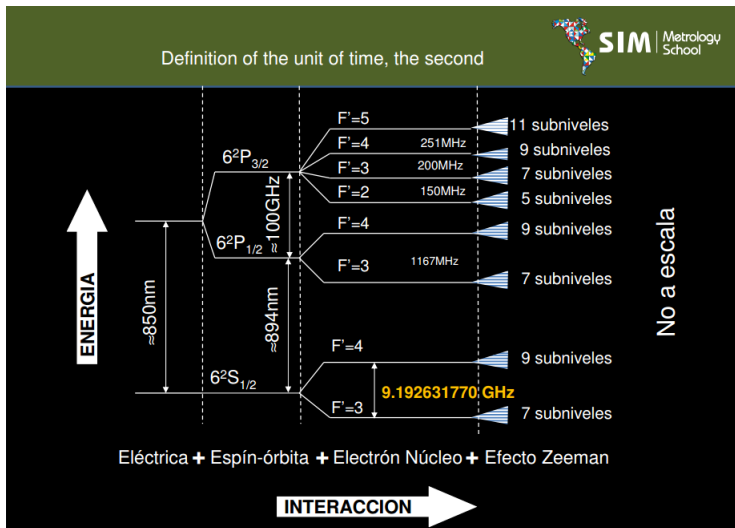
Neutrons: 78

Electrons: 55



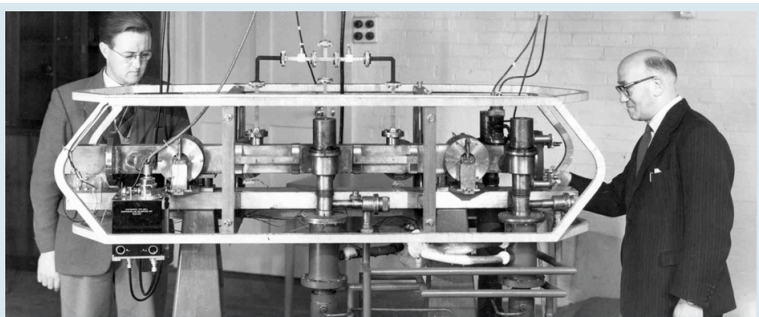
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The International System of Units (SI)



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The International System of Units (SI)



1955 Birth of Atomic Time

Louis Essen and Jack Parry design and build the world's first caesium atomic clock at NPL. Essen invites Director Edward Bullard 'to come and witness the death of the astronomical second and the birth of atomic time'

The International System of Units (SI)

Time and frequency measurements are very important on fundamental research.

Why do we need better and better clocks?

- Measurement of the fundamental constants (c , α , R) and their possibly time variation
- Test the validity of the special and general theory of relativity
- Very high accuracy spectroscopy
- Astronomy, Radio Astronomy and Astrophysics

The International System of Units (SI)

Time and frequency metrology is very important in telecommunication networks, navigation systems among other important technological applications

- Communication
- Satellite Navigation (GPS, GLONASS, GALILEO)
- Time Synchronization for World Financial Markets

The International System of Units (SI)

For these reasons, clock are getting better and better

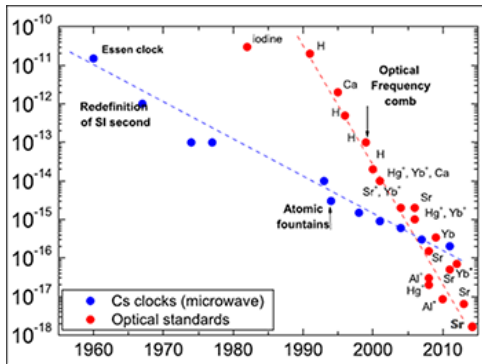


Figure : Evolution of the accuracy of atomic clocks

<http://www.bipm.org/en/news/full-stories/2017-07-definition-second.html>

T & F in the International System (SI)

Two units of measurement in the International System (SI) apply to time and frequency metrology

Second (s)

- The base unit for the quantity *time* is the second
- One of the 7 base SI units
- The symbol for second is s

Hertz (Hz)

- The derived unit for the quantity *frequency* is the hertz
- Defined as events per second
- One of 21 SI units derived from base units
- The symbol for hertz is Hz

Three basic types of time and frequency information

Date and Time-of-Day

The notation used to describe when an event occurred. *I was born on September 3, 1980 at 0800 UTC*

Time Interval

The duration between two events *The record in the 100 metres for men is 9.58 seconds.*

Frequency

The rate of a repetitive event *This device has to be calibrated once a year*

T & F in the International System (SI)

The units of time of day are defined as multiples of the SI second

- 1 minute = 60 seconds
- 1 hour = 60 minutes or 3600 seconds
- The hour and the minute are Non-SI units accepted for use with the SI
- The symbols are h and min. Remember: ~~hs~~ nor ~~mins~~

Hour and minutes are based on the sexagesimal (base 60) system that is around 4000 years old. Days are based on the duodecimal (base 12) system that is at least 3500 years old.

<https://www.bipm.org/en/publications/si-brochure/table6.html>

T & F in the International System (SI)

The units of time interval are defined as fractional parts of the SI second

- millisecond = $1 \times 10^{-3} \text{ s}$
- microsecond = $1 \times 10^{-6} \text{ s}$
- nanosecond = $1 \times 10^{-9} \text{ s}$
- picosecond = $1 \times 10^{-12} \text{ s}$

The sub-second units are all relatively new (within the last few hundred years) and all use the decimal (base 10) system.

T & F in the International System (SI)

The units of frequency are expressed in hertz, or in multiples of the hertz

- hertz (Hz) = one event or cycle per second
- kilohertz (kHz) = 1×10^3 Hz
- megahertz (MHz) = 1×10^6 Hz
- gigahertz (GHz) = 1×10^9 Hz

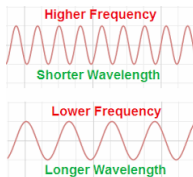
The relationship between frequency and time interval

We can measure frequency to get time interval, or we can measure time interval to get frequency. This is because frequency is the reciprocal of time interval:

$$f = \frac{1}{T} \quad (1)$$

Where T is the period of the signal in seconds and f is the frequency in hertz.

Wavelength

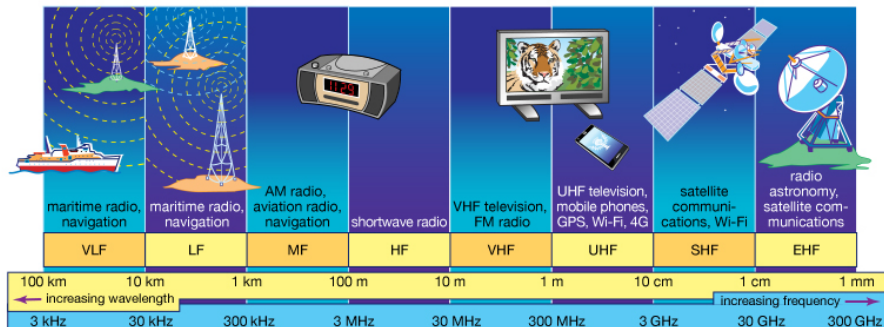


The wavelength is the length of one complete wave cycle, expressed in units of length

$$\lambda = \frac{c}{f} \quad (2)$$

- Where c is the speed of light: a constant of $299\,727\,738\,\text{m s}^{-1}$
- To get λ in meters, it is common to use $\lambda = \frac{300}{f}$, where f is in MHz.
- Wavelength is mostly used in waves propagating in free space

Frequency bands



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Clocks and Oscillators

A clock counts cycles of a frequency and records units of time interval, such as seconds, minutes, hours, and days. A clock consists of an oscillator, a counter, and a display.

A wristwatch is a good example of a typical clock. Most wristwatches contain a quartz oscillator that generates 32 768 cycles per second. After a watch counts 32 768 cycles, it records that one second has elapsed by updating its display.

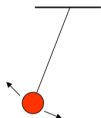
Oscillators are the heart of all clocks. They produce a periodic event that repeats at a nearly constant rate. This rate is called the resonance frequency. The best clocks contain the best oscillators.

Synchronization and Syntonization

- **Synchronization** is the process of setting two or more clocks to the same time.
- **Syntonization** is the process of setting two or more oscillators to the same frequency.

The parts of a clock

**Repeating Motion + Counting Mechanism/Display
(from oscillator)**



Earth Rotation

Pendulum Swing

Quartz Crystal Vibration

Cesium Atomic Vibration



Sundial

Clock Gears and Hands

Electronic Counter

Microwave Counter

Inside a clock

The words “clock” and “oscillator” are often used incorrectly by metrologists

To most people, a clock is a device that displays the time of day. It answers perhaps the world's most common question: What time is it now?

Technically, an oscillator is the reference or “time base” for the ticks of a clock.

However, metrologists often refer to oscillators as clocks. Thus, you will probably hear the term “clock” in this meeting when we are referring to devices that produce frequency, but that do not always keep time or have a display.



Frequencies and periods

- The frequency of the oscillator and the period of the TIC-TAC are related.

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- For example: A pendulum oscillates at 0,5 Hz and the clock generates a TIC each second.

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- For example: A pendulum oscillates at 0,5 Hz and the clock generates a TIC each second.
- Another example: A cesium clock counts 9 192 631 770 oscillations of the radiation associated to an atomic transition, to generate one TIC

Atomic clocks

Standard	Resonator	First Device Built	Time Accuracy of best device (24 h)	Frequency Accuracy of best device (24 h)
Rubidium gas cell	^{87}Rb resonance (6 834 682 911 Hz)	1958	~100 ns	$\sim 1 \times 10^{-12}$
Cesium beam	^{133}Cs resonance (9 192 631 770 Hz)	1952	~1 ns	$\sim 1 \times 10^{-14}$
Hydrogen maser	Hydrogen resonance (1 420 405 752 Hz)	1960	~1 ns	$\sim 1 \times 10^{-14}$
Cesium fountain (not sold commercially)	^{133}Cs resonance (9 192 631 770 Hz)	1991	~10 ps	$\sim 1 \times 10^{-16}$

Rubidium ($f \sim 6,8 \text{ GHz}$) is used as secondary representation of the second.

[HTTP://WWW.BIPM.ORG/EN/PUBLICATIONS/MISES-EN-PRATIQUE/STANDARD-FREQUENCIES.HTML](http://www.bipm.org/en/publications/mises-en-pratique/standard-frequencies.html)

Stability

- In T & F, it indicates how well an oscillator can produce the same frequency over a given period of time. Stability doesn't indicate whether the time or frequency is "right" or "wrong", but only whether it stays the same.
- It is quantified by the Allan Variance

Stability and accuracy

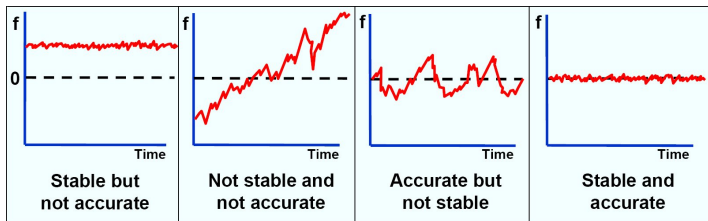
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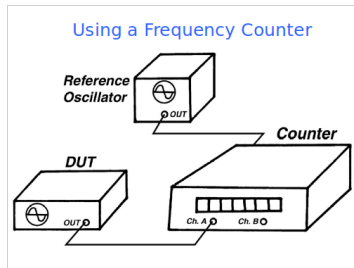
Accuracy

- Indicates how well an oscillator has been set on time or frequency
- Is normally expressed as a dimensionless number (unitless): $\frac{\Delta f}{f}$

Stability and accuracy



Accuracy



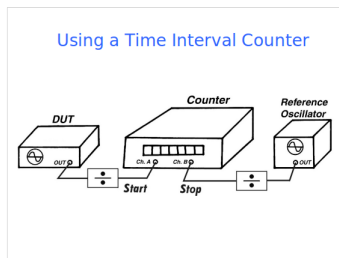
Accuracy evaluation

$$\frac{\Delta f}{f} = \frac{f_{\text{measured}} - f_{\text{nominal}}}{f_{\text{nominal}}}$$

- f_{measured} is the reading of the counter
- f_{nominal} is the nominal frequency of the oscillator under test (10 MHz, for example)

Accuracy in time domain

The same can be done if you measure time difference:



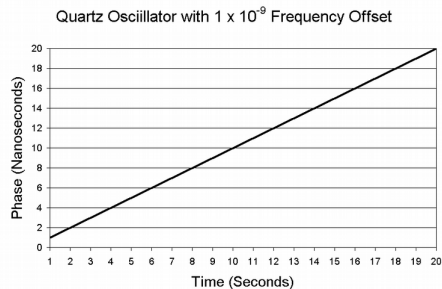
Accuracy evaluation in time domain

$$-\frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}$$

The quantity Δt is the phase change expressed in time units, estimated by the difference of two readings from a time interval counter or oscilloscope. T is the duration of the measurement, also expressed in time units.

Accuracy in time domain

The same can be done if you measure time difference:

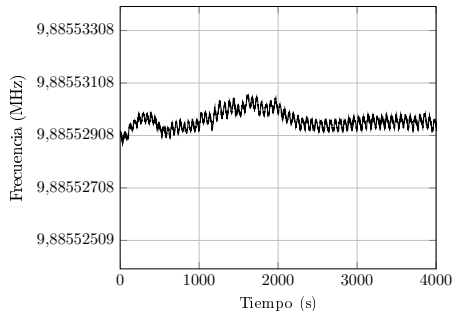
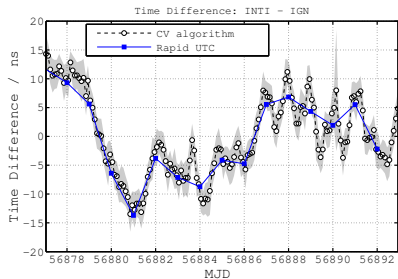


Accuracy evaluation in time domain

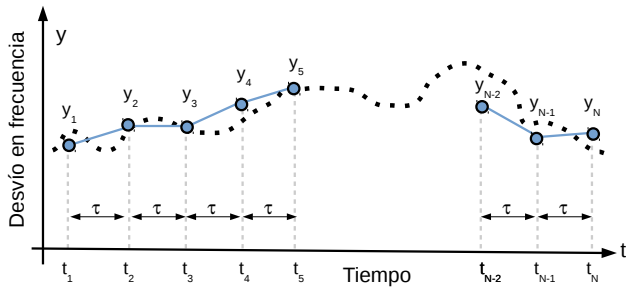
$$-\frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}$$

If you compute the slope of the Phase difference, you obtain $\frac{\Delta t}{T}$ in $\frac{ns}{s}$

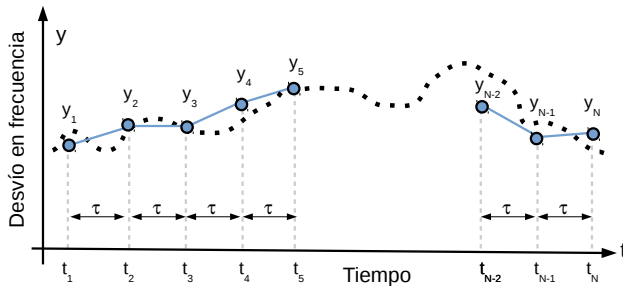
Standard deviation cannot be used in T&F...



Allan Variance

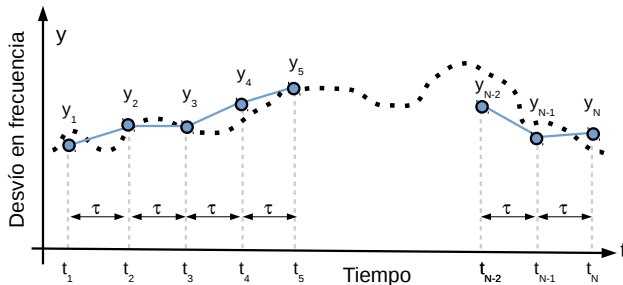


Allan Variance



Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2} \langle (\Delta y)^2 \rangle$

Allan Variance

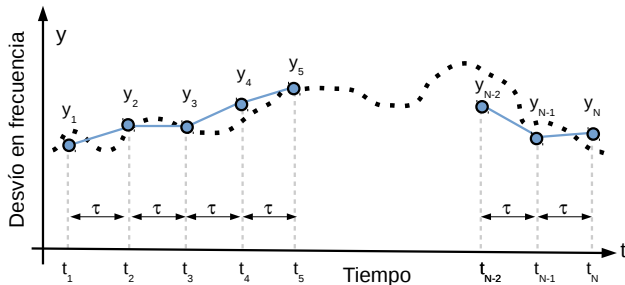


Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2} \langle (\Delta y)^2 \rangle$

Estimator:

$$\hat{\sigma}_y^2(\tau) = \frac{1}{2N} \sum_{i=1}^N (\bar{y}_2 - \bar{y}_1)^2$$

Allan Variance



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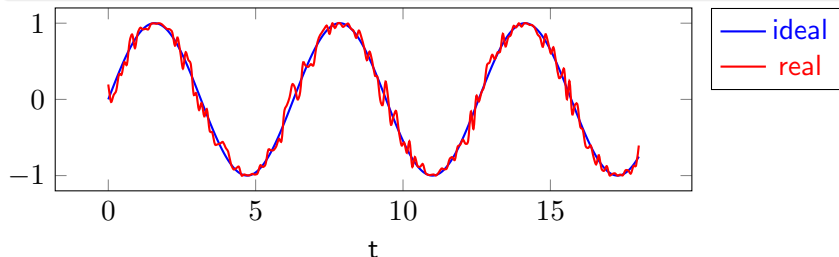
- $\bar{y}_2 - \bar{y}_1 \Rightarrow$ Normal Distribution
- $(\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_1^2$ Distribution
- $\sum_{i=1}^N (\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_N^2$ Distribution

Ideal oscillator

$$V_o \cos(2\pi\nu_o t + \varphi)$$

Ideal oscillator

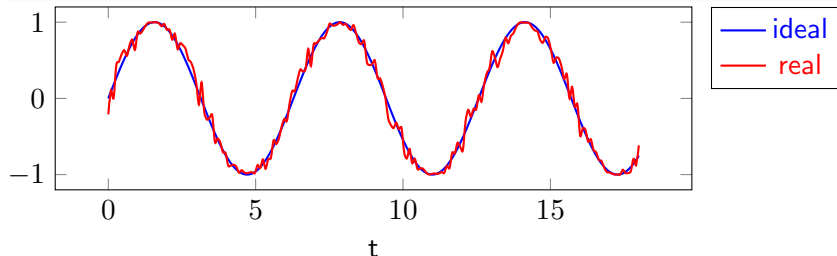
$$V_o \cos(2\pi\nu_o t + \varphi)$$



Stability

Ideal oscillator

$$V_o \cos(2\pi\nu_o t + \varphi)$$



Real Oscillator model

$$[V_o + \varepsilon(t)] \cos(2\pi\nu_o t + \varphi(t)); \quad \varphi(t) : \text{phase noise}$$

$$V_o \cos(2\pi\nu_o t + \underline{\varphi(t)})$$

Instant frequency: $\nu(t)$

$$\nu(t) = \nu_o + \frac{1}{2\pi} \frac{d\varphi(t)}{dt}$$

Frequency deviation: $y(t)$

$$y(t) = \frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2\pi\nu_0} \frac{d\varphi(t)}{dt}$$

$$V_o \cos(2\pi\nu_o t + \underline{\varphi(t)})$$

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$$\nu(t) = \nu_o + \frac{1}{2\pi} \frac{d\varphi(t)}{dt}$$

Frequency deviation: $y(t)$

$$y(t) = \frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2\pi\nu_0} \frac{d\varphi(t)}{dt}$$

$$x(t) \equiv \frac{\varphi(t)}{2\pi\nu_0} \Rightarrow y(t) = \frac{dx}{dt}$$

Time dependence of Allan variance

From PSD to $\sigma_y(\tau)$

$$\sigma_y^2(\tau) = \int_0^\infty S_y(f) \left(2 \frac{\sin^4(\pi \tau f)}{(\pi \tau f)^2} \right) df$$

Time dependence of Allan variance

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Power law noise model

$$S_y(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + \underbrace{h_0f^0}_{\text{White noise}} + h_1f^1 + h_2f^2$$

Time dependence of Allan variance

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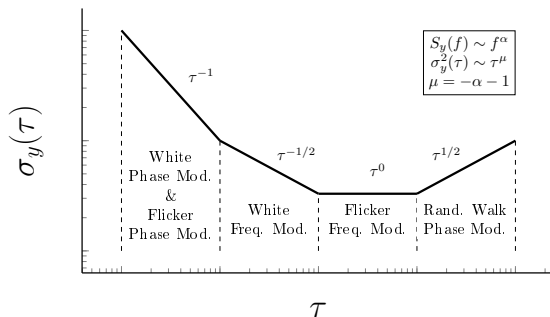
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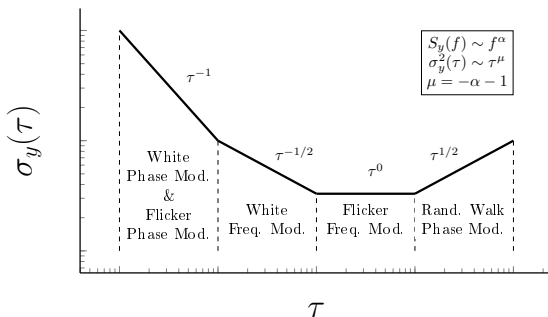
Slope relationships

$$\begin{aligned} S_y(f) &\sim f^\alpha \\ \sigma_y^2(\tau) &\sim \tau^{-\alpha-1} \end{aligned}$$

Time dependence of Allan variance

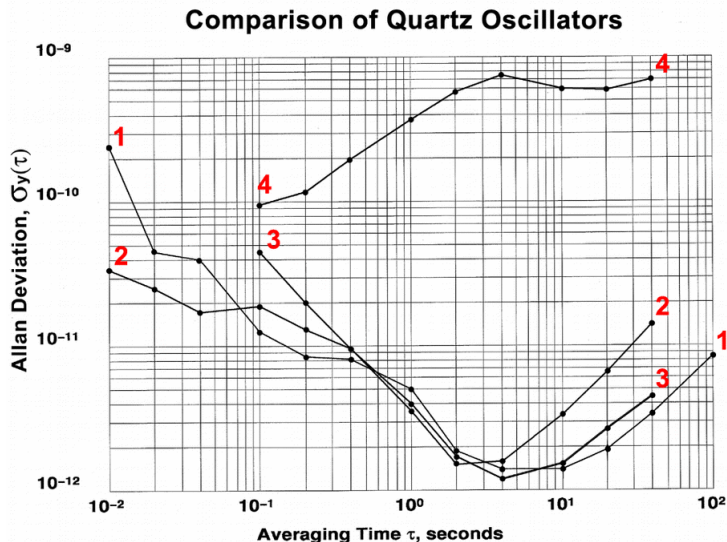


Time dependence of Allan variance

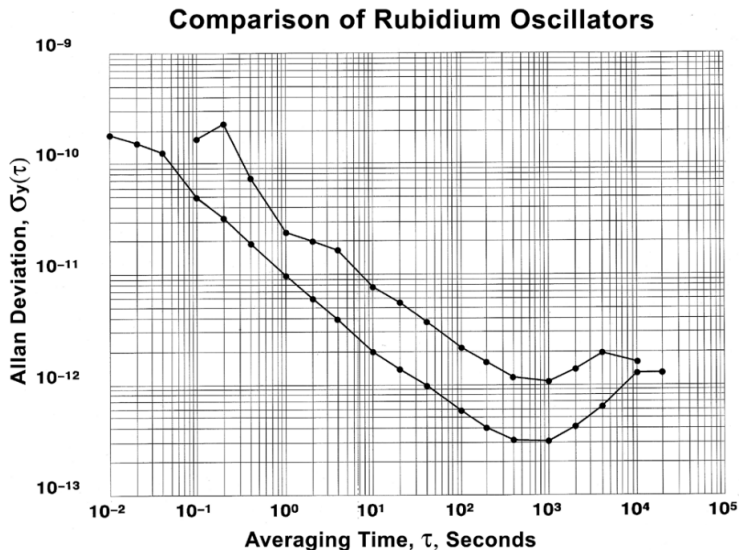


$S_y(f)$	$\sigma_y(\tau)$	Noise Type	Origin
$h_{-2}f^{-2}$	$\tau^{1/2}$	Random Walk F. Mod.	Ambient
$h_{-1}f^{-1}$	τ^0	Flicker Freq. Mod.	Resonator
h_0	$\tau^{-1/2}$	White Freq. Mod.	Thermal noise
h_1f^1	τ^{-1}	Flicker Phase Mod.	Electric noise

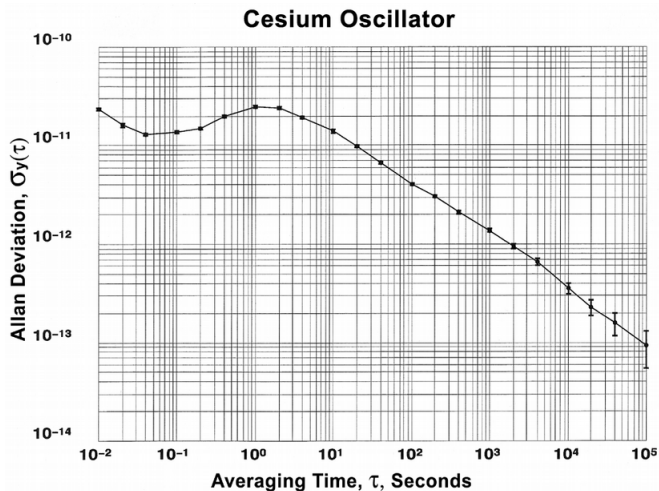
Time dependence of Allan variance



Time dependence of Allan variance



Time dependence of Allan variance

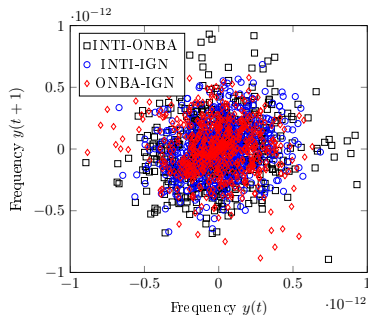
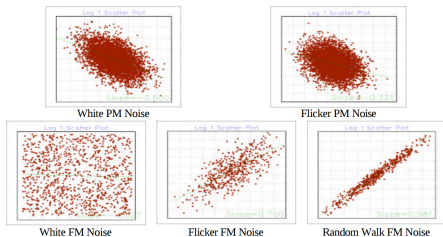


Noise identification: Lag-1

y_1	y_2	y_3	\dots	y_N	\leftarrow Original
y_N	y_1	y_2	\dots	y_{N-1}	\leftarrow Lag-1

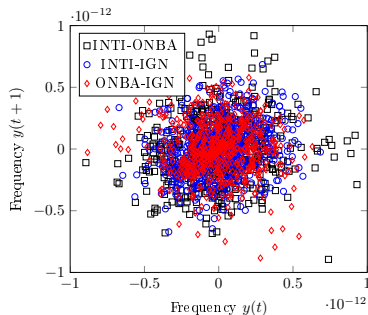
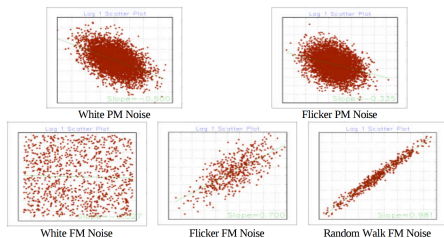
Noise identification: Lag-1

y_1	y_2	y_3	\dots	y_N	← Original
y_N	y_1	y_2	\dots	y_{N-1}	← Lag-1



Noise identification: Lag-1

y_1	y_2	y_3	\dots	y_N	← Original
y_N	y_1	y_2	\dots	y_{N-1}	← Lag-1



” White Frequency Modulation + Flicker Phase Modulation... ”

Types of quartz oscillators

Crystal Oscillator Categories

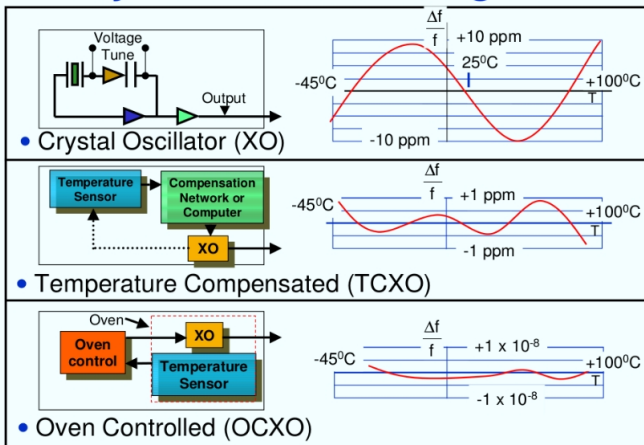


Table A.3. Summary of oscillator types.

Oscillator Type	Quartz (TCXO)	Quartz (MCXO)	Quartz (OCXO)	Rubidium	Cesium	Hydrogen Maser
Primary Standard	No	No	No	No	Yes	No
Intrinsic Standard	No	No	No	Yes	Yes	Yes
Resonance Frequency	Mechanical (varies)	Mechanical (varies)	Mechanical (varies)	6.834682608 GHz	9.19263177 GHz	1.42040575 GHz
Leading Cause of Failure	None	None	None	Rubidium Lamp (15 years or more)	Cesium Beam Tube (3 to 25 years)	Hydrogen Depletion (7 years or more)
Stability, $\sigma_y(\tau)$, $\tau=1s$	1×10^{-9}	1×10^{-10}	1×10^{-12}	5×10^{-11} to 5×10^{-12}	5×10^{-11} to 5×10^{-12}	1×10^{-12}
Noise Floor, $\sigma_y(\tau)$	1×10^{-9} ($\tau = 1$ to 10^2 s)	1×10^{-10} ($\tau = 1$ to 10^2 s)	1×10^{-12} ($\tau = 1$ to 10^2 s)	1×10^{-12} ($\tau = 10^3$ to 10^5 s)	1×10^{-14} ($\tau = 10^5$ to 10^7 s)	1×10^{-15} ($\tau = 10^5$ to 10^8 s)
Aging/year	5×10^{-7}	5×10^{-8}	5×10^{-9}	2×10^{-10}	None	$\sim 1 \times 10^{-13}$
Frequency Offset after warm up	1×10^{-6}	1×10^{-7} to 1×10^{-8}	1×10^{-8} to 1×10^{-10}	5×10^{-10} to 5×10^{-12}	5×10^{-12} to 1×10^{-14}	1×10^{-12} to 1×10^{-13}
Warm-Up Time	< 10 s to 1×10^{-6}	< 10 s to 1×10^{-8}	< 5 min to 1×10^{-8}	< 5 min to 5×10^{-10}	30 min to 5×10^{-12}	24 hours to 1×10^{-12}
Cost	\$100	\$1000	\$2000	\$3000 to \$8000	\$30,000 to \$80,000	\$200,000 to \$300,000

References

- ① Lombardi, M. A. (2008). Selecting a Primary Frequency Standard for a Calibration Laboratory. Cal Lab Magazine: The International Journal of Metrology, 33-39.
- ② Riley, W. J. (2008). Handbook of Frequency Stability Analysis.—National Institute of Standards and Technology (NIST), US Department of Commerce. NIST Special Publication, 1065.
- ③ Lombardi, M. A., Novick, A. N., Neville-Neil, G., & Cooke, B. (2016). Accurate, traceable, and verifiable time synchronization for world financial markets. Journal of research of the NIST, 121, 436-463.

The End



" Never measure anything but frequency "

Arthur Schawlow, Nobel Prize in Physics 1981 "for his contribution to the development of laser spectroscopy"