Introduction to Time and Frequency Metrology concepts

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October 23, 2017
The International System of Units (SI)
The International System of Units (SI)
Second (s): Definition

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Segundo (s): Definición (No oficial!)

El segundo es la duración de 9 192 631 770 períodos de la radiación correspondiente a la transición entre dos niveles hiperfinos del estado fundamental del átomo de cesio 133
Cesium (Cs)

Energy levels: 6
Protons: 55
Neutrons: 78
Electrons: 55

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The International System of Units (SI)

Definition of the unit of time, the second

\[ \begin{align*}
\text{\(6^2P_{3/2}\)} & \approx 100\text{GHz} \\
\text{\(6^2P_{1/2}\)} & \approx 894\text{nm} \\
\text{\(6^2S_{1/2}\)} & \approx 850\text{nm} \\
\end{align*} \]

\[ \begin{align*}
F' &= 5 & \text{11 subniveles} \\
F' &= 4 & \text{9 subniveles} \\
F' &= 3 & \text{7 subniveles} \\
F' &= 2 & \text{5 subniveles} \\
F' &= 4 & \text{9 subniveles} \\
F' &= 3 & \text{7 subniveles} \\
F' &= 3 & \text{9 subniveles} \\
\text{9.192631770}\text{GHz} & \text{7 subniveles} \\
\end{align*} \]

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The International System of Units (SI)

1955
Birth of Atomic Time

Louis Essen and Jack Parry design and build the world’s first caesium atomic clock at NPL. Essen invites Director Edward Bullard ‘to come and witness the death of the astronomical second and the birth of atomic time’
Time and frequency measurements are very important on fundamental research.

Why do we need better and better clocks?

- Measurement of the fundamental constants \((c, \alpha, R)\) and their possibly time variation
- Test the validity of the special and general theory of relativity
- Very high accuracy spectroscopy
- Astronomy, Radio Astronomy and Astrophysics
Time and frequency metrology is very important in telecommunication networks, navigation systems among other important technological applications.

- Communication
- Satellite Navigation (GPS, GLONASS, GALILEO)
- Time Synchronization for World Financial Markets
For these reasons, clock are getting better and better

Figure: Evolution of the accuracy of atomic clocks

Two units of measurement in the International System (SI) apply to time and frequency metrology.

**Second (s)**
- The base unit for the quantity *time* is the second.
- One of the 7 base SI units.
- The symbol for second is s.

**Hertz (Hz)**
- The derived unit for the quantity *frequency* is the hertz.
- Defined as events per second.
- One of 21 SI units derived from base units.
- The symbol for hertz is Hz.
Three basic types of time and frequency information

Date and Time-of-Day

The notation used to describe when an event occurred. I was born on September 3, 1980 at 0800 UTC

Time Interval

The duration between two events. The record in the 100 metres for men is 9.58 seconds.

Frequency

The rate of a repetitive event. This device has to be calibrated once a year.
The units of time of day are defined as multiples of the SI second

- 1 minute = 60 seconds
- 1 hour = 60 minutes or 3600 seconds
- The hour and the minute are Non-SI units accepted for use with the SI
- The symbols are h and min. Remember: hs nor mins

Hour and minutes are based on the sexagesimal (base 60) system that is around 4000 years old. Days are based on the duodecimal (base 12) system that is at least 3500 years old.

The units of time interval are defined as fractional parts of the SI second:

- millisecond = $1 \times 10^{-3}$ s
- microsecond = $1 \times 10^{-6}$ s
- nanosecond = $1 \times 10^{-9}$ s
- picosecond = $1 \times 10^{-12}$ s

The sub-second units are all relatively new (within the last few hundred years) and all use the decimal (base 10) system.
The units of frequency are expressed in hertz, or in multiples of the hertz:

- hertz (Hz) = one event or cycle per second
- kilohertz (kHz) = $1 \times 10^3$ Hz
- megahertz (MHz) = $1 \times 10^6$ Hz
- gigahertz (GHz) = $1 \times 10^9$ Hz
The relationship between frequency and time interval

We can measure frequency to get time interval, or we can measure time interval to get frequency. This is because frequency is the reciprocal of time interval:

\[ f = \frac{1}{T} \]  

(1)

Where \( T \) is the period of the signal in seconds and \( f \) is the frequency in hertz.
Wavelength

The wavelength is the length of one complete wave cycle, expressed in units of length

\[ \lambda = \frac{c}{f} \]  \hspace{1cm} (2)

- Where \( c \) is the speed of light: a constant of 299,727,738 m s\(^{-1}\)
- To get \( \lambda \) in meters, it is common to use \( \lambda = \frac{300}{f} \), where \( f \) is in MHz.
- Wavelength is mostly used in waves propagating in free space
A clock counts cycles of a frequency and records units of time interval, such as seconds, minutes, hours, and days. A clock consists of an oscillator, a counter, and a display.

A wristwatch is a good example of a typical clock. Most wristwatches contain a quartz oscillator that generates 32,768 cycles per second. After a watch counts 32,768 cycles, it records that one second has elapsed by updating its display.

Oscillators are the heart of all clocks. They produce a periodic event that repeats at a nearly constant rate. This rate is called the resonance frequency. The best clocks contain the best oscillators.
Synchronization and Syntonization

- **Synchronization** is the process of setting two or more clocks to the same time.
- **Syntonization** is the process of setting two or more oscillators to the same frequency.
Inside a clock

The parts of a clock

Repeating Motion + Counting Mechanism/Display
(from oscillator)

Earth Rotation
Pendulum Swing
Quartz Crystal Vibration
Cesium Atomic Vibration

Sundial
Clock Gears and Hands
Electronic Counter
Microwave Counter
The words “clock” and “oscillator” are often used incorrectly by metrologists

To most people, a clock is a device that displays the time of day. It answers perhaps the world’s most common question: What time is it now?

Technically, an oscillator is the reference or “time base” for the ticks of a clock.

However, metrologists often refer to oscillators as clocks. Thus, you will probably hear the term “clock” in this meeting when we are referring to devices that produce frequency, but that do not always keep time or have a display.
Frequencies and periods

- The frequency of the oscillator and the period of the TIC-TAC are related.
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- For example: A pendulum oscillates at 0.5 Hz and the clock generates a TIC each second.
Frequencies and periods

- The frequency of the oscillator and the period of the TIC-TAC are related.
- For example: A pendulum oscillates at 0.5 Hz and the clock generates a TIC each second.
- Another example: A cesium clock counts 9,192,631,770 oscillations of the radiation associated to an atomic transition, to generate on TIC
## Atomic clocks

<table>
<thead>
<tr>
<th>Standard</th>
<th>Resonator</th>
<th>First Device Built</th>
<th>Time Accuracy of best device (24 h)</th>
<th>Frequency Accuracy of best device (24 h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubidium gas cell</td>
<td>$^{87}\text{Rb}$ resonance (6 834 682 911 Hz)</td>
<td>1958</td>
<td>$\sim$100 ns</td>
<td>$\sim$1 x $10^{-12}$</td>
</tr>
<tr>
<td>Cesium beam</td>
<td>$^{133}\text{Cs}$ resonance (9 192 631 770 Hz)</td>
<td>1952</td>
<td>$\sim$1 ns</td>
<td>$\sim$1 x $10^{-14}$</td>
</tr>
<tr>
<td>Hydrogen maser</td>
<td>Hydrogen resonance (1 420 405 752 Hz)</td>
<td>1960</td>
<td>$\sim$1 ns</td>
<td>$\sim$1 x $10^{-14}$</td>
</tr>
<tr>
<td>Cesium fountain (not sold commercially)</td>
<td>$^{133}\text{Cs}$ resonance (9 192 631 770 Hz)</td>
<td>1991</td>
<td>$\sim$10 ps</td>
<td>$\sim$1 x $10^{-16}$</td>
</tr>
</tbody>
</table>

Rubidium ($f \sim 6.8 \text{GHz}$) is used as secondary representation of the second. [HTTP://WWW.BIPM.ORG/EN/PUBLICATIONS/MISES-EN-PRATIQUE/STANDARD-FREQUENCIES.HTML](HTTP://WWW.BIPM.ORG/EN/PUBLICATIONS/MISES-EN-PRATIQUE/STANDARD-FREQUENCIES.HTML)
Stability and accuracy

Stability

- In T & F, it indicates how well an oscillator can produce the same frequency over a given period of time. Stability doesn’t indicate whether the time or frequency is ”right” or ”wrong”, but only whether it stays the same.
- It is quantified by the Allan Variance
Stability and accuracy

Stability
- In T & F, it indicates how well an oscillator can produce the same frequency over a given period of time. Stability doesn’t indicate whether the time or frequency is ”right” or ”wrong”, but only whether it stays the same.
- It is quantified by the Allan Variance

Accuracy
- Indicates how well an oscillator has been set on time or frequency
- Is normally expressed as a dimensionless number (unitless): $\frac{\Delta f}{f}$
Accuracy evaluation

\[ \frac{\Delta f}{f} = \frac{f_{\text{measured}} - f_{\text{nominal}}}{f_{\text{nominal}}} \]

- \( f_{\text{measured}} \) is the reading of the counter
- \( f_{\text{nominal}} \) is the nominal frequency of the oscillator under test (10 MHz, for example)
Accuracy in time domain

The same can be done if you measure time difference:

\[- \frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}\]

The quantity \(\Delta t\) is the phase change expressed in time units, estimated by the difference of two readings from a time interval counter or oscilloscope. \(T\) is the duration of the measurement, also expressed in time units.
Accuracy in time domain

The same can be done if you measure time difference:

\[ \frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T} \]

If you compute the slope of the Phase difference, you obtain \( \frac{\Delta t}{T} \) in \( \frac{ns}{s} \).
Standard deviation cannot be used in T&F...

Time Difference: INTI − IGN

<table>
<thead>
<tr>
<th>MJD</th>
<th>Time Difference / ns</th>
</tr>
</thead>
<tbody>
<tr>
<td>56878</td>
<td>-20</td>
</tr>
<tr>
<td>56880</td>
<td>-15</td>
</tr>
<tr>
<td>56882</td>
<td>-10</td>
</tr>
<tr>
<td>56884</td>
<td>-5</td>
</tr>
<tr>
<td>56886</td>
<td>0</td>
</tr>
<tr>
<td>56888</td>
<td>5</td>
</tr>
<tr>
<td>56890</td>
<td>10</td>
</tr>
<tr>
<td>56892</td>
<td>15</td>
</tr>
</tbody>
</table>

CV algorithm
Rapid UTC

Introduction to T&F Metrology concepts

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Allan Variance

\[ \sigma^2_y(\tau) = \frac{1}{2} \langle (\Delta y)^2 \rangle \]

Estimator:
\[ \hat{\sigma}^2_y(\tau) = \frac{1}{2N} \sum_{i=1}^{N} (\bar{y}_2 - \bar{y}_1)^2 \]

\[ (\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi^2_1 \] Distribution

\[ \sum_{i=1}^{N} (\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi^2_N \] Distribution
Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2} \langle (\Delta y)^2 \rangle$
Allan Variance: \( \sigma_y^2(\tau) = \frac{1}{2} \langle (\Delta y)^2 \rangle \)

Estimator:
\[
\hat{\sigma}_y^2(\tau) = \frac{1}{2N} \sum_{i=1}^{N} (y_{2i} - \bar{y}_1)^2
\]
Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2} \langle (\Delta y)^2 \rangle$

Estimator:

$\hat{\sigma}_y^2(\tau) = \frac{1}{2N} \sum_{i=1}^{N} (\bar{y}_2 - \bar{y}_1)^2$

- $\bar{y}_2 - \bar{y}_1 \Rightarrow$ Normal Distribution
- $(\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_1^2$ Distribution
- $\sum_{i=1}^{N} (\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_N^2$ Distribution
Ideal oscillator

\[ V_o \cos(2\pi\nu_o t + \varphi) \]
Stability

Ideal oscillator

\[ V_o \cos(2\pi \nu_0 t + \varphi) \]
Stability

Ideal oscillator

\[ V_0 \cos(2\pi \nu_o t + \varphi) \]

Real Oscillator model

\[ [V_0 + \varepsilon(t)] \cos(2\pi \nu_o t + \varphi(t)); \quad \varphi(t) : \text{phase noise} \]
Stability

\[ V_o \cos(2\pi \nu_o t + \varphi(t)) \]

**Instant frequency:** \( \nu(t) \)

\[ \nu(t) = \nu_o + \frac{1}{2\pi} \frac{d\varphi(t)}{dt} \]

**Frequency deviation:** \( y(t) \)

\[ y(t) = \frac{\Delta \nu(t)}{\nu_0} = \frac{1}{2\pi \nu_0} \frac{d\varphi(t)}{dt} \]
Stability

\[ V_0 \cos(2\pi \nu_0 t + \varphi(t)) \]

**Instant frequency: \( \nu(t) \)**

\[ \nu(t) = \nu_0 + \frac{1}{2\pi} \frac{d\varphi(t)}{dt} \]

**Frequency deviation: \( y(t) \)**

\[ y(t) = \frac{\Delta \nu(t)}{\nu_0} = \frac{1}{2\pi \nu_0} \frac{d\varphi(t)}{dt} \]

\[ x(t) \equiv \frac{\varphi(t)}{2\pi \nu_0} \Rightarrow y(t) = \frac{dx}{dt} \]
From PSD to $\sigma_y(\tau)$

$$\sigma_y^2(\tau) = \int_0^\infty S_y(f) \left( 2 \frac{\sin^4(\pi \tau f)}{(\pi \tau f)^2} \right) df$$
Time dependence of Allan variance

From PSD to $\sigma_y(\tau)$

$$\sigma^2_y(\tau) = \int_0^\infty S_y(f) \left( 2 \frac{\sin^4(\pi \tau f)}{(\pi \tau f^2)} \right) df$$

Power law noise model

$$S_y(f) = h_{-2} f^{-2} + h_{-1} f^{-1} + h_0 f^0 + h_1 f^1 + h_2 f^2$$

White noise
### Time dependence of Allan variance

**From PSD to $\sigma_y(\tau)$**

$$
\sigma_y^2(\tau) = \int_0^\infty S_y(f) \left( 2 \frac{\sin^4(\pi \tau f)}{(\pi \tau f)^2} \right) df
$$

**Power law noise model**

$$
S_y(f) = h_{-2} f^{-2} + h_{-1} f^{-1} + \underbrace{h_0 f^0}_{\text{White noise}} + h_1 f^1 + h_2 f^2
$$

**Slope relationships**

$$
S_y(f) \sim f^\alpha \\
\sigma_y^2(\tau) \sim \tau^{-\alpha-1}
$$
Time dependence of Allan variance

\[ S_y(f) \sim f^\alpha \]
\[ \sigma_y^2(\tau) \sim \tau^\mu \]
\[ \mu = -\alpha - 1 \]
Time dependence of Allan variance

\[ S_y(f) \sim f^{\alpha} \]
\[ \sigma_y^2(\tau) \sim \tau^\mu \]
\[ \mu = -\alpha - 1 \]

**Noise Type**

<table>
<thead>
<tr>
<th>Noise Type</th>
<th>Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Walk F. Mod.</td>
<td>Ambient</td>
</tr>
<tr>
<td>Flicker Freq. Mod.</td>
<td>Resonator</td>
</tr>
<tr>
<td>White Freq. Mod.</td>
<td>Thermal noise</td>
</tr>
<tr>
<td>Flicker Phase Mod.</td>
<td>Electric noise</td>
</tr>
</tbody>
</table>

**Noise Power Spectral Densities**

<table>
<thead>
<tr>
<th>Power Spectral Densities</th>
<th>( f )</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{-2} f^{-2} )</td>
<td>( \tau^{1/2} )</td>
<td></td>
</tr>
<tr>
<td>( h_{-1} f^{-1} )</td>
<td>( \tau^0 )</td>
<td></td>
</tr>
<tr>
<td>( h_0 )</td>
<td>( \tau^{-1/2} )</td>
<td></td>
</tr>
<tr>
<td>( h_1 f )</td>
<td>( \tau^{-1} )</td>
<td></td>
</tr>
</tbody>
</table>
Time dependence of Allan variance

Comparison of Quartz Oscillators

Allan Deviation, $\sigma_y(\tau)$

Averaging Time $\tau$, seconds
Time dependence of Allan variance

Comparison of Rubidium Oscillators

Allan Deviation, $\sigma_y(\tau)$

Averaging Time, $\tau$, Seconds
Time dependence of Allan variance

\[ \sigma_y(\tau) \]

Cesium Oscillator

Allan Deviation, $\sigma_y(\tau)$

Averaging Time, $\tau$, Seconds

$10^{-14}$ $10^{-13}$ $10^{-12}$ $10^{-11}$ $10^{-10}$

$10^{-2}$ $10^{-1}$ $10^0$ $10^1$ $10^2$ $10^3$ $10^4$ $10^5$
Noise identification: Lag-1

<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>…</th>
<th>$y_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_N$</td>
<td>$y_1$</td>
<td>$y_2$</td>
<td>…</td>
<td>$y_{N-1}$</td>
</tr>
</tbody>
</table>

← Original
← Lag-1
Noise identification: Lag-1

\[
\begin{array}{cccccc}
  y_1 & y_2 & y_3 & \ldots & y_N \\
  y_N & y_1 & y_2 & \ldots & y_{N-1}
\end{array}
\]

← Original

← Lag-1

- White Frequency Modulation
- Flicker Phase Modulation
- Random Walk FM Noise

![Graphs showing different types of noise](image)
Noise identification: Lag-1

\[
\begin{array}{cccc}
  y_1 & y_2 & y_3 & \cdots & y_N \\
  y_N & y_1 & y_2 & \cdots & y_{N-1}
\end{array}
\]

← Original
← Lag-1

"White Frequency Modulation + Flicker Phase Modulation..."
Types of quartz oscillators

- **Crystal Oscillator (XO)**
- **Temperature Compensated (TCXO)**
- **Oven Controlled (OCXO)**
<table>
<thead>
<tr>
<th>Oscillator Type</th>
<th>Quartz (TCXO)</th>
<th>Quartz (MCXO)</th>
<th>Quartz (OCXO)</th>
<th>Rubidium</th>
<th>Cesium</th>
<th>Hydrogen Maser</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Standard</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Intrinsic Standard</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Resonance Frequency</strong></td>
<td>Mechanical (varies)</td>
<td>Mechanical (varies)</td>
<td>Mechanical (varies)</td>
<td>6.834682508 GHz</td>
<td>9.19263 177 GHz</td>
<td>1.42040575 GHz</td>
</tr>
<tr>
<td><strong>Leading Cause of Failure</strong></td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>Rubidium Lamp (15 years or more)</td>
<td>Cesium Beam Tube (3 to 25 years)</td>
<td>Hydrogen Deposition (7 years or more)</td>
</tr>
<tr>
<td><strong>Stability, ( \sigma_\tau(\tau) ), ( \tau = 1 ) s</strong></td>
<td>( 1 \times 10^9 )</td>
<td>( 1 \times 10^{10} )</td>
<td>( 1 \times 10^{12} )</td>
<td>( 5 \times 10^{-11} ) to ( 5 \times 10^{-12} )</td>
<td>( 5 \times 10^{-11} ) to ( 5 \times 10^{-12} )</td>
<td>( 1 \times 10^{-12} )</td>
</tr>
<tr>
<td><strong>Noise Floor, ( \sigma_\tau(\tau) )</strong></td>
<td>( 1 \times 10^9 ) (( \tau = 1 ) to ( 10^2 ) s)</td>
<td>( 1 \times 10^{10} ) (( \tau = 1 ) to ( 10^7 ) s)</td>
<td>( 1 \times 10^{12} ) (( \tau = 1 ) to ( 10^8 ) s)</td>
<td>( 1 \times 10^{12} ) (( \tau = 10^3 ) to ( 10^8 ) s)</td>
<td>( 1 \times 10^{14} ) (( \tau = 10^7 ) to ( 10^8 ) s)</td>
<td>( 1 \times 10^{15} ) (( \tau = 10^7 ) to ( 10^8 ) s)</td>
</tr>
<tr>
<td><strong>Aging/year</strong></td>
<td>( 5 \times 10^{-7} )</td>
<td>( 5 \times 10^{-8} )</td>
<td>( 5 \times 10^{-9} )</td>
<td>( 2 \times 10^{-10} )</td>
<td>None</td>
<td>( \sim 1 \times 10^{-13} )</td>
</tr>
<tr>
<td><strong>Frequency Offset after warm up</strong></td>
<td>( 1 \times 10^9 )</td>
<td>( 1 \times 10^9 ) to ( 1 \times 10^{10} )</td>
<td>( 1 \times 10^{10} ) to ( 1 \times 10^{12} )</td>
<td>( 5 \times 10^{12} ) to ( 1 \times 10^{16} )</td>
<td>( 5 \times 10^{12} ) to ( 1 \times 10^{16} )</td>
<td>( 1 \times 10^{12} ) to ( 1 \times 10^{15} )</td>
</tr>
<tr>
<td><strong>Warm-Up Time</strong></td>
<td>(&lt; 10 ) s to ( 1 \times 10^6 )</td>
<td>(&lt; 10 ) s to ( 1 \times 10^8 )</td>
<td>(&lt; 5 ) min to ( 1 \times 10^6 )</td>
<td>(&lt; 5 ) min to ( 1 \times 10^6 )</td>
<td>(&lt; 5 ) min to ( 5 \times 10^{12} )</td>
<td>(&lt; 5 ) min to ( 5 \times 10^{12} )</td>
</tr>
<tr>
<td><strong>Cost</strong></td>
<td>$100</td>
<td>$1000</td>
<td>$2000</td>
<td>$3000 to $8000</td>
<td>$30,000 to $80,000</td>
<td>$200,000 to $300,000</td>
</tr>
</tbody>
</table>
References


"Never measure anything but frequency"

Arthur Schawlow, Nobel Prize in Physics 1981 "for his contribution to the development of laser spectroscopy"