Introduction to Time and Frequency Metrology concepts

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The International System of Units (SI)

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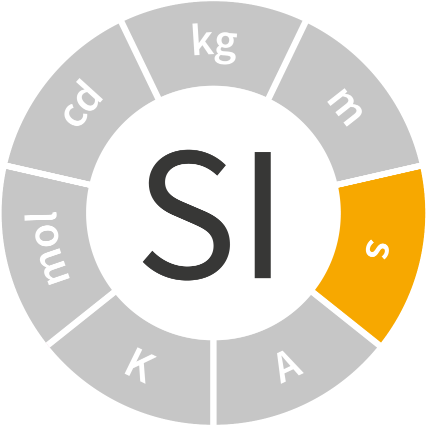


T & F in the International System (SI)

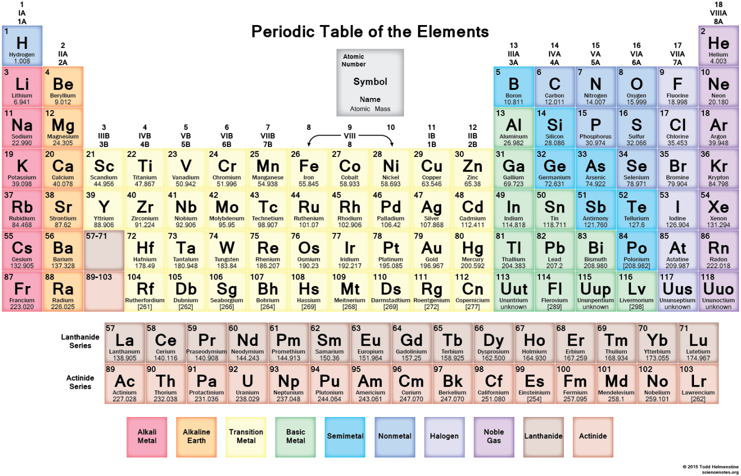
Second (s): Definition

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Segundo (s): Definición (No oficial!)

El segundo es la duración de 9 192 631 770 períodos de la radiación correspondiente a la transición entre dos niveles hiperfinos del estado fundamental del átomo de cesio 133

The International System of Units (SI)



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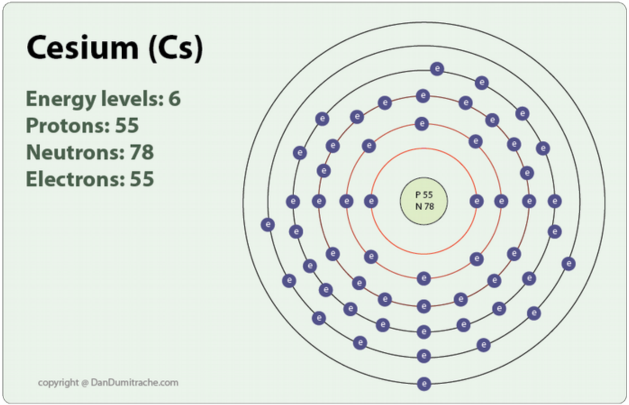

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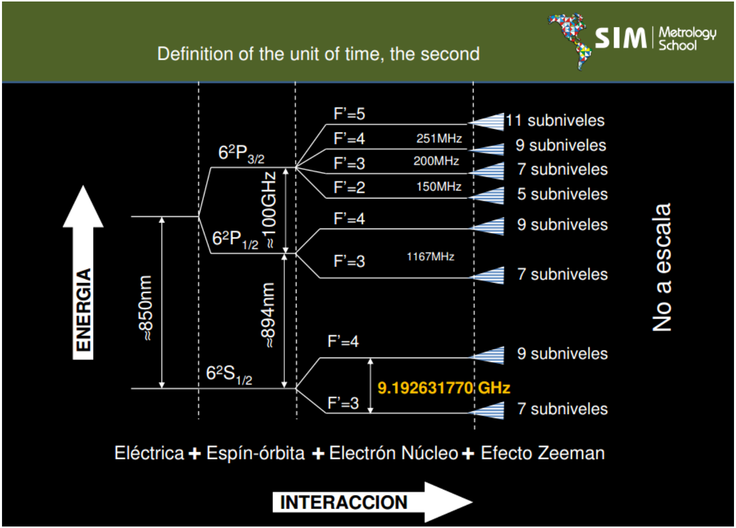

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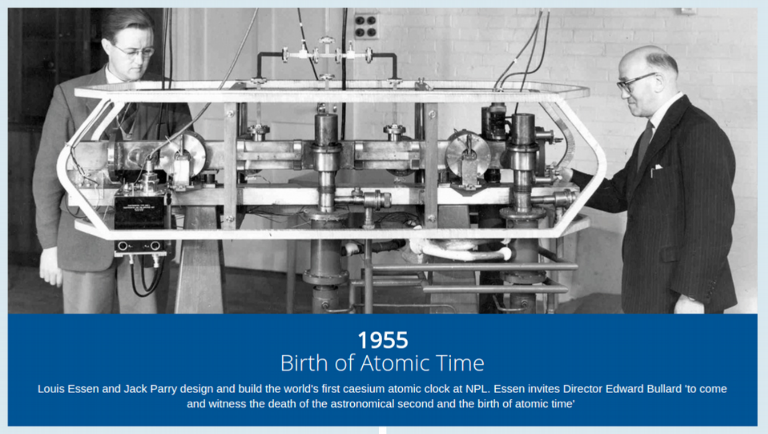
The International System of Units (SI)



The International System of Units (SI)

Time and frequency measurements are very important on fundamental reseach.
Why do we need better and better clocks?

- Measurement of the fundamental constants (c, α, R) and their possibly time variation
- Test the validity of the special and general theory of relativity
- Very high accuracy spectroscopy
- Astronomy, Radio Astronomy and Astrophysics

The International System of Units (SI)

Time and frequency metrology is very important in telecommunication networks, navegation systems among other important technological applications

- Communication
- Satellite Navigation (GPS, GLONASS, GALILEO)
- Time Synchronization for World Financial Markets

For these reasons, clock are getting better and better

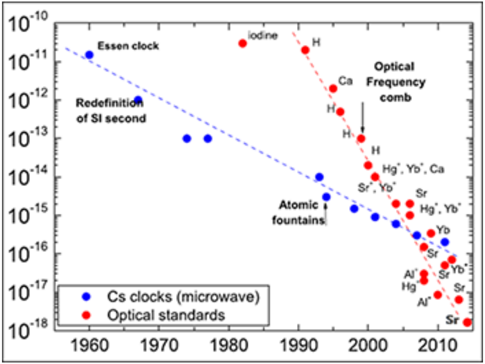


Figure : Evolution of the accuracy of atomic clocks

http://www.bipm.org/en/news/full-stories/2017-07-definition-second.html

T & F in the International System (SI)

Two units of measurement in the International System (SI) apply to time and frequency metrology

Second (s)

- The base unit for the quantity *time* is the second
- One of the 7 base SI units
- The symbol for second is s

Hertz (Hz)

- The derived unit for the quantity *frequency* is the hertz
- Defined as events per second
- One of 21 SI units derived from base units
- The symbol for hertz is Hz

Three basic types of time and frequency information

Date and Time-of-Day

The notation used to describe when an event occurred. *I was born on September 3, 1980 at 0800 UTC*

Time Interval

The duration between two events *The record in the 100 metres for men is 9.58 seconds.*

Frequency

The rate of a repetitive event *This device has to be calibrated once a year*

T & F in the International System (SI)

The units of time of day are defined as multiples of the SI second

- 1 minute = 60 seconds
- 1 hour = 60 minutes or 3600 seconds
- The hour and the minute are Non-SI units accepted for use with the SI
- The symbols are h and min. Remember: ~~hs~~ nor ~~mins~~

Hour and minutes are based on the sexagesimal (base 60) system that is around 4000 years old. Days are based on the duodecimal (base 12) system that is at least 3500 years old.

`https://www.bipm.org/en/publications/si-brochure/table6.html`

T & F in the International System (SI)

The units of time interval are defined as fractional parts of the SI second

- millisecond $= 1 \times 10^{-3}$ s
- microsecond $= 1 \times 10^{-6}$ s
- nanosecond $= 1 \times 10^{-9}$ s
- picosecond $= 1 \times 10^{-12}$ s

The sub-second units are all relatively new (within the last few hundred years) and all use the decimal (base 10) system.

T & F in the International System (SI)

The units of frequency are expressed in hertz, or in multiples of the hertz

- hertz (Hz) $=$ one event or cycle per second
- kilohertz (kHz)$= 1 \times 10^3$ Hz
- megahertz (MHz)$= 1 \times 10^6$ Hz
- gigahertz (GHz) $= 1 \times 10^9$ Hz

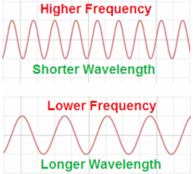
The relationship between frequency and time interval

We can measure frequency to get time interval, or we can measure time interval to get frequency. This is because frequency is the reciprocal of time interval:

$$f = \frac{1}{T} \tag{1}$$

Where T is the period of the signal in seconds and f is the frequency in hertz.

Wavelength

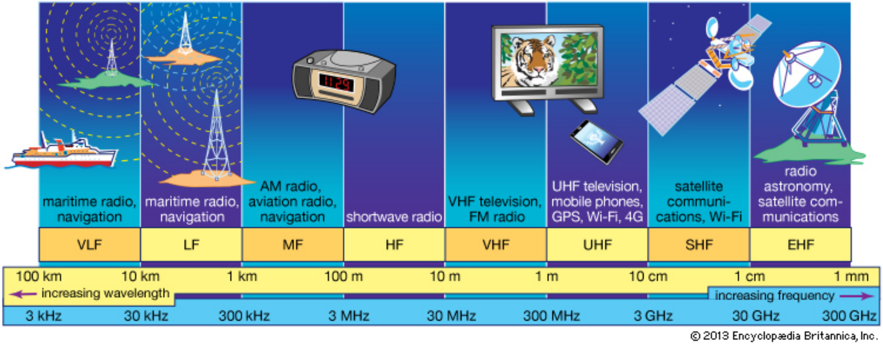


The wavelength is the length of one complete wave cycle, expressed in units of length

$$\lambda = \frac{c}{f} \tag{2}$$

- Where c is the speed of light: a constant of $299\,727\,738\,\mathrm{m\,s^{-1}}$
- To get λ in meters, it is common to use $\lambda = \frac{300}{f}$, where f is in MHz.
- Wavelength is mosly used in waves propagating in free space

Frequency bands



Clocks and Oscillators

A clock counts cycles of a frequency and records units of time interval, such as seconds, minutes, hours, and days. A clock consists of an oscillator, a counter, and a display.

A wristwatch is a good example of a typical clock. Most wristwatches contain a quartz oscillator that generates 32 768 cycles per second. After a watch counts 32 768 cycles, it records that one second has elapsed by updating its display.

Oscillators are the heart of all clocks. They produce a periodic event that repeats at a nearly constant rate. This rate is called the resonance frequency. The best clocks contain the best oscillators.

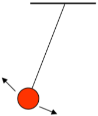
Synchronization and Syntonization

- **Synchronization** is the process of setting two or more clocks to the same time.
- **Syntonization** is the process of setting two or more oscillators to the same frequency.

Inside a clock

The parts of a clock

Repeating Motion + (from oscillator) | **Counting Mechanism/Display**



Repeating Motion (from oscillator)	Counting Mechanism/Display
Earth Rotation	Sundial
Pendulum Swing	Clock Gears and Hands
Quartz Crystal Vibration	Electronic Counter
Cesium Atomic Vibration	Microwave Counter

Inside a clock

The words “clock” and “oscillator” are often used incorrectly by metrologists

To most people, a clock is a device that displays the time of day. It answers perhaps the world’s most common question: What time is it now?

Technically, an oscillator is the reference or “time base” for the ticks of a clock.

However, metrologists often refer to oscillators as clocks. Thus, you will probably hear the term “clock” in this meeting when we are referring to devices that produce frequency, but that do not always keep time or have a display.

Frequencies and periods

- The frequency of the oscillator and the period of the TIC-TAC are related.

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- For example: A pendulum oscilates at 0,5 Hz and the clock generates a TIC each second.

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- For example: A pendulum oscilates at 0,5 Hz and the clock generates a TIC each second.
- Another example: A cesium clock counts 9 192 631 770 oscillations of the radiation asociated to an atomic transition, to generate on TIC

Atomic clocks

Standard	Resonator	First Device Built	Time Accuracy of best device (24 h)	Frequency Accuracy of best device (24 h)
Rubidium gas cell	^{87}Rb resonance (6 834 682 911 Hz)	1958	~100 ns	~1 x 10^{-12}
Cesium beam	^{133}Cs resonance (9 192 631 770 Hz)	1952	~1 ns	~1 x 10^{-14}
Hydrogen maser	Hydrogen resonance (1 420 405 752 Hz)	1960	~1 ns	~1 x 10^{-14}
Cesium fountain (not sold commercially)	^{133}Cs resonance (9 192 631 770 Hz)	1991	~10 ps	~1 x 10^{-16}

Rubidium (f $\sim$ 6,8 GHz) is used as secondary representation of the second.
HTTP://WWW.BIPM.ORG/EN/PUBLICATIONS/MISES-EN-PRATIQUE/STANDARD-FREQUENCIES.HTML

Stability and accuracy

Stability

- In T & F, it indicates how well an oscillator can produce the same frequency over a given period of time. Stability doesn't indicate whether the time or frequency is "right" or "wrong", but only whether it stays the same.
- It is quantified by the Allan Variance

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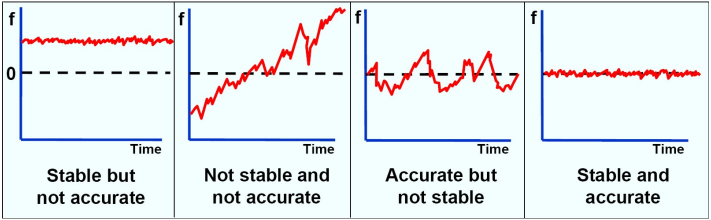
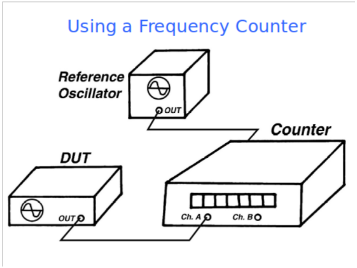
Accuracy

- Indicates how well an oscillator has been set on time or frequency
- Is normally expressed as a dimensionless number (unitless): $\frac{\Delta f}{f}$

Stability and accuracy



Accuracy



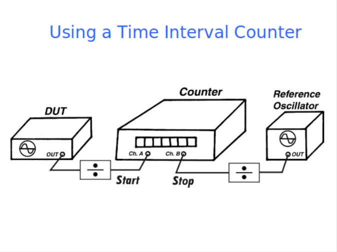
Accuracy evaluation

$$\frac{\Delta f}{f} = \frac{f_{measured} - f_{nominal}}{f_{nominal}}$$

- $f_{measured}$ is the reading of the counter
- $f_{nominal}$ is the nominal frequency of the oscillator under test (10 MHz, for example)

Accuracy in time domain

The same can be done if you measure time difference:



Accuracy evaluation in time domain

$$-\frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}$$

The quantity Δt is the phase change expressed in time units, estimated by the difference of two readings from a time interval counter or oscilloscope. T is the duration of the measurement, also expressed in time units.

Accuracy in time domain

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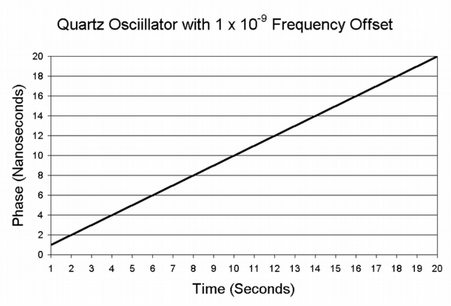
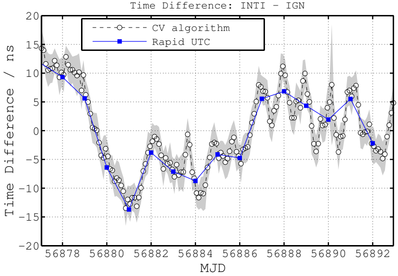


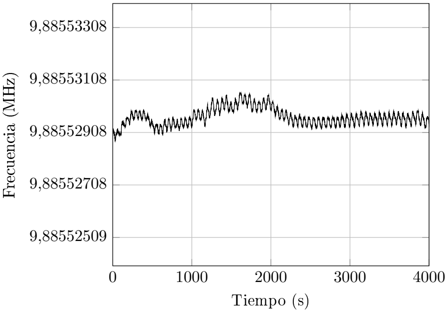
Accuracy evaluation in time domain

$$-\frac{\Delta f}{f} = \frac{\Delta t}{T} = \frac{TIC_2 - TIC_1}{T}$$

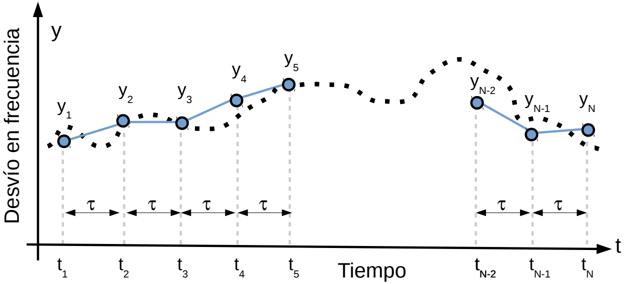
If you compute the slope of the Phase difference, you obtain $\frac{\Delta t}{T}$ in $\frac{ns}{s}$

Standard deviation cannot be used in T&F...





Allan Variance



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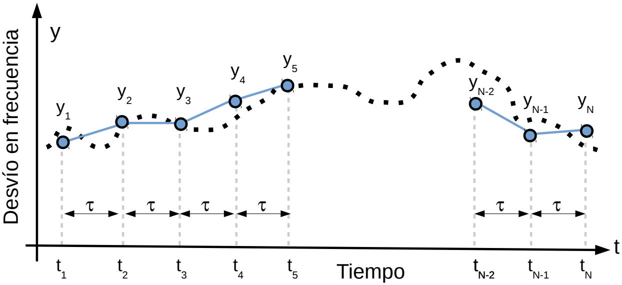
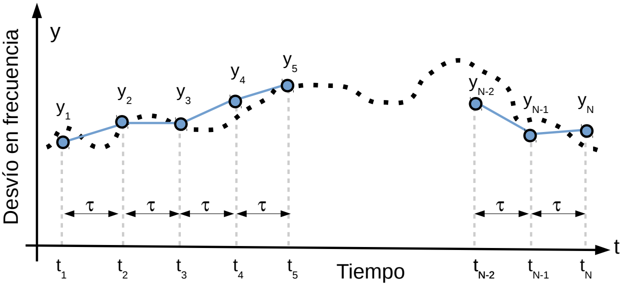


Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2}\langle(\Delta y)^2\rangle$

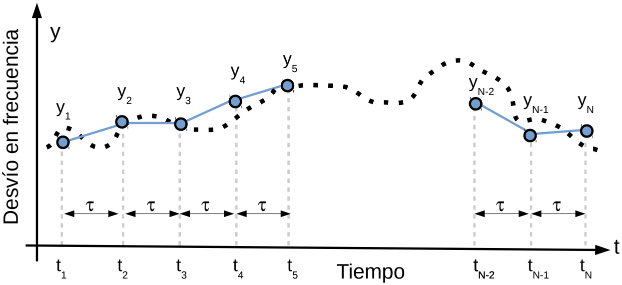
Allan Variance



Allan Variance: $\sigma_y^2(\tau) = \frac{1}{2}\langle(\Delta y)^2\rangle$

Estimator:

$$\hat{\sigma}_y^2(\tau) = \frac{1}{2N}\sum_{i=1}^{N}(\bar{y}_2 - \bar{y}_1)^2$$



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- $\bar{y}_2 - \bar{y}_1 \Rightarrow$ Normal Distribution
- $(\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_1^2$ Distribution
- $\sum_{i=1}^{N}(\bar{y}_2 - \bar{y}_1)^2 \Rightarrow \chi_N^2$ Distribution

Stability

Ideal oscillator

$$V_o \cos(2\pi\nu_o t + \varphi)$$

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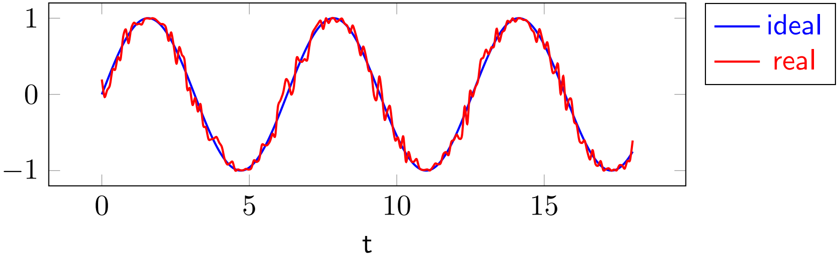
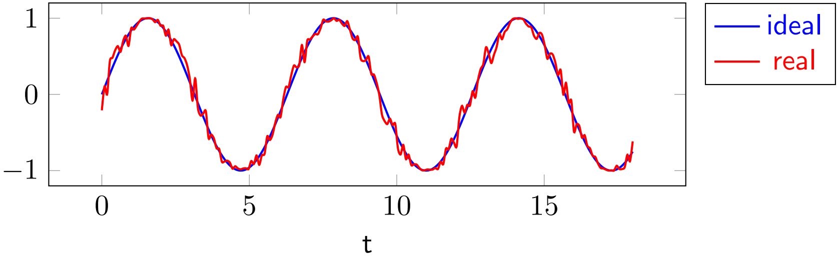


Ideal oscillator

$$V_o \cos(2\pi\nu_o t + \varphi)$$



Real Oscillator model

$$[V_o + \varepsilon(t)] \cos(2\pi\nu_o t + \varphi(t));$$

$\varphi(t)$: phase noise

Stability

$$\boxed{V_o \cos(2\pi\nu_o t + \underline{\varphi(t)})}$$

Instant frequency: $\nu(t)$

$$\nu(t) = \nu_o + \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

Frequency deviation: $y(t)$

$$y(t) = \frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2\pi\nu_0}\frac{d\varphi(t)}{dt}$$

Stability

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$$y(t) = \frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2\pi\nu_0}\frac{d\varphi(t)}{dt}$$

$$\boldsymbol{x(t) \equiv \frac{\varphi(t)}{2\pi\nu_0} \Rightarrow y(t) = \frac{dx}{dt}}$$

Time dependence of Allan variance

From PSD to $\sigma_y(\tau)$

$$\sigma_y^2(\tau) = \int_0^\infty S_y(f) \left(2 \frac{\sin^4(\pi \tau f)}{(\pi \tau f)^2} \right) df$$

Time dependence of Allan variance

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Power law noise model

$$S_y(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + \underbrace{h_0 f^0}_{\text{White noise}} + h_1 f^1 + h_2 f^2$$

Time dependence of Allan variance

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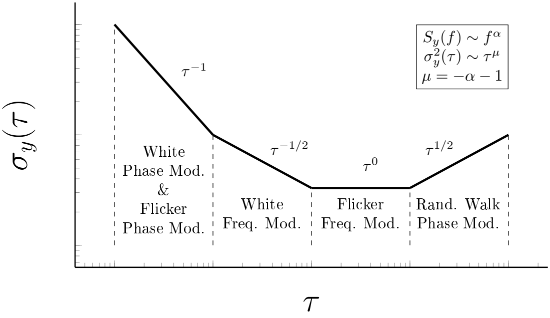
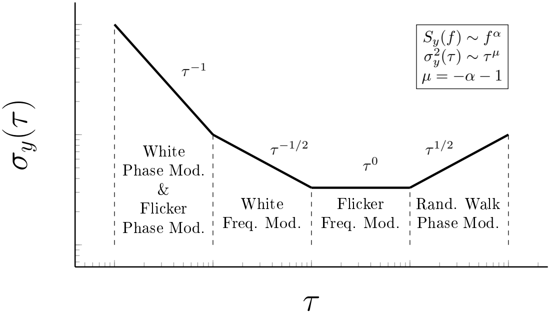
Slope relationships

$$S_y(f) \sim f^\alpha$$
$$\sigma_y^2(\tau) \sim \tau^{-\alpha-1}$$

Time dependence of Allan variance

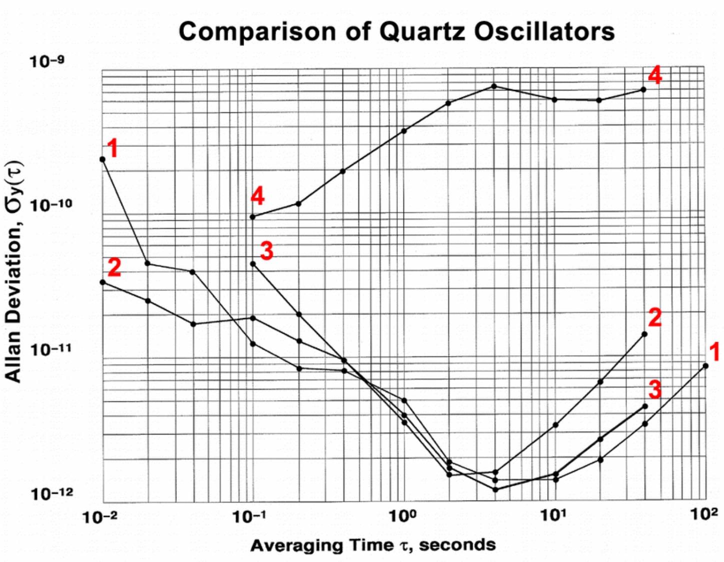


Time dependence of Allan variance



$S_y(f)$	$\sigma_y(\tau)$	**Noise Type**	**Origin**
$h_{-2}f^{-2}$	$\tau^{1/2}$	Random Walk F. Mod.	Ambient
$h_{-1}f^{-1}$	τ^0	Flicker Freq. Mod.	Resonator
h_0	$\tau^{-1/2}$	White Freq. Mod.	Thermal noise
$h_1 f^1$	τ^{-1}	Flicker Phase Mod.	Electric noise

Time dependence of Allan variance



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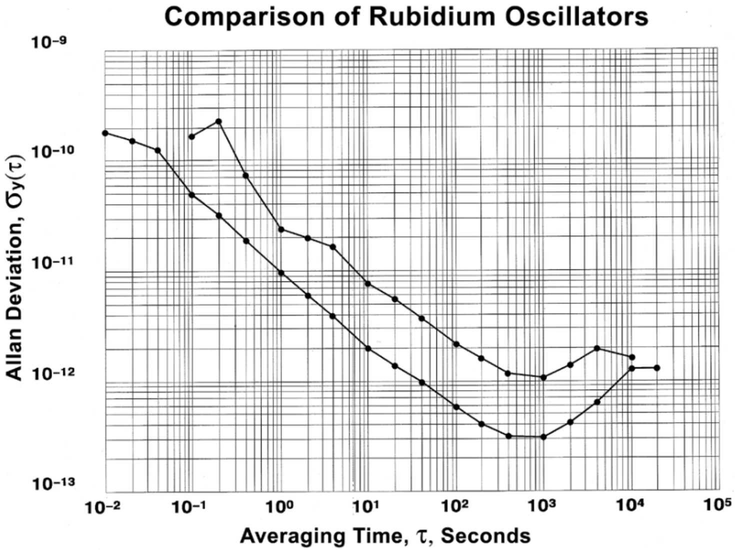
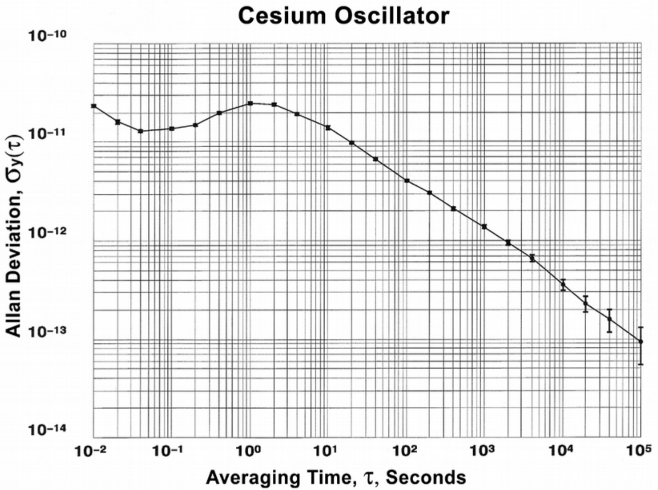


Time dependence of Allan variance



Noise identification: Lag-1

y_1	y_2	y_3	$\dots$	y_N	$\leftarrow$ Original
y_N	y_1	y_2	$\dots$	y_{N-1}	$\leftarrow$ Lag-1

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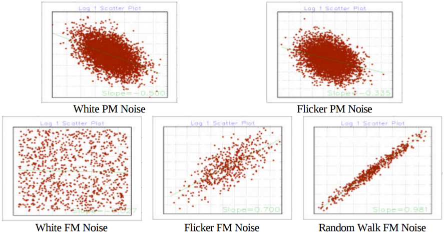

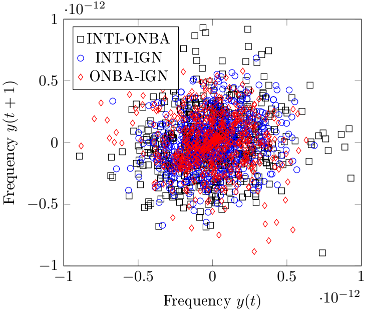

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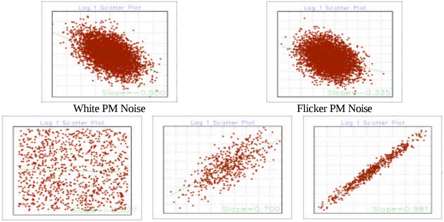


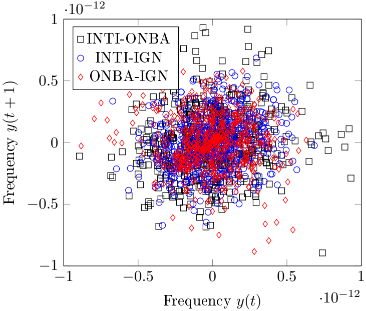
White PM Noise

Flicker PM Noise

White FM Noise

Flicker FM Noise

Random Walk FM Noise



" White Frequency Modulation + Flicker Phase Modulation... "

Types of quarz oscillators

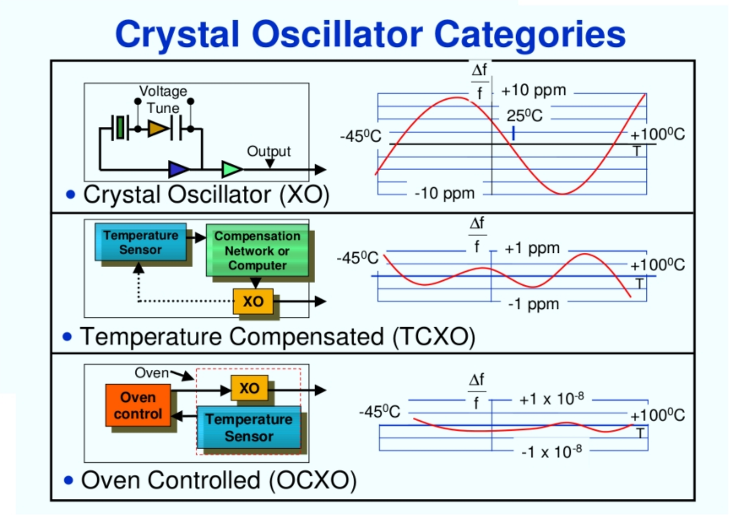


Table A.3. Summary of oscillator types.

Oscillator Type	Quartz (TCXO)	Quartz (MCXO)	Quartz (OCXO)	Rubidium	Cesium	Hydrogen Maser
Primary Standard	No	No	No	No	Yes	No
Intrinsic Standard	No	No	No	Yes	Yes	Yes
Resonance Frequency	Mechanical (varies)	Mechanical (varies)	Mechanical (varies)	6.834682608 GHz	9.19263177 GHz	1.42040575 GHz
Leading Cause of Failure	None	None	None	Rubidium Lamp (15 years or more)	Cesium Beam Tube (3 to 25 years)	Hydrogen Depletion (7 years or more)
Stability, $\sigma_y(\tau)$, τ=1s	1×10^{-9}	1×10^{-10}	1×10^{-12}	5×10^{-11} to 5×10^{-12}	5×10^{-11} to 5×10^{-12}	1×10^{-12}
Noise Floor, $\sigma_y(\tau)$	1×10^{-9} ($\tau = 1$ to 10^2 s)	1×10^{-10} ($\tau = 1$ to 10^2 s)	1×10^{-12} ($\tau = 1$ to 10^2 s)	1×10^{-12} ($\tau = 10^3$ to 10^5 s)	1×10^{-14} ($\tau = 10^5$ to 10^7 s)	1×10^{-15} ($\tau = 10^3$ to 10^5 s)
Aging/year	5×10^{-7}	5×10^{-8}	5×10^{-9}	2×10^{-10}	None	$\sim 1 \times 10^{-13}$
Frequency Offset after warm up	1×10^{-6}	1×10^{-7} to 1×10^{-8}	1×10^{-8} to 1×10^{-10}	5×10^{-10} to 5×10^{-12}	5×10^{-12} to 1×10^{-14}	1×10^{-12} to 1×10^{-13}
Warm-Up Time	< 10 s to 1×10^{-6}	< 10 s to 1×10^{-8}	< 5 min to 1×10^{-8}	< 5 min to 5×10^{-10}	30 min to 5×10^{-12}	24 hours to 1×10^{-12}
Cost	$100	$1000	$2000	$3000 to $8000	$30,000 to $80,000	$200,000 to $300,000

References

1. Lombardi, M. A. (2008). Selecting a Primary Frequency Standard for a Calibration Laboratory. Cal Lab Magazine: The International Journal of Metrology, 33-39.
2. Riley, W. J. (2008). Handbook of Frequency Stability Analysis.–National Institute of Standards and Technology (NIST), US Department of Commerce. NIST Special Publication, 1065.
3. Lombardi, M. A., Novick, A. N., Neville-Neil, G., & Cooke, B. (2016). Accurate, traceable, and verifiable time synchronization for world financial markets. Journal of research of the NIST, 121, 436-463.

The End

" Never measure anything but frequency "

Arthur Schawlow, Nobel Prize in Physics 1981 "for his contribution to the development of laser spectroscopy"