Sisyphus cooling of a bound atom

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Received May 28, 1991; revised manuscript received September 6, 1991

Cooling that results from optical dipole forces is considered for a bound atom. Through optical pumping, the atom can be made to feel decelerating optical dipole forces more strongly than accelerating optical dipole forces. This effect, which has previously been realized for free atoms, is called Sisyphus cooling. A simple model for a bound atom is examined in order to reveal the basic aspects of cooling and heating when the atom is confined in the Lamb–Dicke regime. Results of semiclassical and quantum treatments show that the minimum energy achieved is near the zero-point energy and can be much lower than the Doppler cooling limit. Two practical examples that approximate the model are briefly examined.

1. INTRODUCTION

Interest in laser-cooled atoms and atomic ions has continued to increase over the past few years. The simplest cooling scheme that has been proposed is called Doppler cooling, in which the atom experiences a damping force owing to spontaneous scattering of light.[1] This cooling applies to a two-level system. That is, it can be explained by a consideration of scattering between a single ground state and a single optically excited state. It leads to minimum temperatures (the Doppler cooling limit) $T_D \simeq \hbar\Gamma/2k_B$ when $\hbar\Gamma \gg R$. Here $2\pi\hbar$ is Planck's constant, Γ is the decay rate from the excited state, k_B is Boltzmann's constant, and $R = (\hbar k)^2/2m$ is the recoil energy, where $\lambda = 2\pi/k$ is the wavelength of the exciting radiation and m is the mass of the atom. For strongly allowed (large-Γ) electric dipole transitions in atoms and ions, this leads to temperatures of approximately 1 mK.

In 1988 an important experiment by the group of Phillips at the National Institute of Standards and Technology, Gaithersburg, Maryland,[2] showed that temperatures of laser-cooled sodium atoms were substantially below the minimum temperature predicted theoretically for two-level systems. A theoretical analysis and subsequent experiments[3–8] showed that the new cooling mechanisms responsible for these low temperatures were based on a combination of several effects such as optical pumping between more than one ground-state sublevel, light shifts, and polarization or intensity gradients.

The idea of the new cooling mechanisms is that laser beam electric fields **E** create a polarization **d** in an atom that depends on the ground-state sublevel and on the laser polarization. This dipole interacts with the laser field, and the reactive part of this interaction induces an energy shift of the ground-state sublevels, which varies from one ground-state sublevel to the other. This shift is called the light shift or the ac Stark shift. If the laser intensity or the laser polarization varies in space, these light shifts, which are now position dependent, give rise to a dipole or ponderomotive force equal in magnitude to the gradient of the atomic energy. Optical pumping takes place between the sublevels, so if the laser polarization/intensity varies in space, the relative populations of the ground-state sublevels are also position dependent. For an atom moving arbitrarily slowly in a spatially periodic light field, the averaged force is equal to zero, because the deceleration force experienced by the atom in the ascending parts of the potential curves (associated with the position-dependent light shifts) is offset by the acceleration felt in the descending parts. However, optical pumping takes a finite time to occur (the pumping time τ_P is inversely proportional to the laser intensity I). Therefore, in general, for a moving atom, it is possible to arrange the laser tuning and the polarization/intensity gradients so that the atom has time to run up one of the dipole potential hills (thereby losing kinetic energy) before being optically pumped into another ground-state sublevel where the dipole potential has less effect on the kinetic energy. After a while, the atom is returned by optical pumping (or by some other relaxation mechanism) to the original ground state, and the process is repeated. Since the atom appears to be running up the potential hills more than down, this cooling has been called Sisyphus cooling after the Greek myth[9] (see also Refs. 10 and 11). Cooling that is due to static electric and magnetic fields in combination with optical pumping has also been considered.[12,13]

The Sisyphus cooling mechanism was first introduced for a two-level atom moving in a high-intensity laser standing wave.[9] The required multilevel structure of the atom was then provided by the dressed states of the atom in the strong laser field. However, the time lag appearing between the dressed-state populations of a moving atom and the corresponding populations of an atom at rest is, for a two-level atom, of the order of the radiative lifetime $\tau_R = \Gamma^{-1}$ of the atomic excited state. At low intensity the optical pumping time τ_P for a multilevel atom can be much longer than τ_R. Because of this, the new cooling mechanisms based on optical pumping and light shifts can lead

0740-3224/92/010032-11$05.00

to friction coefficients as high as Sisyphus cooling for two-level atoms but at lower intensity, since, with longer time lags ($\tau_P \gg \tau_R$), smaller velocities can give rise to comparable decelerations.[3]

Doppler cooling exhibits the damping of the spontaneous scattering force, whereas Sisyphus cooling uses the decelerating effect of the dipole force combined with a dissipation of potential energy by spontaneous Raman processes. For completeness, it is useful to identify a third kind of cooling, which may be regarded as due to optical pumping of the atom into a low-energy state that no longer interacts with the laser. Velocity-selective coherent population trapping[8,14] and resolved sideband cooling of trapped ions[15,16] are of this category. In both cases the atoms (ions) are pumped into low-energy states that interact only weakly with the laser. This interaction becomes weaker the smaller the velocity becomes or, in the case of sideband cooling, the narrower the cooling transition is. Related schemes, which put atoms into a low-energy state that no longer interacts (or interacts weakly) with the laser light, have been proposed.[17]

Sisyphus or polarization-gradient cooling[8] has been an important development for cooling and manipulation of atoms. For free atoms Sisyphus cooling gives temperatures down to values where the kinetic energy of the atoms is approximately equal to the depth of the potential wells associated with the position-dependent light shifts. This limit applies down to the point where the dipole well energies are of the order of the recoil energy R, in which case the limiting energy is approximately equal to R.

In this paper we investigate the possibility of extending the Sisyphus cooling mechanism to trapped atoms or ions. We propose a specific scheme and show that it can lead to temperatures much lower than the Doppler limit. Instead of permitting the atom to move freely in space, we assume that the atom is bound to a localized region of space by some external potential. We assume that the intensity gradient is due to a standing-wave laser field. By a localized region of space we mean a region whose spatial extent is much smaller than the wavelength of the cooling laser divided by 2π (e.g., k^{-1}). That is, the atom is confined in the Lamb–Dicke regime. For simplicity of discussion, the atom is assumed to be bound in one dimension by a harmonic well characterized by the oscillation frequency ω_v. The spread of the minimum-energy state is given by $x_0 = (\hbar/2m\omega_v)^{1/2}$, where m is the atomic mass. The spatial extension of the wave functions then scales as x_0, and the condition $x_0 <$ (spatial extent) $\ll 1/k$ is equivalent to $\hbar\omega_v \gg R$. When $R \gg \hbar\omega_v$, the atom, even when it is near its cooling limit, will always sample many peaks and valleys of the standing-wave laser field, and the cooling limit should be adequately described by the theory for free atoms.

For a two-level bound atom the Doppler cooling limit is achieved when the natural width Γ of the optical transition is much larger than the oscillation frequency ω_v[1] and $R/\hbar$. The opposite condition, $\Gamma \ll \omega_v$, corresponds to what is called resolved sideband cooling. In this regime it has been shown[15,16] that it is possible to achieve conditions in which the difference between the minimum energy and the zero-point energy is less than the recoil energy as long as $\omega_v \gg R/\hbar$. (For this case, however, $k_BT > \hbar\Gamma$.) In order to compare Doppler cooling and Sisyphus cooling in the same conditions, we will therefore assume here that $\Gamma \gg \omega_v$. The main result of this paper is that, when $\Gamma \gg \omega_v$ and $\hbar\omega_v \gg R$, Sisyphus cooling leads to minimum temperatures T_s such that $k_BT_s \simeq \hbar\omega_v$, whereas Doppler cooling leads to temperatures T_D such that $k_BT_D \simeq \hbar\Gamma$, that is, higher than T_s by a factor of Γ/ω_v.

To illustrate the basic ideas, we consider a simple model system, which is described in Section 2. More-complicated cases can be generalized from this example. From a semiclassical treatment (atomic motion treated classically) given in Section 3, we will be able to derive cooling rates and limits. From this treatment we will see that the minimum energies are approximately equal to $\hbar\omega_v$. At these energies we might expect quantum effects of the motion to be important, so we will examine the cooling limits, treating the motion quantum mechanically in Section 4. In Section 5 we briefly compare the cooling rates and limits achieved here with those obtained from Doppler cooling. Although the model was chosen to be simple enough to show the basic effects, in Section 6 we examine a couple of possible experimental examples that closely approximate the simple model.

2. MODEL

A. Atomic Level Structure and Laser Configuration

We consider Sisyphus cooling of a single bound atom. We assume that the atom's internal structure is represented by three levels as shown in Fig. 1. We label these levels g for ground state, e for excited state, and r for reservoir state. We assume that the transition $g \rightarrow e$ is an electric dipole transition excited by a standing-wave laser beam whose frequency ω is tuned above the resonance frequency ω_0 for this transition by an amount δ (where all frequencies are expressed in radians per second). Level e spontaneously decays at rate Γ. It decays to level r with branching fraction β and to level g with branching fraction $1 - \beta$. We assume that the laser's intensity and detuning are such that the fraction of time the atom spends in the excited state is negligible. We assume that level r is transferred back to level g at a rate $R_{r\rightarrow g} = 1/T_r$, where the duration of the transfer process is so short ($\ll T_r$) that one can consider the position of the ion as remaining unchanged during the transfer. We assume that the atom is confined to one dimension (the x direction) by a harmonic well U_0 [Fig. 1(b)] characterized by the atom oscillation (or vibration) frequency ω_v. Here, U_0 is assumed to be an external potential that is independent of the laser beam. When the atom spontaneously decays from level e, the direction of photon emission is assumed parallel (or antiparallel) to the atom's motion. We assume that $E_e - E_g \gg |E_r - E_g|$, so that the effect of recoil heating is mathematically simple. These simplifying assumptions will not qualitatively affect the results for other, more complicated cases.

A key element in Sisyphus cooling is that the energies of the dressed states or of the light-shifted ground-state sublevels vary along the direction of the atomic motion. Such a situation will be achieved here if we take a laser intensity that varies linearly with x over the range of atomic motion. We are concerned mainly with the case in which this intensity gradient is due to a laser standing wave along the x direction (see also Appendix A). We then

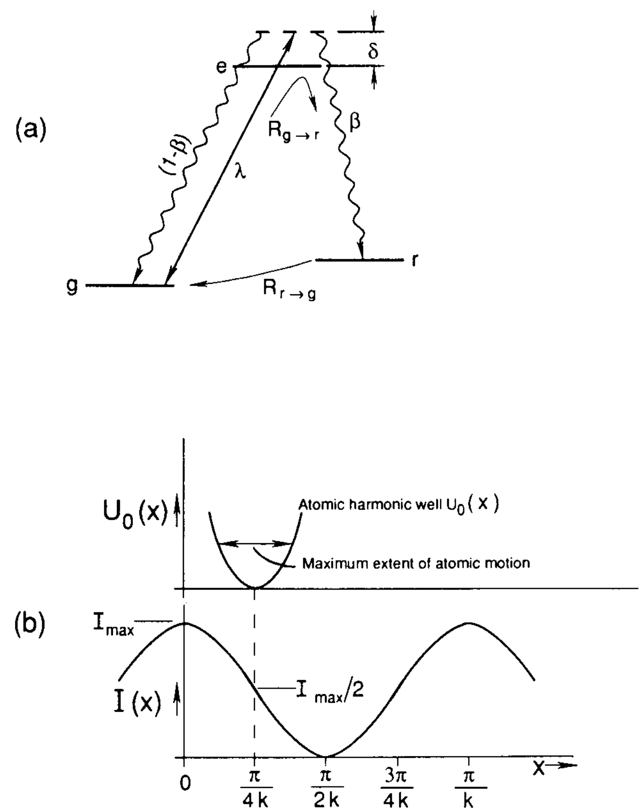

Fig. 1. Model system. (a) The atom has three internal energy levels: g, e, and r. A standing-wave laser beam drives $g \to e$ above its resonance frequency. Level e decays with branching fraction β to level r and with branching fraction $1 - \beta$ to level g. Transfer from level r to g occurs at a rate $R_{r\to g}$. (b) The atom is assumed to be confined by a harmonic well in the x direction. The maximum extent of the atom's motion is assumed to be less than $\lambda/2\pi$ (Lamb–Dicke regime).

write the laser electric field (classically) as

$$\mathbf{E} = (\mathbf{E}_{\max}/2)[\cos(kx - \omega t) + \cos(kx + \omega t)] = \mathbf{E}_{\max} \cos \omega t \cos kx\,, \tag{1}$$

where x is the atom's position and the wave vector $k = 2\pi/\lambda$, where λ is the laser wavelength. According to Eq. (1), the laser intensity $I(x)$ varies as $\cos^2(kx)$, as shown in Fig. 1(b). Sisyphus cooling will be most efficient when the intensity gradient is a maximum; this occurs at the half-intensity point of the standing wave. Therefore we assume that the atom's average position is near $x = \pi/4k$ [Fig. 1(b)]. The linear variation of the light shift of level g with x over the range of the atom's motion follows from the localization assumption: The extent of the atom's motion is less than $\lambda/2\pi$ (Lamb–Dicke regime).

In this paper we focus on the regime

$$R_{g\to r}, R_{r\to g} \ll \omega_v \ll \Gamma \ll \delta\,, \tag{2}$$

where $R_{g\to r}$ and $R_{r\to g}$ are, respectively, the rates of transfer from g to r when the atom is in level g and from r to g when the atom is in level r. We also assume that the laser intensity is weak enough to avoid any saturation of the $g \to e$ transition. That is,

$$s \equiv \frac{2{\omega_1}^2}{4\delta^2 + \Gamma^2} \ll 1\,. \tag{3}$$

In Eq. (3), s is the saturation parameter and $\omega_1 = \langle e|\mathbf{d}|g\rangle \cdot \mathbf{E}_{\max}/\hbar$ is the Rabi frequency. In this low-saturation regime the population of the excited state is small, and we can derive rate equations for the evolution of the lower states r and g.

The condition $\Gamma \ll \delta$ implies that the light shift of g [Eqs. (13) below] is larger than the contribution to the width of this level that is due to the photon scattering rate. The condition $\omega_v \ll \Gamma$ has already been discussed in Section 1 as defining the regime in which the Doppler limit is achieved for a two-level bound atom. In a point of view in which the atomic motion is treated classically (semiclassical approach), the condition $R_{g\to r}, R_{r\to g} \ll \omega_v$ means that the bound atom makes several oscillations in the harmonic binding potential before being transferred from g to r or from r to g. It is then possible to neglect the variation with space of the populations of levels g and r and to consider only their average over an oscillation period. This greatly simplifies the calculation of the rate of variation of the atomic external energy in the semiclassical approach. From a fully quantum point of view, in which the atomic motion is quantized, the condition $R_{g\to r}$, $R_{r\to g} \ll \omega_v$ means that the width of the vibrational levels in the binding potential, owing to the excitation rates from g or r, is smaller than the spacing $\hbar\omega_v$ between these levels. This introduces a similar simplification into the calculations, since it permits a secular approximation to be used. The secular approximation consists in neglecting, in the master equation describing the evolution of the atomic density matrix σ, any coupling between the populations of the vibrational levels and the off-diagonal elements of σ. In this way we can obtain a set of equations involving only the populations of the vibrational levels, which has a simple interpretation in terms of transition rates. This is an example of a situation in which the rate of variation of atomic internal variables ($R_{g\to r}, R_{r\to g}$) is slower than the rate of variation of external variables (ω_v). A similar example was recently studied in connection with the quantization of atomic motion in optical molasses.[18]

B. Qualitative Explanation of the Cooling

For $\delta > 0$, as shown in Fig. 1(a), the electric field of the laser shifts the level g to higher energy. Because $s \ll 1$, the atom is primarily in g or r. When it is in the state g (more precisely, in the dressed state, which contains the larger admixture of g), it is subject to an additional potential energy $U_L(x)$, which is a function of x proportional to the laser intensity. For the conditions of Fig. 1(b) the effect of this potential for the limited range of the atom's motion is to give an additional force in the $+x$ direction.

A crude explanation of the Sisyphus cooling is the following: As the atom oscillates back and forth (with frequency ω_v) in the well $U_0(x)$, it experiences an additional potential hill $U_L(x)$ sloping down toward the $+x$ direction. The atom is more likely to be transferred to level r by spontaneous Raman scattering when it is in regions of higher intensity. Therefore the predominant effect is that, after the atom runs up the laser hill $U_L(x)$ in the $-x$ direction (therefore losing kinetic energy), it is transferred to level r, where $U_L(x) = 0$. After a time of order T_r it is transferred back to level g, where on the average it must run up the hill $U_L(x)$ again before being transferred to level r. This leads to a net cooling effect.

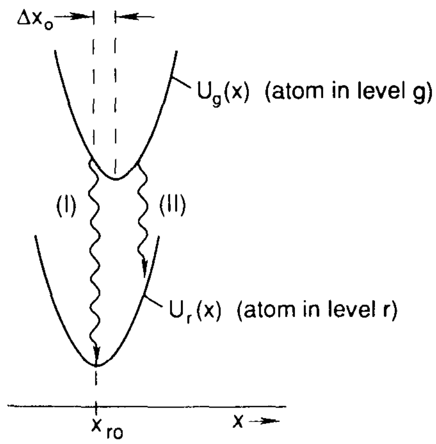

Fig. 2. Qualitative explanation of the cooling. When the atom is in state r, it experiences the confining potential $U_r(x)$, which is unperturbed by the laser fields. When the atom is in level g, the potential well is shifted in the $+x$ direction by the dipole force of the laser. This shifted potential is represented by $U_g(x)$. The laser excitation is assumed to be weak enough that the atom spends a negligible amount of time in level e. Therefore the spontaneous Raman transitions $g \to e \to r$ can be represented by transitions between g and r such as those indicated by (I) and (II) in the figure. Process (I) is favored more than process (II) because, when the atom is on the left-hand side of the well U_g, the chance of $g \to e$ excitation is higher since the laser intensity is higher [see Fig. 1(b)]. Process (I) leads to cooling because the atom drops to a lower part of the well U_r. Process (II) causes heating and, along with the heating resulting from recoil, eventually balances the cooling, resulting in a minimum energy.

A more accurate explanation is the following: When the atom is in the state g, it experiences the combined potential $U_0 + U_L = U_g$, which is a well with frequency ω_v shifted by an amount Δx_0 in the $+x$ direction (upper part of Fig. 2). When the atom oscillates in the $-x$ direction in the well U_g, it is more likely to be transferred to level r by spontaneous Raman scattering because the laser intensity is higher. It can therefore drop into a lower part of the unshifted well $U_r(x) = U_0(x)$, keeping the same position x since the duration Γ^{-1} of the transfer process is much smaller than the oscillation period ω_v^{-1} in the binding potential. It therefore loses potential energy [process (I) of Fig. 2]. After a time of order T_r (which is independent of the position x), the atom is transferred back to level g and the process is repeated. Heating, as illustrated by process (II) in Fig. 2, in which the atom gains potential energy, can also occur but is less favored because the laser intensity is less in the $+x$ direction.

Spontaneous Raman scattering ($g \to e \to r$) occurs at a rate proportional to the intensity, which according to Eq. (1) can be expressed as

$$R_{g\to r} \propto \frac{{E_{\max}}^2}{2}\cos^2(kx) \simeq I\left(x = \frac{\pi}{4k}\right)\left[1 - 2k\left(x - \frac{\pi}{4k}\right)\right]. \tag{4}$$

The term $-2kxI(x = \pi/4k)$ is responsible for the cooling. The term that is independent of x causes heating because of two effects. First, for each scattering event, photon recoil causes an average increase in energy $2R$.[19] Second, when the atom changes from level g to level r, the well switches from U_g to U_r. As we will see in Section 3, the corresponding potential energy change, when it is averaged over one oscillation period, leads to heating. A similar effect occurs in the transitions from r to g. The balance of the cooling with these sources of heating yields the minimum kinetic energy for the atom. Since $\delta > 0$, there is some additional Doppler or spontaneous force antidamping that tends to increase the atom's energy; this is shown in Subsection 5.B to be negligible for the conditions assumed here.

The cooling is the same when the atom's well is in a region of positive slope [e.g., $x = 3\pi/4k$ in Fig. 1(b)]. The cooling effect goes to zero when the well is centered at points of zero slope [e.g., $x = 0$, $\pi/2k$, and π/k in Fig. 1(b)].

3. SEMICLASSICAL TREATMENT

In this section we treat the external motion of the atom classically. Such treatment, often called semiclassical, is valid if the atomic de Broglie wavelength is small compared with the length scale (the light wavelength λ) over which the field varies.[20] In principle, it should also require that the extension of the atomic wave packet be much larger than the size of the ground state of the confining potential. Actually, we will see that the latter condition can be dropped, and the semiclassical and quantum results are identical even if the atom is mostly in the ground state of the well.

A. Internal Atomic Dynamics

The internal states of the atom are treated quantum mechanically. In the low-intensity domain of interest here [relation (3)], the optical Bloch equations, giving the evolution of the atomic density operator, can be simplified by an adiabatic elimination of the optical coherences (off-diagonal matrix elements of the density operator between ground and excited states). We are then left with a set of rate equations for the populations π_j ($j = g, r, e$) of the atom's three internal levels[20,21]:

$$\dot{\pi}_g = -R_{g\to e}(x)(\pi_g - \pi_e) + R_{r\to g}\pi_r + (1 - \beta)\Gamma\pi_e, \tag{5a}$$

$$\dot{\pi}_e = R_{g\to e}(x)(\pi_g - \pi_e) - \Gamma\pi_e, \tag{5b}$$

$$\dot{\pi}_r = -R_{r\to g}\pi_r + \beta\Gamma\pi_e, \tag{5c}$$

where

$$R_{g\to e}(x) = (\Gamma s/2)\cos^2(kx). \tag{6}$$

Since we have assumed that $s \ll 1$, the scatter rate $R_{g\to e}(x)$ is much smaller than Γ and the population of the excited state, π_e, is small compared with 1. For times long compared with Γ^{-1}, we have therefore approximately

$$\pi_e = \pi_g(s/2)\cos^2(kx), \tag{7}$$

$$\dot{\pi}_g = -R_{g\to r}(x)\pi_g + R_{r\to g}\pi_r, \tag{8a}$$

$$\dot{\pi}_r = -\dot{\pi}_g, \tag{8b}$$

where

$$R_{g\to r}(x) = \beta R_{g\to e}(x) = \beta(\Gamma s/2)\cos^2(kx). \tag{9}$$

In the limit $R_{r\to g}, R_{g\to r} \ll \omega_v$, and when we use $\pi_g + \pi_r \simeq 1$, the steady-state solution of Eqs. (8) is simply

$$\pi_g^{\text{st}} = \frac{R_{r\to g}}{\langle R_{g\to r}\rangle + R_{r\to g}}, \tag{10a}$$

$$\pi_r^{\text{st}} = \frac{\langle R_{g\to r}\rangle}{\langle R_{g\to r}\rangle + R_{r\to g}}, \tag{10b}$$

where $\langle R_{g\to r}\rangle$ stands for the average of $R_{g\to r}(x)$ over the atom oscillation period for the atom in level g (recall that we have assumed that $R_{r\to g}$ does not depend on x). The time constant τ_{int} for the relaxation of these internal variables is given by

$$1/\tau_{\text{int}} = \langle R_{g\to r}\rangle + R_{r\to g}. \tag{11}$$

B. External Atomic Dynamics

When the atom is in level r, or when the atom is in level g in the absence of the laser, the center of the atom's well is denoted x_{r0}. Therefore the atom's harmonic well can be described by

$$U_r(x) = U_0(x) = \tfrac{1}{2}m\omega_v^2(x - x_{r0})^2. \tag{12}$$

When the atom is in level g in the presence of the laser field, the center of the atom's well is shifted by the laser light. Using the dressed-state formalism,[9] we find that the energy of the state g is shifted by an amount

$$U_L(x) = \frac{\hbar}{2}[\omega_1^2\cos^2(kx) + \delta^2]^{1/2} - \frac{\hbar}{2}\delta \simeq U_{L0}\cos^2(kx), \tag{13a}$$

where

$$U_{L0} = \hbar\omega_1^2/4\delta \tag{13b}$$

is the maximum value of U_L. Equations (13) are valid for $\delta \gg \Gamma, \omega_1$. The internal state $|g\rangle$ changes to the dressed state $|g\rangle'$, but $|g\rangle' \simeq |g\rangle$ in the limit $\omega_1 \ll \delta$. The center of the atom's well, x_{g0}, when it is shifted by the dipole potential, corresponds to the spatial minimum of the total potential for the atom in level g:

$$U_g(x) = U_r(x) + U_L(x). \tag{14}$$

Since the cooling will be a maximum where the intensity gradient is maximum, we choose $kx_{r0} = \pi/4$. Assuming that the shift Δx_0 between the two wells is small compared with $\lambda/2\pi$, we linearize $U_L(x)$ around x_{r0} to obtain

$$U_L(x) \simeq U_{L0}/2 - kU_{L0}(x - x_{r0}). \tag{15}$$

The shift Δx_0 is then

$$\Delta x_0 = x_{g0} - x_{r0} = 2\xi(kx_0)x_0, \tag{16a}$$

where

$$\xi = U_{L0}/\hbar\omega_v \tag{16b}$$

and $x_0 = (\hbar/2m\omega_v)^{1/2}$ is the spread of the zero-point wave function in the harmonic well. We show in Subsection 3.D that the minimum kinetic energy is achieved for $U_{L0} \simeq \hbar\omega_v$, that is, for $\xi \simeq 1$. Therefore, since we have assumed the Lamb–Dicke criterion $kx_0 \ll 1$, the shift Δx_0 of the well $U_g(x)$ that is due to the dipole potential is, for minimum kinetic energy, much smaller than x_0 and thus much smaller than $\lambda/2\pi$, as we assumed in deriving Eq. (16a).

C. Cooling Rate

The variation of the external (kinetic + potential) energy E_x can be expressed by the equation

$$\dot{E}_x = \pi_g\langle R_{g\to r}(x)[U_r(x) - U_g(x)]\rangle + \pi_r R_{r\to g}\langle U_g(x) - U_r(x)\rangle + \pi_g\langle R_{g\to e}(x)\rangle 2R. \tag{17}$$

The first two terms stand for the average change in potential energy as the atom goes from g to r and from r to g, respectively. They therefore contain both Sisyphus cooling and the heating that is due to the sudden switching of the wells. Recoil heating for the $g \to r$ transfer is contained in the third term of Eq. (17), and spontaneous force Doppler antidamping (heating) can be neglected (Subsection 5.B). The averages are taken over one oscillation period in the initial potential [$U_g(x)$ for the first term of Eq. (17) and $U_r(x)$ for the second].

As in relation (4), we linearize the rate $R_{g\to r}(x)$ [Eq. (9)] around x_{r0}:

$$R_{g\to r}(x) \simeq \beta(\Gamma s/4)[1 - 2k(x - x_{r0})], \tag{18}$$

and we use the equipartition of energy for the atom oscillating in $U_g(x)$:

$$E_x = 2\left\langle \frac{1}{2}m\omega_v^2(x - x_{g0})^2\right\rangle. \tag{19}$$

For times long compared with τ_{int}, we then obtain, using Eqs. (10) for π_g and π_r,

$$\dot{E}_x = -(1/\tau_s)(E_x - E_{x0}), \tag{20a}$$

where the steady-state energy E_{x0} is

$$E_{x0} = \frac{m\omega_v^2(\Delta x_0)}{2k} + \frac{R}{k\beta(\Delta x_0)} = \frac{U_{L0}}{2}\left(1 + \frac{1}{\xi^2\beta}\right) = \frac{\hbar\omega_v}{2}\left(\xi + \frac{1}{\xi\beta}\right) \tag{20b}$$

and the Sisyphus cooling time constant τ_s is given by

$$\frac{1}{\tau_s} = \frac{4\langle R_{g\to r}\rangle R_{r\to g}}{\langle R_{g\to r}\rangle + R_{r\to g}}\left(\frac{R}{\hbar\omega_v}\right)\xi. \tag{20c}$$

For the conditions of interest ($\xi \simeq 1$, $R/\hbar\omega_v \ll 1$), one can check that $\tau_s \gg \tau_{\text{int}}$.

D. Cooling Limit

In steady state the atom's external energy is reduced to E_{x0} [Eq. (20b)]. E_{x0} is minimized when the laser intensity is adjusted to make $\xi \equiv U_{L0}/\hbar\omega_v = \beta^{-1/2}$, in which case

$$E_{x0}^{\min} = \hbar\omega_v/\sqrt{\beta} = U_{L0}. \tag{21}$$

From the second expression in Eq. (21) the minimum external energy is equal to the depth of the wells created by the standing-wave laser beam. This agrees with the result of Sisyphus cooling of free atoms.[8] When $\beta \simeq 1$, this predicts an energy near the zero-point energy for the ion in its well. In such a situation we might question the validity of this approach, which treats the atomic motion classically. Therefore it will be useful to compare the cooling limit derived here with the results from a quantum-mechanical treatment of the atom's motion.

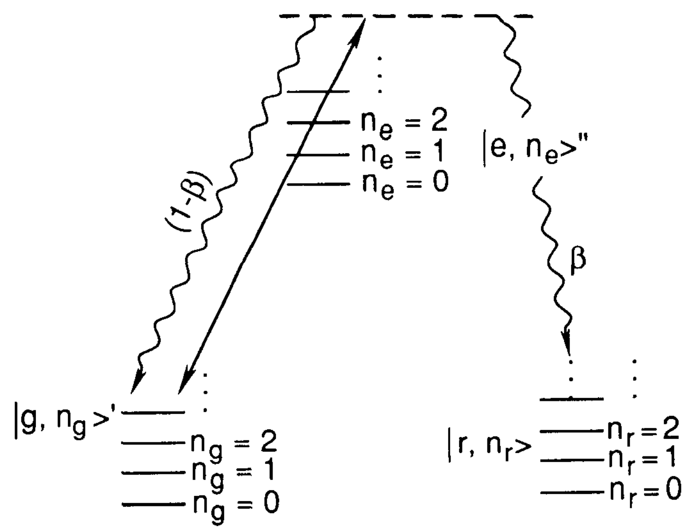

Fig. 3. Diagram for the cooling when the motion is quantized. Each electronic state has a sublevel structure of harmonic-oscillator levels labeled by the quantum numbers n_j, where $j = g$, e, or r. Cooling is represented by scattering processes of the form $n_g \to n_e \to n_r$, where $n_r < n_g$. This may be viewed as due to spontaneous Raman scattering off states $|g, n_g\rangle'$ and $|e, n_e\rangle''$, which are dressed by the laser.

4. QUANTUM TREATMENT

A. Transition Rates

In this section we treat the atom's motion in its well quantum mechanically. We describe the cooling process as shown in Fig. 3, where laser photons undergo spontaneous scattering from the dressed states, which now include the perturbed harmonic-oscillator states (denoted by primes and double primes).

Cooling results from spontaneous Raman scattering processes of the form $|g, n_g\rangle' \to |e, n_e\rangle'' \to |r, n_r\rangle$, where $n_r < n_g$. States $|g, n_g\rangle'$ and $|e, n_e\rangle''$ are assumed to be dressed by the standing-wave laser field. We can calculate the cooling and the cooling limit as was done in Ref. 19. For practical purposes the cooling rate is adequately represented by the semiclassical treatment of Subsection 3.C. Here we are interested primarily in the cooling limit, since the semiclassical treatment predicts a minimum energy approximately equal to $\hbar\omega_v$. We can calculate the minimum energy, using Eq. (27) of Ref. 19. It will be more instructive, however, to calculate the rates for each process $n_g \to n_r$ separately. According to Ref. 19, these rates can be written as

$$\Gamma_{g,n_g\to r,n_r} = C\left|\sum_{n_e}\frac{\langle n_r|\exp(-i\mathbf{k}_s\cdot\mathbf{X}|n_e\rangle''\{\langle n_e|''\}f(X)|n_g\rangle'}{\delta-(E_{n_e}-E_{n_g})/\hbar+i\Gamma/2}\right|^2, \tag{22}$$

where C is a factor that includes the laser intensity (at the position $x = \pi/4k$) and matrix elements for the $g \to e$ transition, $\mathbf{X}$ is the atomic position operator, and $\mathbf{k}_s$ is the wave vector for the scattered photon. For simplicity we will assume the scattered photon to be in either the $+x$ or the $-x$ direction, so that $\mathbf{k}_s \cdot \mathbf{X} = \pm kX$. $f(X)$ is a function representing the x dependence of the laser's electric field amplitude near $x = \pi/4k$; from Eq. (1), $f(X) \simeq 1 - k(X - x_{r0})$. E_{n_e} and E_{n_g} are the energies of the individual harmonic-oscillator levels for the excited and ground states, respectively. The sum is performed over all possible excited harmonic-oscillator states.

Since we have assumed that $\delta \gg \omega_v$, the denominator in the sum in Eq. (22) can be approximated by δ. This amounts to neglecting Doppler antidamping; the validity of this approximation is discussed in Subsection 5.B. With this approximation and the closure relation over the states $|n_e\rangle''$, Eq. (22) simplifies to

$$\Gamma_{g,n_g\to r,n_r} = \beta(C/\delta^2)|\langle n_r|\exp(-i\mathbf{k}_s\cdot\mathbf{X})f(X)|n_g\rangle'|^2. \tag{23}$$

When $\beta \neq 1$, we must also consider scattering directly back to the ground state from the excited state. These direct rates give rise to additional recoil heating for the $g \to e \to g$ scattering process:

$$\Gamma_{g,n_g\to g,n_g^*} = (1-\beta)(C/\delta^2)|\{\langle n_g^*|'\}\exp(-i\mathbf{k}_s\cdot\mathbf{X})f(X)|n_g\rangle'|^2, \tag{24}$$

where n_g^* is the final harmonic-oscillator state in the direct scattering process. Finally, we must consider the change in harmonic-oscillator energy levels in the transitions $r \to g$. As in the semiclassical treatment, this leads to a heating resulting from the sudden switching of the harmonic well (even without any spatial variation of the transfer rate $R_{r\to g}$). This process is described by the rate

$$\Gamma_{r,n_r\to g,n_g} = C'|\{\langle n_g|'\}1|n_r\rangle|^2, \tag{25}$$

where C' is a constant characterizing the rate of the $r \to g$ process. In Subsection 4.D these rates will be used in a master equation for the populations of the harmonic-oscillator levels. Before that is discussed, two preliminary steps are required. First we must find an expression for the perturbed harmonic-oscillator wave functions. Then we will obtain simple expressions for the rates in Eqs. (23)–(25).

B. Dressed Harmonic-Oscillator Wave Functions

As we discussed in Section 1, the dipole force acts uniformly over the extent of the wave functions of interest in the Lamb–Dicke limit. Therefore the effect of this dipole force is simply to shift the center of the harmonic-oscillator well for the g state to $x_{r0} + \Delta x_0$, the value calculated semiclassically. (This can be explicitly verified by the use of second-order perturbation theory to dress the wave functions.) The wave functions $|n_g\rangle'$ are simply obtained by using the spatial translation operator

$$|n_g\rangle' = \exp(-i\Delta x_0 P_x/\hbar)|n_g\rangle, \tag{26}$$

where P_x is the x component of the atomic momentum operator.

In the following, we will require both the position and momentum operators X and P_x in terms of raising and lowering operators $a^\dagger$ and a:

$$X = x_0(a^\dagger + a), \tag{27}$$

$$P_x = ix_0 m\omega_v(a^\dagger - a). \tag{28}$$

C. Evaluation of the Rates

Using Eq. (26) in Eqs. (23)–(25), we find that

$$\Gamma_{g,n_g\to r,n_r} = \beta(C/\delta^2)|\langle n_r|A_1|n_g\rangle|^2, \tag{29a}$$

$$\Gamma_{g,n_g\to g,n_g^*} = (1-\beta)(C/\delta^2)|\langle n_g^*|A_2|n_g\rangle|^2, \tag{29b}$$

$$\Gamma_{r,n_r\to g,n_g} = C'|\langle n_g|A_3|n_r\rangle|^2, \tag{29c}$$

where

$$A_1 = \exp(-ik_s X) f(X) \exp(-i\Delta x_0 P_x/\hbar), \quad (30a)$$

$$A_2 = \exp(i\Delta x_0 P_x/\hbar)\exp(-ik_s X) f(X)\exp(-i\Delta x_0 P_x/\hbar), \quad (30b)$$

$$A_3 = \exp(-i\Delta x_0 P_x/\hbar) \quad (30c)$$

and where $k_s = \pm k$, depending on the direction of the spontaneously emitted photon (assumed to be along the $+x$ or the $-x$ direction).

We now replace the position and momentum operators X and P_x in A_1, A_2, and A_3 by their expressions in terms of a and $a^\dagger$ [Eqs. (27) and (28)]. It is convenient to change the origin of the x axis to x_{r0}, in which case $f(X) = 1 - kX$. Since $kx_0(n+1)^{1/2} \ll 1$ (Lamb–Dicke criterion), we keep only terms in first order in $kx_0 a$ or $kx_0 a^\dagger$. This gives

$$A_1 \simeq 1 - kx_0[a^\dagger(1 - \xi + ik_s/k) + a(1 + \xi + ik_s/k)], \quad (31a)$$

$$A_2 \simeq 1 - kx_0(a^\dagger + a)(1 + ik_s/k), \quad (31b)$$

$$A_3 \simeq 1 + \xi k x_0(a - a^\dagger). \quad (31c)$$

Therefore, in the Lamb–Dicke limit, the important rates are those for which the vibrational quantum number changes by 1, and we have

$$\Gamma_{g,n\to r,n} = \beta\frac{C}{\delta^2} \equiv R_{g\to r}, \quad (32a)$$

$$\Gamma_{g,n\to r,n+1} = R_{g\to r}(kx_0)^2(n+1)[1 + (1-\xi)^2], \quad (32b)$$

$$\Gamma_{g,n\to r,n-1} = R_{g\to r}(kx_0)^2 n[1 + (1+\xi)^2], \quad (32c)$$

$$\Gamma_{r,n\to g,n} = C' \equiv R_{r\to g}, \quad (32d)$$

$$\Gamma_{r,n\to g,n+1} = R_{r\to g}(kx_0)^2(n+1)\xi^2 = \left(\frac{n+1}{n}\right)\Gamma_{r,n\to g,n-1}, \quad (32e)$$

$$\Gamma_{g,n\to g,n+1} = \frac{1-\beta}{\beta}R_{g\to r}2(kx_0)^2(n+1) = \left(\frac{n+1}{n}\right)\Gamma_{g,n\to g,n-1}. \quad (32f)$$

D. Rate Equations and Cooling Limit

A master equation for the populations takes the form

$$\dot{\pi}_{g,n} = \sum_{n'=n-1}^{n+1}\sum_{i=g,r}(-\Gamma_{g,n\to i,n'}\pi_{g,n} + \Gamma_{i,n'\to g,n}\pi_{i,n'}), \quad (33a)$$

$$\dot{\pi}_{r,n} = \sum_{n'=n-1}^{n+1}(-\Gamma_{r,n\to g,n'}\pi_{r,n} + \Gamma_{g,n'\to r,n}\pi_{g,n'}), \quad (33b)$$

where $\pi_{g,n}$ denotes the population with internal state g and external state n, etc. As expected, in steady state these equations show, after a little algebra, that the total population going from n to $n+1$ is equal to the total population going from $n+1$ to n. That is,

$$(\Gamma_{g,n\to r,n+1} + \Gamma_{g,n\to g,n+1})\pi_{g,n} + \Gamma_{r,n\to g,n+1}\pi_{r,n} = (\Gamma_{g,n+1\to r,n} + \Gamma_{g,n+1\to g,n})\pi_{g,n+1} + \Gamma_{r,n+1\to g,n}\pi_{r,n+1}. \quad (34)$$

In order to exploit this result, we now look for an approximate expression for $\pi_{g,n}$ and $\pi_{r,n}$ as a function of the total population of level n ($\pi_n = \pi_{g,n} + \pi_{r,n}$). Equations (33), written for steady state, give a relation between $\pi_{g,n}$, $\pi_{r,n}$, $\pi_{g,n\pm1}$, and $\pi_{r,n\pm1}$. If we assume that ξ is not large compared with 1 and recall that $kx_0(n+1)^{1/2} \ll 1$, then Eqs. (32) and (33) give $\Gamma_{g,n\to r,n}\pi_{g,n}/\Gamma_{r,n\to g,n}\pi_{r,n} \simeq 1 + O[(n+1)(kx_0)^2]$, so

$$\pi_{gn} \simeq \frac{\Gamma_{r,n\to g,n}}{\Gamma_{g,n\to r,n} + \Gamma_{r,n\to g,n}}\pi_n, \quad (35a)$$

$$\pi_{rn} \simeq \frac{\Gamma_{g,n\to r,n}}{\Gamma_{g,n\to r,n} + \Gamma_{r,n\to g,n}}\pi_n. \quad (35b)$$

Substituting relations (35) into Eq. (34) and using Eqs. (32), we obtain

$$\frac{\pi_{n+1}}{\pi_n} = \frac{1 + \beta\xi(\xi-1)}{1 + \beta\xi(\xi+1)}, \quad (36)$$

where we have neglected terms of order $(n+1)(kx_0)^2$ and higher. For $\xi \ll 1$, we have $\pi_{n+1}/\pi_n \to 1$. π_{n+1}/π_n is minimized for $\xi = U_{L0}/\hbar\omega_v = \beta^{-1/2}$, in which case

$$\left(\frac{\pi_{n+1}}{\pi_n}\right)_{\min} = \frac{2 - \sqrt{\beta}}{2 + \sqrt{\beta}}. \quad (37)$$

Since, according to Eq. (36), π_{n+1}/π_n is independent of n, we can write

$$\frac{\pi_{n+1}}{\pi_n} = \exp\left(-\frac{\hbar\omega_v}{k_B T}\right), \quad (38)$$

which shows that the system reaches a thermodynamic equilibrium. For such a thermodynamic equilibrium, $\langle n_v\rangle = [\exp(\hbar\omega_v/k_B T) - 1]^{-1}$, in which case

$$\langle n_v\rangle = [1 + \beta\xi(\xi-1)]/2\beta\xi. \quad (39)$$

This is minimized for $\xi = \beta^{-1/2}$, in which case

$$\langle n_v\rangle = \beta^{-1/2} - 1/2. \quad (40)$$

This corresponds to a minimum energy

$$E_{x0}^{\min} = \hbar\omega_v(\langle n_v\rangle + 1/2) = \hbar\omega_v\beta^{-1/2}, \quad (41)$$

in agreement with the semiclassical answer [Eq. (21)]. This remarkable agreement between the semiclassical and quantum theories can be interpreted in the following way. The motion of a quantum particle in a harmonic well is governed by the same basic equations as those for the classical motion. (This is particularly clear in the Wigner function approach and is a particular feature of the harmonic potential.) Consequently, the only place where Planck's constant plays a role in the problem here is in the recoil heating, where $R = \hbar^2k^2/2m$, and this recoil is handled in the same way in both treatments. Therefore it should not be surprising that both approaches lead to the same minimum energy, even though this energy is close to the zero-point energy in the quantum well.

5. DISCUSSION

A. Cooling Rate

It is interesting to compare the rate for the Sisyphus cooling discussed here with the rate for Doppler cooling. Doppler cooling can be represented in Fig. 1 by the conditions $\beta = 0$ and $\delta < 0$. The rate for Doppler cooling has been calculated by many authors; for example, from Eq. (19a) of Ref. 22 the cooling rate is

$$\tau_D^{-1} = -2R_{SD}\hbar k^2\delta_D\{m[(\Gamma/2)^2 + \delta_D^2]\}^{-1}, \tag{42}$$

where R_{SD} is the total scattering rate, the subscripts D stand for Doppler cooling, and the intensity is assumed to be below saturation ($s \ll 1$). For minimum temperature using Doppler cooling, we want $\delta_D = -\Gamma/2$. For this case,

$$\tau_D^{-1} = 4R_{SD}R/\hbar\Gamma. \tag{43}$$

According to Eq. (20c), assuming, for example, that $R_{r\to g} \gg \langle R_{g\to r}\rangle$ (that is, $\pi_g \simeq 1$), the ratio of rates for Sisyphus cooling and Doppler cooling is

$$\frac{\tau_s^{-1}}{\tau_D^{-1}} = \left(\frac{\Gamma}{\omega_v}\right)\left(\frac{\langle R_{g\to r}\rangle}{R_{SD}}\right)\xi. \tag{44}$$

Therefore, for the minimum-temperature case ($\xi \simeq 1$), the ratio of Sisyphus cooling to Doppler cooling is approximately given by the ratio $(\Gamma/\omega_v)(\langle R_{g\to r}\rangle/R_{SD})$. For $\langle R_{g\to r}\rangle/R_{SD} \simeq 1$, Sisyphus cooling is faster by Γ/ω_v.

Comparing the Sisyphus cooling rate with the cooling rate in the resolved sideband limit[15,16,19] is not so straightforward because the conditions in which each applies are different. For efficient sideband cooling (in a two-level atom), we want $\delta = -\omega_v$ and $\Gamma \ll \omega_v$, and these conditions are incompatible with relation (2). Therefore a comparison of cooling rates depends on the specific atomic system considered. The practical advantages of one cooling scheme over the other will also depend on the specific system investigated; this will depend, for example, on the availability and the ease of operation of the required lasers.

B. Contribution of Doppler Antidamping in Sisyphus Cooling

For the model considered here ($\delta > 0$), Doppler cooling is turned into an antidamping term. We can use Eq. (42) to check that this heating effect can be neglected compared with Sisyphus cooling. For the situation of interest here ($\delta \gg \Gamma$), we have

$$\tau_D^{-1} = \left(\frac{4\pi_g\langle R_{g\to r}\rangle}{\beta}\right)\frac{R}{\hbar\delta} = \left(\frac{4}{\beta}\right)\left(\frac{\langle R_{g\to r}\rangle R_{r\to g}}{\langle R_{g\to r}\rangle + R_{r\to g}}\right)\left(\frac{R}{\hbar\delta}\right). \tag{45}$$

By comparing with Eq. (20c), we find that

$$\frac{\tau_D^{-1}}{\tau_S^{-1}} = \left(\frac{1}{\beta\xi}\right)\frac{\omega_v}{\delta} \ll 1. \tag{46}$$

Therefore we may neglect the effects of Doppler antidamping compared with Sisyphus cooling except when $\beta \ll 1$.

C. Cooling Limit

The minimum energy for Doppler cooling is $\hbar\Gamma/2$. Therefore, according to Eq. (21) or (41), the minimum energy for Sisyphus cooling will be smaller than that for Doppler cooling by the approximate ratio $\omega_v/\Gamma \ll 1$. Consequently, the Sisyphus cooling rate is more efficient (at low temperatures, where the Lamb–Dicke criterion is satisfied) and the Sisyphus cooling limit is smaller than that for Doppler cooling. The minimum energy for sideband cooling and Sisyphus cooling is approximately the same, since the ion can be cooled to near the zero-point energy ($\hbar\omega_v/2$) in either case. However, for some applications it may be desirable to cool to $\langle n_v\rangle \ll 1$, in which case sideband cooling appears to be the appropriate choice.

6. POSSIBLE EXPERIMENTAL CONFIGURATIONS

To give an idea of how Sisyphus cooling might be employed, we consider the example of a single, trapped $^{24}Mg^+$ ion in a magnetic field **B**. To make the problem tractable, we will consider only the cooling of one degree of freedom, namely, the axial oscillation in a Paul rf trap or a Penning trap. To be consistent with the notation above, we call this the x degree of freedom. We will also make the simplifying assumption that the scattering rate for cooling the other degrees of freedom is adjusted so that, on the average, the recoil heating of the axial degree of freedom appears to be due only to reemission along the $+x$ or the $-x$ direction from the laser that cools the axial degree of freedom. This is consistent with the assumption made in the calculations of Sections 3 and 4.

The relevant internal energy levels are shown in Fig. 4(a). We assume that a linearly polarized standing-

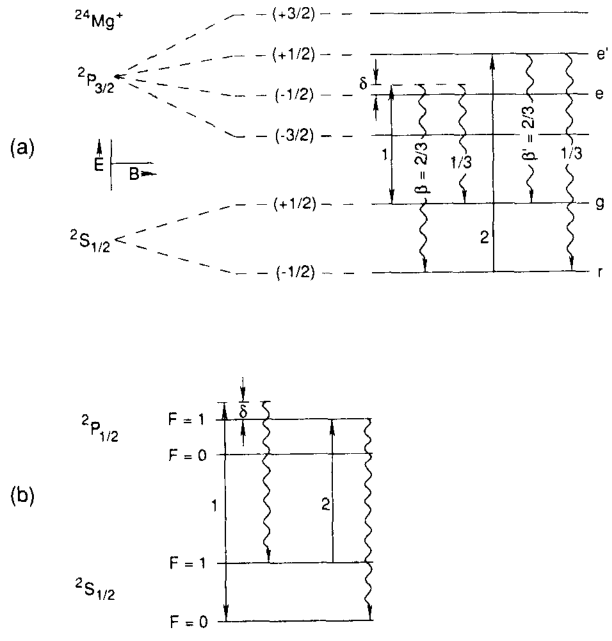

Fig. 4. The cooling described in the text could be approximately realized by a trapped $^{24}Mg^+$ ion in a magnetic field. A specific case is illustrated for the levels labeled g, e, and r in (a). Transfer from level r to level g could be realized with microwave radiation tuned to the $r \to g$ transition. This transfer could also be accomplished by spontaneous Raman transitions $r \to e' \to g$ by using a traveling-wave laser beam (laser 2 in the figure) tuned to the $r \to e'$ transition. In this case additional recoil heating must be accounted for (see the text). Another case that would apply to an ion or an atom with an outer unpaired electron and a spin 1/2 nucleus (and no intermediate electronic states) is shown in (b). Here the magnetic field is assumed to be small enough that the Zeeman structure is unresolved. Transfer from r to g through spontaneous Raman transitions (laser 2) is indicated in the figure.

wave laser beam (**k** vectors parallel and antiparallel to **x**) is tuned above the $^2S_{1/2}(m_J = +1/2) \rightarrow {}^2P_{3/2}(m_J = -1/2)$ transition frequency by an amount δ [laser 1 in Fig. 4(a)]. We assume that **B** (parallel to **x**) is large enough that $\hbar\delta \ll 2\mu_B B/3$, where μ_B is the Bohr magneton. This ensures that there will be negligible excitation on transitions other than those of interest. One situation for approximate realization of the cooling limit described in Subsections 3.D and 4.D is the following: First we leave laser 1 on long enough that the ion is pumped to level r with high probability [from Fig. 4(a), $\beta = 2/3$)]. We then turn laser 1 off and transfer ions from level r to g with a microwave π pulse. (To accomplish the transfer, we need not as-sume that the duration of the π pulse is less than $\omega_v{}^{-1}$.) Laser 1 is then turned on again, and the process is repeated. When laser 1 is turned back on, we assume that it is accomplished in a time long compared with $\omega_v{}^{-1}$ but short compared with $R_{g\rightarrow r}{}^{-1}$, so n_g is unchanged as $|g\rangle$ becomes dressed. As far as the cooling limit goes, this situation is actually somewhat better than the model that we have assumed in Section 2 because we avoid the heating resulting from well switching in the $r \rightarrow g$ transition. Mathematically, this amounts to setting $A_3 = 1$ in Eq. (29c). In this case the steady-state energy would be given by $\langle n_v\rangle = [2 + \beta\xi(\xi - 2)]/4\beta\xi$. This would be minimized for $\xi = (2/\beta)^{1/2}$, in which case $\langle n_v\rangle_{\min} = (2\beta)^{-1/2} - \frac{1}{2}$. For $\beta = 2/3, \langle n_v\rangle_{\min} \simeq 0.37$.

Experimentally, it might be easier to accomplish the $r \rightarrow g$ transfer with a second, traveling-wave laser beam tuned near the $^2S_{1/2}(m_J = -1/2) \rightarrow {}^2P_{3/2}(m_J = +1/2)$ transition [laser 2 in Fig. 4(a)]. (Additional cooling could be obtained with a second, standing-wave laser beam, but this would be more difficult to arrange experimentally.) The $r \rightarrow g$ transfer is then accomplished by spontaneous Raman scattering with probability $\beta' = 2/3$ for each scattering event from laser 2. This basic scheme could also be realized on $^2S_{1/2} \rightarrow {}^2P_{1/2}$ transitions in $^{24}Mg^+$ or other ions (atoms) with similar energy-level structure.

Using laser 2 will lead to a slightly higher minimum energy than our model predicts because of the additional recoil heating in the $r \rightarrow g$ transfer. To account for this heating properly, we must modify Eq. (17). Therefore we write

$$\dot{E}_x' = \dot{E}_x + \pi_r R_{r\rightarrow e'}(2R), \tag{47}$$

where $\dot{E}_x$ is given by Eq. (17), $R_{r\rightarrow e'}$ is the excitation rate from r to e' [e' is the $^2P_{3/2}(m_J = +1/2)$ level] that is due to the second laser, and $R_{r\rightarrow g} = 2R_{r\rightarrow e'}/3$. In steady state this amounts to doubling the recoil heating term in Eq. (17). For this case, $E_{x0} = \hbar\omega_v(\xi + 3/\xi)/2$, which is minimized for $\xi = \sqrt{3}$, yielding $E_{x0} = \sqrt{3}\,\hbar\omega_v$ or $\langle n_v\rangle = \sqrt{3} - 1/2 \simeq 1.23$.

For $^{24}Mg^+$ we take $\Gamma/2\pi = 43.0$ MHz and $\lambda = 279.64$ nm. If we assume that $\omega_v/2\pi = 2$ MHz, the conditions of relation (2) are reasonably satisfied for $\delta/2\pi = 1$ GHz and $\omega_1/2\pi = 0.118$ GHz (obtained from $\xi = \sqrt{3}$), in which case $R_{g\rightarrow e} \simeq 4.68 \times 10^5\ \mathrm{s}^{-1}$. For these conditions and laser 2 adjusted to make $R_{r\rightarrow e'} = R_{g\rightarrow e}$, we have $\tau_s = 17.4\ \mu$s. For the transitions of interest the resonant cross section is $\lambda^2/4\pi$. If laser 2 is tuned to resonance the required intensity is 5.34 mW/cm^2. For one of

$2\mu_B B/3$

the beams in the standing-wave laser beam, we require an intensity of 5.78 W/cm^2. To satisfy the condition $\hbar\delta \ll 2\mu_B/3$ we require that $B \gg 0.107T$. The effective temperature of the ion is given by

$$T = \hbar\omega_v/[k_B \ln(1 + \langle n_v\rangle^{-1})]. \tag{48}$$

For $\langle n_v\rangle \simeq 1.23$, $T \simeq 0.16$ mK. For the same transition the Doppler cooling limit is $\langle n_v\rangle \simeq 10$, corresponding to $T \simeq 1$ mK. For $\langle n_v\rangle < 1$, Eq. (48) shows that the temperature is only logarithmically dependent on $\langle n_v\rangle$. To achieve lower temperatures, we require ω_v to be lower, but if we make ω_v too low, we violate the condition $kx_0 \ll 1$ (in the example above, $kx_0 \simeq 0.23$).

Another scheme that might be used to cool an ion in a rf trap in a low magnetic field is illustrated in Fig. 4(b). This would work for $^{199}Hg^+$ ions, for example. Laser 1 provides the Sisyphus cooling, and laser 2 [or microwave radiation between the $^2S_{1/2}$ ($F = 0$) and ($F = 1$) levels] provides the repumping into level g. Similar schemes exist in other ions or atoms.

Experimentally, it may be difficult to make the center of the atom's well coincide with the point of maximum intensity gradient. One way around this problem for the standing-wave configuration would be to permit the standing wave to run over the center of the atom's position. This could be accomplished by the construction of a moving standing wave composed of two counterpropagating laser beams separated by frequency ω_m. When $R_{g\rightarrow r} \ll \omega_m \ll \omega_v$, the above expressions hold, with the magnitude of the intensity gradient replaced by its value averaged over the standing wave.

7. SUMMARY

We have examined Sisyphus cooling for a bound atom for the condition in which the radiative linewidth Γ is much larger than the oscillation frequency ω_v of the atom in its well. We have studied the strong-binding limit, which is given by the condition that the recoil energy be much less than the zero-point energy of the atom in the well. We have examined the cooling, using a simple three-level model for the internal states of the atom. We have assumed that the atom is confined in one dimension and cooled by a standing-wave laser beam. This model can be generalized to more-complicated cases. Since the minimum energies are near the zero-point energy of the atom in its well, we have examined the cooling limit, treating the atom's motion both classically and quantum mechanically. Within the approximations of the model the cooling limits are found to be the same.

We found that the minimum energy of the atom occurs when the depth of the dipole potential well created by the standing wave is approximately equal to the zero-point energy of the atom. In this case the minimum energy of the atom is given by the condition that the mean occupation number $\langle n_v\rangle$ of the atom in its well be approximately equal to 1. If we compare Sisyphus cooling with Doppler cooling, which operates on the same type of broad transition ($\Gamma \gg \omega_v$), we find that Sisyphus cooling is much more efficient for both the cooling rate (as long as we are in the Lamb–Dicke limit) and the cooling limit. Sisyphus cool ing leads to a temperature nearly as low as that resulting from sideband cooling without the requirement of a narrow atomic transition. We have briefly examined experimental examples that assume atomic ions stored in traps. The same ideas should apply, however, for neutral atoms confined by magnetic traps, optical dipole traps, etc.

APPENDIX A: COOLING IN OTHER INTENSITY GRADIENTS

In the main text we have treated Sisyphus cooling in a standing-wave laser beam. The main ingredient in the cooling is that the atom experiences an intensity gradient along its direction of motion. Therefore it is useful to consider cooling in a more general intensity gradient. The cooling treatment of Section 4 applies, except that we must replace k in relations (4) and (15), Eq. (16a), and relation (18) by $[dI/dx_i]/2I$ evaluated at the equilibrium position of the ion, $x_i(0)$. If we define

$$\zeta = \frac{1}{k}\left|\frac{[dI/dx_i]_{x_i(0)}}{2I[x_i(0)]}\right|, \tag{A1}$$

then we find that the cooling rate is equal to the cooling rate of Eqs. (20) multiplied by ζ^2, and the cooling limit (from the condition $\dot{E}_x = 0$) is given by

$$E_{x_0} = U_{L_0}\frac{1 + 1/\xi^2\zeta^2\beta}{2}. \tag{A2}$$

This limiting energy is minimized for $\xi = (2/\beta\zeta^2)^{1/2}$, in which case

$$E_{x_0}{}^{\min} = \hbar\omega_v/\zeta\sqrt{\beta}. \tag{A3}$$

As an example, consider cooling in the intensity gradient of a focused traveling-wave Gaussian laser beam whose intensity in the direction transverse to the axis of the beam is given by $I(x_i) = I_0 \exp(-2x_i^2/w_0^2)$, where w_0 is the beam waist. If we assume that the atom is nominally localized to the position where $I[x_i(0)] = I_0/2$, then $x_i(0) = \pm[(\ln 2)/2]^{1/2}w_0$. For this condition, $\zeta = [(\ln 2)/2\pi^2]^{1/2}(\lambda/w_0)$. For a given ω_v, the cooling rate is reduced by the factor ζ^2, and the minimum energy is increased by the factor ζ^{-1}. However, in certain experiments this cooling might still be useful, because the region over which this intensity gradient is linear is larger and a standing-wave laser beam is not required. Therefore the condition $\hbar\omega_v \gg R$ need not be satisfied. On the other hand, the minimum achievable energy is larger than the recoil energy. This can be shown with the fact that the relative variation of $I(x)$ over the spatial distribution corresponding to Eq. (A3) should remain much less than 1.

ACKNOWLEDGMENTS

D. J. Wineland thanks the U.S. Office of Naval Research and the U.S. Air Force Office of Scientific Research for support. J. Dalibard warmly thanks the National Institute of Standards, Gaithersburg, for hospitality and partial support for the period during which this research was completed. He also acknowledges financial help from Direction des Recherches, Etudes et Techniques. We thank W. M. Itano and W. D. Phillips for helpful suggestions on the manuscript.

The Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure is associated with the Centre National de la Recherche Scientifique and the Université Pierre et Marie Curie.

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