In Search of the Best Clock – an Update

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Because of the increased need for better clock performance than is currently available, this paper addresses some fundamental questions regarding clock metrology. Heretofore, most work has focussed on improving the clocks to meet the increased need. Though this is fundamental, we will show that significant gains are also available through the algorithms (computational methods for optimally combining the information) which process the readings of the clocks and through international comparisons now available via satellite. Proper algorithms for processing seem to be more important than the proportionate attention generally given them. In fact, to date, the only way we have been able to investigate some of the outstanding time predictability in long-term of the millisecond pulsar, PSR 1937+21, is by using such optimization algorithms.

Introduction

In the search for the best clock, we explicitly mean the clock with the best long-term frequency stability, which in turn can be shown to lead to the minimum long-term time dispersion [1]. This paper shows that such a clock is not a single physical device, but rather an optimal computed combination of information from a set of clocks and frequency standards. There are three important steps in time and frequency metrology for producing a best clock. Step one is to have an accurate primary frequency standard; the better the frequency accuracy, the better will be the achievable long-term frequency stability. Step two is to have a precise and accurate method of accumulating the time intervals defined by the available frequency standards. Step three is the optimum use by some computer algorithm of the time differences among all the clocks involved and of the frequency calibrations with the primary frequency standards. The deviations of the optimally computed time and frequency are less than those of the best contributing member clock [2-4].

There are now sufficient data on the long-term stability of clocks from high accuracy satellite time and frequency comparison techniques that we can view most of the best clocks in the world as if they were in the same laboratory. Therefore, we are now at a point in international time and frequency metrology where we have never been before. We can combine the time differences and frequency calibrations from the best clocks and frequency standards in the world without long-term deterioration and generate a "world's best clock" using an optimization algorithm. The clocks studied are primary cesium beam frequency standards, hydrogen masers, mercury ion frequency standards and various commercial atomic clocks. All of the primary timing centers' outputs, which are available to NBS via the Global Positioning System's common-view measurements, were used in the study. The millisecond pulsar signal from PSR 1937+21 was used as well. The efforts to search for the "best clock" in this study have the advantage that post analysis better helps us characterize and deal with systematic deviations.

2. Current Clock Performance and Some Future Considerations

The stochastic behaviors of some of the main frequency references are plotted in Figure 1 [5]. The frequency instabilities resulting from systematic and environmental perturbations are also of basic importance. The causes of these instabilities have to be considered in order to approach "best clock" performance [6,7].

Figure 2 illustrates an idea that uses technology which is nearly available and which needs a digital servo or computer processor to appreciate the theoretical estimation of frequency stability of 1 part in 10^{16} for an averaging time of about

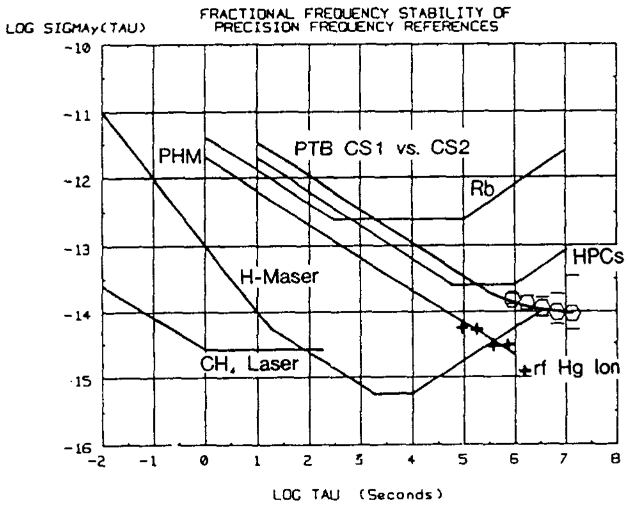

Figure 1. A $\sigma_y(\tau)$ versus τ plot of the measured stability of some of the best frequency references. The hexagons are for PTB CS1 versus PTB CS2. Rb is for a commercial rubidium gas-cell frequency standard. Better long-term stability performance has been observed [10,13]. The "HPCs" stand for a high performance commercial cesium-beam frequency standard and "PHM" for a passive hydrogen maser. The "+" symbols are compliments of the USNO for their rf trapped mercury-ion standard versus their active hydrogen maser, M19, for 36 days of data. "H-Maser" stands for an active hydrogen maser with the measurement system bandwidth, f_h, set at 500 Hz; the stability shown is $1/\sqrt{2}$ times the stability between a pair -- under the assumption of equal contribution. "CH_4" is for methane-stabilized laser.

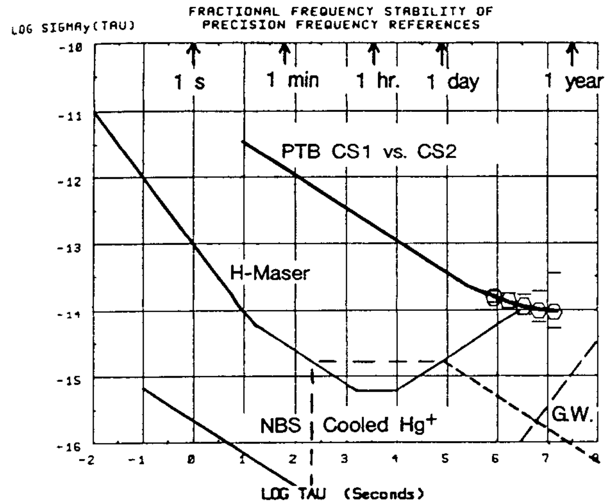

Figure 2. An illustration of a digital and/or computer servo technique to achieve a stability of one part in 10^{16} at τ = 1 year.

1 year. The idea is to use very high accuracy (10^{-16} or better) frequency standards that will soon be available in the optical or near optical region of the spectrum. A frequency divider and/or multiplier chain would be needed for operation for a few hundred seconds once a day in order to calibrate the rate of an active hydrogen maser, which could then carry the memory of that frequency day by day. If the uncertainties in each calibration were limited primarily by the instability of the maser, and these uncertainties were random and uncorrelated from day to day, then the maser could be made to track the frequency of the optical standard (at least in a software sense). The knowledge of the optical standard's frequency would then improve as $\tau^{-1/2}$ down to the accuracy limit of the optical standard itself.

3. Ensemble Time

We give below some examples of applications of ensemble time concepts.

A. The NBS ensemble algorithm from which UTC(NBS) is derived features two weighting factors per clock [2]. In theory, the algorithm's free-running output, NBS(AT1), should have better short-term and long-term stability than any member clock(s) in short-term and/or in long-term stability. Another feature of the NBS algorithm is that a clock's weighting factors are adaptive, conforming to the performance of each clock as it ages -- sometimes getting better and sometimes getting worse toward the end of clock life. This algorithm's output has been tested using other time scale data and using simulated data. The tests appear to be consistent with the theory.

B. Recently, a global Kalman ensemble time was generated using the data from all the principal timing centers available via GPS [8]. Figure 3 is a plot of the

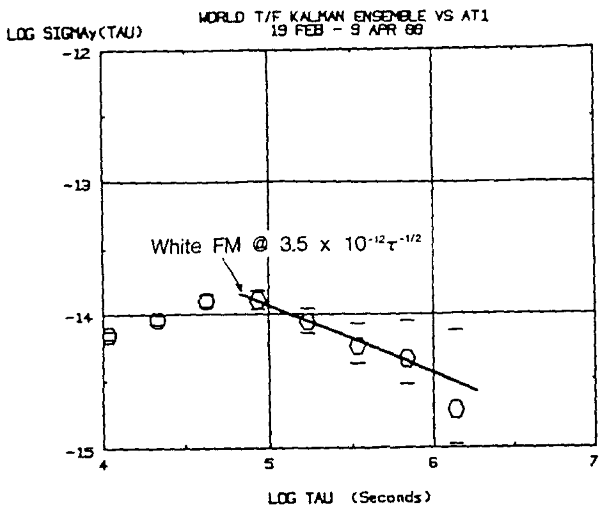

Figure 3. Frequency stability of NBS(AT1) or UTC(NBS) with respect to a Global ensemble taken via GPS in common-view [8]. The Kalman smoother characteristics cause the stability values to be too optimistic at τ = 1/8, 1/4, 1/2 day.

frequency stability between this ensemble time and NBS(AT1). For sample times shorter than one day the time/frequency Kalman smoother biases the stability values which are too low. For τ greater than or equal to one day the plotted values obviously fit the theoretical curve very well.

C. The Global Position System is anticipating the use of ensemble time both at the Master Control Center and for the satellite vehicle (SV) clocks. To show the quality of such an approach we employed the NBS time scale algorithm used in generating AT1 and applied it to the SV clocks [9]. This set of space clocks also gave us an independent reference significantly decoupled from the usual environmental disturbances. The stability plot of $\sigma_y(\tau)$-versus-τ of this space ensemble against NBS(AT1) gave rise to a performance which was better than that of most of the major timing centers in the world [9,10]. The long-term stability was well modeled by a flicker noise FM ($\sigma_y(\tau)$ was independent of the sample time, τ) for 10 days < τ < 1/2 year at a level of about 2 X 10^{-14}.

D. TTBIPM88 ensemble time, as generated by Guinot at BIPM, is based on a an algorithm with special weight functions for the set of about 160 clocks in order to meet the most stringent requirements for long-term stability, such as for the studies of millisecond pulsars [5]. Most of the clocks in TTBIPM88 are commercial cesium clocks, a few are primary cesium clocks, and a few are hydrogen masers. TTBIPM88 ensemble time along with the USNO master clock (with the coordinate time steering removed to make it an independent ensemble time), the compensated arrival times of the millisecond pulsar (PSR1937+21) and the NBS(AT1) ensemble time were used in an N-cornered-hat relationship to estimate the stability of each of the four. The estimated stabilities are plotted in Figure 4.

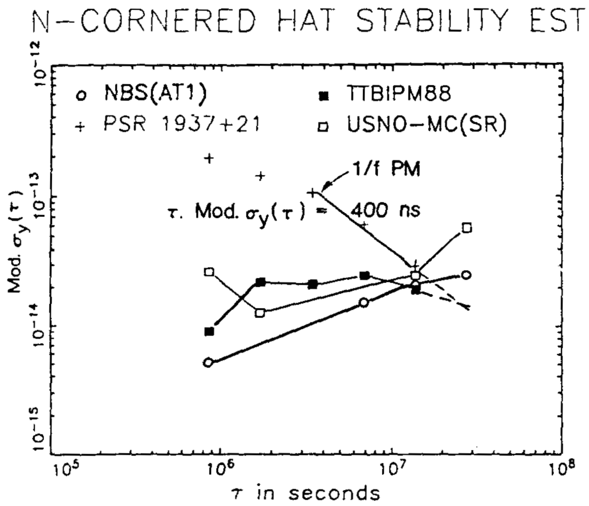

Figure 4. An N-cornered hat frequency stability estimate with four nominally independent contributing clocks: millisecond pulsar PSR 1937+21, NBS(AT1), TTBIPM88 and UTC(USNO SR) (The USNO master clock with steering removed). Because of correlations and apparent correlations, due to finite data lengths (this data set covered 1060 days), negative variances (not plotted) may result.

Because TTBIPM88 shows annual variations relative to some other time scales; for example, TAI, and because it so highly dependent in long term on the stabilities of the PTB primary cesium standards, a study was performed to determine whether long-term correlations in long term exist between the PTB Cs 1 and the PTB Cs 2 standards. Changes in humidity have been identified as one probable driving mechanism [10-12]. The correlation analysis was done by observing these two standards against other presumably independent time scales, and by using the cross-correlation technique used in reference [10]. Figure 5 is a time residual plot of

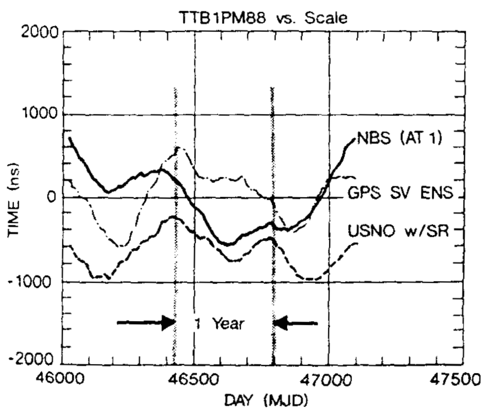

Figure 5. A plot of the time residuals of TTBIPM88 versus three other independent time scales - showing an apparent correlation in the annual variations (changes in slopes).

TTBIPM88 with respect to three other time scales and shows evidence of annual correlations. That one of the time scales was the GPS SV ensemble would rule out a humidity-coupling mechanism between it and the other two. Figure 6 shows the results of the cross-correlation analysis using NBS(AT1) and the GPS SV ensemble as the other independent scale to test for correlation between PTB Cs 1 and PTB Cs 2. The two independent reference scales were interchanged in their roles, and the correlation curve remained essentially the same as shown in Figure 6.

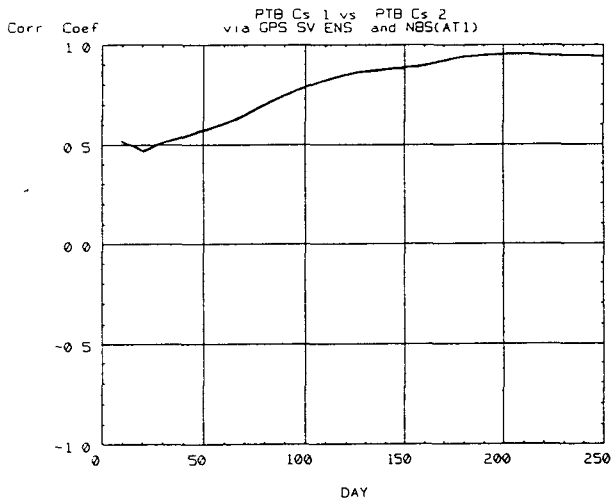

Figure 6. A cross-correlation analysis between PTB Cs 1, PTB Cs 2, NBS(AT1) and GPS SV ensemble time - showing evidence of annual correlations between the two primary cesium beam standards.

4. Conclusions

A few of the more significant time and frequency metrology advantages which arise from using appropriate processing algorithms have been demonstrated. Without the use of clock ensemble algorithms it would have been impossible to estimate the stability of the millisecond pulsar PSR 1937+21 at one year sample time at the current level (Mod.$\sigma_y(\tau$ = 360 days) $\lesssim 1.5 \times 10^{-14}$). Within about 10 years we project that, using advanced clocks which are now being developed and the ensemble methods described here, the level of stability 1×10^{-15} for τ = 1 year will be achieved. At this level of atomic-time stability there is a possibility for observing gravitational waves in comparisons with millisecond pulsars, that is, assuming that uncertainties in millisecond pulsar time due to earth orbit uncertainties are sorted out.

The time scale TTBIPM88 is a scale constructed in retrospect and appears to be the best long-term stable time scale yet constructed. In the long term PTB's primary cesium clocks are given high weights and are credited with making a major contribution toward this excellent long-term stability (sampling times of about a year and longer). We have shown some evidence that the random fluctuations between PTB Cs1 and Cs2 may not be random and uncorrelated; i.e., there may be some annual correlated movement of the two primary cesium beam standards, which in turn could induce an annual term in TTBIPM88. The size of the deviation seems to be of the order of 300 ns to 400 ns in half a year. This tentative conclusion needs further study for confirmation.

Better algorithms are being studied and insights into environmental effects on clocks are being gained. Future algorithms will need to make better use of information theory, maximum likelihood estimation techniques, Kalman filters, efficient statistical processing, models for systematic effects and of ways to compensate for environmental perturbations. With continued data sharing and international cooperative effort, all time scales stand to benefit. The output of such cooperation can be the "best clock" possible.

5. References

1. D. W. Allan, "Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction of Precision Clocks and Oscillators," IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, UFFC-34, 647-654, 1987.
2. D. W. Allan, J.E. Gray, and H.E. Machlan, "The National Bureau of Standards Atomic Time Scale: Generation, Stability, Accuracy and Accessibility," Chapter 9 in Time and Frequency: Theory and Fundamentals, ed. B.E. Blair, NBS Monograph 140 (Out of Print); D.W. Allan and M.A. Weiss, "The NBS Atomic Time Scale, AT1," NBS Technical Note 1316, 1988.
3. D. M. Granveaud and B. Guinot, "Atomic Time Scales," IEEE Trans. Instr. Meas., IM-21, 396, 1972.
4. B. Guinot, "Some Properties of Algorithms for Time Scales," Metrologie, 24, 195-198, 1987.
5. Private communication: Dr. DeBoer of the PTB, Dr. Winkler of the USNO Dr. Wineland of the NBS, Dr. Drullinger of the NBS, and Dr. B. Guinot of BIPM.
6. CCIR Report 364-5, "Performance of standard-frequency generators", Standards Frequencies and Time Signals, Vol. VII, 120-129, 1986.
7. F. L. Walls, "Frequency Stability of a Locked Oscillator," to be published in Proceedings of Fourth Symposium in Frequency Standards and Metrology, Ancona, Italy, September 6-9. 1988.
8. M.A. Weiss, "The Design of Kalman Smoothers for Global Positioning Data," submitted to CPEM Digest, 1988.
9. D.W. Allan and T.K. Peppler, "Ensemble Time and FRequency STability of GPS Satellite Clocks," to be published in Proc. of 42nd Annual Symposium on Frequency Control, Baltimore, MD, June 1-4, 1988.
10. D.W. Allan, "A Study of Long-Term Stability of Atomic Clocks," Proceedings of the Precise Time & Time Interval Planning Meeting, December 1987.

11. E. Bava, F. Cordara, V. Pettiti, and P. Tavella, "Analysis of the Seasonal Effects on a Cesium Clock to Improve the Long-Term Stability of a Time Scale, to be published in the Proceedings of the 19th Annual Precise Time and Time Interval Planning Meeting, Redondo Beach, CA, December 1-3, 1987.

12. J. E. Gray, H. E. Machlan and D. W. Allan, "The Effect of Humidity on Commercial Cesium Beam Atomic Clocks," to be published, Proc. of the 42nd Annual Symposium on Frequency Control, 1988.

13. F. Danzy and W. J. Riley, "Stability Test Results for GPS Rubidium Clocks," to be published in the Proceedings of the 19th Annual Precise Time and Time Interval Planning Meeting, Redondo Beach, CA, December 1-3, 1987.