Precise Phase Noise Measurements of Oscillators and Other Devices from 1 MHz to 20 GHZ

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Abstract

In this talk the commonly used measures of phase noise are briefly defined and their relationships explained. Techniques for making precise measurements of phase noise in oscillators, multipliers, dividers, amplifiers, and other components are discussed. Particular attention is given to methods of calibration which permit accuracies of 1 dB or better to be achieved. Common pitfalls to avoid are also covered. It is shown that the double balanced mixer approach is the most versatile of these techniques. Phase noise floors (precisions) in excess of -170 dB relative to 1 radian^2 per hertz are achievable for carrier frequencies from well below 1 MHz to the GHz range. The disadvantage for precise source measurements is the need for a reference source of comparable or better performance. This limitation does not apply to the measurement of amplifiers, multipliers, dividers, etc. Other techniques avoid this requirement by using a delay line or cavity to generate a pseudo reference generally with some sacrifice in noise floor near the carrier. Analogues of these techniques are used for carrier frequencies from a few Hz to 10^{15} Hz.

I. Introduction

The output of an oscillator can be expressed as

$$V(t) = [V_o + \varepsilon(t)] \sin(2\pi\nu_o t + \phi(t)) \qquad (1$$

where V_o is the nominal peak output voltage, and ν_o is the nominal frequency of the oscillator. The time variations of amplitude have been incorporated into $\varepsilon(t)$ and the time variations of the actual frequency, $\nu(t)$, have been incorporated into $\phi(t)$. The actual frequency can now be written as

$$\nu(t) = \nu_o + \frac{d[\phi(t)]}{2\pi dt} \qquad (2$$

The fractional frequency deviation is defined as

$$y(t) = \frac{\nu(t) - \nu_o}{\nu_o} = \frac{d[\phi(t)]}{2\pi\nu_o dt} \qquad (3$$

Power spectral analysis of the output signal V(t) combines the power in the carrier ν_o with the power in $\varepsilon(t)$ and $\phi(t)$ and therefore is not a good method to characterize $\varepsilon(t)$ or $\phi(t)$.

Since in many precision sources understanding the variations in $\phi(t)$ or y(t) are of primary importance, we will confine the following discussion to frequency-domain measures of y(t), neglecting $\varepsilon(t)$ except in cases where it sets limits on the measurement of y(t). The amplitude fluctuations, $\varepsilon(t)$, can be reduced using limiters whereas $\phi(t)$ can be reduced in some cases by the use of narrow band filters.

Spectral (Fourier) analysis of y(t) is often expressed in terms of $S_\phi(f)$, the spectral density of phase fluctuations in units of radians squared per Hz bandwidth at Fourier frequency (f) from the carrier ν_o, or $S_y(f)$, the spectral density of fractional frequency fluctuations in a 1 Hz bandwidth at Fourier frequency f from the carrier ν_o [1]. These are related as

$$S_\phi(f) = \frac{\nu_o^2}{f^2} S_y(f) \qquad \mathrm{rad}^2/\mathrm{Hz} \qquad 0<f<\infty \tag{4}$$

It should be noted that these are single-sided spectral density measures containing the phase or frequency fluctuations from both sides of the carrier.

Other measures sometimes encountered are $\mathcal{L}(f)$, dBC/Hz, and $S_{\Delta\nu}(f)$. These are related by [1,2]

$$S_{\Delta\nu}(f) = \nu_o^2 S_y(f) \qquad \mathrm{Hz}^2/\mathrm{Hz}$$

$$\mathcal{L}(f) = (1/2)S_\phi(f) \quad f_1<|f|<\infty \quad \text{for} \int_{f_1}^{\infty} S_\phi(f)df < 1\ \mathrm{rad}^2 \tag{5}$$

$$\mathrm{dBC/Hz} = 10 \log \mathcal{L}(f)$$

$\mathcal{L}(f)$ and dBC/Hz are single sideband measures of phase noise which are not defined for large phase excursions and are therefore measurement system dependent. Because of this an IEEE subcommittee on frequency stability recommended the use of $S_\phi(f)$ which is well defined independent of the phase excursion [1]. This distinction is becoming increasingly important as users require the specification of phase noise near the carrier where the phase excursions are large compared to 1 radian. Single sideband phase noise can now be specified as $(1/2)S_\phi(f)$.

The above measures provide the most powerful (and detailed) analysis for evaluating types and levels of fundamental noise and spectral density structure in precision oscillators and signal handling equipment as it allows one to examine individual Fourier components of residual phase (or frequency) modulation. On the other hand this analysis is extremely detailed and one often needs an analysis of the long-term average performance.

II. Methods of Measuring Phase Noise

Figure 1 shows the block diagram for a typical scheme used to measure the phase noise of a precision source using a double balanced mixer and a reference source. Fig. 2 illustrates a similar technique for measuring only the added phase noise of multipliers, dividers, amplifiers, and passive components. The output voltage of the mixer as a function of phase deviation, $\Delta\phi$, between the two inputs is normally given by

$$V_{out} = K \cos \Delta\phi \qquad (6$$

Near quadrature this can be approximated by

$$V_{out} = K_d \delta\phi, \text{ where } \delta\phi \equiv [\Delta\phi - \frac{2n-1}{(\;2\;)}\pi] < .1 \qquad (7$$

where n is the integer to make $\delta\phi \sim 0$. The phase to voltage conversion ratio sensitivity, K_d, is dependent on the frequency, the drive level, and impedance of both input signals, and the IF termination of the mixer [7]. The combined spectral density of phase noise of both input signals including the mixer and amplitude noise from the IF amplifiers is given by

$$S_\phi(f) = \left(\frac{V_n}{G(f)K_d}\right)^2 \frac{1}{BW} \qquad (8$$

where V_n is the RMS noise voltage at Fourier frequency f from the carrier measured after IF gain G(f) in a noise bandwidth BW. Obviously BW must be small compared to f. This is very important where $S_\phi(f)$ is changing rapidly with f, e.g., $S_\phi(f)$ often varies as f^{-3} near the carrier. In Fig. 1, the output of the second amplifier following the mixer contains contributions from the phase noise of the oscillators, the mixers, and the post amplifiers for Fourier frequencies much larger than the phase-lock loop bandwidth. In Figure 2, the phase noise of the oscillator cancels out to a high degree (often more than 20 dB). Termination of the mixer IF port with 50 ohms maximizes the IF bandwidth, however, termination with reactive loads can reduce the mixer noise by ~ 6 dB, and increase K_d by 3 to 6 dB as shown in Fig. 3. [3] Accurate determination of K_d can be achieved by allowing the two oscillators to slowly beat and measuring the slope of the zero crossing in volts/radian with an oscilloscope or other recording device. The time axis is easily calibrated since one beat period equals 2π radians. Estimates of K_d obtained from measurements of the peak to peak output voltage sometimes introduce errors as large as 6 dB in $S_\phi(f)$ [3]. By comparing the level of an IF signal (a pure tone is best), on the spectrum analyzer used to measure V_n with the level recording device used to measure K_d, the accuracy of $S_\phi(f)$ can be made independent of the accuracy of the spectrum analyzer voltage reference. Some care is necessary to assure that the spectrum analyzer is not saturated by spurious signals such as the line frequency and its multiples. Sometimes aliasing in the spectrum analyzer is a problem. Typical best performance is shown in Fig. 4. This measurement system exceeds the performance of almost all available oscillators from 0.1 MHz to 10 GHz and is generally the technique of first choice because of its versatility and simplicity. The use of specialized mixers with multiple diodes per leg increases the phase to voltage conversion sensitivity, K_d and therefore reduces the contribution of IF amplifier noise [4] as shown in Fig. 4. The resolution of the above systems can be greatly enhanced (typically 20 dB) using correlation techniques to separate the phase noise from the device under test from the noise in the mixer and IF amplifier [4].

For example consider the scheme illustrated in Figure 5. At the output of each double balanced mixer there is a signal which is proportional to the phase difference, $\Delta\phi$, between the two oscillators and a noise term, V_N, due to contributions from the mixer and amplifier. The voltages at the input of each bandpass filter are

$$V_1(\text{BP filter input}) = G_1\ \Delta\phi(t) + C_1 V_{N1}(t) \qquad (9$$

$$V_2(\text{BP filter input}) = G_2\ \Delta\phi(t) + C_2 V_{N2}(t)$$

where $V_{N1}(t)$ and $V_{N2}(t)$ are substantially uncorrelated. Each bandpass filter produces a narrow band noise function around its center frequency f:

$$V_1(\text{BP filter output}) = G_1[S_\phi(f)]^{1/2}\ B_1{}^{1/2} \cos\ [2\pi ft + \psi(t)]$$

$$+C_1[S_{VN1}(f)]^{1/2}\ B_1{}^{1/2} \cos\ [2\pi ft + n_1(t)] \qquad (10$$

$$V_2(\text{BP filter output}) = G_2[S_\phi(f)]^{1/2}\ B_2{}^{1/2} \cos\ [2\pi ft + \psi(t)]$$

$$+\ C_2[S_{VN2}(f)]^{1/2}\ B_2{}^{1/2} \cos\ [2\pi ft + n_1(t)]$$

where B_1 and B_2 are the equivalent noise bandwidths of filters 1 and 2 respectively. Both channels are bandpass filtered in order to help eliminate aliasing and dynamic range problems. The phases $\psi(t)$, $n_1(t)$ and $n_2(t)$ take on all values between 0 and 2π with equal likelihood. They vary slowly compared to $1/f$ and are substantially uncorrelated. When these two voltages are multiplied together and low pass filtered only one term has finite average value. The output voltage is

$$V^2_{out} = 1/2\ G_1 G_2 S_\phi(f)\ B_1{}^{1/2} B_2{}^{1/2} + D_1\langle\cos[7(t)]\rangle \qquad (11$$

$$+\ D_2 \cos[\psi(t) - n_2(t)]\rangle + D_3\langle\cos[n_1(t) - n_2(t)]\rangle$$ so that $S_\phi(f)$ is given by

$$S_\phi(f) = \frac{(2)V_N^2}{G_1 G_2 \sqrt{B_1 B_2}} \quad (12$$

For times long compared to $B_1^{-1/2} B_2^{-1/2}$ the noise terms D_1, D_2 and D_3 tend towards zero as $\sqrt{t}$. Limits in the reduction of these terms are usually associated with harmonics of 60 Hz pickup, dc offset drifts, and nonlinearities in the multiplier. Also if the isolation amplifiers have input current noise then they will pump current through the source resistance. The resulting noise voltage will appear coherently on both channels and can't be distinguished from real phase noise between the two oscillators. One half of the noise power appears in amplitude and one half in phase modulation.

Obviously the simple single frequency correlator used in this illustration can be replaced by a fast digital system which simultaneously computes the correlated phase noise for a large band of Fourier frequencies. Typical results show a reduction in noise floor of order 20 dB over the noise floor of a single channel (See Fig. 4). The great power of this technique is that it can be applied at any carrier frequency where one can obtain double balanced mixers. The primary limitations come from the bandwidth and nonlinearities in the cross correlator.

Another method of determining $S_\phi(f)$ uses phase modulation of the reference oscillator by a known amount. The ratio of the reference phase modulation to the rest of the spectrum then can be used for a relative calibration. This approach can be very useful for measurements which are repeated a great many times.

It is sometimes convenient to use a high-Q resonance directly as a frequency discriminator as shown in Fig. 6.

The oscillator is typically tuned 1/2 linewidth ($\nu_o/2Q$) away from line center yielding a detected signal of the form

$$V_{out} = G(f)kQdy(f)\ [V + \varepsilon(t)] \quad (13$$

Note that this approach mixes frequency fluctuations between the oscillator and reference cavity with the amplitude noise of the transmitted signal. By using amplitude control (e.g. by processing to normalize the data), one can reduce the effect of amplitude noise. [5] The measured noise at the detector is then related to the oscillator reference cavity phase fluctuation by

$$S_\phi(f) = \left(\frac{\nu_o V_N}{f \nu_o Q k G(f)}\right)^2 \frac{1}{BW} \quad (14$$

This approach has the limitations that $\Delta\nu$ must be small compared to the linewidth of the cavity, and removing the effect of residual amplitude noise is difficult; however, no reference source is needed.

Differential techniques can be used to measure the inherent frequency (phase) fluctuations of two High-Q resonators as shown in Fig. 7 [6]. The output voltage is of the form $V_{out} = 2QK_d\, dy(f)$. The phase noise spectrum of the resonators is then obtained using equation 4.

$$S_\phi(f) = \left(\frac{\nu_o V_N}{2QfK_d}\right)^2 \frac{1}{BW} \qquad (15$$

The phase noise in the source can cancel out by 20 to 40 dB depending on the similarity of resonate frequencies Q's and the transmission properties of the two resonators. This approach was first used to demonstrate that the inherent frequency stability of precision quartz resonators exceeds the performance of most quartz crystal controlled oscillators [6].

A still different approach uses a delay line to make a pseudo reference which is retarded relative to the incoming signal [7-10] as shown in Fig. 8.

The mixer output is of the form

$$V_{out} = 2\pi t_d K_d \nu_o dy \qquad (16$$

and the oscillator phase noise is given by

$$S_\phi(f) = \left(\frac{V_n}{2\pi f \tau_d G(f)\, K_d}\right)^2 \frac{1}{BW} \qquad f < \frac{1}{\tau_d} \qquad (17$$

This approach is often used at microwave frequencies when only one oscillator is available. However the phase noise close to the carrier becomes virtually unresolvable for a finite delay line. For example if $f = 1$ Hz and $\tau_d = 500$ ns, then, $(2\pi f \tau_d)^2 \sim 10^{-11}$. The noise floor of this technique is 110 dB higher at $f = 1$ Hz than that of the two oscillator method and it decreases as $1/f^2$. The noise floor can be reduced by ~ 20 to 40 dB using the correlation techniques described above. [10]

The use of frequency multipliers (or dividers) between the oscillators and the double balanced mixer increases (decreases) the phase noise level [11] as

$$S_\phi\, \nu_2(f) = \left(\frac{\nu_2}{\nu_1}\right)^2 S_\phi\, \nu_1(f) \qquad (18$$

Figure 4 shows the noise of a specialized 5 to 25 MHz multiplier referred to the 5 MHz input. A potential problem with the use of the multiplier approach comes from exceeding the dynamic range of the mixer. Once the phase excursion, $\Delta\phi$, exceeds about 0.1 radian, nonlinearities start to become important and at $\Delta\phi \sim 1$ radian, the measurement is no longer valid [11].

References

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Figure Captions

Fig. 1. Precision phase measurement system using a spectrum analyzer. Calibration requires a recording device to measure the slope at the zero crossing. The accuracy is better than 0.2 dB from dc to 0.1 ν_0 Fourier frequency offset from the carrier ν_0. Carrier frequencies from a few Hz to 10^{10} Hz can be accommodated with this type of measurement system. [3]

Fig. 2. Precision phase measurement system featuring self calibration to 0.2 dB accuracy from dc to 0.1 ν_0 Fourier frequency offset from carrier. This system is suitable for measuring signal handling equipment, multipliers, dividers, frequency synthesizers, as well as passive components. [3]

Fig. 3. Double-balanced mixer phase sensitivity at 5 MHz as a function of Fourier frequency for various output terminations. The curves on the left were obtained with 10 mW drive while those on the right were obtained with 2 mW drive. The data demonstrate a clear choice between constant, but low sensitivity or much higher, but frequency dependent sensitivity. [3]

Fig. 4.

Curve A. The noise floor $S_\phi(f)$ (resolution) of typical double balanced mixer systems (e.g. Fig. 1 and Fig. 2) at carrier frequencies from 1 to 100 MHz. Similar performance possible to 20 GHz. [4]

Curve B. The noise floor, $S_\phi(f)$, for a high level mixer. [4]

Curve C. The correlated component of $S_\phi(f)$ between two channels using high level mixers. [4]

Curve D. The equivalent noise floor $S_\phi(f)$ of a 5 to 25 MHz frequency multiplier.

Curve E. Approximate phase noise floor of Figure 8 using a 500 ns delay line.

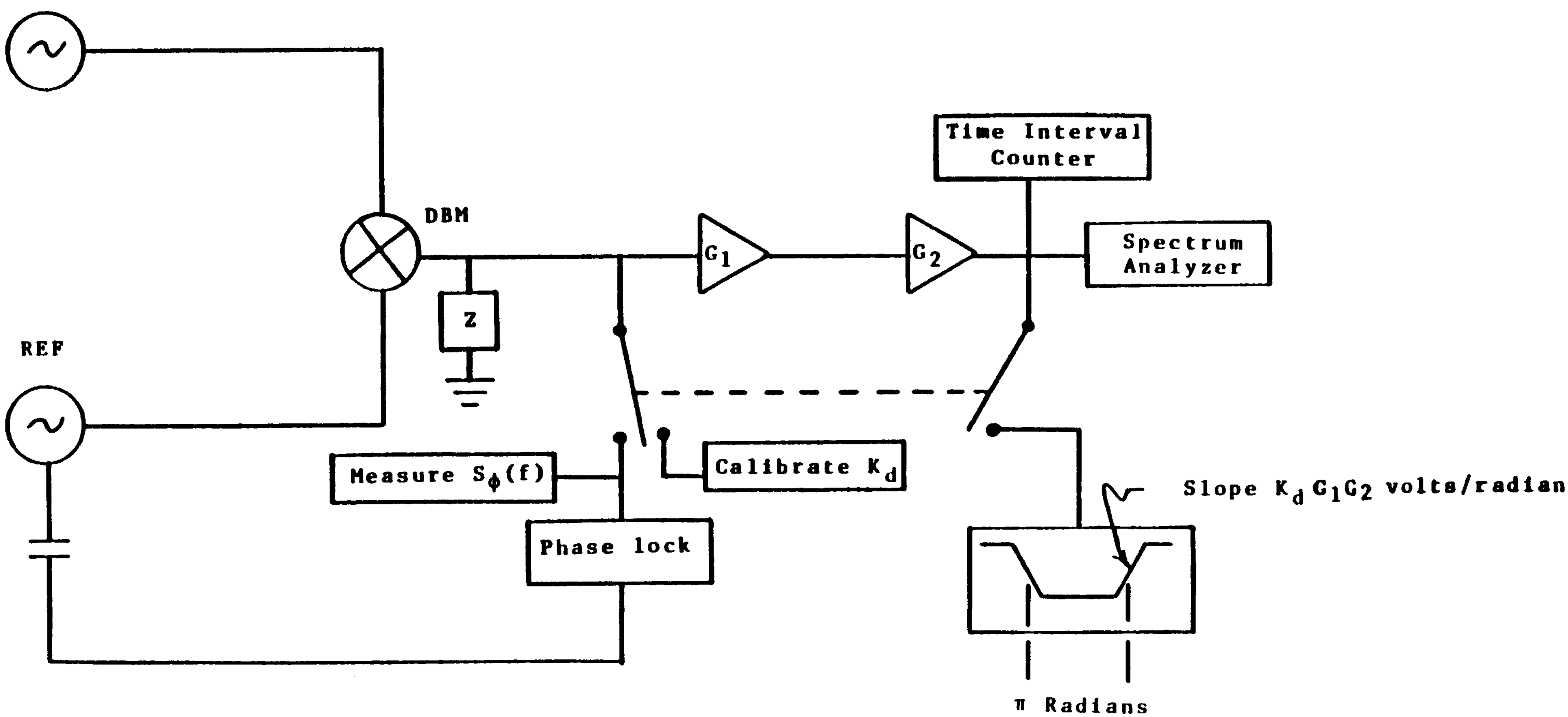
Fig. 5. Correlation phase noise measurement system.

Fig. 6. High-Q resonance used as a frequency discriminator.

Fig. 7. Differential cavity frequency discriminator.

Fig. 8. Delay line frequency discriminator.

Fig 1



DBM
REF
Z
Measure $S_\phi(f)$
Phase lock
Calibrate K_d
G_1
G_2
Time Interval Counter
Spectrum Analyzer
Slope $K_d G_1 G_2$ volts/radian
π Radians

Fig 2

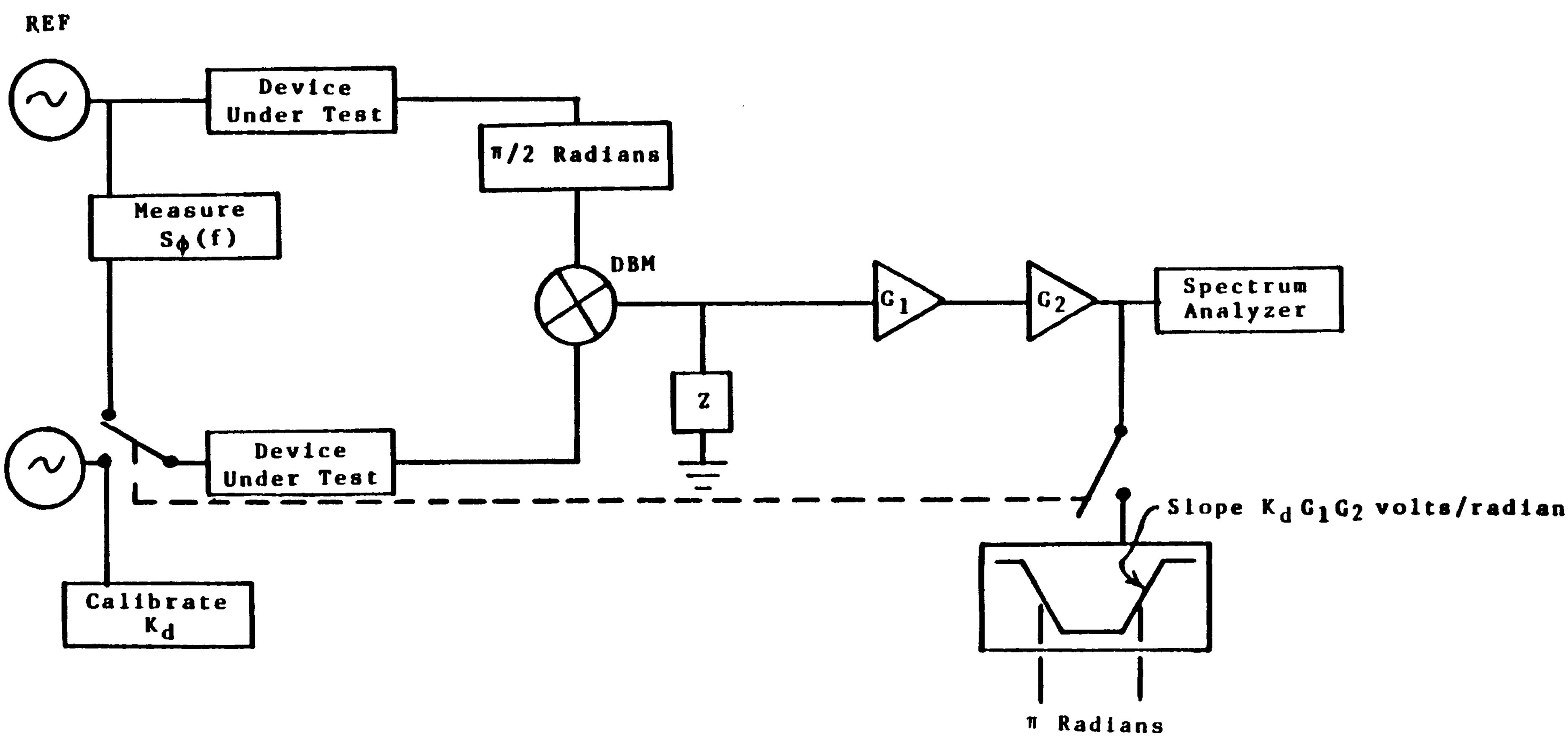
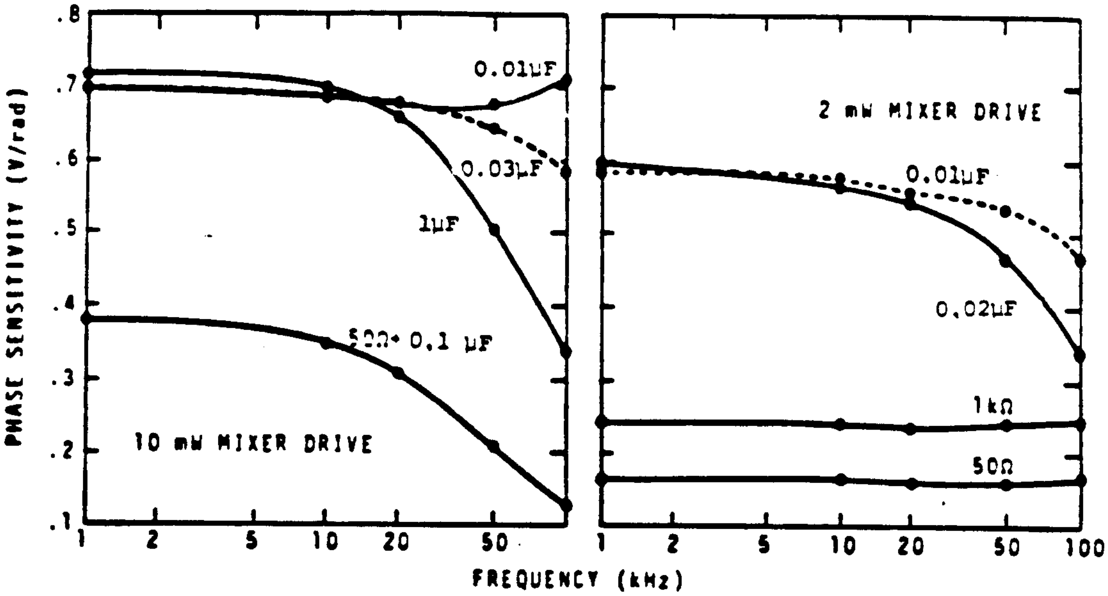

REF
Device Under Test
π/2 Radians
Measure $S_\phi(f)$
DBM
G_1
G_2
Spectrum Analyzer
Z
Device Under Test
Slope $K_d G_1 G_2$ volts/radian
Calibrate K_d
π Radians

Fig 3



PHASE SENSITIVITY (V/rad)
.8
.7
.6
.5
.4
.3
.2
.1
0.01μF
0.03μF
1μF
50Ω+0.1 μF
10 mW MIXER DRIVE
2 mW MIXER DRIVE
0.01μF
0.02μF
1kΩ
50Ω
1
2
5
10
20
50
100
FREQUENCY (kHz)

Fig 4

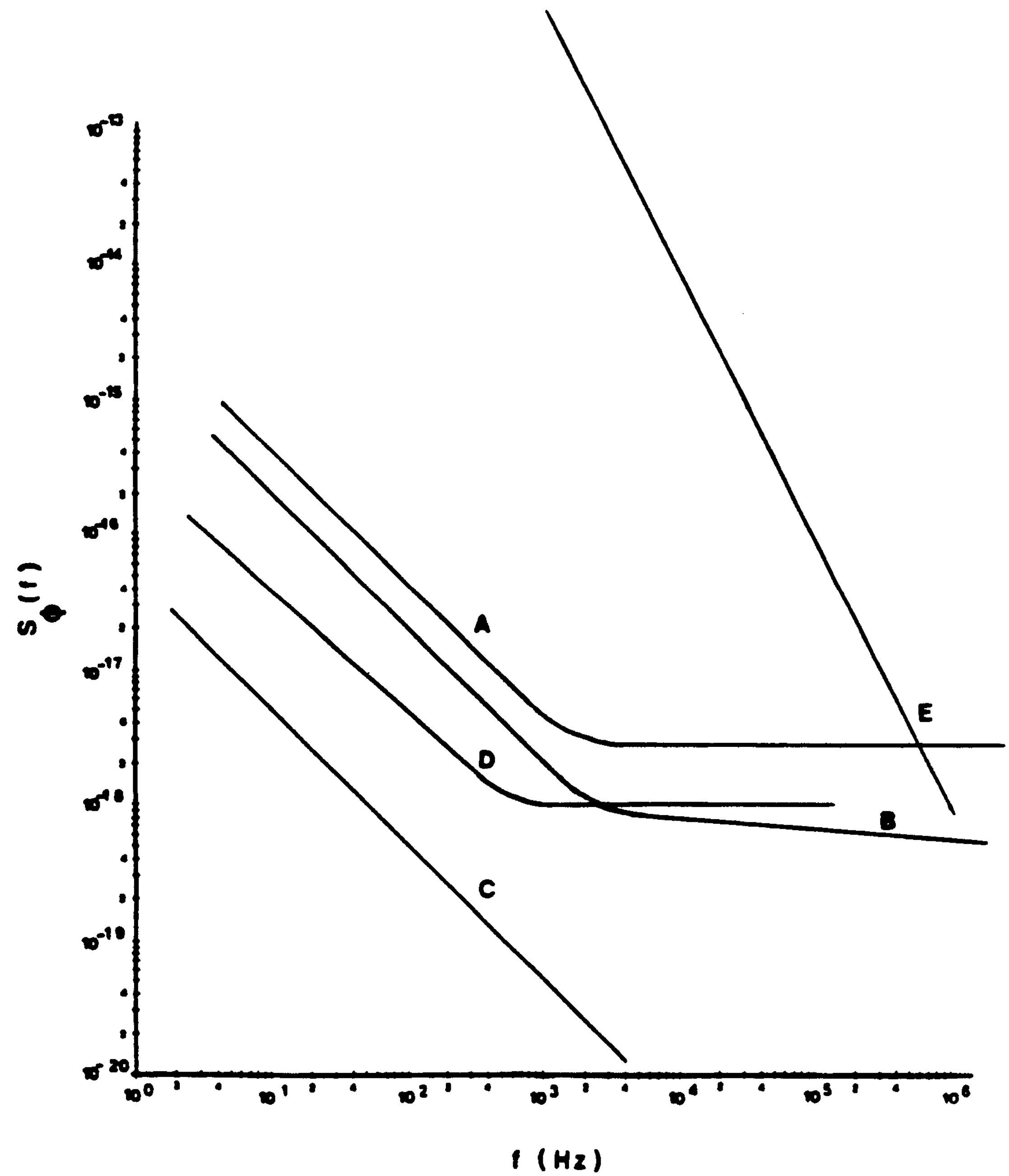

10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
$S_{\Phi}(f)$
10^0
10^1
10^2
10^3
10^4
10^5
10^6
f (Hz)
A
B
C
D
E

Fig 5

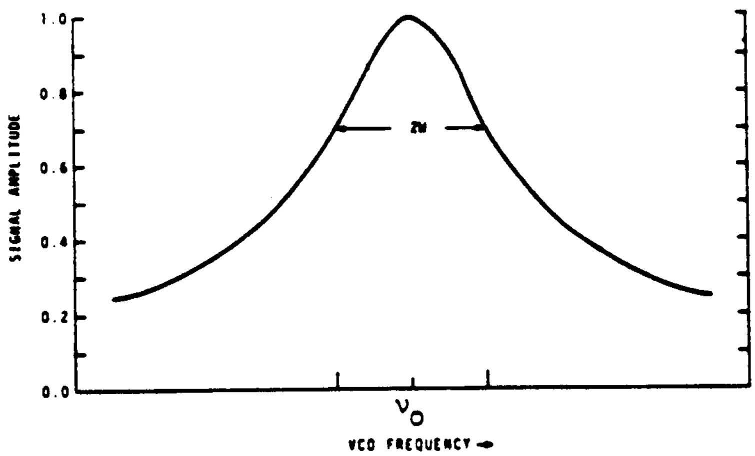
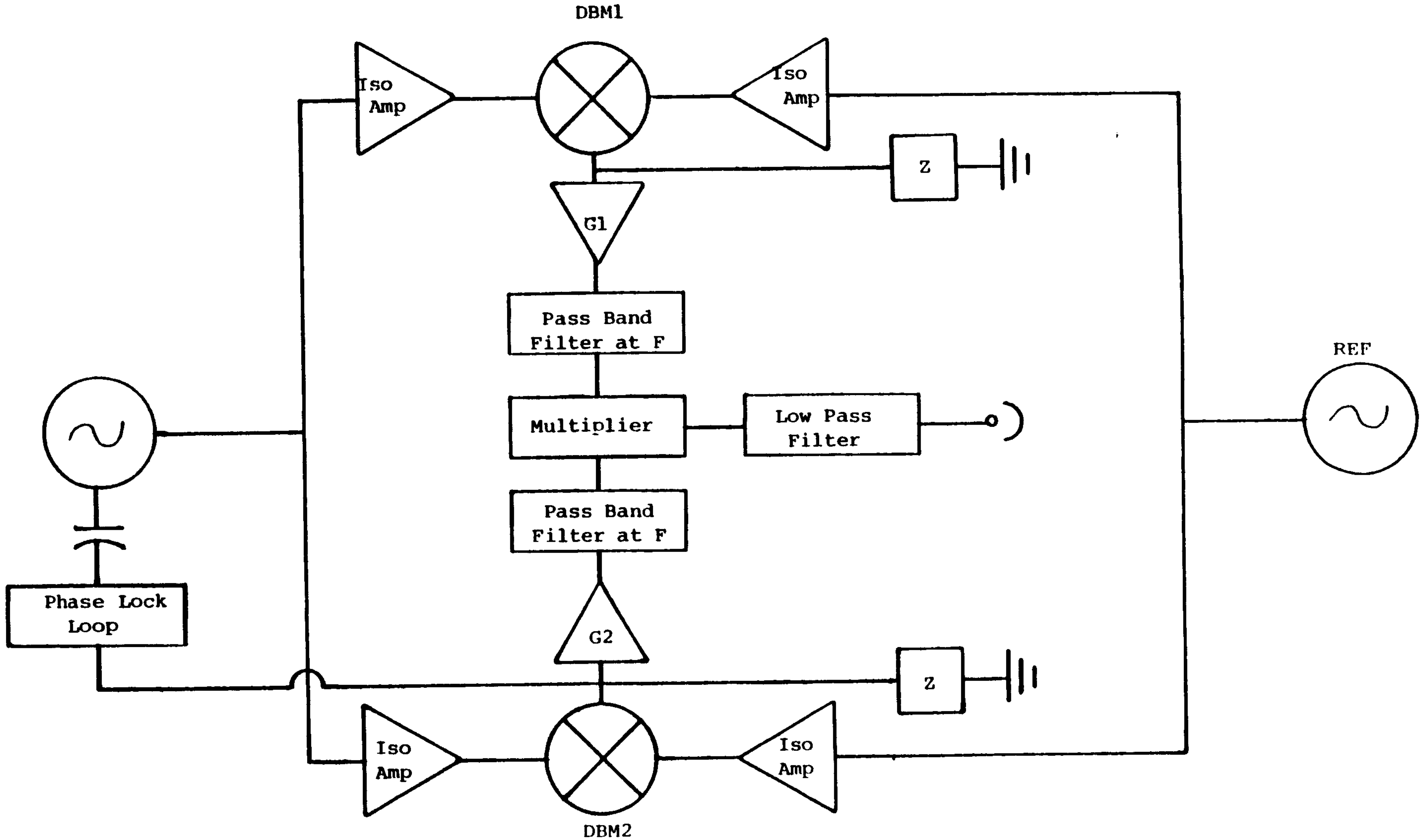
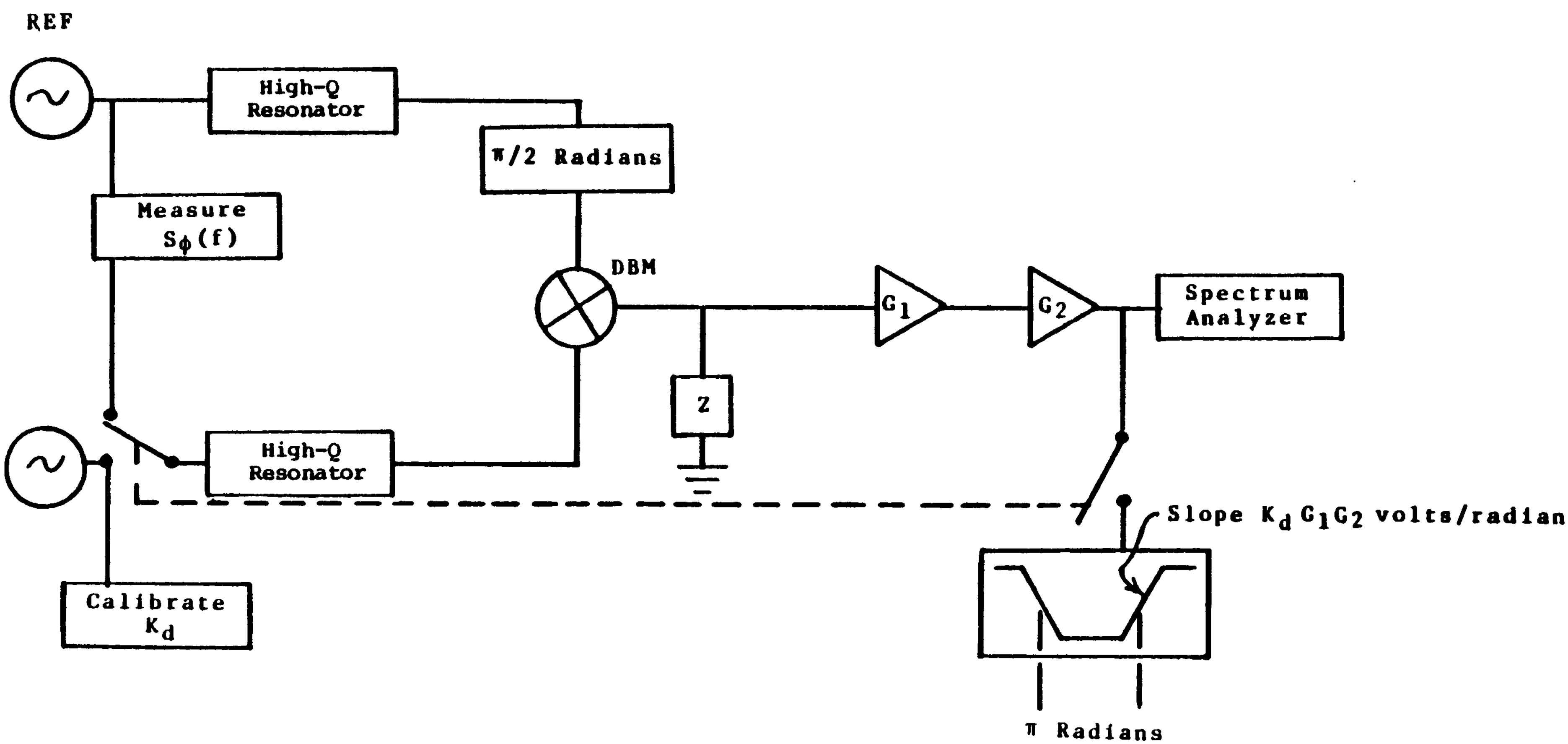


Fig 6



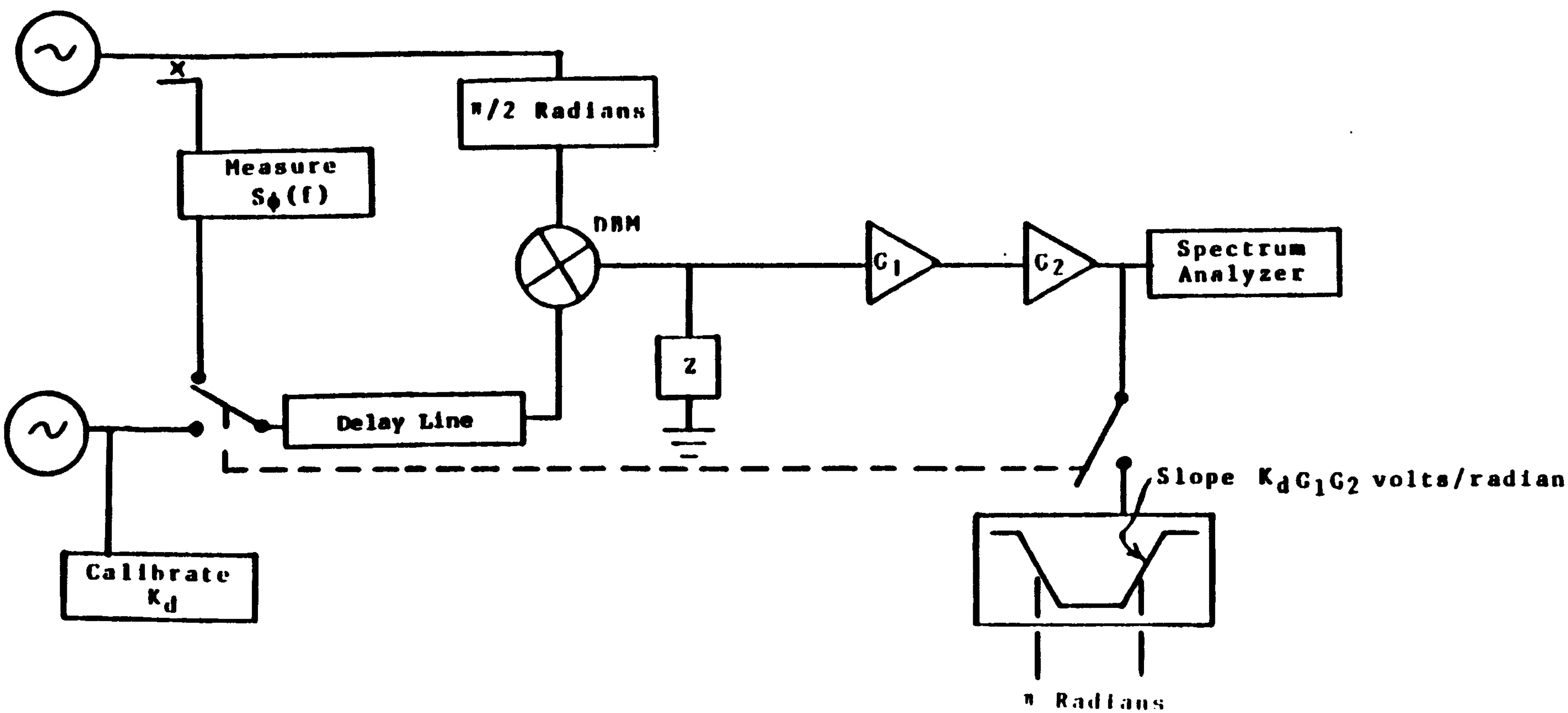
DBM1
Iso Amp
Iso Amp
Z
G1
Pass Band Filter at F
Multiplier
Low Pass Filter
REF
Pass Band Filter at F
Phase Lock Loop
G2
Z
Iso Amp
Iso Amp
DBM2

Fig 1



REF
High-Q Resonator
π/2 Radians
Measure $S_\phi(f)$
DBM
High-Q Resonator
Calibrate K_d
Z
G_1
G_2
Spectrum Analyzer
Slope $K_d G_1 G_2$ volts/radian
π Radians

Fig 8



π/2 Radians
Measure
$S_\phi(f)$
DBM
G_1
G_2
Spectrum
Analyzer
Z
Delay Line
Slope $K_d G_1 G_2$ volts/radian
Calibrate
K_d
π Radians