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GRAVITY TIDE MEASUREMENTS WITH A FEEDBACK GRAVITY METER

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Abstract. We have analyzed gravity tide data obtained using a calibrated gravity meter with electrostatic feedback. We find good agreement between the measured amplitude and phase of the major semidiurnal components and the corresponding values to be expected using current earth models and ocean load calculations. Both local and global barometric pressure changes make significant contributions to the power in the tidal bands and are included in the fitting function. The admittance estimates at diurnal frequencies can be used to determine the frequency and to set a lower bound on the dissipation of the nearly diurnal resonance in the tidal response. These estimates are in reasonable agreement with results obtained by other methods but are somewhat different than the values to be expected on the basis of theoretical considerations.

Introduction

Measurements of the amplitudes and phases of the major frequency components of the earth tides can be used to study various properties of the earth such as the resonant behavior in the diurnal band resulting from the coupling between the core and the mantle or the modification of the tidal admittance due to mantle anelasticity. Comparisons between theory and experiment can be made more meaningful if the absolute amplitudes of the tidal constituents can be determined, and this in turn requires a knowledge of the transfer function of the instrument used. The stability and linearity of the transfer function are also important, but as we show below, slow changes in the calibration or small nonlinearities can be incorporated into the analysis without great difficulty. The residual uncertainties do not dominate the error budget of the measurement.

In this paper we discuss the acquisition and analysis of gravity tide data. We also compare our results with earlier analyses of strain tides [Levine and Harrison, 1976; Levine, 1978], and we find good agreement between the two estimates of the frequency of the diurnal resonance.

Instrument Design

The data were acquired using a modified LaCoste and Romberg model D gravity meter. (Note that the use of the manufacturer's name is for identification purposes only and does not imply endorsement or a certification of suitability by the U.S. Government.) This instrument measures the acceleration of gravity by balancing the gravitational force on a small proof mass with a force exerted by a calibrated spring. As normally used, the suspension force exerted by the spring is varied manually by moving the upper end point of the spring using a lever system driven by a calibrated screw until the beam is centered at its null position. The position of the beam may be determined either optically using a telescope attached to the lid of the instrument or electronically using a galvanometer driven by the output of a capacitive bridge circuit. This method of bringing the proof mass to its equilibrium position is somewhat difficult to perform automatically since it would require a servomotor to turn the measuring screw automatically. It is far more convenient to use electrostatic feedback to keep the beam at its null position. This also minimizes the hysteresis that may accompany changes in the length of the spring.

Block and Moore [1966] and Weber and Larson [1966] discuss methods for applying electrostatic feedback to these instruments. In these methods the relationship between feedback voltage and gravitational force is usually nonlinear and can be linearized only with some difficulty using a mechanical adjustment of the feedback electrodes [Moore and Farrell, 1970].

Harrison and Sato [1984] developed a different method for using feedback with LaCoste and Romberg survey gravity meters. In their method the conventional electrostatic feedback system is modified so that the voltages applied to the two outer plates are no longer equal. The error signal, which is proportional to the deviation of the beam from its null position, is integrated, and the result is applied through two amplifiers to the two outer plates. The gains of the two amplifiers are different and are adjusted so as to linearize the relationship between the gravitational force on the beam and the integrated error

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voltage. The details of the adjustment process are discussed in their paper.

Calibration of the Gravity Meter

LaCoste and Romberg calibrate the gravity meter by recording the position of the support point for the spring (as read from the dial indicator on the drive screw) at places where g is known. The nonlinearity of the screw is estimated by recording the change in the position of the support point when small, known masses are added to the proof mass at about a dozen different equilibrium positions of the screw. These methods yield a calibration that relates changes in gravitational force to the motion of the screw. The simplest way to calibrate and linearize the electrostatic feedback system is to transfer this calibration to the feedback system using the relationship between the screw position and g provided by the manufacturer. This method was used several times during the course of our work. In addition to providing a calibration relating voltage to gravitational force, this method also is used to linearize the feedback system by adjusting the gains of the outer plate drive amplifiers until equal and opposite deviations of the screw from a nominal equilibrium position produce equal and opposite changes in the output voltage of the feedback system.

The process of linearizing the electrostatic feedback system assumes that the screw and the spring are perfectly linear. As the experiment proceeded, the instrument drifted somewhat, and the operating point was adjusted periodically to compensate for this drift. Subsequent tests of the stability of the calibration and the linearity of the feedback were therefore carried out at different positions of the screw.

The linearity of the screw on our instrument was subsequently studied by Larson and coworkers [Larson and Harrison, 1985; Larson et al., 1984]. They found that our instrument, D43, had a screw nonlinearity that was significantly larger than we originally assumed.

The calibration of the screw was evaluated by moving the instrument vertically (at the Washington Monument) and by measuring the changes in the feedback voltage for equal and opposite changes of 0.5 counter units in the screw position at 44 different equilibrium positions spaced 0.4 counter units apart. These measurements were repeated in the laboratory using the range reset screw to vary the initial position of the beam. The two methods gave identical results. The comparisons were extended to the full range of the meter (200 counter units). This process depends only on a knowledge of the vertical gravity gradient and need not be performed at a site where the absolute value of g is well known.

Unfortunately, the results of this work were not available to us during the measurement period, and the calibrations at that time were done assuming a linear screw. As a result, the nonlinearities of the screw resulted in small nonlinearities in the electrostatic feedback as well as a small change in the calibration factor. These deviations from a simple linear relationship between gravitational force and output voltage were removed during the data reduction, but it is likely that some residual problems remain in the data. We estimate these effects below.

Data Acquisition and Preliminary Analysis

The gravity meter was located in one of our laboratories in Boulder, Colorado. The laboratory is isolated from vibration of the rest of the building and is located about 5 m below ground level.

The output of the gravity meter was sampled and digitized 10 times per hour with a resolution of 12 bits (1 part in 4096). The least significant bit of the digitizer corresponded to a change in the acceleration of gravity of about 0.8×10^{-8} m/s^2 (0.8 μGal).

We acquired data for 1 year but did not use the last part of the record because it was heavily contaminated with high-frequency noise generated by other experiments in the same laboratory. The length of the quiet part of the record was about 6200 hours with only a few very short gaps. The gravity meter drifted continuously during this period. The instrument was rezeroed whenever the drift had offset the equilibrium position by about 500×10^{-8} m/s^2. This corresponded to limiting the magnitude of the feedback voltage to less than 3 V. A zero adjustment was required about every 3 weeks. This drift is essentially purely instrumental. A second gravity meter located nearby recorded no significant secular change in g during the same time period.

The first step in the analysis is to convert the recorded data from volts to changes in acceleration. This was done using the various calibrations that were made during the course of the measurements. These calibrations and linearity checks were done using the screw as a reference and assuming it to be perfectly linear. As a result, subsequent analyses with the final screw model showed that small nonlinearities were present. We express the result of a calibration in the form

$$g = aV + bV^2$$

where g is in units of 10^{-8} m/s^2 and V is in volts. The average value of a is -176.97. The maximum change in a over the measurment period was 1.61 or about 0.9%. If we had totally ignored all of the subsequent calibrations in our analysis and just used the first calibration to convert the data from volts to gravitational acceleration, the resulting maximum error in the amplitude of the tides would thus have been of the order of 0.9%. Since the admittance represents a time average of the amplitude, the error in the admittance would have been of the order of one half of this value.

The value of b changes from calibration to calibration, and the fact that it is nonzero is due to the nonlinearities in the screw which we did not know about when the calibrations were performed. The maximum value of the magnitude of b is 0.22. The maximum contribution of the quadratic term, given that the magnitude of V was not allowed to exceed 3 V is thus about 2×10^{-8} m/s^2. When V is 3 V (the worst case), the value of the first term is about 530×10^{-8}, so that the quadratic term contributes about 0.4% to the computation. If we had totally ignored the quadratic term, we would thus expect that the admittances would be wrong by no more than about 0.2%. The uncertainties introduced by totally ignoring the quadratic term are thus smaller than those introduced by ignoring the change in the linear screw factor. In fact we applied both corrections as well as we could, although some residual effects no doubt remain. We feel that our ignorance of the temporal changes of a and b does not exceed 10% of the total measured change during the experiment, so that we feel confident in the calibration at the level of about 0.1%.

The next step in the analysis was to remove the offsets produced by the rezeroing and the linear drift of the instrument. The offsets were removed by subtracting an appropriate value from the data before the rezero to make it join smoothly onto the data following the rezero. The screw was always adjusted between sample points, and the motion of the screw was completed well before the next 6-min sample time. We did not see any effects of hysteresis during these adjustments.

The slope of the linear trend was quite constant and averaged 36.4×10^{-8} m/s^2/d. We did not obtain a significantly different value by removing the trend in a piecewise fashion using consecutive subsets of the data. The secular change in g reported by the instrument over the entire measurement period was approximately 9400×10^{-8} m/s^2. The residual secular change in the data after the straight line was removed was less than 10×10^{-8} m/s^2, so that the secular change can be modeled as a simple linear drift to within our calibration uncertainty. The level of the residual secular effect is consistent with the data recorded by other instruments at the same location.

Interpolation Across Gaps in the Record

The record contained a total of about 95 hours of gaps due to all causes. The average length of a gap was 7 hours. The gaps were never caused by a failure of the instrument itself so that the thermal equilibrium of the instrument was never disturbed during the experiment. The gaps were patched using the interpolation process that we have discussed previously [Levine, 1978]. A spherical harmonic expansion of the tidal potential [Munk and Cartwright, 1966] is fit to the data on either side of the gap, and the resulting coefficients are used to construct a patch using the same type of expansion of the potential for the time period of the failure. The expansion uses 12 terms: five from degree 2 and seven from degree 3. Small secular drifts across the gap were removed by adding a small secular term to the patch. The maximum deviation of this term from the global linear trend was 0.8×10^{-8} m/s^2.

The adequacy of the patching process was tested both by applying it to various test cases and by examining the residuals for the real data. Based on our simulations, we think that the patch is consistent with the rest of the data to about 0.25% or about 0.5×10^{-8} m/s^2. Since the total extent of all of the patches is about 1.5% of the data, the uncertainty in the admittances due to the gaps is well below 0.1%.

Final Data Analysis

The estimate of the tidal admittance uses the tidal potential derived from the tables of Cartwright and Tayler [1971] and Cartwright and Edden [1973]. For each component we construct two time series of the form $a_k\cos(2\pi f_k t+\phi_k+\alpha_k)$ and $a_k\sin(2\pi f_k t+\phi_k+\alpha_k)$, where a_k is the amplitude given by Cartwright and Tayler and by Cartwright and Edden, f_k is the frequency in cycles per hour computed from the Doodson number of the component, ϕ_k is the phase at the start of the analysis epoch, and α_k is -90° if the term in question arises from a spherical harmonic Y_n^m in which (n + m) is odd and is zero otherwise. Each term is multiplied by the appropriate spherical harmonic of the station colatitude and east longitude and by the best estimate for the gravimetric factor at Boulder [Wahr, 1981]. The gravimetric factor g_m for all of the n = 2 tides is 1.1518. The theoretical admittance for the n = 2 components is thus $2Gg_m/R$, where G, the gravitational constant, has the value 9.7985 m/s^2 and R, the radius of the earth, is 6371 km. The gravimetric factor for n = 3 is 1.07, and the theoretical admittance is $3Gg_m/R$.

Both physical intuition and mathematical stability argue that it is unwise to fit these terms to the data directly. Many of these terms have frequencies which differ from each other by less than one cycle per year, and the amplitudes of such terms cannot be estimated reliably using our data. We therefore form a series of sums, each sum containing the contributions due to all terms having frequencies which differ from each other by less than one cycle per year. This process yields 55 frequency bands: 3 long period, 31 diurnal, 19 semidiurnal and 2 terdiurnal. Each band has two adjustable constants: the amplitude of the sum of the theoretical series (the terms using the cosine function) and the amplitude of the quadrature series (using the sine function). We can interpret these constants as an absolute amplitude and phase of the tidal admittance as a function of frequency. In our usage, the admittance at a given frequency is the complex ratio of the observed amplitude to the theoretical body tide to be expected at the station. The admittance is averaged over frequencies which differ from each other by one cycle per year or less. It is also averaged over the different spherical harmonics, although the contributions of terms with n = 2 dominate the potential for all but the terdiurnal frequencies.

Atmospheric Pressure Effects

We also incorporate several other series into the fitting process. Two very important noise sources in the gravity record are the direct and indirect effects of changes in barometric pressure, and inclusion of these effects is very important.

The direct effect is simply the changing gravitational attraction between the proof mass and the atmosphere, while the indirect effects result from the elastic response of the earth to the changing atmospheric pressure. The magnitude of the direct effect, assuming a simple plane-parallel geometry, is -0.42×10^{-10} m/s^2/Pa (-0.42 μGal/mbar), the negative sign implying that an increase in pressure results in a decrease in the apparent downward gravitational force on the proof mass. The elastic response of the earth tends to offset this effect somewhat, since an increase in pressure should compress the earth. We initially incorporated the local barometric pressure variations into the fitting process directly. The barometric pressure series is digitized on the same time mesh as the gravity data and is simply incorporated into the fitting function. The admittance calculated by the fitting process is smaller than the value quoted above and is of the order of $-0.38 \pm 0.07 \times 10^{-10}$ m/s^2/Pa. We attribute the difference between the admittance obtained in this way and the expected value of -0.42 to the response of the earth and to a deviation from the plane parallel geometry assumed in the model. Warburton and Goodkind [1977] found an admittance of -0.33×10^{-10} m/s^2/Pa using data from Pinon Flat and La Jolla. The topography at their sites is very different from the topography of Boulder and the stations are much closer to the ocean, so that their results are not directly comparable with our result. These effects are also discussed by Spratt [1982].

The fractional error in the admittance to the local barometric pressure (19%) is somewhat larger than the uncertainty to be expected if we assume that the uncertainty results from residual nonbarometrically induced noise. The admittance is also not stationary in time, and a significant part of the quoted uncertainty in our estimate is due to the fact that different subsets of the data yield different admittances. The model does, however, improve the quality of the fit. Using a single value for the admittance, calculated by fitting the local barometric pressure to the entire data set using a least squares fit, reduces the variance in the residuals by 32%. The local barometric pressure is most effective in removing noise with periods of from 3 to 10 days. The effect on the tidal admittances is of the order of 0.6% in the diurnal band and 0.3% in the semidiurnal band. In both cases, the effect of including the barometric pressure improves the agreement between the experimental tidal admittances and the theory, although the corrections are systematically too small.

These discrepancies are not too surprising. The plane-parallel model will fail if the pressure varies significantly over distances of the order of the scale height of the atmosphere, and such variations are quite common in a region such as Boulder where the topography deviates significantly from that assumed in the simplified model. The model also does not incorporate the loading distortions resulting from global pressure changes.

A more complete model of the effects of barometric pressure has been suggested by T. M. Van Dam and J. M. Wahr (unpublished manuscript, 1986). They have computed the effects of local and global pressure fluctuations on gravity measurements. They use twice daily global pressure data obtained from NCAR. The effects are computed separately: the direct effect of local barometric pressure changes and the direct and indirect effects of the global pressure fluctuations. The effects are converted to changes in effective gravitational force using the appropriate Green's function for the indirect loading effects and a direct calculation for the direct effects. The results are calculated every 12 hours.

The rms amplitude of the signal due to the local effects is 3.2×10^{-8} m/s^2, while the global contribution has a mean amplitude of 1.2×10^{-8} m/s^2. The power spectra of both contributions decrease approximately as the inverse square of the frequency except near one cycle per day where both series have a broad peak.

The effect of the global time series is important at long periods and near one cycle per day. The global contribution near one cycle per day is 39% of the local contribution. At other frequencies the global contribution to the variance usually is 10% or less of the total pressure effect. Since the computations are performed every 12 hours, we cannot compute a global contribution at semidiurnal periods. Although this is unfortunate, the direct effect is quite small there, and neglecting the global contribution is probably not too serious.

The coherence between the local and global time series is small at all frequencies, and the global effects therefore can not be included by simply adjusting the scale factor of the local series. At diurnal periods the contributions of the two series will enter with a varying phase difference, so that the apparent diurnal admittance, calculated using only the local contribution, will appear to be nonstationary in time by some appreciable fraction of the amplitude of the global contribution.

The admittances to local and global barometric pressure are handled differently. The global pressure effect arises from a convolution of the worldwide pressure data with the appropriate Green's function. The resulting gravity series is used directly with a fixed admittance of unity. The admittance to local barometric pressure fluctuations is estimated as before, using the gravity time series of Van Dam and Wahr for all frequencies less than one cycle per day and a suitably band-passed local barometer for frequencies greater than one cycle per day.

Using this procedure, we find that the admittance of the gravity time series calculated by Van Dam and Wahr is 1.0 ± 0.07, and the admittance to the semidiurnal barometric pressure is $0.39 \pm 0.04 \times 10^{-10}$ m/s^2/Pa. Both values are consistent with our noise estimates and are not significantly changed if the analysis is repeated using the data divided into subsets.

Error Estimates

Three other series are computed and are incorporated into the fitting function. The first two series use diurnal and semidiurnal frequencies which do not appear in the tidal spectrum. The two frequencies are 1.0283 cycles/d (Doodson number 171555) and 1.94596 cycles/d (Doodson number 260555). The third series has a frequency of 3.8645 cycles/d (Doodson number 455555). The first two series provide estimates of the nontidal power in the diurnal and semidiurnal bands, respectively, while the third series, at double the M2 frequency, serves as a test for nonlinearity. We expect the admittance of these series to be zero, and the calculated values provide an estimate of the noise in the data. We find that the amplitudes of the first two series are of the order of 0.04×10^{-8} m/s^2, and this provides a first estimate of the uncertainties to be expected in our calculations. The fractional uncertainty in the amplitudes for the larger tidal components would be about 0.08%. The amplitude of the 4 cycle/d term is 0.02×10^{-8} m/s^2, a value consistent with the background noise and

TABLE 1. Ocean Load at Boulder, Colorado

Component	Amplitude	Phase	Body Tide	Phasor, deg
O1(S)	0.786	69.9	35.42	1.008, 1.20
O1(P)	0.789	77.4	35.42	1.005, 1.25
P1(S)	0.369	56.4	16.49	1.013, 1.06
K1(S)	1.282	55.3	49.82	1.015, 1.19
K1(P)	1.392	63.0	49.82	1.013, 1.40
M2(S)	0.522	109.8	50.80	0.997, 0.56
M2(P)	0.581	128.2	50.80	0.993, 0.52
S2(S)	0.270	15.4	23.68	1.011, 0.17
S2(P)	0.308	9.1	23.68	1.013, 0.12

S, for Schwiderski model; P, for Parke model. Phasor is computed as (body + load)/body. All amplitudes are in units of 10^{-8} m/s^2 (μGal); all phases are in degrees, positive angles imply phase lead.

implying a residual quadratic response of less than 0.04% of the first-order linear response. This estimate is an upper bound to the quadratic response.

A second method for estimating the variance of the admittance estimate is to examine the power spectrum of the residuals at frequencies near the tidal bands. The noise power at semidiurnal periods is 1.6×10^{-15} g^2/cycle/h, where g is measured in m/s^2. The power spectrum increases toward lower frequencies approximately as f^{-2}. This value for the noise predicts that amplitude estimates will be biased by about 0.05×10^{-8} m/s^2. This value is slightly higher than the uncertainty estimated from the amplitudes of the test frequencies. We have adopted this more conservative estimate of the noise power since we feel it is likely to be more representative of the average noise power. The fractional uncertainty in the admittance estimates for the larger components due to the broadband noise contribution is thus of the order of 0.1%, a value that is of the same order as the other uncertainties.

We think that the noise power could be reduced somewhat further especially at semidiurnal periods using more frequent calibrations and a more comprehensive model for the barometric pressure. In particular, it is likely that our model does not fully account for the anisotropic spatial variations in the barometric pressure which are likely to be present in Boulder.

Ocean Loading

The ocean load tides were computed for us by D. Agnew (private communication, 1985). His results are presented in Table 1. Calculations using the Schwiderski [1980] and Parke [1978] models are shown for comparison where they are available. We take the difference between the two as a rough estimate of how far either is likely to be from the correct value. The two estimates for M2, for example, differ by about 0.4%. We have only listed loads which are statistically significant given our noise level of about 0.05×10^{-8} m/s^2.

Semidiurnal Admittance

The results at M2 serve as the most precise comparison between the experiment and the earth models. The amplitude of the M2 component of the body tide at our site is 50.8×10^{-8} m/s^2. The ocean load tide can be combined with the body tide and the result is then normalized by the body tide as shown in Table 1. The Parke and Schwiderski models differ by about 10% in amplitude, but the two phasors only differ by about 4 parts in 1000. These phasors define the theoretical admittance as a function of frequency for our site.

The first experimental determination of the M2 admittance simply compares the observed amplitude to the body tide. We find that the amplitude is 1.0052 and the phase is 0.8°. When the local barometric pressure series is added to the fitting function, the admittance, defined as above as the ratio of the observed tide to the body tide, becomes 1.002 and the phase becomes 0.65°. If the full barometric pressure correction is used, the admittance becomes 0.999 with a phase of 0.61°. The formal uncertainties in these values are ±0.002 in amplitude and ±0.7° in phase. These results are in somewhat better agreement with the Schwiderski model than with Parke's calculation. We note that these values might change slightly if global semidiurnal pressure data become available.

We have examined the admittance of all of the remaining semidiurnal components, but none of them provides as significant a comparison between the data and the model estimates. We find no components that differ from an admittance of unity by a statistically significant factor, but we have had to estimate the ocean load contribution for some of the smaller components. There is some evidence that the admittance at L2 is somewhat too large, although that conclusion depends on an empirical estimate of the ocean loading at that frequency. The admittance estimate at S2 is also not very robust due to contamination by thermal effects. We find no anomaly at S2, but the uncertainty in our estimate is 1.1%.

Diurnal Admittance

The frequency dependence of the diurnal admittance can provide an estimate of the nearly diurnal resonance resulting from the coupling of the core and the mantle. The largest diurnal components are O1, P1, and K1. Thermal effects are quite large at S1, and the admittance at that frequency is quite unreliable. The Parke and Schwiderski models predict phasors at O1 that differ by about 0.3%. The diurnal admittances are calculated in the same way as was done for the semidiurnal band. The measured amplitude and phase for the O1 admittance is 1.000 and 1.1° if no barometric pressure correction is used. When the local pressure is included in the fit, the values become 1.006 and 1.3° and when the full barometric pressure correction is incorporated, the values become 1.009 and 1.25°. As with the semidiurnal admittance, the results are in somewhat better agreement with the Schwiderski model.

The effect of the nearly diurnal resonance is most easily estimated by computing the ratio of the admittances for the other components to the admittance for O1. In each case, we compare the phasor obtained by adding the barometric pressure and ocean load to the body tide to the phasor obtained by fitting the potential to the gravity data. Note that the ocean loads at O1 and K1 are significantly different in all models (Table 1), so that the measured body tide admittances cannot be compared directly. We then normalize the admittance at P1 and K1 by the admittance at O1. Although the Parke and Schwiderski models predict phasors that differ, the normalization reduces the significance of the difference. The result for K1/O1 is 0.984 and 0.07°, while for P1/O1 the values are 0.995 and -0.003°.

These values may be interpreted in terms of the model of the resonance. We model the resonance effect on the gravimetric factor K_f at some frequency f in terms of the gravimetric factor at frequency O1, K_{O1}, and a strength parameter K_1. Specifically,

$$K_f = K_{O1} + K_1 \ (f - f_{O1})/(f_1 - f)$$

where $f_{O1} = 0.9295357$ cycles/d and f_1 is the center frequency of the resonance. The center frequency of the resonance is most easily expressed in terms of its offset from one cycle per sidereal day:

$$f_1 = (1 + 1/r) \ f_{K1}$$

where the sidereal frequency f_{K1} is 1.0027379 cycles/d. The value of r is of the order of 400.

The frequency of the P1 component is quite far removed from the resonance, so that the resonance function at that frequency does not depend strongly on r. Using a frequency of 0.9972621 cycles/d for P1, the value of the resonance function is 8.484 if r = 400 and 8.791 if r = 450. Using the relative admittance of 0.995 for P1, the relative strength of the resonance, K_1/K_{O1} is found to be 5.89×10^{-4} if r = 400 and 5.69 10^{-4} if r = 450. The value from Wahr's calculation is 5.32×10^{-4}. The fractional uncertainty in our estimate of the resonance strength is about 60%, given that the uncertainty in the admittances is of the order of 0.1%. Note that the estimates of the amplitude ratios are independent of the calibration.

If we use Wahr's value for the resonance strength, we can estimate the resonance frequency using the K1/O1 admittance ratio. We find that r = 412. The uncertainty of 0.1% in the admittance implies an uncertainty of 10% in the value for r, but this estimate does not include any uncertainty in the ocean load and is therefore somewhat too small. Values of r within 15% of the value quoted would not be inconsistent with our data.

Although we cannot get a significant estimate of the resonance dissipation, we may place an upper bound on its magnitude. The estimated uncertainty in the phase of the admittance is about 0.06°, assuming that the uncertainty in the amplitude of the admittance arises from sources that are incoherent with the tides. The phase angle residuals of all of the admittances are not significantly larger than this value, so that all of our estimates are consistent with no dissipation. Given our uncertainty level, we would consider any phase residual less than about 0.1° as probably due to noise, and this may be used to set an upper bound on the ratio of the imaginary and real parts of the resonance function. The ratio must be less than 0.2%. The ratio of the imaginary part to the real part may also be specified in terms of the Q of an equivalent oscillating system. The Q is proportional to the damping time expressed in units of the period. Our data imply that Q > 1000.

These estimates may be compared with the results of Neuberg and Zurn [1986]. They do not quote a final value, but a resonant frequency of 1.0051 ±0.0001 cycles/d appears to us to be consistent with their data. Their value of r would then be 426 ± 14. They also estimate values for the imaginary part of the admittance. Their results show considerable scatter, but their estimates for Q are all greater than 1000 (and in some cases are much greater than this value).

Gwinn et al. [1986] have obtained estimates of the resonant frequency and dissipation by analyzing data obtained using very long baseline radio interferometry. They obtain 434 ± 2 for r and 0.01 ± 0.004 for the ratio of the imaginary part of the frequency to the real part.

We have used our improved knowledge of the ocean loads to reexamine our previous estimate of the diurnal resonance using data from a laser strain meter [Levine, 1978]. The estimate for strain data is more complicated than for gravity, since the strain tide on a given aziumuth depends in a complicated way on the Love numbers [Levine and Harrison, 1976]. The resonance model has four parameters: the frequency and the effects on the three Love numbers h, k, and l. The diurnal ocean loads are quite large, amounting to 10-15% of the body tide, but the frequency dependence is not great enough to account for the very low admittance for Q1 that we reported previously. The ratios of the admittances for K1, P1, and O1 provide the most robust estimate of the frequency and dissipation of the resonance. If we take Wahr's estimate for the strength of the resonance and if we insert the current ocean load corrections into our previous results, we find that the effects on K1 and P1 imply that r = 430 ± 100 and Q > 300. The relatively large uncertainties are due to the rather high level of noise at diurnal periods. Our previous work included the effect of local barometric pressure, but thermoelastic strains were not adequately estimated and probably make a significant contribution to the measurements. Although the estimates derived from strain data have quite large uncertainties, the effect of the resonance on strain tides is much larger than for gravity, and in the future such data may provide the most accurate estimates of the parameters of the resonance.

Conclusions

We have analyzed data from a modified Lacoste and Romberg gravity meter, and we find good agreement between our data and the earth model proposed by Wahr. The admittance at semidiurnal frequencies agrees with a model including ocean and barometric pressure loads to better than the experimental uncertainties, and the effects of local and global barometric pressure changes are statistically significant.

The diurnal admittance provides an estimate of the strength and frequency of the resonance and an upper bound on the dissipation. The center frequency differs somewhat from the theoretical estimate but is in good agreement with other measurements.

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References

Block, B., and R. D. Moore, Measurements in the earth mode frequency range by an electrostatic sensing and feedback gravimeter, J. Geophys. Res., 71, 4361-4375, 1966.

Cartwright, D. E., and A. C. Edden, Corrected tables of tidal harmonics, Geophys. J. R. Astron. Soc., 33, 253-264, 1973.

Cartwright, D. E., and R. J. Tayler, New computations of the tide-generating potential, Geophys. J. R. Astron. Soc., 23, 45-74, 1971.

Gwinn, C. R., T. A. Herring, and I. I. Shapiro, Geodesy by radio interferometry: Studies of the forced nutations of the Earth, 2, Interpretation, J. Geophys. Res., 91, 4755-4765, 1986.

Harrison, J. C., and T. Sato, Implementation of electrostatic feedback with a LaCoste-Romberg model G gravity meter, J. Geophys. Res., 89, 7957-7961, 1984.

Larson, J. V., and J. C. Harrison, An improved analysis of the electrostatic feedback of LaCoste and Romberg gravity meters, Eos Trans. AGU, 66, 857, 1985.

Larson, J. V., W. T. Dewhurst, and R. E. Moose, Use of electrostatic feedback for determining calibration errors in LaCoste and Romberg G and D meters, Eos Trans. AGU, 65, 860, 1984.

Levine, J., Strain tide spectroscopy, Geophys. J. R. Astron. Soc., 54, 27-41, 1978.

Levine, J., and J. C. Harrison, Earth tide strain measurements in the Poorman Mine near Boulder, Colorado, J. Geophys. Res., 81, 2543-2555, 1976.

Moore, R. D., and W. E. Farrell, Linearization and calibration of electrostatically feedback gravity meters, J. Geophys. Res., 75, 928-932, 1970.

Munk, W. H., and D. C. Cartwright, Tidal spectroscopy and prediction, Proc. R. Soc. London Ser. A, 259, 533-581, 1966.

Neuberg, J., and W. Zurn, Investigation of the nearly diurnal resonance using gravity, tilt and strain data simultaneously, in Proceedings of the Tenth International Symposium on Earth Tides, in press, 1986

Parke, M. E., Open ocean tide modeling, in Proceedings of the Ninth Geophysics Conference, pp. 289-297, Ohio State University, Columbus, 1978.

Schwiderski, E. W., On charting global ocean tides, Rev. Geophys., 18, 243-268, 1980.

Spratt, R. S., Modelling the effect of atmospheric pressure variations on gravity, Geophys. J. R. Astron. Soc., 71, 173-186, 1982.

Wahr, J. M., Body tides on an elliptical, rotating, elastic and oceanless Earth, Geophys. J. R. Astron. Soc., 64, 677-703, 1981.

Warburton, R. J., and J. M. Goodkind, The influence of barometric pressure on gravity, Geophys. J. R. Astron. Soc., 48, 281-292, 1977.

Weber, J., and J. V. Larson, Operation of LaCoste and Romberg gravimeter at sensitivity approaching the thermal fluctuation limits, J. Geophys. Res., 71, 6005-6009, 1966.

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