Measurements of Frequency Stability

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Invited Paper

The characterization of frequency stability in the time domain and frequency domain are briefly defined and their relationships explained. Techniques for making precise measurements of frequency fluctuations in oscillators, multipliers, dividers, amplifiers, and other components are discussed. Particular attention is given to methods of calibration which permit accuracies of 1 dB or better to be achieved when measuring in the frequency domain. Common pitfalls to avoid are also covered, and efficient time-domain techniques are explained.

I. INTRODUCTION

The output of an oscillator can be expressed as

$$V(t) = [V_o + \epsilon(t)] \sin(2\pi\nu_0 t + \phi(t)) \tag{1}$$

where V_o is the nominal peak output voltage, and ν_0 is the nominal frequency of the oscillator. The time variations of amplitude have been incorporated into $\epsilon(t)$ and the time variations of the actual frequency, $\nu(t)$, have been incorporated into $\phi(t)$. The actual frequency can now be written as

$$\nu(t) = \nu_0 + \frac{d[\phi(t)]}{2\pi\, dt} \tag{2}$$

the fractional frequency deviation is defined as

$$y(t) = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{d[\phi(t)]}{2\pi\nu_0\, dt}. \tag{3}$$

Power spectral analysis of the output signal $V(t)$ combines the power in the carrier ν_0 with the power in $\epsilon(t)$ and $\phi(t)$ and, therefore, is not a good method to characterize $\epsilon(t)$ or $\phi(t)$.

Since in many precision sources understanding the variations in $\phi(t)$ or $y(t)$ are of primary importance, we will confine the following discussion to frequency- and time-domain measures of $y(t)$, neglecting $\epsilon(t)$ except in cases where it sets limits on the measurement of $y(t)$. The amplitude fluctuations $\epsilon(t)$ can be reduced using limiters whereas $\phi(t)$ can be reduced in some cases by the use of narrow-band filters.

Spectral (Fourier) analysis of $y(t)$ is often expressed in terms of $S_\phi(f)$, the spectral density of phase fluctuations in units of radians squared per hertz bandwidth at Fourier frequency (f) from the carrier ν_0, or $S_y(f)$, the spectral density of fractional frequency fluctuations in a 1-Hz bandwidth at Fourier frequency f from the carrier ν_0[1]. These are related as

$$S_\phi(f) = \frac{\nu_0^2}{f^2} S_y(f)\ (\mathrm{rad}^2/\mathrm{Hz}), \qquad 0 < f < \infty. \tag{4}$$

It should be noted that these are single-sided spectral density measures containing the phase or frequency fluctuations from both sides of the carrier.

Other measures sometimes encountered are $\mathcal{L}(f)$, dBC/Hz, and $S_{\Delta\nu}(f)$. These are related by [1], [2]

$$\begin{aligned} S_{\Delta\nu}(f) &= \nu_0^2 S_y(f)\ (\mathrm{Hz}^2/\mathrm{Hz}) \\ \mathcal{L}(f) &= (1/2) S_\phi(f), \qquad \text{for } \int_{f_1}^{\infty} S_\phi(f)\, df < 1\ \mathrm{rad}^2 \\ \mathrm{dBC/Hz} &= 10 \log \mathcal{L}(f). \end{aligned} \tag{5}$$

$\mathcal{L}(f)$ and dBC/Hz are single-sideband measures of phase noise which are not defined for large phase excursions and are therefore measurement system dependent. Because of this, an IEEE Subcommittee on Frequency Stability recommended the use of $S_\phi(f)$ which is well defined independent of the phase excursion [1]. This distinction is becoming increasingly important as users require the specification of phase noise near the carrier where the phase excursions are

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large compared to 1 rad. Single-sideband phase noise can now be specified as $1/2\ S_\phi(f)$.

The above measures provide one of the most powerful (and detailed) analysis for evaluating types and levels of fundamental noise and spectral density structure in precision oscillators and signal handling equipment as it allows one to examine individual Fourier components of residual phase (or frequency) modulation. On the other hand, this analysis is extremely detailed and one often needs an analysis of the long-term average performance.

Time-domain analysis is often much more efficient in providing meaningful measures of long-term performance. Time-domain measures are, of course, related to the frequency-domain measures by Fourier transforms. Useful relationships between the frequency domain and time domain are given in Sections III and IV for some special but important cases.

The time deviation $x(t)$ of the signal given in (1) can be expressed as

$$x(t) = \frac{\phi(t)}{2\pi\nu_0} \tag{6}$$

illustrating that the time deviation is proportional to the phase deviation. This can also be expressed as

$$x(t) = \int_0^t y(t')\,dt'$$

or

$$y(t) = \frac{dx(t)}{dt} \tag{7}$$

which relates frequency, time, and phase deviation.

The frequency deviations of precision signal sources (i.e., a precision oscillator driving amplifiers, frequency multipliers, and dividers, etc.) typically fall into two categories: systematic and random deviations. The systematics usually have the form of frequency and phase (or time) offset, frequency and phase (or time) drift, modulation sidebands, and dependence on environmental influences. The random deviations are categorically nondeterministic in character although the level may be deterministic and are often well described stochastically by power law spectral processes

$$S_y(f) = \sum_{\alpha=-2}^{+2} h_\alpha f^\alpha. \tag{8}$$

In some cases f may be band limited for particular values of α.

It has been shown [3] that the classical variance (square of the standard deviation) diverges with the data length for $\alpha \leqslant -1$, and hence is not a good measure of a source's frequency stability. In 1971, an IEEE subcommittee recommended a time-domain measure of frequency stability of a source which is convergent for all of the above kinds of power law spectra [1]

$$\sigma_y^2(\tau) = 1/2\langle \bar{y}_{k+1} - \bar{y}_k^2 \rangle \tag{9}$$

where the $\bar{y}_{k+1} + \bar{y}_k$ are adjacent measurements of the fractional frequency each averaged over a sample time τ. The expectation brackets "$\langle\ \rangle$" imply taking all possible values of k—infinite time average. This measure is often called the "Allan variance," or "two-sample variance" and has been shown to have a sound theoretical base as well as being practical and efficient in measuring the frequency stability of precision signal sources where power law spectra are the applicable models, which is the case for essentially all precision sources. In nonprecision oscillators, systematics typically dominate the frequency instability. In that case the systematics should be measured directly [4].

For a finite set of N sequential adjacent samples of the frequency, each averaged over a sample time τ, one may estimate $\sigma_y^2(\tau)$

$$\sigma_y^2(\tau) \cong \frac{1}{2(N-1)} \sum_{k=1}^{N-1} (\bar{y}_{k+1} - \bar{y}_k)^2. \tag{10}$$

If the data are a set of $N+1$ time or phase deviation readings, then

$$\sigma_y^2(\tau) = \frac{1}{2\tau^2(N-1)} \sum_{k=1}^{N-1} (x_{k+2} - 2x_{k+1} + x_k)^2 \tag{11}$$

since $\bar{y}_k = (x_{k+1} - x_k)/\tau$ where the spacing of the x_k time deviation readings is τ. If a data set of $N+1$ time deviation readings, spaced by an interval τ_0, are measured and stored in a computer array, then one can calculate $\sigma_y^2(\tau)$ for any $\tau = n\tau_0$, where $n = 1, 2, 3, \cdots, N/2$. In general, one may write

$$\sigma_y^2(\tau) = \frac{1}{2\tau^2(N+1-2n)} \sum_{k=1}^{N+1-2n} (x_{k+2n} - 2x_{k+n} + x_k)^2. \tag{12}$$

This equation gives high data utilization and the best confidence of the estimate, which is also unbiased [5]. It is sometimes called the "overlapping estimate" of the two-sample or Allan variance.

For power law spectra $\sigma_y^2(\tau)$ can be shown to be proportional to τ^μ, where μ has a nominally unique value for each value of α in (8). Hence, a $\log \sigma_y(\tau)$ versus $\log \tau$ plot indicates the type of noise via the slope (the value of $\mu/2$), while the level indicates the intensity of noise present. See Section IV for examples and relationships.

Some cautions are as follows in using $\sigma_y(\tau)$ as a measure of frequency stability: Discrete frequency sidebands on the carrier ν_0 yield anomalous contributions to $\sigma_y(\tau)$ for τ in the range of and larger than $1/(2f_0)$ where f_0 is the Fourier frequency of the sideband. This perturbation generally falls off as $1/\tau$ ($1/\tau^2$ in power) [6]. Frequency drift of D (D is the fractional frequency over the interval τ, dimensions s^{-1}), causes a contribution term to $\sigma_y(\tau)$ proportional to τ^{+1}

$$\sigma_y(\tau) = D\tau/\sqrt{2}. \tag{13}$$

II. FREQUENCY-DOMAIN MEASUREMENTS OF FREQUENCY STABILITY

Fig. 1 shows the block diagram for a typical scheme used to measure the phase noise of a precision source using a double balanced mixer and a reference source. Fig. 2 illustrates a similar technique for measuring only the added phase noise of multipliers, dividers, amplifiers, and passive components. The output voltage of the mixer as a function of phase deviation, $\Delta\phi$, between the two inputs is normally given by

$$V_{\text{out}} = K \cos \Delta\phi. \tag{14}$$

Near quadrature this can be approximated by

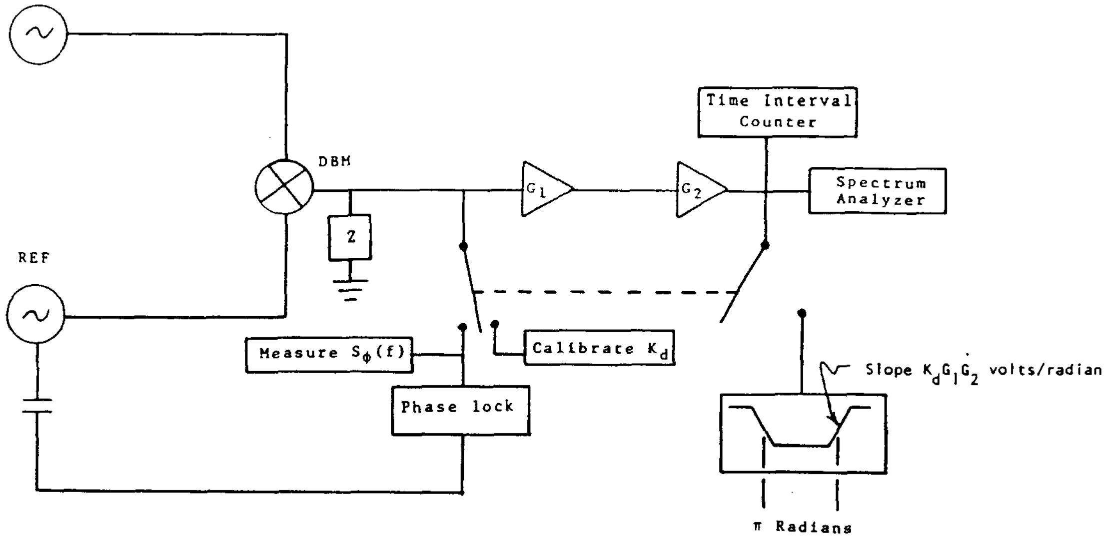

Fig. 1. Precision phase measurement system using a spectrum analyzer. Calibration requires a recording device to measure the slope at the zero crossing. The accuracy is better than 0.2 dB from dc to $0.1\nu_0$ Fourier frequency offset from the carrier ν_0. Carrier frequencies from a few hertz to 10^{10} Hz can be accommodated with this type of measurement system. The time interval counter can be used to measure frequency offset and $\sigma_y(\tau)$ using the heterodyne method. The results for $\sigma_y(\tau)$ may be biased by dead time [2], [3], [6].

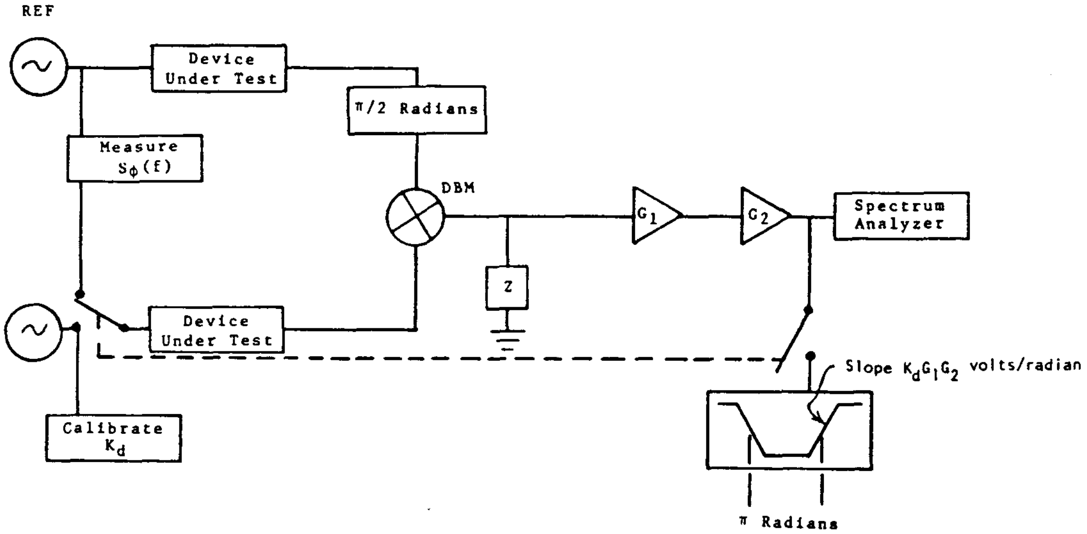

Fig. 2. Precision phase measurement system featuring self-calibration to 0.2-dB accuracy from dc to 100-kHz Fourier frequency offset from carrier. This system is suitable for measuring signal handling equipment, multipliers, dividers, frequency synthesizers, as well as passive components [7].

$$V_{\text{out}} = K_d\,\delta\phi, \qquad \text{where } \delta\phi \equiv \left[\Delta\phi - \frac{2n-1}{(2)}\pi\right] < 0.1 \tag{15}$$

where n is the integer to make $\delta\phi \sim 0$. The phase-to-voltage conversion ratio sensitivity K_d is dependent on the frequency, the drive level, and impedance of both input signals, and the IF termination of the mixer [7]. The combined spectral density of phase noise of both input signals including the mixer and amplitude noise from the IF amplifiers is given by

$$S_\phi(f) = \left(\frac{V_n^2}{G(f)K_d}\right)\frac{1}{BW} \tag{16}$$

where V_n is the rms noise voltage at Fourier frequency f from the carrier measured after IF gain $G(f)$ in a noise bandwidth BW. Obviously BW must be small compared to f. This is very important where $S_\phi(f)$ is changing rapidly with f, e.g., $S_\phi(f)$ often varies as f^{-3} near the carrier. In Fig. 1, the output of the second amplifier following the mixer contains contributions from the phase noise of the oscillators, the mixers, and the post amplifiers for Fourier frequencies much larger than the phase-lock loop bandwidth. In Fig. 2, the phase noise of the oscillator cancels out to a high degree (often more than 20 dB). Termination of the mixer IF port with 50 Ω maximizes the IF bandwidth, however, termination with reactive loads can reduce the mixer noise by ~ 6 dB, and increase K_d by 3 to 6 dB as shown in Fig. 3

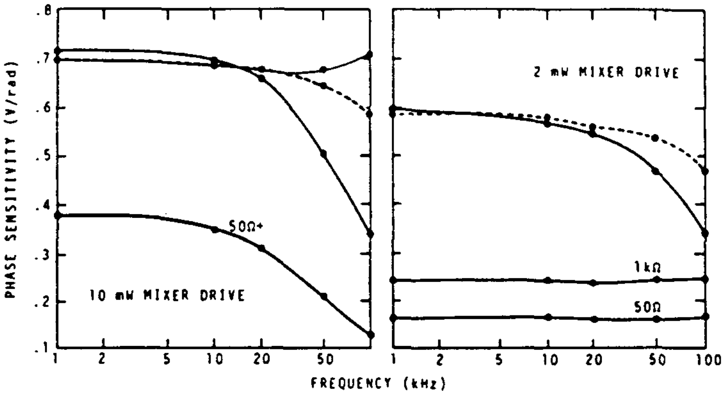

Fig. 3. Double-balanced mixer phase sensitivity at 5 MHz as a function of Fourier frequency for various output terminations. The curves on the left were obtained with 10-mW drive while those on the right were obtained with 2-mW drive. The data demonstrate a clear choice between constant, but low sensitivity or much higher, but frequency-dependent sensitivity [7].

[7]. Accurate determination of K_d can be achieved by allowing the two oscillators to slowly beat and measuring the slope of the zero crossing in volts/radian with an oscilloscope or other recording device. The time axis is easily calibrated since one beat period equals 2π radians. Estimates of K_d obtained from measurements of the peak-to-peak output voltage sometimes introduce errors as large as 6 dB in $S_\phi(f)$ [7]. By comparing the level of an IF signal (a pure tone is best) on the spectrum analyzer used to measure V_n and the recording device used to measure K_d, the accuracy of $S_\phi(f)$ can be made independent of the accuracy of the spectrum analyzer voltage reference. Some care is necessary to assure that the spectrum analyzer is not saturated by spurious signals such as the line frequency and its multiples. Sometimes aliasing in the spectrum analyzer is a problem. Typical best performance is shown in Fig. 4. This measurement system exceeds the performance of almost all available oscillators from 0.1 MHz to 10 GHz and is generally the technique of first choice because of its versatility and simplicity.

The use of specialized mixers with multiple diodes per leg increases the phase-to-voltage conversion sensitivity K_d and therefore reduces the contribution of IF amplifier noise [8] as shown in Fig. 4. The resolution of the above systems can be greatly enhanced (typically 20 dB) using correlation techniques to separate the phase noise from the device under test from the noise in the mixer and IF amplifier [8].

The use of frequency multipliers (or dividers) between the oscillators and the double balanced mixer increases (decreases) the phase noise level [9] as

$$S_\phi \nu_2(f) = \left(\frac{\nu_2}{\nu_1}\right)^2 S_\phi \nu_1(f) \tag{17}$$

where $S_\phi \nu_2(f)$ explicitly designates that the phase noise spectrum is at carrier frequency ν_2 and similarly for $S_\phi \nu_1(f)$. The use of multipliers reduces the sensitivity of the measurement of phase noise to added noise in the mixer and IF amplifiers. However, this is only helpful when the multiplier noise is less than the mixer and IF noise. Fig. 4 shows the noise of a specialized 5- to 25-MHz multiplier referred to the 5-MHz input. A potential problem with the use of the multiplier approach comes from exceeding the dynamic range of the mixer. Once the phase excursion $\delta\phi$ exceeds about 0.1 rad, nonlinearities start to become important and at $\delta\phi \sim 1$ rad, the validity of the measurement has virtually vanished [9].

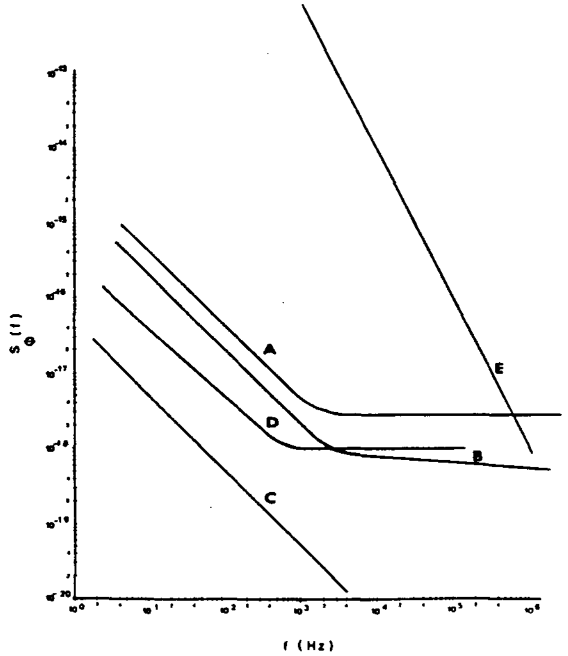

Fig. 4. Curve A—The noise floor $S_\phi(f)$ (resolution) of typical double balanced mixer systems (e.g., Figs. 1 and 2) at carrier frequencies from 1 to 100 MHz. Similar performance possible to 20 GHz [8]. Curve B—The noise floor $S_\phi(f)$ for a high level mixer [8]. Curve C—The correlated component of $s_\phi(f)$ between two channels using high level mixers. Curve D—The equivalent noise floor $S_\phi(f)$ of a 5- to 25-MHz frequency multiplier. Curve E—Approximate noise floor of a delay line FM discriminator system with a 500-ns delay [14].

Another method of determining $S_\phi(f)$ uses phase modulation of the reference oscillator by a known amount. The ratio of the reference phase modulation to the rest of the spectrum then can be used for a relative calibration. This approach can be very useful for measurements which are repeated a great many times.

It is sometimes convenient to use a high-Q resonance directly as a frequency discriminator. If the oscillator is tuned to the side of the resonance, the *amplitude* of the transmitted (or reflected) signal depends on the frequency difference between the oscillator and the resonance as well as the amplitude of the oscillator. By using amplitude control (e.g., a limiter stage) between the oscillator and the resonance or a fast processor to normalize the data, one can reduce the effect of amplitude noise [10]. This approach has the limitations that $\Delta\nu$ must be small compared to the linewidth of the cavity, and removing the effect of residual amplitude noise is difficult; however, no reference source is needed. Differential techniques can be used to measure the inherent frequency fluctuation of high-Q resonances [11]. The resulting error signal then is of the form $\Delta V(f) = K\Delta\nu(f)$ where K is the frequency to detected voltage conversion factor. One can calculate $S_\phi(f)$ using (4).

A still different approach uses a delay line to make a pseudo-reference which is phase incoherent relative to the incoming signal [12]–[14]. The advantage here is that one does not need a reference oscillator in order to measure the phase noise far from the carrier. However, the close-in phase noise is unresolvable for a finite delay line; see Fig. 4.

III. TIME-DOMAIN MEASUREMENTS OF FREQUENCY STABILITY

Historically, the measurement of frequency stability in the time domain has often involved heterodyne techniques (beat-frequency method) using a scheme similar to that shown in Fig. 1 [1], [2]. Although very simple to implement, this approach suffers from dead-time biases [3] and the inability to measure sources which are very close in frequency. A much more accurate and general approach is shown in Fig. 5 [15]. This Dual Mixer Time Difference System has no dead-time errors and can easily measure sources that are on the same frequency as well as those offset in frequency, and the measurement precision is state of the art. Recently, this approach has been extended to permit the comparison of a large number of sources against a common clock or against one another [16]. Using this technique the time difference between a pair of clocks is operationally measured with a resolution of the order of 1 ps or better when comparing the phase of the 5-MHz outputs from the clocks. In general, the resolution can be improved by increasing the comparison frequencies. The range of nominally achievable and operational resolutions are given by $((1 \times 10^{-6}$ and $20 \times 10^{-6})/\nu_0)$ seconds, respectively, where ν_0 is the input frequency. The achievable and operational frequency resolution is of the order $(1 \times 10^{-6}$ and $20 \times 10^{-6})/(\nu_0\tau)$, respectively, where τ is the measurement integration time. The dual-mixer time difference technique allows one to measure either x_k or $\bar{y}_k$ with state-of-the-art precision.

One caution in using the dual-mixer time difference technique is that the actual date of when a time measurement is made can have discontinuities in repetition rate of one period of the beat frequency involved. This can be dealt with by interpolating to a date which gives a nominally constant repetition rate.

IV. RELATIONSHIP BETWEEN FREQUENCY-DOMAIN AND TIME-DOMAIN MEASURES

Cutler has shown [1], [17] that

$$\sigma_y^2(\tau) = 2\int_0^\infty df\, S_y(f)\, \frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}. \tag{18}$$

For the specific power-law spectra often occurring in the random deviations of precision signal sources, amplifiers, multipliers, etc., one can construct the following table to relate the time-domain and frequency-domain measures (see Table 1).

In general, one can write

$$\sigma_y^2(\tau) \cong \begin{cases} \sim \tau^{-\alpha-1}, & \alpha \leqslant +1 \\ \sim \tau^{-2}, & \alpha \geqslant +1 \end{cases}. \tag{19}$$

Hence, the kind of noise (value of α) can be deduced from the τ-dependence if $\alpha \leqslant +1$. Fortunately, this range of α is the usual range for most oscillators if $\tau \geqslant 1$ s. For $\tau < 1$ s, it is often more informative to characterize the oscillator using $S_\phi(f)$, as sidebands and modulation are often important to measure in this region. The ambiguity associated with $\alpha \geqslant +1$ can be resolved by modulating the measure-

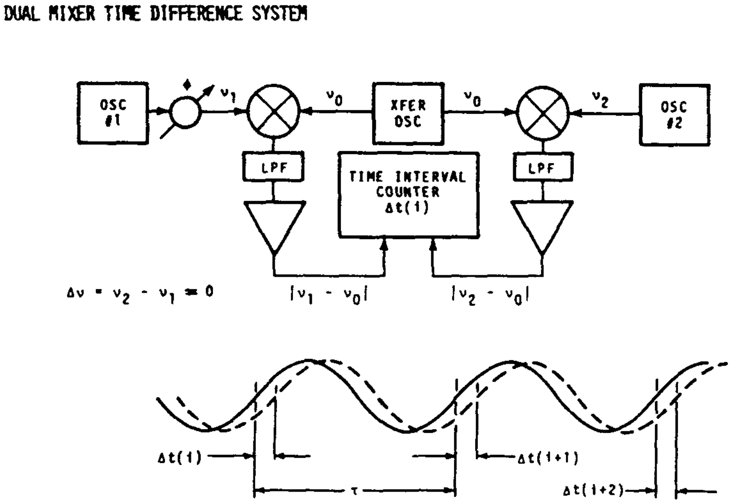

Fig. 5. Precision frequency (time) measurement system capable of measuring state-of-the-art signal sources. This system has no dead-time bias and can measure sources which are at the same or slightly different frequencies ($|y| \leqslant 10^{-6}$). Minimum τ is $1/|\nu_1 - \nu_0|$ [15].

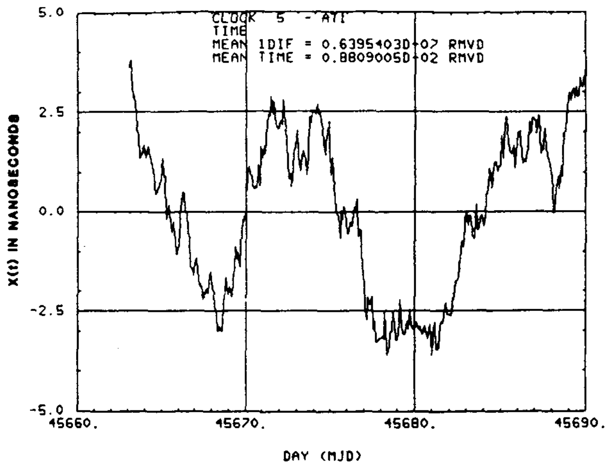

Fig. 6. Time difference $x(t)$ between a passive hydrogen maser and the NBS cesium-based time scale [19].

Table 1

Name of Noise	α	$S_y(f) =$	$\sigma_y^2(\tau) =$
White phase	2	$h_2 f^2$	$\dfrac{3 f_h h_2}{(2\pi)^2 \tau^2}$
Flicker phase	1	$h_1 f$	$\dfrac{(1.038 + 3 \ln(\omega_h \tau)) h_1}{(2\pi)^2 \tau^2}$
White frequency	0	h_0	$\dfrac{h_0}{2\tau}$
Flicker frequency	−1	$h_{-1} f^{-1}$	$2 \ln(2) h_{-1}$
Random-walk frequency	−2	$h_{-2} f^{-2}$	$\dfrac{(2\pi)^2 h_{-2} \tau}{6}$

Note: $\omega_h/2\pi = f_h$ is the measurement system bandwidth—often called the high-frequency cutoff. $\ln = \log_e$.

ment bandwidth [3], [18], [20]. In this region, $\sigma_y^2(\tau)$ is proportional to $\omega_B^{\alpha-1}$, where $\omega_B/2\pi = f_B$ is the effective measurement system bandwidth. So if ω_B can be changed in the hardware or software, it is easy to tell the difference between white noise phase modulation (PM) ($\alpha = 2$) and flicker noise PM ($\alpha = 1$). If the predominate noise is white PM, significant improvements in the precision of $\sigma_y(\tau)$ can be obtained by reducing the measurement bandwidth consistent with the measurement time.

As an example of a clock stability analysis see the plot of $x(t)$ for a passive hydrogen maser against the NBS clock ensemble as shown in Fig. 6. These are the random variations. If (18) is used to estimate $\sigma_y(\tau)$, one obtains the result in Fig. 7. Notice the apparent $\alpha = 0$ and $\alpha = -1$ models deducible from the $\tau^{-1/2}$ and τ^0 behavior, respectively.

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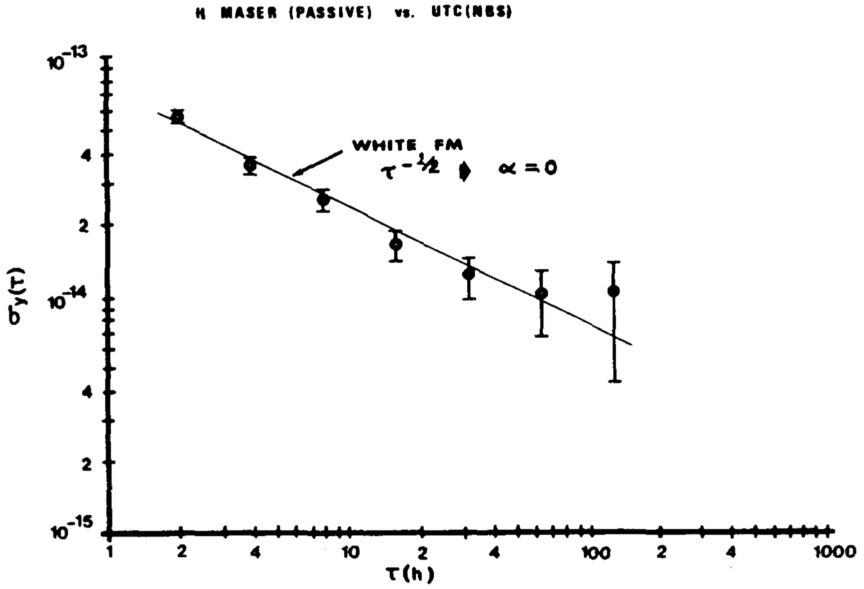

Fig. 7. Fractional frequency difference $\sigma_y(\tau)$ calculated from the time error of Fig. 6.

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