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# POSITION LOCATION WITH THE NBS/GPS TIME-TRANSFER SYSTEM*

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## Abstract

The NBS/GPS time transfer system is a low-cost receiver of the GPS C/A code for anyone with a high quality clock interested in referencing that local clock to UTC(NBS) at state-of-the-art time and frequency accuracies using the common-view technique. In particular it is used in comparing primary time standards around the world for international time and frequency coordination. In order to achieve the highest accuracy for time and frequency transfer the local user must know the coordinates of the receiver antenna within the GPS coordinate system. For this purpose we included a fully automatic position location program in the receiver software.

The ability of the receiver to perform absolute and differential positioning was evaluated in experiments over three baselines: short (26 m - 77 m), medium (131 km), and ling (240 km). Solutions from the receiver were compared with WGS-72 first order survey points. Absolute positioning error varied from 4.1 m to 10.2 m except during periods when the GPS space vehicle (SV) health was bad. We find a 7.2 mean absolute positioning error over the eight separate results in the four experiments with the GPS healthy. The short and medium baseline differential positioning error varied from 1.2 m to 2.5 m, reflecting a limitation due to multipath delays.

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## Discussion

The NBS/GPS time transfer system is designed for time comparisons between remote locations in the common-view mode (1). For this purpose the receiver needs its location in the GPS coordinates. The receiver can solve for its location by measuring the delay between its reference 1 pps and that of the GPS via four satellites in different known locations spanning a good geometric formation. These four measurements are used in the usual way (2) to solve for the four unknowns: 3 coordinates of position, and the time offset of the reference from GPS time. A single measurement consists of tracking an SV for a number of minutes. We make pseudo-range measurements every second, apply range delay and clock corrections to every fifteen of these, and finally reduce these with a single linear fit. Therefore, each measurement has an RMS associated with it based on a linear fit to a number of points equal to four times the number of minutes. Confidence in the measurement may be estimated by taking the RMS and dividing by the square root of the number of points. The receiver makes only one measurement at a time, thus relying for its solution on the stability of the local reference, the stability of GPS time, and the stationarity of the receiver during the period between measurements. Further, since we are solving for a fixed position, more solutions may be taken over a period of time and averaged.

There are various factors that influence the quality of the solution. The geometric formation of the various satellites above the horizon defines a factor called the geometric dilution of precision (GDOP) which multiplies any range error's contribution to the position solution. This factor is a function only of the satellite positions relative to the receiver. The best geometry would be three satellites evenly spaced around the horizon and one directly overhead, though low elevations cause increased atmospheric noise. Data was taken so that a satellite was in the same location each day, thus keeping geometric and multipath conditions constant. The noise of the receivers, the instabilities of the references, and their relative drifts contributed no more than 1 ns to the pseudo-range measurements used for the position solution, depending on the gain of the antenna used and the track length. When multiplied by the GDOP of 3 to 4, this translates to an error of about 1 m in position.

This, then, is the minimum standard deviation we expect in a data set. Since these effects have a white noise spectrum we expect a decrease in their error contribution with the square root of the averaging time (3).

Other significant effects on the solutions are due to drift in the GPS system clock relative to the reference clock, multipath delays and the quality of the satellite ephermerides and clock correction errors. The effect of drift in the GPS system clock on the solutions is directly related to the time the receiver takes to acquire the four measurements for the solution. The receiver has two modes of acquiring data for position location. The slow method uses data acquired from normal time transfer tracks taken over any interval in a day. The fast method requires there to be four SV's up at once in a good geometry. This method spends two minutes on each satellite, hence taking 8 minutes to acquire data. When this method was used any effect of GPS clock drift was negligible. Only in one experiment was the effect of GPS system clock drift significant, and there only for absolute position measurements. Differential common-view measurements tend to cancel various sources of error to the extent they are the same at both locations. This depends largely on the similarity of satellite geometry at the two locations, a function of the length of baseline. This applies to drift in the GPS system clock, SV clock correction errors, errors in the ephemerides, and common propagation delay errors such as ionospheric modeling errors. Instabilities not common to two sites, such as local ionospheric effects, receiver noise, etc., have been shown to have a white noise spectrum. by appropriate averaging techniques the relative rates of clocks at the two sites can be measured to a part in $10^{14}$ at 8 days when used in the common-view mode (3).

Multipath is a significant problem with a clear access or C/A code receiver. Since the C/A code is transmitted at a 1.023 Mps rate a reflected signal delayed half a bit, which would integrate with the locked signal, would have a delay of approximately 500 ns or 150 ns and bias the lock point a fraction of this amount. According to Spilker (4), a P-code receiver experiences a worst-case multipath error of 8 ns from a reflected signal with 0.6 relative amplitude. For the C/A code this translates to 80 ns or 24 m, since the bit rate for the C/A code is 0.1 that of the P-code. In our experiments we attempted to isolate the antennas from other objects as much as possible. But, the best geometry of satellites for position location requires tracking some of them at low angles. In this case reflections off the earth and low objects are to be expected. Even with high elevation satellites and a high-gain antenna, we still found evidence of multipath.

The quality of satellite ephemerides and clock correction errors will be analyzed via the separation of variance technique developed at NBS for studying the GPS system (5). This is a technique for separating the magnitudes of different noise and error instabilities in GPS clock minus local reference clock data. These components are: the local reference, the GPS system clock, the SV clocks, the correction from SV clocks to GPS system time, and the ephemeris plus propagation delay to the SV's in the same location in the sky every day. Separation of variance yields worst case estimates of frequency fluctuations for these noise components. Since they are nominally white (3), the variance in time equals one third the frequency variance times the time interval, in this case one sidereal day. Relevant to this paper are the SV clock correction, ephemeris error contribution to range error, and any systematic propagation delay errors such as the ionospheric correction.

## Results

#1a) Short baselines measurements were conducted from MJD 45523 to 45531, i.e., July 8, 1983 to July 16, 1983, with no data on MJD's 45529 and 45530. Two omni-directional antennas were placed on the roof NBS, Boulder, Colorado separated a measured distance of 26 m. Eight slow-lock tracks a day were done of SV's 5, 6, 8, and 9 by the two receivers simultaneously, with a solution after the first four and then after each subsequent track. Thus there were five solutions a day on seven days, only two of which per day used a completely different set of data. The spatial position GDOPs for the five solutions were: 3.71, 3.71, 3.91, 2.97, 4.85. Each track was 8 minutes long, hence each measurement was based on 32 points (each point being a 15 s average). The RMS deviations of the individual track RMS deviations was 9.7 ns, or 2.9 m, hence, dividing by $\sqrt{32}$, yields a confidence in the range measurements of 0.5 m. Multiplying by the average GDOP, 3.85, we find a position uncertainty due to noise on single tracks of 1.9 m.

During this period the GPS system clock was drifting relative to UTC(NBS) between -20 and -26 ns/day. The longest time for four tracks, giving a single solution, was 44 min. Using the largest drift, 26 ns/day, we find a maximum range error due to this drift of 0.24 m. The clock correction error we estimate from the separation of variance technique. We find a frequency instability of 4.4 parts in $10^{13}$ at 1 sidereal day as the root sum square over the four SV's used. This implies a range error of 6.6 m due to SV clock correction error. Similarly, for position plus ephemeris error we find an instability of 1.5 parts in $10^{13}$, implying a range error of 2.2 m. The root sum square of these three error sources is 7.0 m deviation in range. Multiplying by the average GDOP of 3.8 we find an estimate of 26.5 m which is somewhat larger than the 11.6 m and 12.5 m standard deviations to the absolute positioning to the absolute positioning data for the two receivers. We see here that most of the absolute positioning error was due to errors in the data words from the SV's, and in particular, these absolute positioning errors are dominated by clock correction errors.

Since the antennas were only 26 m apart, all these errors cancel for the common-view differential solutions. The only errors contributing here are due to multipath, and receiver noise. To separate these we may average each differential solution separately over the seven days. Since the tracks are done with the same geometry each day, multipath conditions are constant each day for a given track. The standard deviation around the mean of data from a fixed track each day we expect reflects only receiver noise. The root mean square of these standard deviations over all five solutions was for $\Delta X$, $\Delta Y$, and $\Delta Z$, respectively: 0.7 m, 0.9 m, and 1.0 m, yielding a root sum square of 1.5 m as the total deviation due to receiver noise. We expect this data would improve with averaging, yielding a mean differential position reflecting the fixed multipath for each solution with greater confidence. On the other hand, the deviation around the mean over the five fixed multipath solutions reflects the differences in multipath disturbances. This gives us a lower bound on the multipath effect. These standard deviations around the total means for

ΔX, ΔY, and ΔZ, respectively, were 0.5 m, 1.6 m, and 0.6 m yielding a root sum square of 1.8 m as a minimal measure of the fluctuation in differential position due to multipath. The statistics over the entire data set are given in the table below. The actual differential error of 1.2 m seems quite reasonable in view of the size of the disturbances.

#1b) A second short baseline experiment was conducted from MJD 45822 to 45850, May 3 to 31, 1984, after all the other experiments were done. Here we used two high-gain antennas separated by a measured 77 m baseline. Six slow-lock tracks were done each day of SV's 6, 8, 9, and 11 by the two receivers simultaneously, yielding three solutions each day, though they were not completely independent. One of the high gain antenna packages was not functioning properly during this experiment. There was no data from MJD 45825 through 45836, and missing points on various other days, giving a total of 36 points with 16 of them independent. The GDOP's were 4.0, 3.1, and 3.1 giving an average of 3.42. Each track was 6 minutes long consisting of 24 points. The RMS deviation of the individual tracks was under 5.1 ns or 1.53 m giving a confidence of 0.31 m on the individual range measurements after dividing by $\sqrt{24}$. This implies a confidence of 1.06 m in position after multiplying by the average GDOP.

The frequency offset of GPS system time relative to UTC(NBS) was between -5. and +15 ns/day during this interval. The maximum time interval for a solution was 2 hr. 19 min. This gives a maximum range error of 0.44 m. From the separation of variance technique we find a clock correction instability of 3.98 m in range, and similarly a propagation plus ephemeris instability of 2.01 m in range. Combining these three we have a root sum square of 4.5 m range instability or 15.3 m position instability after multiplying by the average GDOP. We again see that this is somewhat larger than the actual 9.0 m and 9.2 m standard deviations. We see here also in this last experiment that clock correction errors are much larger than propagation plus ephemeris errors.

As in 1a) the only errors contributing in differential common-view positioning are multipath and receiver noise. Unfortunately, with only three solutions per day, all solutions being dependent, there are no independent multipath configurations. Averaging over fixed multipath solutions exhibits only small variations, since they are dependent. thus we cannot estimate the effect of multipath in this way. We do note that the error between differential solution and measurement of 1.5 m is comparable to the case using omni-directional antennas. We conclude that there is little improvement with the high gain antennas keeping in mind that one of them was working poorly.

#2) The medium-range baseline experiment was performed from MJD 45544 to 45553, July 28 to August 7, 1983, between Boulder, CO and Cheyenne, WY. A receiver with a high gain antenna was placed in each of these locations, programmed to take common-view data from SV's 6, 8, and 9, and do positioning in the fast-lock mode. The GDOP was never more than 3.7, and always close to this value. The first day, MJD 45544, there were 12 solutions, the next day 7, and the third day 12. Then from 45547 through 45553 there were 9 each day with the exception of none on 45549, and 7 on 45550. On a given day, only every fourth solution was independent. So there were a total of 83 data solutions, 25 of which were independent. The RMS for 90 seconds of data was about 3.2 ns, which gives us a confidence of 1.3 ns or .4 m for each range measurement since 6 points were used in each linear fit. Multiplying by the 3.7 GDOP we find a 1.4 m positioning uncertainty due to noise on individual tracks.

Because data was taken in the fast-lock mode a solution was given every 8 minutes. Therefore any effect of drift in the GPS time scale on position location was negligible. Clock correction, ephemeris, and propagation instabilities estimated from the separation of variance technique were approximately the same as for the short baseline experiment #1a) discussed above. Thus we estimate 7.0 m standard deviation in range due to clock correction, propagation, and ephemeris, yielding an estimate of 25.9 m deviation in position error, from multiplying by the GDOP. Again, this is larger that the actual sample standard deviation for absolute positioning. We also again see that these errors are dominated by SV clock correction errors.

With a 131 km baseline these clock correction, propagation, and ephemeris errors must cancel to a great extent with common view differential positioning data. The range to the satellites is on the order of 2500 km. Thus the ratio of the 131 km baseline to the two ranges is about 200 to 1. Again we assume differential positioning error to be almost entirely due to receiver noise and multipath though ionospheric effects may also be present. To estimate receiver noise and multipath contributions to error we use the technique as above on the data from 45547 to 45553 when there were nine solutions each day with a fixed geometry. Taking separately the standard deviations over the 6 days around the means for each daily solution, hence each fixed multipath configuration, we find an RMS of 1.5 m due to receiver. On the other hand, the deviations of the 9 fixed geometry means around the total mean yield a value of 1.7 m as a minimal measure of deviation due to multipath. Thus we see a significant multipath contribution to the 2.5 m differential error even with the high gain antenna.

#3a) The first long baseline experiment was performed from MJD 45605 to 45623, September 28 to October 16, 1983, with a high gain antenna in Washington and an omni-directional antenna in Boulder. Data was taken in the fast-lock mode from SV's 5, 6, 8, and 9 yielding 5 solutions per day, only the first and last being truly independent. There was no data on MJD's 45616, 45619 and 45621. So there were 16 days of data, 80 total with 32 independent. The GDOP was never more than 4.0 at either location, but always close to this. The RMS for 90 seconds of data, a fit to 6 points, was about 10.0 ns, giving a confidence of 4.1 ns or 1.2 m on the range measurements. Multiplying by the GDOP we have a 4.8 m position uncertainty due to noise on individual tracks.

As in #2) above, we find no effect of GPS clock drift on positioning since data was taken in the fast-lock mode. Separation of variance again gives us approximations for error due to clock corrections and propagation plus ephemeris. The square root of the sum of squares of instabilities due to clock corrections over the different SV's gives a 2.4 m range error for this period. The propagation plus ephemeris error we estimate at 2.1 m. The root sum square of these is 3.2 m in range, multiplied by a GDOP of 4.0 yields 12.9 m in position error. Here we find excellent agreement with the 13.5 m and 11.2m absolute positioning sample standard deviations. We conclude here that though the absolute positioning error is primarily caused by errors in the transmitted data, these errors are balanced between clock correction errors and propagation plus ephemeris errors.

The 2405 km baseline for the long range experiments is about one tenth the range to the satellites. Hence the geometry is significantly different at the two locations so range errors contribute differently to positioning. Differential positioning therefore has error contributions due to errors in the SV data words as well as multipath and receiver noise. This effect can be seen in the increase in the standard deviation of the differential measurement data. We note the surprising result that for this experiment the differential positioning error is larger that either of the absolute positioning errors, and indeed larger than the sample standard deviation for differential data.

#3b) The second long baseline experiment was performed from 45733 to 45743, February 3 to 13, 1984, with a high gain antenna at both locations and data taken in the slow lock mode from SV's 6, 8, 9, and 11. We had three solutions per day, only one independent, with only two solutions on MJD 45736. Thus there were 32 data points, 11 independent. The GDOP was under 4.1. The RMS deviation on the range measurement fits to 6 minutes, 24 points, of data was approximately 4.8 ns. Dividing by the square root of 24, we have a confidence of about 1 ns or 0.3 m on the range measurements, which translates via the GDOP to a position uncertainty of 1.2 m due to noise on individual tracks.

The GPS clock had little drift during this period, hence any effect was negligible. Separation of variance predicts errors due to clock correction, ephemeris, and propagation errors. During this interval the Cesium clock on SV 9 was on the verge of failure. We find a range error of 4.3 m predicted due to clock correction error, mostly due to SV 9. The range error due to ephemeris plus propagation error we estimate at 1.9 m. These give us a root sum square of 4.7 m. Multiplying by the 4.1 GDOP factor we estimate a 19.3 m deviation in position. We see this agrees fairly well with the 17.0 m and 14.5 m sample deviations for absolute positioning data.

We note that in this experiment the absolute positioning data seems significantly larger than the sample deviations. Examining the errors we see that in both locations the largest part of the error is vertical, the solution being lower than the WGS-72 survey. At NBS, 19.5 m of the 21.0 m RMS error was vertical, also with the solution lower than the survey.

Again, as in 3a), because of the long 2405 km baseline we expect error contributions to differential positioning due to errors in the SV data words in addition to multipath and receiver noise.

#3c) A third experiment was conducted between NBS, Boulder, CO and USNO, Washington, DC from MJD 45775 to 45792, March 16 to April 2, 1984. Data was taken in the slow lock mode from SV's 6, 8, 9, and 11 using high gain antennas. There were three solutions per day with no data on 45790 and only one point on 45779 and 45781. There were 47 data points, with 17 independent. The average GDOP was under 4.7. The RMS on the range fits to 6 minutes, 24 points, was 4.1 ns, implying a confidence of 0.8 ns or 0.24 m. With the GDOP we have a 1.1 m position uncertainty due to noise on individual tracks.

During this interval there was again no significant effect of GPS clock drift on positioning. Again, we find clock correction, ephemeris, and propagation delay errors from separation of variance. During this interval SV 9 was using its Rubidium clock which seemed to have only small clock correction error, and a somewhat large ephemeris error as compared to the other SV's. The predicted range instability due to clock correction error was 4.6 m, and that due to ephemeris plus propagation was 2.7 m, yielding a root sum square of 5.3 m. Via a 4.3 GDOP we estimate a 23.0 deviation in position, which again is somewhat larger than the 13.8 m and 14.2 m sample deviation for absolute positioning data.

It is interesting to note that most of the absolute positioning error is vertical, as for 3b) but with the sign reversed. Of the 15.6 m error at USNo 14.8 m is vertical, with the solution higher than the WGS-72 survey. At NBS, the error was 13.2 m with 12.9 of it vertical, the solution higher than the survey. Apparently there have been systematic errors in the SV data words.

|  |  |  | Vector |  | root |  | Standard |  | root |
|---|---|---|---|---|---|---|---|---|---|
|  |  |  | Differential |  | sum |  | Deviations |  | sum |
| MJD | Rcvr | $\Delta X$ | $\Delta Y$ | $\Delta Z$ | square | $\sigma_X$ | $\sigma_Y$ | $\sigma_Z$ | square |
| #1a: slow lock, 35 data public, 14 ind |  |  |  |  |  |  |  |  |  |
| 45523-45531 | Abs: NBS09 | 1.5 | 0.3 | 5.8 | 6.0 | 3.5 | 6.2 | 9.1 | 11.6 |
|  | Abs: NBS07 | 2.3 | -0.1 | 6.6 | 7.0 | 4.0 | 7.0 | 9.6 | 12.5 |
| 26 m Diff: | NBS09-NBS07 | -0.8 | 0.3 | -0.8 | 1.2 | 0.9 | 1.8 | 1.2 | 2.4 |
| #1b: slow lock, 36 data pts, 16 ind |  |  |  |  |  |  |  |  |  |
| 45822-45850 | Abs: NBS11 | 4.1 | 4.0 | -6.6 | 8.7 | 3.8 | 5.8 | 6.0 | 9.2 |
|  | Abs: NBS03 | 5.1 | 4.7 | -7.5 | 10.2 | 3.8 | 5.6 | 6.0 | 9.0 |
| 77 m Diff: | NBS11-NBS03 | -0.1 | -0.7 | 0.9 | 1.5 | 0.6 | 0.8 | 0.9 | 1.4 |
| #2: fast lock, 83 data pts, 25 ind |  |  |  |  |  |  |  |  |  |
| 45544-45553 | Abs: NBS03 | 0.2 | 4.0 | 3.3 | 5.2 | 1.5 | 8.7 | 12.4 | 15.2 |
|  | Abs: CHEY | 1.4 | 6.2 | 3.0 | 7.0 | 1.5 | 8.9 | 12.8 | 15.7 |
| 130 km Diff: | NBS03-CHEY | -1.3 | -2.2 | 0.4 | 2.5 | 0.7 | 1.4 | 1.5 | 2.1 |
| #3a: fast lock, 80 data pts, 32 ind |  |  |  |  |  |  |  |  |  |
| 45605-45623 | Abs: USNO11 | 3.0 | -2.6 | 0.6 | 4.1 | 2.9 | 7.6 | 10.8 | 13.5 |
|  | Abs: NBS03 | -3.6 | 3.1 | 8.2 | 9.5 | 3.1 | 6.8 | 8.4 | 11.2 |
| 2405 km Diff: | USNO-NBS03 | 6.6 | -5.7 | -7.6 | 11.6 | 3.9 | 4.5 | 5.9 | 8.4 |
| #3b: slow lock, 32 data pts, 11 ind |  |  |  |  |  |  |  |  |  |
| 45733-45743 | Abs: USNO07 | 2.5 | -17.7 | 10.0 | 20.1 | 3.4 | 12.1 | 11.4 | 17.0 |
|  | Abs: NBS03 | 3.5 | -14.6 | 7.5 | 16.8 | 3.1 | 10.1 | 10.0 | 14.5 |
| 2405 km Diff: | USNO07-NBS03 | -1.0 | -3.1 | 2.5 | 4.1 | 1.4 | 3.1 | 3.4 | 4.9 |
| #3c: slow lock, 47 data pts, 17 ind |  |  |  |  |  |  |  |  |  |
| 45775-45792 | Abs: USNO07 | 2.1 | 13.0 | -8.4 | 15.6 | 9.2 | 8.3 | 5.9 | 13.8 |
|  | Abs: NBS03 | 0.9 | 8.3 | -10.2 | 13.2 | 8.1 | 9.1 | 7.2 | 14.2 |
| 2405 km Diff: | USNO07-NBS03 | 1.2 | 4.7 | 1.9 | 5.2 | 1.8 | 3.0 | 2.8 | 4.5 |