EXCESS NOISE IN QUARTZ CRYSTAL RESONATORS

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Summary

Frequency and phase noise in quartz crystal resonators are studied as a function of the driving power. At low power, where the crystal behaves linearly, 1/f fluctuations of the resonance freuqency are observed. At medium power the nonlinearities of the crystal significantly increase the phase fluctuations at low Fourier frequencies. At high power, thermal instabilities and chaotic behavior occur characterized by the generation of high level white noise.

Introduction

The short and medium term frequency stabilities of quartz crystal oscillators are limited by additive and parametric noises of the electronics and by parametric noise of the quartz resonator.[1] In an oscillator it is not always possible to distinguish between these different contributions and thereby determine the level of frequency fluctuations of the resonator. A significant advance in solving this problem came with the introduction of a new technique to test quartz crystals independently of the oscillator electronics.[2] This enabled measurements of flicker and random walk frequency noises of various pairs of crystals at different frequencies and of different origins.[2,3] A second advance on the same problem came with the introduction of the concept of a dual crystal system, in which a quartz oscillator with good spectral purity is phase locked to a passive quartz resonator.[4] In this approach, the frequency stability in medium and long term is dominated by the passive quartz resonator and hence its stability becomes observable. Such measurement systems can be used for comparing quartz crystal instabilities and for trying to understand the origins and the mechanisms of these frequency fluctuations.

The previous measurements[2,3] have shown that the resonance frequency of quartz resonators exhibits flicker (1/f) fluctuations and random walk ($1/f^2$) fluctuations. Random walk frequency noise has been shown to be correlated to temperature fluctuations.[3,5] The mechanisms contributing to the generation of 1/f noise are not well understood yet.

When the resonator is excited at[9] a moderately high drive level a large increase of the noise level is observed at low Fourier frequencies. It will be shown that this excess noise is due to the nonlinearities of the crystal. At very high drive levels, the resonator can exhibit chaotic behavior producing high level white noise.

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Measurements at Low Power

The measurement system, presented for the first time in ref. 2, will be briefly described. As schematically shown in Fig. 1, two identical resonators (or two that adjusted to be nearly identical) are driven at their resonance frequency by a frequency source with high spectral purity. When the two resonators are at the same frequency (frequencies are tuned with adjustable capacitors) and with equal Q-factors (Q-factors are balanced with additional resistors), the frequency fluctuations of the source are rejected by 50 to 60 dB. The double balanced mixer (DBM) operated at its quadrature point by means of the 90° phase shifter detects the phase fluctuations $d\phi$ induced by the frequency fluctuations of each resonator, following the relation

$$d\phi = -2Q\frac{(d\omega_1 - d\omega_2)}{\omega_0} \qquad (1)$$

where Q is the resonator loaded Q-factor and $d\omega_1$($d\omega_2$) is the instantaneous frequency difference of the resonance frequency of resonator 1 (2) from the driving frequency ω_0. Equation (1) is valid for Fourier frequencies within the resonator's bandwidth and for low drive powers (typically 10 µW were used). Outside this bandwidth it must be corrected to take into account the filtering effect of the resonators. The mixer output is amplified and processed by a spectrum analyzer, yielding the power spectral density $S_v(f)$ of the output voltage noise. This is related to the frequency spectrum of the resonator $S_{y_0}(f)$,

$$S_v(f) = (G\mu)^2 S_Q(f) = (2G\mu Q)^2\ S_{y_0}(f), \qquad (2)$$

where μ is the mixer sensitivity, and G is the low noise amplifier gain. The corresponding resonance frequency fluctuations of the resonators are deduced from relation (1) (again for Fourier frequencies within the resonator's bandwidth). Each resonator contributes to the total noise, which is measured. If the two resonators are identical the noise amplitude of each of them is obtained by dividing by $\sqrt{2}$.

Great care must be taken to minimize the spurious influence of temperature changes. The measurements reported below were obtained using a digitally controlled oven. Temperature was measured with a LC-cut quartz sensor. The temperature control loop had a 10^{-4} K resolution. Additional thermal filtering, with copper mass and thermal discontinuities, reduced the residual temperature fluctuations of the crystals under test down to a few µK over a few sec.[6] The two resonators were in the same oven and thermally coupled in order to reduce random frequency walk noise. This does not perturb 1/f noise measurements, because 1/f noise is not correlated with temperature fluctuations.[3,5] Over a day, temperature stability was of the order of 5×10^{-5} K. The resonators generally were operated at their turn-over temperature. This measurement system was used to test 5 MHz, 10 MHz, and 2.5 MHz resonators.

1) 5 MHz resonators. Six AT-cut crystals (fifth overtone) were studied. Two were commercial BVA[7] resonators and four were standard commercial high quality resonators. A typical spectrum (pair of BVA crystals) is shown in Fig. 2. The noise is essentially 1/f frequency noise. The $1/f^3$ noise is due to the filtering effect of the resonator; this occurs at its half bandwidth. For three pairs of resonators the frequency noise, $S_{y_0}(f)$, measured 1 Hz from the carrier was equal to $2.5\times10^{-13}/\sqrt{Hz}$, $2.5\times10^{-13}/\sqrt{Hz}$ and $3.10^{-13}/\sqrt{Hz}$. The raw data were divided by $\sqrt{2}$ under the assumption that the noise of each resonator in a given pair was identical. Obviously by doing all possible pairs one can obtain a more accurate value for each resonator independent of this assumption. All these resonators had unloaded Q-factors in the range $2.5\times10^6 < Q_0 < 2.7\times10^6$.

2) 10 MHz resonators. Two kinds were measured: AT-cut and BT-cut crystals. Both were third overtone resonators, but with very different Q-factors, typically 3.10^5 for AT-cut and 1.7 10^6 for BT-cut. BT-cut crystals have much higher Q-factors because this crystallographic orientation has lower internal losses for the thickness-shear mode.[8] Noise spectra are shown

in Fig. 2. A large difference in the noise levels can be observed between the AT and BT cuts. At 1 Hz from the carrier these levels are of the order of $7\times10^{-13}/\sqrt{Hz}$ for the BT resonators to $3.2\times10^{-11}/\sqrt{Hz}$ for the AT resonators. These results show a strong dependence between the 1/f noise level and Q-factor.

3) 2.5 MHz resonators. Two pairs of fifth overtone, AT-cut resonators were tested with unloaded Q-factors close to 4×10^6. Although these pairs had almost the same Q-factors, the noise levels differed by almost one order of magnitude. The presence of $1/f^2$ noise in the spectra of $S_{y_0}(f)$ indicates that the sensitivity to temperature fluctuations are very large for this type of resonator. The 1/f noise power 1 Hz from the carrier, evaluated for each individual crystal, was plotted as a function of the unloaded Q-factors in Fig. 3. A linear regression among these experimental points gives

$$S_{y_0}(1\ \text{Hz}) \cong \frac{2}{Q^4} \qquad (3)$$

These results are also summarized in Table I.

The data show a clear dependence of 1/f noise on the resonator's unloaded Q-factors, following a $1/Q^4$ law. The only exception occurs with the 2.5 MHz (#2) crystals, which show excessive noise most likely due to thermal transient effects.

If these crystals are used in an oscillator, the 1/f spectrum will give a flicker floor in time domain, whose corresponding values are given in Table I, and which corresponds to the best stability achievable in that case with an oscillator.

Measurements at Medium Power

When the drive level of the crystal is increased it exhibits nonlinear effects due mainly to the higher order elastic constants. This is the well known amplitude frequency effect where distortions appear in the amplitude and phase resonance curves and even hysteresis can be developed.[9]

The nonlinear behavior of a resonator, driven in transmission can be represented by the phenomenological relation,

$$\frac{d^2i}{dt^2} + \frac{\omega_0}{Q}\frac{di}{dt} + \omega_0^2[1 - 2\varepsilon(\Omega)\cos\Omega t]\, i(1 + ki^2) = F\cos\omega t, \qquad (4)$$

where i is the current through the crystal, Q the loaded Q-factor, and F the amplitude of the driving force. k is the nonlinear coefficient (related to the next higher order elastic constants). $\varepsilon(\Omega)\cos\Omega t$ introduces a modulation of the resonance angular frequency ω_0, which represents the frequency noise of the resonator. Solving this equation with a perturbation method gives the phase noise of the output signal.

Let $S_{y_0}(\Omega)$ be the frequency noise spectrum of the crystal. When driving it at low power, in the the linear range, the corresponding phase spectrum is given by

$$S_\phi(\Omega) = S_{y_0}(\Omega)\,\frac{\omega_0^2}{\Omega^2 + \omega_0^2/4Q^2} \quad . \qquad (5)$$

At medium levels, when the resonator is driven near the jump frequency, the phase spectrum becomes

$$S_\phi(\Omega) = S_{y_0}(\Omega)\,\frac{\omega_0^2(\Omega^2 + \omega^2/4Q^2)}{\Omega^2\,(\Omega^2 + \omega_0^2/Q^2)} \quad . \qquad (6)$$

Thus the ratio between the phase noises at medium and low powers is

$$\frac{S_\phi(\text{medium power})}{S_\phi(\text{low power})} = \frac{(\Omega^2 + \omega_0^2/4Q^2)^2}{\Omega^2\,(\Omega^2 + \omega_0^2/Q^2)} \quad . \qquad (7)$$

This ratio goes to unity for $\Omega \gg \omega_0/Q$ and is equal to $\omega_0^2/16Q^2\Omega^2$ for $\Omega \ll \omega_0/4Q$. Therefore the induced phase noise for the lower Fourier frequency components can be greatly increased by the crystal nonlinearities. Such a noise was experimentally observed on a 5 MHz (5th overtone AT-cut) resonator driven at 2.5 mW, as shown on Fig. 5.

It should be noted that higher drive levels were once considered as a means to improve the (white phase noise) signal/noise ratio and therefore the short term stability in oscillators. However, our results indicate this is not in general a practical solution since the nonlinear response of the resonator increases the close in noise at the same time.

Measurements at High Power

For still higher levels, quartz resonators exhibit large instabilities. Such phenomena are known in nonlinear systems as chaotic behavior. Chaos has been observed in many different systems[10], for example phase locked loops[11] and Josephson junctions[12,13] which can be considered as low Q-factor resonators. The main difference between these systems and quartz crystal resonators stem from the high Q-factor and thermal effects, which strongly modify the behavior of the crystal at high power.

The dissipated power induces large temperature rises. Positive or negative frequency shifts follow according to the sign of temperature coefficient of the resonator, i.e. to the location of the operating temperature on the frequency-temperature characteristics.

Near room temperature, the temperature coefficient is negative (of the order - 4 Hz/K). A temperature rise will induce a negative frequency shift, which can be large enough to pull the crystal to the frequency where the down jump phenomenon occurs (Fig. 4). Thus the amplitude becomes much smaller decreasing the dissipated power. Temperature therefore decreases causing the frequency to increase until it reaches the second jump and so on. This gives a cycling with large amplitude and phase perturbations, as shown in Fig. 6. This phenomenon occurs when the amplitude-frequency effect and the temperature coefficient have opposite signs.

Above the turn-over temperature the sign of the temperature coefficient becomes positive. In this case the temperature rises, inducing a positive frequency change, which corresponds to an additional amplitude-frequency effect. This is equivalent to increasing the nonlinearities of the crystal. Fig. 7 illustrates the amplitude resonance curve of a 5 MHz (AT-cut, fifth overtone) crystal excited with a 7 V rms driving signal. As the drive frequency increases the crystal goes from stable state to chaotic states. The transition is sudden, and no set of cascading subharmonic bifurcations is observed, as would be the case in low-Q resonators. The absence of subharmonics is explained by the high Q of the quartz crystal, which filters all the subharmonic frequencies, which are out of its linewidth.

The sideband frequency noise of the transmitted signal was measured in the same conditions. In Fig. 8 the power spectral density of frequency fluctuations shows a f^2 dependence versus Fourier frequency. This corresponds to a white phase spectrum, characteristic of the generation of white noise in the crystal resonator by the chaotic behavior (white noise was also observed in other chaotic systems.[10-13] The level of this phase noise is S_ϕ = -34 dB (rad^2/Hz), which is about 80 dB larger than the phase noise levels of Fig. 5.

Conclusion

The noise behavior of a quartz crystal resonator is strongly influenced by the amplitude of the oscillation and the corresponding nonlinearities. At low power 1/f frequency noise is observed. At medium power the nonlinear response of the crystal can amplify this noise and give larger phase fluctuations. At high power, thermal effects become an additional source of instability. At sufficiently high drive power chaotic states take place which generate very large white phase noise. The problem of chaos is of most interest however, from the basic physics point of view. The theory of cascading bifurcations is a good explanation of how a deterministic system can have stochastic solutions although it does not directly apply in the case of quartz crystal, because of the high Q-factors.

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References

1. L. S. Cutler and C. L. Searle, "Some aspects of the theory and measurement of frequency fluctuations in frequency standards", Proc. IEEE, 54, 2, p. 136 (1966).

2. F. L. Walls and A. E. Wainwright, "Measurement of the short-term stability of quartz crystal resonators and the implications for quartz oscillator design and applications, IEEE Trans. Instrum. Meas., IM-24, p. 15 (1975).

3. J. J. Gagnepain, "Fundamental noise studies of quartz crystal resonators", Proc. 30th Ann. Freq. Cont. Symp., p. 84 (1976).

4. S. R. Stein, C. M. Maney, Jr., F. L. Walls, and J. E. Gray, "A systems approach to high performance oscillators", Proc. 32nd Ann. Freq. Cont. Symp., p. 484 (1981).

5. Y. Noguchi, Y. Teramachi, F. Musha, "1/f fluctuations of a quartz crystal oscillator", Proc. 35th Ann. Freq. Cont. Symp., p. 484 (1981).

6. G. Marianneau, J. J. Gagnepain, "Digital temperature control for ultrastable quartz oscillators", Proc. 34th Ann. Freq. Cont. Symp., p. 52 (1980).

7. R. J. Besson, "A new piezoelectric resonator design", Proc. 30th Ann. Freq. Cont. Symp., p. 78 (1976) "A new electrodeless resonator design", Proc. 31st Ann. Freq. Cont. Symp., p. 147 (1977).

8. J. Lamb and J. Richter, "Anisotropic acoustic attenuation with new measurements for quartz at room temperatures:, Proc. Roy. Soc. A, vol. 293, p. 479 (1966).

9. J. J. Gagnepain and R. Besson, "Nonlinear effects in piezoelectric quartz crystals", Physical Acoustics, vol. XI, p. 245, W. P. Mason, R. N. Thurston Eds, Academic Press (1975).

10. "Evolution of order and chaos:, Proc. Int. Symp. on Synergetics, H. Haken Ed. Springer-Verlag (1982).

11. D. D'Humieres, M. Bearley, B. Huberman, A. Libchaber, "Chaotic states and routes to chaos in the forced pendulum:, Ginzton Lab. Report 33429, Stanford University (1982).

12. B. A. Hubertson, J. P. Cratchfield, "Chaotic states of anharmonic systems in periodic fields", Phys. Rev. Letters, 43, n^{o} 23, p. 1743 (1979).

13. N. Petersen, A. Davidson, "Chaos and noise rise in Josephson junctions", Appl. Phys. Lett., 39, n^{o} 10, p. 830 (1981).

resonator	2.5 MHz # 1	2.5 MHz # 2	5 MHz BVA	5 MHz	5 MHz	10 MHz	10 MHz	10 MHz	10 MHz	10 MHz
crystal cut	AT	AT	AT	AT	AT	AT	AT	BT	BT	BT
Q_o	4.10^6	4.10^6	$2.6.10^6$	$2.7.10^6$	$2.4.10^6$	$2.5.10^6$	3.10^5	$1.5.10^6$	$1.8.10^6$	$1.9.10^6$
S_y (1 Hz)	$2.5.10^{-26}$	$2.4.10^{-24}$	$6.2.10^{-26}$	$5.8.10^{-26}$	9.10^{-26}	1.10^{-21}	$6.5.10^{-22}$	$2.6.10^{-24}$	2.10^{-24}	$5.0.10^{-25}$
flicker floor	$1.9.10^{-13}$	$1.5.10^{-12}$	$2.9.10^{-13}$	$2.8.10^{-13}$	$3.5.10^{-13}$	$3.7.10^{-11}$	$3.0.10^{-11}$	$1.9.10^{-12}$	$1.7.10^{-12}$	$8.3.10^{-13}$

Table I

1/f noise level at 1 Hz from the carrier, and corresponding flicker floor, for the different samples (2.5 MHz # 2 crystals were not taken into account in the linear regression of Fig. 3)

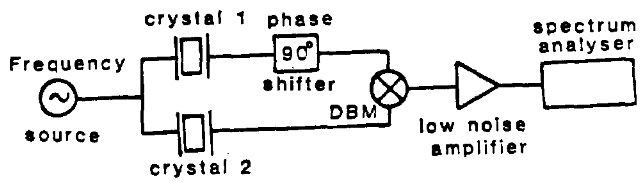

Fig. 1 : Passive frequency noise measurement system :
Relative phase fluctuations induced by the two resonator's resonance frequency fluctuations are measured with a phase bridge composed of a 90° phase shifter and double balanced mixer (DBM)
Mixer sensitivity : 0.15 V/rad at 5 MHz
Amplifier gain : 500

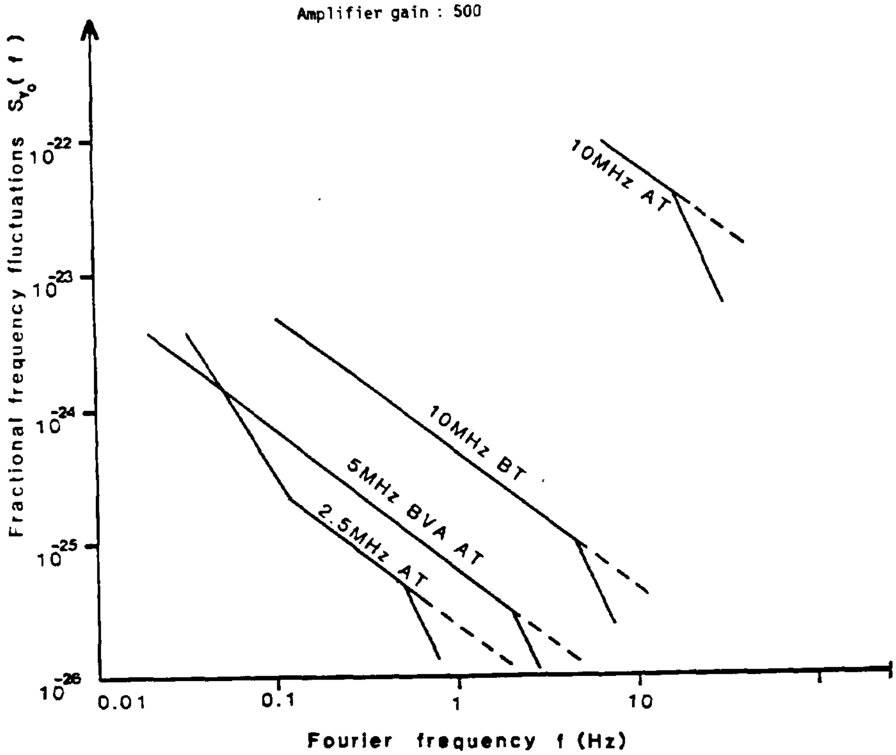

Fig. 2 The solid curve shows the apparent frequency noise spectrum of 2.5 MHz, 5 MHz, and 10 MHz resonators. At Fourier frequencies above the half bandwidth, the spectrum must be corrected for the filtering effect of the resonators yielding the dashed lines.

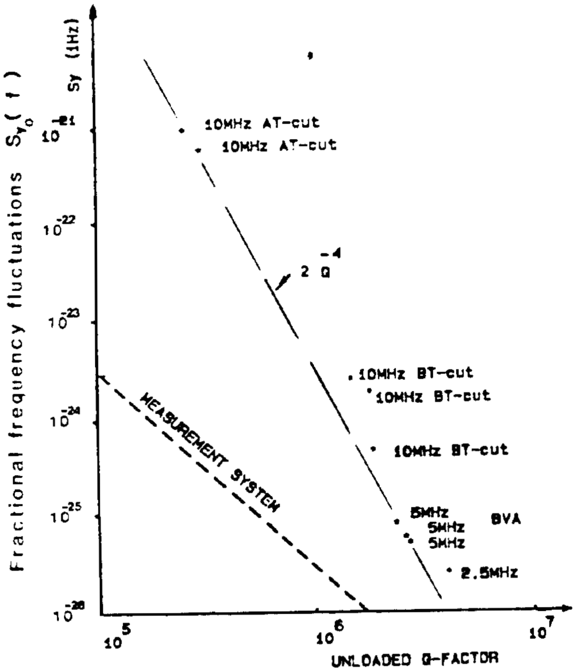

Fig. 3. 1/f noise level, measured at 1 Hz from the carrier as a function of the unloaded Q-factor for the different resonators tested. The measurement system noise is indicated by ----.

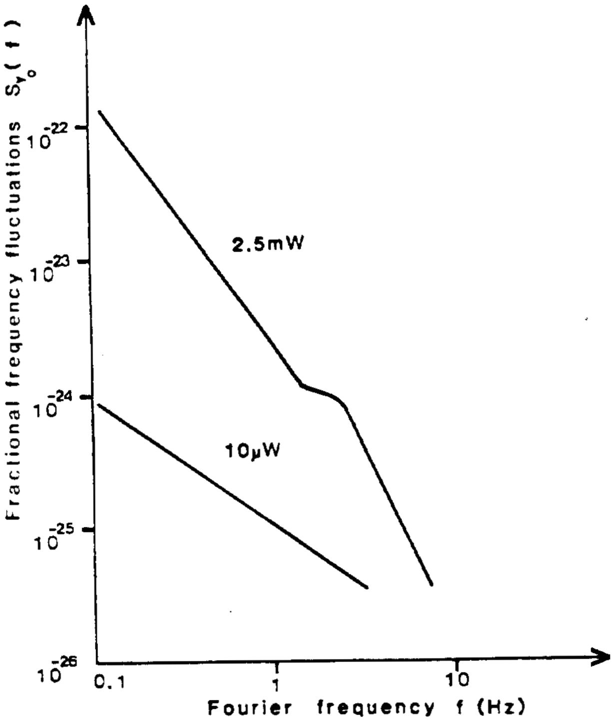

Fig. 5. Frequency noise spectrum of a 5 MHz driven at low and medium power.

Fig. 6. Amplitude perturbation due to thermal effects at high power.

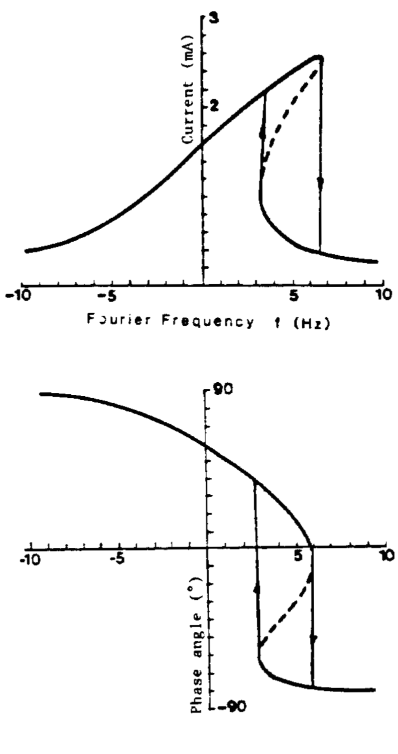

Fig. 4. Amplitude and phase resonance curves of quartz resonators

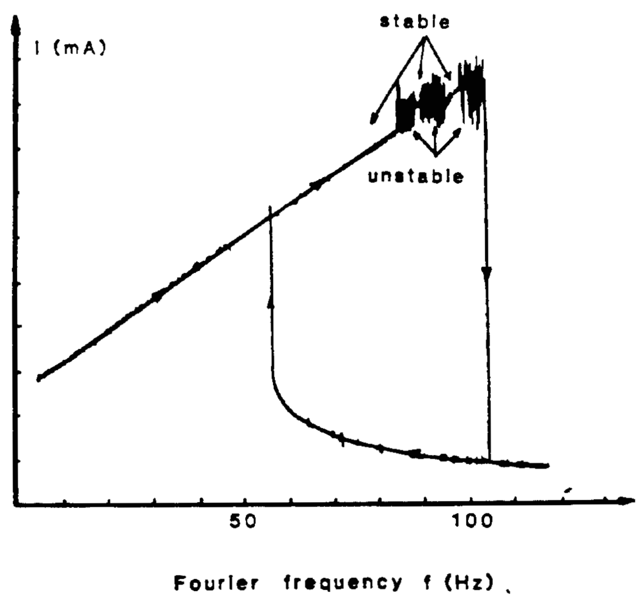

Fig. 7. Stable and unstable states of a 5 MHz resonator driven at high power.

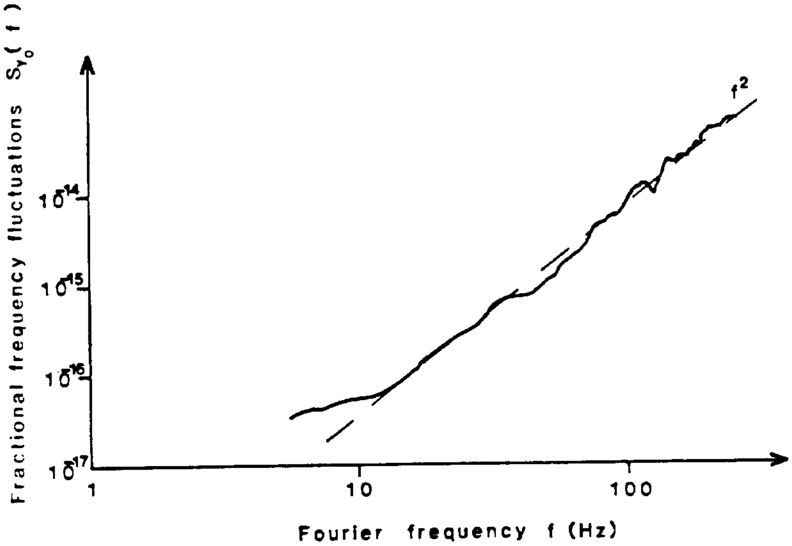

Fig. 8. Frequency fluctuations of a resonator driven at 7 VRMS illustrating the chaotic state of the crystal.