A REVIEW OF FLICKER NOISE FREQUENCY INSTABILITIES IN PRECISION FREQUENCY STANDARDS

David W. Allan
Frequency & Time Standards Section
National Bureau of Standards
Boulder, Colorado 80302 USA

I. INTRODUCTION, BACKGROUND, AND METROLOGY TOOLS

Flicker noise has limited the performance of precision oscillators since their advent as a technological tool. Flicker noise in its common usage is defined as a noise process which has a spectral density which is inversely proportional to the Fourier frequency f ($S_y(f) = h_{-1}/f$) where y(t) is a time varying process whose fluctuations are being characterized, h_{-1} is the proportionality constant (noise level).

Following the notation of reference [1] one can denote the voltage output of an oscillator as

$$V(t) = [V_o + \varepsilon(t)] \sin [2\pi\nu_o t + \phi(t)], \tag{1}$$

where $\varepsilon(t)$ and $\phi(t)$ are the amplitude and phase deviations from nominal respectively, and ν_o is the nominal carrier frequency. Compared to V_o and $2\pi\nu_o t$ respectively, these deviations will be small for precision oscillators. In fact, in many instances $\varepsilon(t)$ is ignored. A note of caution is appropriate at this point, because there are cases where amplitude noise can adversely influence the measurements; e.g., amplitude noise to phase noise conversion occurs in some electronics (often in non-ideal frequency multipliers, and often in non-ideal phase sensitive detectors, e.g., frequency mixers operating away from quadrature). Again following reference [1] one may write the fractional instantaneous frequency deviation as

$$y(t) = \frac{\dot{\phi}(t)}{2\pi\nu_o}, \tag{2}$$

where $\dot{\phi}(t) = \frac{d\phi}{dt}$. For the discrete case and/or the frequency averaged over some time τ, one may write

$$\bar{y}(t_k,\tau) = \frac{1}{\tau}\int_{t_k}^{t_k+\tau} y(t)\, dt \tag{3}$$

$$\equiv y_k$$

Experimentally, of course, one can never measure the true instantaneous frequency because of the high frequency cut-off f_h of the measurement system. So the fluctuations in equation (3) are experimentally more meaningful in assessing noise performance.

For a process perturbed by flicker noise frequency modulation (FM) the classical variance or standard deviation taken on this process does not converge in the infinite time average. A more meaningful time-domain measure which does converge for flicker noise FM as well as for most noise processes perturbing precision oscillators has been recommended [1] and generally adopted by the frequency and time metrology community. This measure (sometimes called the two-sample variance or Allan variance or pair variance) is defined as follows:

$$\sigma_y^2(\tau) = \frac{1}{2} \left\langle (y_{k+1} - y_k)^2 \right\rangle , \tag{4}$$

where $\langle\rangle$ denotes infinite time average, and $t_{k+1} = t_k + \tau$. In practice eq. (4) may be well approximated from a finite data set of M values of y_k ($M \gtrsim 100$ for $\sim 10\%$ confidence interval [2]) as follows:

$$\sigma_y(\tau) \simeq \left[\frac{1}{2(M-1)} \sum_{k=1}^{M-1} (y_{k+1} - y_k)^2 \right]^{1/2} \tag{5}$$

which will herein be called the two-sample deviation (TSD). For flicker noise FM $\sigma_y(\tau)$ has the interesting property of being independent of τ; i.e. $\sigma_y(\tau) = \sqrt{2h_{-1} \ln 2}$, a constant [1,3]. The frequency instabilities of many precision oscillators are characteristically represented for increasing τ (See Fig. 1) on a log $\sigma_y(\tau)$ versus log τ diagram with a decreasing $\sigma_y(\tau)$ until a bottoming occurs ($\sigma_y(\tau)=\sqrt{2h_{-1} \ln 2}$, often called the flicker floor); then $\sigma_y(\tau)$ typically increases due to mechanisms other than flicker noise (typically long-term departures). This paper will review the levels of the flicker floors and long-term stabilities in some precision oscillators as well as some of the causes and some possible ways to decrease the deleterious effects of these noise processes.

Appropriate methods (including the hardware necessary to make precision measurements) are well outlined in the literature [4-9]. At the current time the happy situation exists that despite the vast improvement in the state-of-the-art of precision oscillators over the last few decades one need not be measurement noise limited.

II. HISTORICAL PERSPECTIVE AND STATE-OF-THE-ART

The fractional frequency deviations of precision oscillators are typically well modelled by the following:

$$y(t) = y_o + D\,t + y_r(t), \tag{6}$$

where y_o is the oscillator frequency offset at t=0, D is its nominal frequency drift and $y_r(t)$ is typically a random variation about the first two systematic terms in equation (6). If a TSD is taken on $y_r(t)$ for some state-of-the-art frequency standards, one obtains the stability results shown in Fig. 2 [10], Fig. 3 [11], Fig. 4 [12], Fig. 5 [13], and Fig. 6 [14]. The flicker floors are apparent for most of the oscillators. Only in the case of H-passive is there some question on a reliable measure of the flicker floor; i.e. how much better it will be in stability with increased averaging time.

A. Quartz Crystal Oscillators

A classic example of flicker noise FM is from Attkinson, et.al.'s work [15], where the Fourier frequency extends to below 1 cycle per month (shown in Fig. 7). Making quartz crystal oscillators (QZO) with low flicker floors has been an art as well as a science. As a result the lowest flicker floors achieved have not changed materially over the past couple of decades [16,17,18]. Specifically, in 1966 Barnes reported $\sqrt{2 h_{-1} \ln 2} = 5.4 \times 10^{-13}$ for a quartz crystal oscillator [17], in 1970 Brandenberger reported 2.7×10^{-13} (see Fig. 8) [18] and the best levels achieved to date are about the same [19]. Good science, however, has had impact on the percentage of QZO's that can be produced with low flicker floors; i.e., two decades ago such were hand picked and very special.

Clearly if one could reduce the flicker floor in QZO's this would directly impact their usefulness. Some significant work is presently being done in this area [20, 21].

In the signals from QZO's one also observes flicker noise phase modulation (PM) (see Fig. 1, Fig. 2, Fig. 3). Significant reductions (by 2 orders of magnitude) in the level of this noise have occurred over the last decade [18]. This came about by, first, the recognition of the source of the flicker noise phase modulation (PM) arising in the PN junctions of the electronics associated with the oscillator circuitry, and second, development of a method to reduce the effects of this noise by applying a massive amount of RF negative feedback around the junction.

B. Atomic Frequency Standards

In contrast to QZO's where the flicker floors appear to be resonator limited, atomic frequency standards have shown great improvement in the reduction of the flicker floor where the limitations have been electronics related.

The flicker floor on commercial cesium beam frequency standards (Cs) has improved by about an order of magnitude over the last decade as shown in Fig. 9a; i.e., typical values were a few parts in 10^{13} for $\sqrt{2h_{-1} \ln 2}$, whereas now, one often sees a few parts in 10^{14}. About the same order of improvement has occurred in the flicker floors of H-masers (H), but about one order lower than for cesium; i.e., from 10^{-14} to 10^{-15} as shown in Fig. 9b. Rubidium gas cell frequency standards (Rb) have also shown marked improvement. In 1966 several Rb's were carefully measured to see if they would meet the specifications for a set of tracking stations. Several such units were measured and exhibited flicker floors of about 2×10^{-12}. Currently commercial standards have flicker levels of about 1×10^{-13} (see Fig. 9c).

Other less common atomic frequency standards are: rubidium-masers where flicker levels of about 1×10^{-13} have been achieved [22]; passive H-masers where the best stability to date of any standard [12] has been achieved at a 4 day sample time, τ, of about 3×10^{-15} with no conclusive evidence of a flicker floor. Excellent atomic and molecular frequency standards are now available in the infrared portion of the spectrum using, as examples, resonances in CH_4, CO_2, and I_2. Flicker floors respectively of 3×10^{-14} [23,24], 1×10^{-13} [24,25], and 10^{-12} [24], have been achieved. There is not yet a good way to compare frequency stabilities at these levels between the microwave and infrared portions of the spectrum, but that appears to be forthcoming [26, 27, 28].

C. Superconducting Cavity Stabilized Oscillator (SCSO)

To date the best achieved flicker floor of any precision frequency standard has been achieved using as the resonator the high-Q achievable ($\sim 10^{10}$) in a niobium superconducting cavity tuned at about 10 GHz. A flicker floor of 6×10^{-16} for $10 \text{ s} < \tau < 100 \text{ s}$ ($f_h = 10$ Hz) has been achieved [27]. Though environmental effects degrade the stability for longer sample times in a SCSO, the stability plus and minus a few decades around 1 second is unsurpassed making it an excellent candidate for frequency multiplication into the infrared portion of the spectrum as well as for VLBI applications.

III. SOME FLICKER NOISE SOURCES AND SOME POSSIBLE WAYS TO AVOID DELETERIOUS EFFECTS

A. Quartz Crystal Oscillators

In most cases the resonator appears to limit the flicker FM level. Some of the mechanisms in the resonator which cause flicker FM are thought to be: migration of impurities in the crystal lattice [29] which appears to be dependent upon the amount of power used to excite the crystal resonator [30]; stress relief associated with the crystal mounting [31]; and of course indirectly the cut of the crystal, crystal-Q factor, and the particular overtone used.

Experiments are now being conducted on low stress mountings [20] and on stress compensated (SC) cuts to provide crystals with greatly reduced stress release problems [3]. Another interesting approach that is being studied [32] is frequency locking a QZO to a passive crystal resonator. One can obtain excellent short-term ($\tau \sim$ ms) stability from a QZO whose crystal is driven with about 10 microwatts [33]. Frequency locking (with appropriate attack time) such a QZO with a passive quartz resonator, where one second stabilities of less than one part in 10^{13} have been observed [34], could lead to an overall QZO with an unexcelled flicker floor and also with excellent short-term stability [32].

B. Atomic Frequency Standards

In this case, as was mentioned before, it is generally true that the limitations in the achievable flicker floor have been due to the electronics and hardware configured to the atomic resonance. The flicker noise driving mechanisms arise in basically two areas: First, environmental parameters (e.g., temperature, pressure, magnetic field) often have a flicker like behavior. If an atomic standard is sensitive to one or more of these environmental parameters then its design will undoubtedly feature some shielding against changes in those possible perturbing parameters. The shielding, of course, is never perfect and what happens in practice is that the effects that would have been induced are reduced by a shielding filter, but at some level the perturbing effects still perturb the final frequency output of the standard. Second, the flicker noise driving mechanisms are internal to the hardware and/or the electronics configured to the atomic resonance. Examples of these mechanisms might be: PN junction noise, resonant cavity and material changes, general semi-conductor noise as it shows up in the dc and ac drifts in operational amplifiers, multipliers, modulators, and demodulators, changes in the efficiency of either the source of atoms and/or in the detection system.

In the rubidium gas cell frequency standard there is a strong pressure sensitivity via the partial pressure of the buffer gas intrinsic to the these standards. If the temperature changes, this induces a pressure change via the gas law. A series of experiments were conducted on a commercial Rb standard in which better shielding was sequentially provided against temperature fluctuations, pressure fluctuations, and finally magnetic field fluctuations. Each time the stability improved (see Fig. 10) [13].

For sample times of the order of a day or longer, $\sigma_y(\tau)$ typically increases for Rb standards where a higher flicker floor or frequency drift predominates. The driving mechanisms in this case are more probably internal to the standard, and evidence supports long-term frequency instabilities being caused by such things as changes in the optical pumping spectrum (light shift) [24,35,36], gradual changes in the composition of the buffer gas [24] and perhaps changes in the microwave power [37]. This longer term flicker floor is usually several parts in 10^{13}.

Reduction of the flicker floor in Rb standards seems possible by purposely attacking at least the above mentioned causes. However, the overall improvement in performance will probably not be significant because the short-term stability is limited by other mechanisms, only a nominal reduction in the flicker floor will leave the short-term instabilities and the long-term drift as the predominate effects. The long-term drift and instabilities, of course, could most probably also be reduced; in which case flicker noise would again probably predominate over a significant range of Fourier frequencies.

The cause of flicker noise FM in cesium beam frequency standards may arise from several different mechanisms. The output frequency of a cesium standard is generally dependent upon -- to mention some -- the microwave power, the Ramsey cavity phase shift, the magnetic field, microwave spectrum impurities and asymmmetry, distortion of the modulation frequency, ac and dc instabilities in the servo demodulator circuits, degradation in the servo loop gain, pulling from adjacent spectral lines, cesium beam optics, changes in the cesium detector and the velocity of the cesium atoms. Any one or more of the above mechanisms could induce flicker noise in the standard's output frequency.

The main environmental sensitivities are temperature, magnetic field, and electromagnetic interference (excluded for flicker noise considerations are the more violent environmental sensitivities; e.g., shock, vibration and radiation). The temperature fluctuations which often look like flicker noise, usually affect the output frequency as it transduces via one of the above mechanisms -- very often via microwave power [14]. Figure 11 is taken from reference [14] and clearly shows the correlations of the microwave power and output frequency with the environmental temperature. Now, the Ramsey spectrum as may be seen from the detector of a cesium beam standard has secondary peaks whose displacement from the primary peak is directly dependent on microwave power but in a far more sensitive way than in the frequency of the primary peak. Figure 12 (also taken from ref. [14]) shows the high degree of correlation between the frequency of this secondary peak and the microwave power. Figure 13 is a plot of the stability of the frequency offset of the secondary peak which has a flicker behavior and would translate into a flicker floor for the output frequency locked to the primary peak of a few parts in 10^{14} which is what is observed on this particular commercial standard. Hence, one may conclude that in this particular standard the flicker floor is probably caused by flicker temperature transducing via microwave power to flicker frequency of the output.

There is clearly the possibility of other mechanisms which could induce flicker noise FM on the output of cesium beam frequency standards which may give rise to flicker floors of a few parts in 10^{14}, but it appears that better control of the microwave power may significantly reduce the current flicker floor in some cesium beam standards.

The flicker floor in a hydrogen maser is often caused by cavity pulling [12,24]. Other possible flicker inducing mechanisms are magnetic field, microwave spectrum and power, pressure tuning of the cavity, instabilities in the servo electronics, spin exchange due to changes in the hydrogen flux, etc. The passive hydrogen maser approach of reference [12] is a novel method of controlling the frequency of the cavity to significantly reduce the flicker floor over prolonged periods of time (compare figure 2 and figure 4). Preliminary measurements are very encouraging [12].

C. Superconducting Cavity Oscillator (SCO)

Though SCO's have produced the best flicker floor to date the possibility of extending it to longer sample times or of decreasing it appears very difficult, because the flicker like behavior is most probably caused by the dimensional instabilities of the cavity. Other flicker noise inducing mechanisms, of course, exist which must be adequately addressed. These may be associated with the interrogation of the frequency of the superconducting cavity and/or with the associated frequency lock system. In this regard, the problems are similar to those with quartz crystal oscillators, but the SCO's have the advantages of a much higher Q and higher resonante frequency, i.e., 5 MHz vs X-band.

IV. FUTURE PROGNOSIS AND CONCLUSION

Quartz crystal oscillators, cesium beam frequency standards, and hydrogen masers are on the brink of producing frequency signals whose flicker floors are significantly reduced as compared to the past. One may identify the mechanisms causing the flicker floors in many precision oscillators and hence, in principle, know where to apply effort in order to reduce these flicker floors. Generally, in quartz crystal oscillators the flicker floor is limited by the quartz resonator. In atomic frequency standards the flicker floor is generally limited not by the atomic resonance but by the interrogation system and environmental influences on the same. As a result of the above insights into the mechanisms limiting the flicker floors of some important precision oscillators, one expects with a reasonable expenditure of effort to see some significant advancements in the near future in this area. The flicker floors potentially achievable in the fairly near future seem to be less than 10^{-13} ($1\text{s} < \tau < 10^3\text{s}$) for QZO, and a few parts in 10^{15} for cesium and hydrogen based standards (few days $\leq \tau \leq$ month).

Several novel approaches for new standards are being worked on which show promise [24], but it is too early to tell what they will produce in terms of flicker floors.

V. ACKNOWLEDGMENTS

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REFERENCES

[1] J.A. Barnes, et, al., "Characterization of Frequency Stability," IEEE Trans. on Instrum. & Meas., IM-20, No.2, pp.105-120 (May 1971).

[2] P. Lesage and C. Audoin, "Characterization of Frequency Stability: Uncertainty due to the Finite Number of Measurements," IEE Trans. on Instrum. & Meas., IM-22, No.2, pp.157-161 (June 1973).

[3] D.W. Allan, "Statistics of Atomic Frequency Standards," Proc. IEEE, 54, No.2, pp.221-230 (February 1966).

[4] L. Cutler and C. Searle, "Some Aspects of the Theory and Measurement of Frequency Fluctuations in Frequency Standards," Proc. IEEE, 54, No.2, pp.136-154 (February 1966).

[5] D.W. Allan, "The Measurement of Frequency and Frequency Stability of Precision Oscillators," NBS Tech Note 669 (July 1975).

[6] D. Howe, "Frequency Domain Stability Measurements: A Tutorial Introduction," NBS Tech Note 679 (March 1976).

[7] J. Rutman and G. Sauvage, "Measurement of Frequency Stability in Time and Frequency Domains via Filtering of Phase Noise," IEEE Trans. on Instrum. & Meas., IM-23, No.4 (December 1974).

[8] J. Groslambert, "A New Spectrum Analyzer using both Analog and Digital Filtering via Hadamard Variance," IEEE Trans. on Instrum. & Meas., IM-25, No.4 (December 1976).

[9] D.G. Meyer, "A Test set for Measurement of Phase Noise on High Quality Signal Sources," IEEE Trans. on Instrum. & Meas., IM-19, No.4, pp.215-227 (November 1970).

[10] USA CCIR Study Group 7, Revision of Report 364 on Performance of Standards - FM7/107 (1977).

[11] USA CCIR Study Group 7, Revision of Report 580 on Frequency Stability-FM7/102 (1977).

[12] F. Walls, "Design and Results from a Prototype Passive Hydrogen Maser Frequency Standard," Precise Time and Time Interval Applications and Planning Meeting (December 1976).

[13] H. Hellwig and A.E. Wainwright, "A Portable Rubidium Clock for Precision Time Transport," Proc. of the 7th Annual Precise Time and Time Interval Applications and Planning meeting (December 1975).

[14] D.W. Allan, et.al., "Some Causes and Cures of Frequency Instabilities (Drift & Noise) in Cesium Beam Frequency Standards." (June 1977).

[15] W.R. Attkinson, et.al., "Spectrum Analysis of Extremely Low Frequency Variations of Quartz Oscillators," Proc. of IEEE, No.2 (February 1963).

[16] Private communication with Dr. W. Markovitz regarding their best quartz oscillator then contributing in the USNO Time Scale.

[17] J.A. Barnes, "Atomic Timekeeping and the Statistics of Precision Signal Generators," Proc.IEEE, Vol.54, No.2,pp.207-220 (February 1966).

[18] H. Brandenberger, et.al., "High Quality Quartz Crystal Oscillators: Frequency Domain and Time Domain Stability," Proc. of Tthe Symposium on Frequency Control, pp.266-230 (April 1971).

[19] J.R. Norton, "An Ultrastable Low Power 5MHz Quartz Oscillator Qualified for Space Usage," Proc. of the Symposium on Frequency Control, pp.275-278 (1976).

[20] R. Besson, "A New 'Electrodeless' Resonator Design", E.N.S.C.B.M. Proc. of 31st Annual Symposium on Frequency Control (June 1977).

[21] J.A. Kusters, et.al, "Thermal Trancien Compensated Crystal Cuts - TTC's, Hewlett Packard, Proc. of 31st Annual Symposium on Frequency Control (June 1977).

[22] M. Têtu, et.al., "Short-Term Frequency Stability of the Rb^{87} Maser," IEEE Trans. on Instrum. & Meas. IM-22, No.3 (September 1973).

[23] J. Hall, "Fundamental & Applied Laser Physics," Proc. of Esfahan, Edited by M.S. Feld, A. Javan, N. Kurnit. Printed by John Wiley, p. 463 (1973).

[24] C. Audoin, "Atomic Frequency Standards and Clocks," Journal of Physics E: Scientific Instruments, Vol.9 (1976.

[25] C. Freed, "Advances in Co_2 Laser Stabilization Using the 4.3 μm Flourescence Technique," Proc. of the 2nd Frequency Standards and Metrology Symposium (July 1976).

[26] S. Stein, "Electronic Tuning and Phase - Lock Techniques for Optically Infrared Lasers," Proc. of 31st Annual Symposium on Frequency Control (June 1977).

[27] S. Stein and J.P. Turneaure, Electron. Lett. 8, 321 (1972).

[28] A. Risley, "A Study of Point-Contact Josephson Junctions for use in Frequency Synthesis," Proc. of 31st Annual Symposium on Frequency Control (June 1977).

[29] T. Musha, "1/f Resonant Frequency Fluctuation of a Quartz Crystal," Proc. of the 29th Annual Symposium on Frequency Control (1975).

[30] P. Bryan, "A rapid and Simple in Process Test Method Designed to Improve Quality of Quartz Resonators in Current Demand Today," Proc. of the 29th Annual Symposium on Frequency Control (1975).

[31] E.P. EerNisse, "Quartz Resonator Frequency Shifts Arising From Electrode Stress," Proc. of the 29th Annual Symposium on Frequency Control (1975).

[32] F. Walls, "New Horizons and Old Pitfalls in Frequency and Time Metrology," Proc. of the 2nd Frequency Standards and Metrology Symposium (1976).

[33] D.J. Glaze, Calibration measurement made at NBS and published by commercial vendor.

[34] F. Walls, Private Communication.

[35] G. Busca, "Cavity Tuning and Light-Shift in the Rb^{87} Maser," Proc. of the 27th Annual Symposium on Frequency Control (1973).

[36] H. Oyamada, "A Consideration of Rubidium Lamp Stability for Rubidium Frequency Standard," Proc. of the 28th Annual Symposium on Frequency Control (1974).

[37] A. Risley, Recent work done at NBS on the frequency dependance of a Rb gas cell frequency standard as it depends on microwave power.

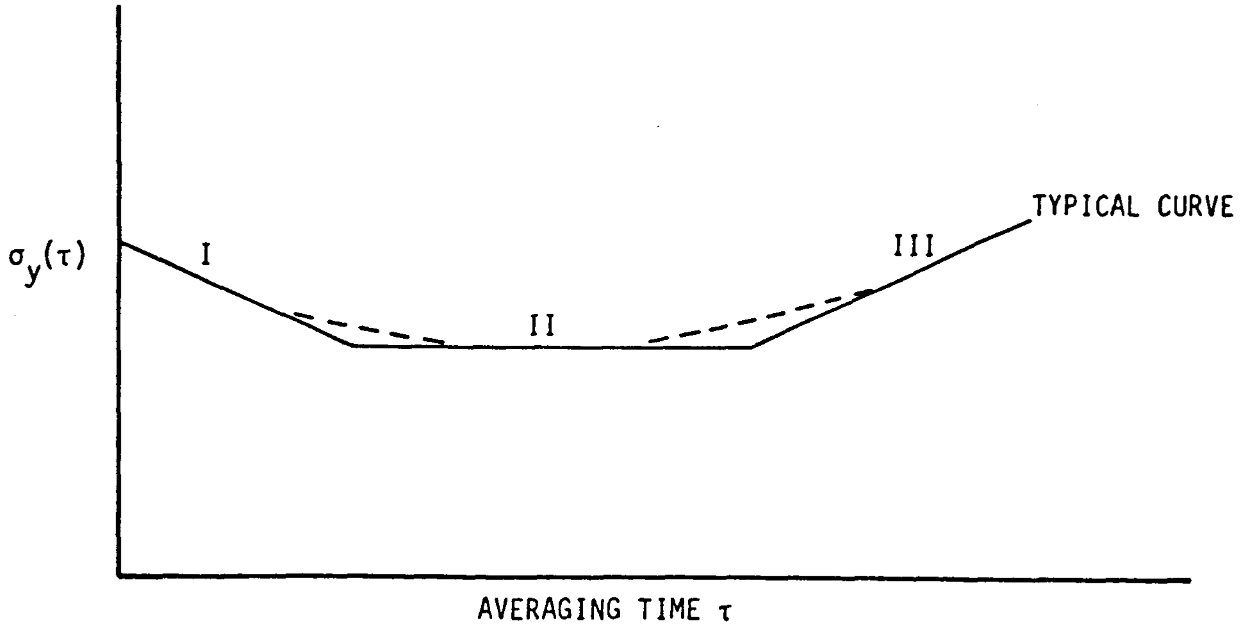


Figure 1 Typical stability characterization plot which serves as an appropriate model for many precision frequency standards

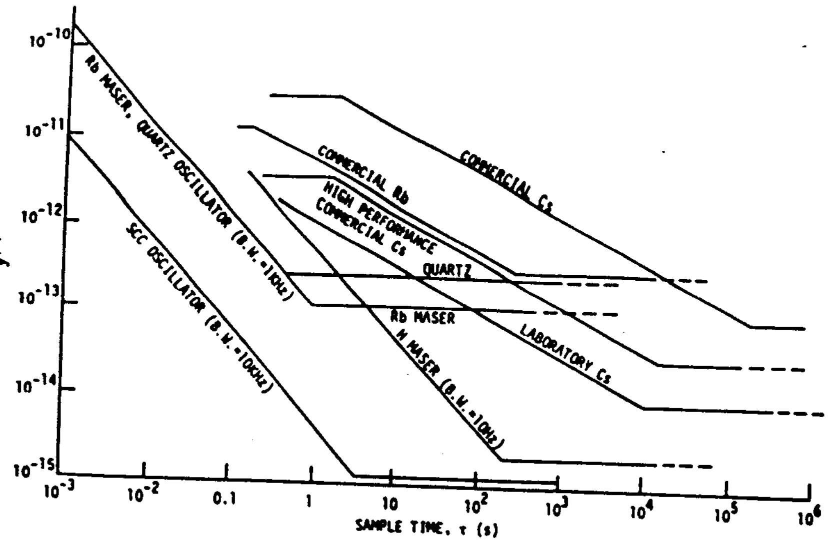


Figure 2 Stability characterization of most kinds of state-of-the-art frequency standards

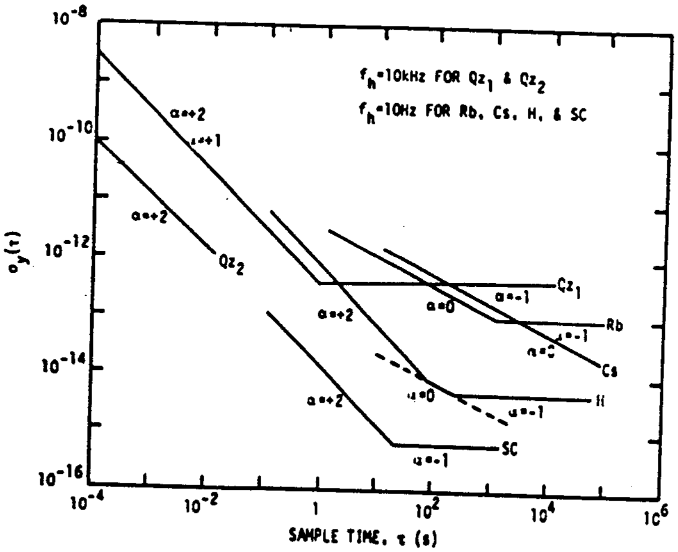


Figure 3a Time domain (a) and frequency domain (b) characterization of a set of state-of-the-art precision frequency standards. Fig. 3(b) shows the region of Fourier frequency where flicker noise (f^{-1}) is of concern.

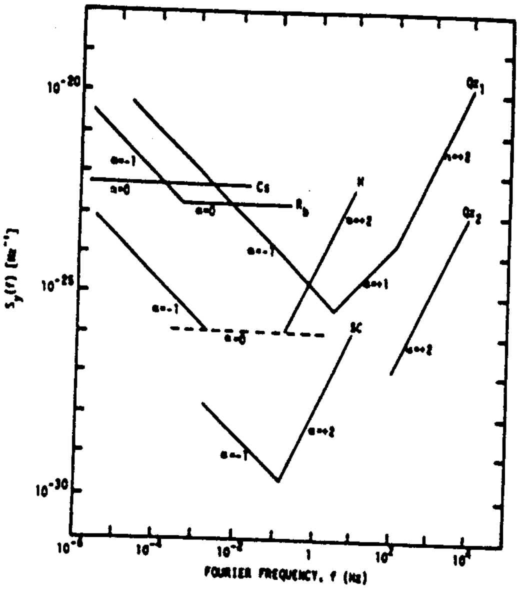


Figure 3b Time domain (a) and frequency domain (b) characterization of a set of state-of-the-art precision frequency standards. Fig. 3(b) shows the region of Fourier frequency where flicker noise (f^{-1}) is of concern

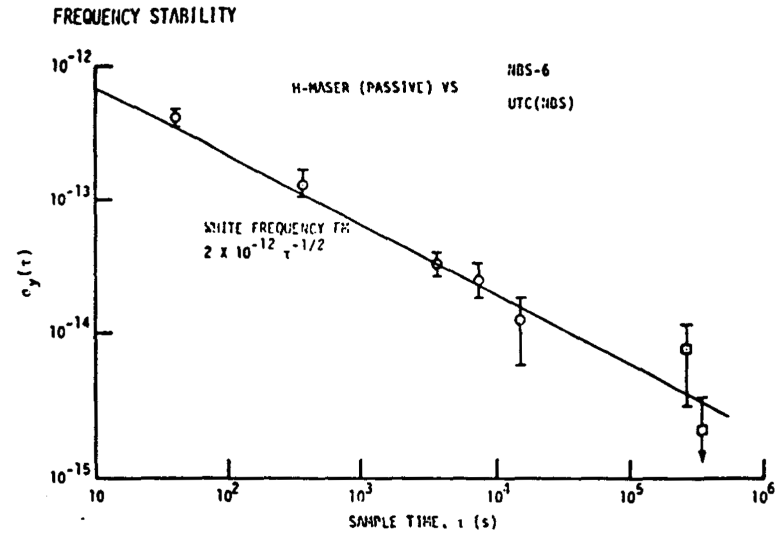


Figure 4 Frequency stability measurements of an NBS prototype passive hydrogen maser frequency standard -- showing no apparent flicker floor at well below 10^{-14}.

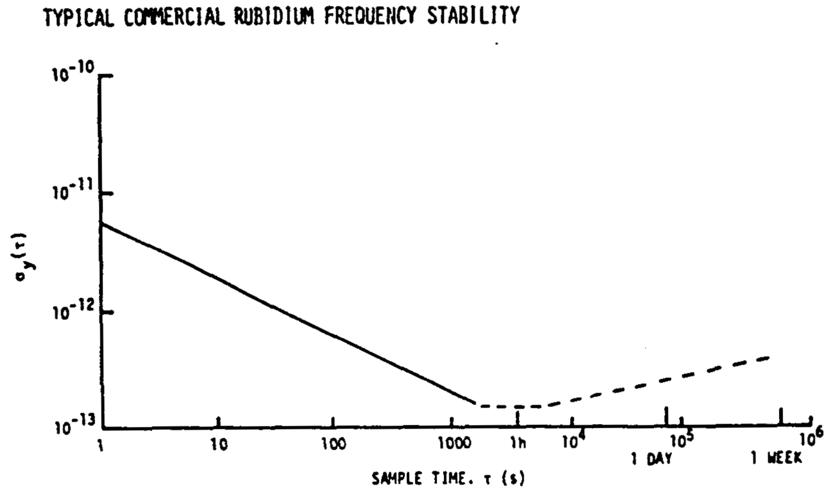


Figure 5 Frequency stability of a commercial rubidium gas cell frequency standard with frequency drift subtracted

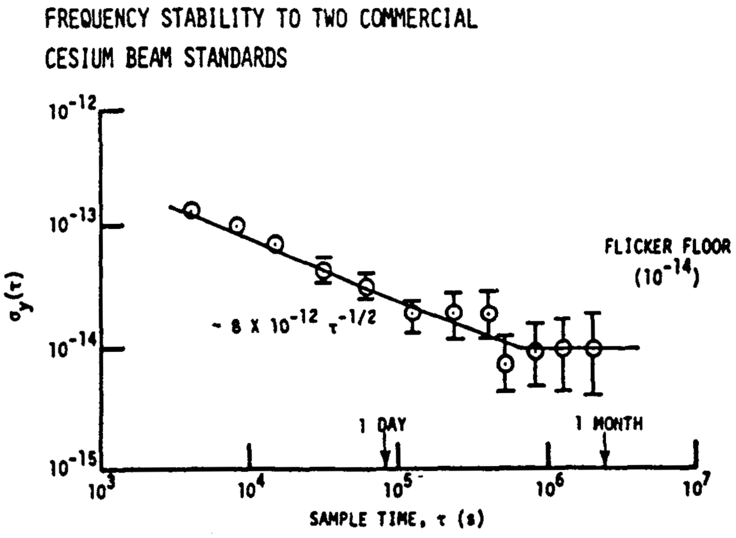


Figure 6 Frequency stability as measured between two state-of-the-art commercial cesium frequency standards

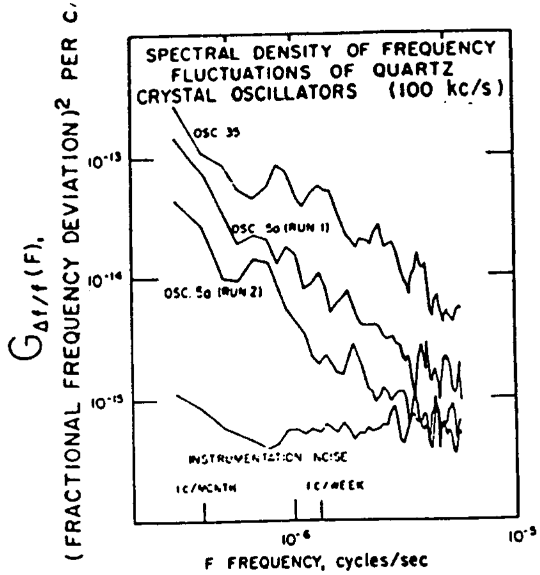


Figure 7 Frequency stability of a quartz crystal oscillator

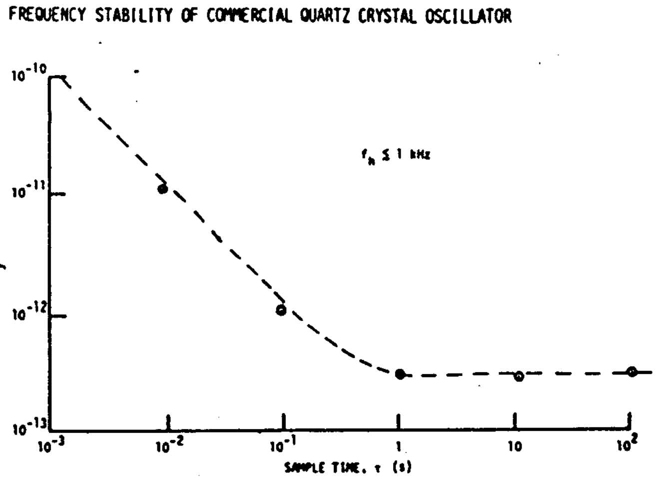


Figure 8 Spectral density of frequency fluctuations of quartz crystal oscillators over a prolonged measurement interval showing the predominance flicker noise over Fourier frenuencies as low as one cycle per month

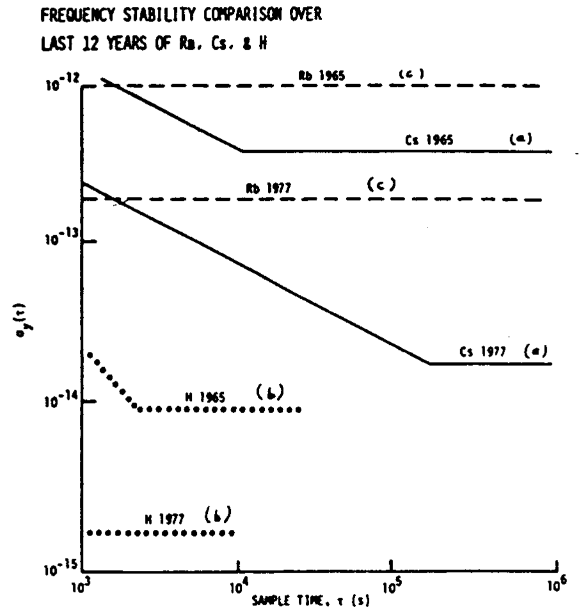


Figure 9 Comparison of the frequency stability from 1965 with 1977 of commercial beam frequency standards (Cs) and commercial rubidium gas cell frequency standards (Rb) and of hydrogen masers (H).

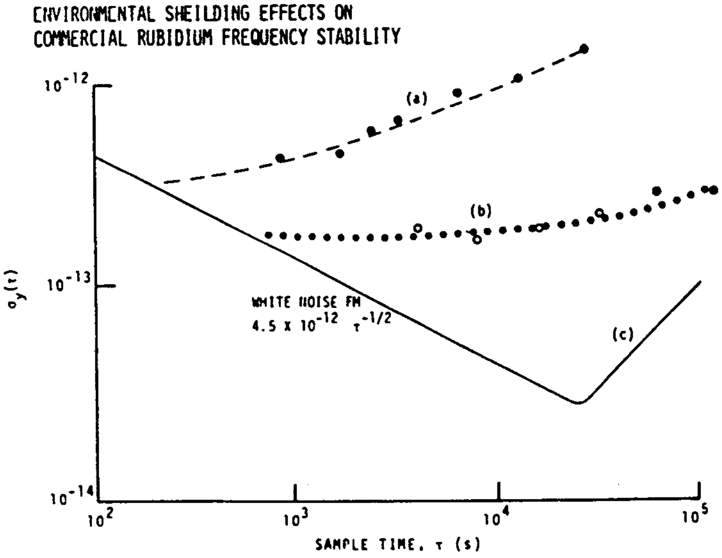
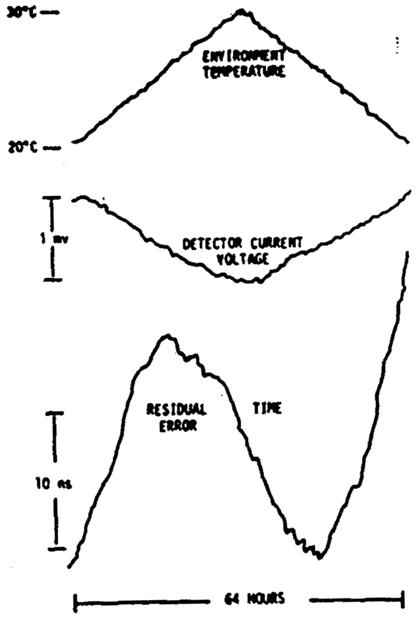


Figure 10 Frequency stability of a commercial rubidium frequency standard: (a) in a laboratory environment; (b) in an environmental chamber with less than 0.1°C control and with additional magnetic shielding; and (c) with a barometric seal around it to reduce pressure sensitivity



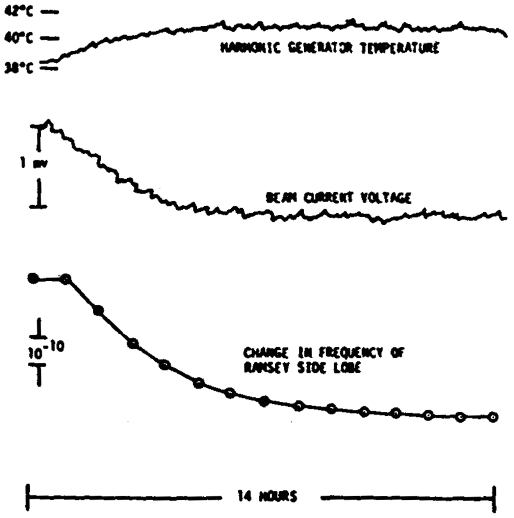


Figure 12 Dependence of the frequency of the Ramsey side lobe, which can be shown to be a function of the microwave power, on the temperature of a commercial cesium beam frequency standard

11 Dependence of the frequency (slope on the x(t) curve) of a commercial cesium beam frequency standard as a function of the environmental temperature. Note the change in beam intensity--probably caused by a change in the microwave power--which is also dependent on the environmental temperature.

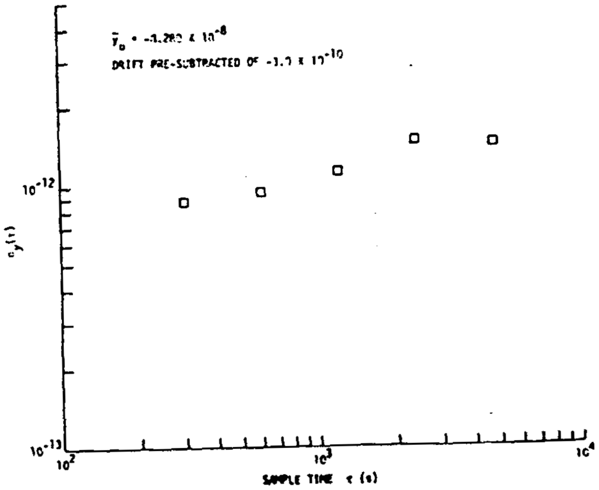


Figure 13 Frequency stability of the Ramsey side lobe at constant temperature. This flicker-type behavior can transduce into flicker of the output frequency of this commercial cesium beam frequency standard.