The earth tides

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For centuries, people living along coastlines have noticed the diurnal and semidiurnal fluctuations in the height of the sea.

The connection between the moon and these tides was obvious, and, even before the formulation of any theory, quite satisfactory predictions of the ocean tides were published. Tidal tables were constructed by various undivulged methods, and these methods were often passed from father to son.

It was less widely appreciated that the earth itself is subjected to tidal stresses and undergoes tidal deformations. It is the purpose of this paper to investigate these tidal stresses and to see how the deformations may be measured.

The origin of the tides

It is customary to discuss tidal effects in terms of a tidal potential, but we can also derive the essential features using the force laws alone.

We suppose initially that the moon is fixed directly over the earth's equator and that the moon does not move (Fig. 1). We imagine the earth to be made up of a large number of point masses bound together. From Newton's law of gravitation, we find that each point of the earth is attracted to the moon by a force given by

$$F = GMm/r^2$$

where G is a constant, M is the mass of the moon, m is the mass of the point, and r is the distance from the moon to the point. This force is always attractive as is shown in the figure.

It is useful to break these forces into two parts: a constant attractive force acting at the center of mass of the earth and a differential force which varies from point to point. The constant force is simply the gravitational attraction of the moon and the earth considered as point objects, while the differential force arises from the fact that different parts of the earth experience attractions of different magnitudes and directions.

It is the constant force that binds the moon and earth together. An observer on the earth detects this force by observing the orbits of the moon and the earth. However, this force is the same for all parts of the earth and produces no internal stresses (we are neglecting small effects such as the stress induced in the earth by the variation of the centripetal acceleration from point to point).

Since the differential force changes from point to point, it induces stresses in the earth, and it is these stresses that are responsible for the tides both in the oceans and in the solid earth. These forces are shown in Fig. 2. The side of the earth closest to the moon experiences a greater attraction than the average force at the center of mass, and the differential force is attractive. The side opposite the moon experiences a force weaker than that experienced by the center of mass, and the differential force therefore points in the opposite direction. The differential forces along a line perpendicular to the earth-moon axis have magnitudes that are comparable to the magnitude of the average force, but since their directions converge on the moon, they have components that point inwards towards the center of the earth.

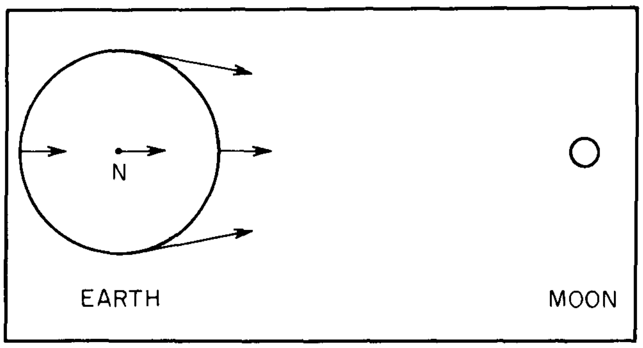

Fig. 1. A simple model of the interaction between the earth and the moon. The moon is assumed to be stationary in the earth's equatorial plane. The view is that of an observer located in space above the North Pole. The gravitational attraction at several points is denoted by the arrows.

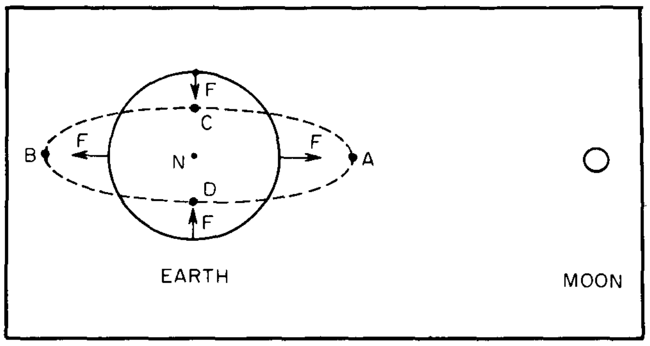

Fig. 2. The gravitational forces acting on the earth due to the moon after a constant force equal to the force acting at the center of mass has been subtracted. The dotted line shows the general shape (exaggerated) of the distortion produced by these forces. The earth is deformed with points A and B moving away from the center of the earth and points C and D moving closer. The view is that of an observer situated above the North Pole.

The detailed evaluation of the variation of the force from point to point depends on the inverse square nature of the gravitational interaction. Since Coulomb's law is also inverse square in nature, we expect that the tidal interaction is analogous to electrostatic polarization when only one type of charge is present in an extended body placed in a nonuniform electric field.

The forces shown in Fig. 2 would tend to deform the earth into a shape indicated schematically and with great exaggeration by the dotted line. As the earth rotates on its axis, the deformation remains always pointed at the moon. From the point of view of an observer on the earth, a given point would trace out the dotted curve once per day and hence every point on the earth would see two tidal peaks per day: one when the moon was at its zenith and a second peak twelve hours later.

The tidal deformation shown in Fig. 2 varies with latitude. The compression of the earth along a line perpendicular to the earth-moon axis in the equatorial plane is accompanied by a compression along the rotation axis of the earth (i.e., along a line connecting the North and South Poles). This axis is perpendicular to the plane of the figure. At the instant of time depicted in the figure, observers at points A and B are displaced outward from the center of the earth, while those at C and D are displaced inward. Observers at the North or South Poles would also be closer to the center of the earth. Since the tidal distortion is positive on the equator and negative at the poles, there must be a latitude where it is zero. Likewise, since it is positive directly under the moon and negative along a line perpendicular to this direction there must be a longitude where the distortion is zero.

The moon is, in fact, rarely over the equator. Although this fact does not alter the tidal bulge from the point of view of an observer on the moon (who, in this simple model, always sees a cigar-shaped tidal bulge pointed straight at him), it greatly changes the tidal deformation as measured by an observer on the surface of the earth.

If the moon is over the Northern Hemisphere, for example, an observer in the Northern Hemisphere will see a larger tidal deformation when the moon is at its zenith than twelve hours later since the latter deformation will be decreased by the latitude effect (twelve hours later the line joining the earth and the moon intersects the observer's side of the earth in the Southern Hemisphere). (See Fig. 3.) This observer would conclude that a diurnal component is also present. This diurnal component augments the semidiurnal distortion when the moon is near the zenith and diminishes the distortion twelve hours later. These two components have a constant phase relationship and both reach their maxima at the same time (in this model). This situation is shown in Fig. 3. An observer at position A on the earth measures the tidal distortion to be $A - A'$ when the moon is at its zenith. Twelve hours later he measures the much smaller tidal distortion $B - B'$.

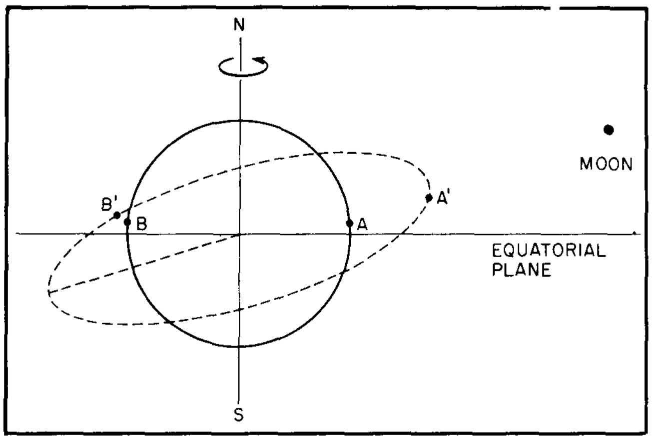

Fig. 3. The distortion induced by the moon when it is not exactly in the equatorial plane. The axis of rotation of the earth is vertical. An observer at latitude A sees a tidal distortion $A - A'$ when the moon is overhead and a smaller distortion $B - B'$ twelve hours later.

The analysis we have done for the effect of the moon must be done for the effect of the sun as well. If we assume a sun fixed at one point in space over the equator, the conclusion is exactly the same as in the lunar case. The sun produces a tidal bulge that points along the earth-sun axis. From the point of view of an observer on the earth, this tidal bulge appears to rotate producing two tidal peaks per day. The tidal deformation will depend on the latitude of the observer just as in the lunar case. When the sun and the observer are not at the equator, there will be a diurnal solar distortion as well.

It is not too difficult to show that the greater mass of the sun is more than offset by its greater distance so that the solar tidal effect is somewhat smaller than the effect induced by the moon. The tidal distortion is produced by the variation in the gravitational attraction, and we can estimate the magnitude of the effect in the simple situation of Fig. 2. The difference between the gravitational attractions at points A and B is simply

$$\Delta F = F_A - F_B = GMm\,(1/r_A{}^2 - 1/r_B{}^2)$$

where G is the gravitational constant, M is the mass of the tide-generating body and m is the mass of a point on the earth at A or B. The distances between the tide-generating body and the points A and B are r_A and r_B, respectively. The difference may be approximated by

$$\Delta F = 2GMm(D/R^3)$$

where D is the diameter of the earth and R is the mean distance from the earth to the tide-generating body. The tides generated by different sources are proportional to the quantity M/R^3 which is approximately $1.16 \times 10^6\,\mathrm{kg/m^3}$ for the moon; the corresponding value for the sun is approximately $6 \times 10^5\,\mathrm{kg/m^3}$ or about one-half of the lunar value.

The tidal distortions produced by other bodies in the solar system are negligible.

The effect of orbital motion

Our simple model becomes more complicated when we include orbital motions. The orbital motion of the earth and the moon cause the axes of the tidal deformations to rotate in space. From the point of view of an observer measuring time with respect to the stars (sidereal time), the orbital motions introduce phase modulations on the simple diurnal and semidiurnal effects.

In addition to these phase modulations, there are amplitude modulations produced by the changing distances between the moon, the earth, and the sun since the orbits are elliptical.

Finally, the apparent latitudes of both the moon and the sun change with time, and this results in a variation of the ratio of the semidiurnal and diurnal amplitudes. This variation depends on the latitude of the observer.

The effect of these amplitude and phase modulations is generally expressed as a Fourier expansion of the tidal time series into its constituent frequency components; we can understand the general conclusion from a simple model, however.

If the amplitude of a tidal constituent varies with time, the resultant time series is expressible as a product of two sinusoidal functions: the first giving the time dependence of the amplitude and the second giving the time dependence of the force. A product of two sinusoidal functions is always expressible as a sum of two sinusoidal functions: one with a frequency equal to the sum of the frequencies of the tidal term and the amplitude variation and one with a frequency equal to the difference. (This effect is termed amplitude modulation and is well known in radio transmission.) Although it is more difficult to prove, a periodic modulation of the phase of a sine wave produces the same sort of sidebands: one at the sum of the frequencies and one at the difference. The amplitudes of the tides change slowly with time; the characteristic periods are the lunar month and the solar year. Since both of these periods are much longer than a solar day, the resultant sum and difference frequencies differ by only small amounts from the nominal of once per day and twice per day of the simple theory.

The actual orbits of the earth and the moon also have longer-period effects such as the precession of the equinoxes, etc. Each of these effects produces an amplitude or a phase modulation of the tides; each is handled in the same way.

When we consider all of these modulations, we obtain a series of sidebands superimposed on the simple two-line spectrum (f = 1/day and f = 2/day), and we are led to expect a cluster of frequencies or sidebands near one cycle/day and a cluster near two cycles/day. In both cases we would expect a series of sideband frequencies offset from the basic diurnal and semidiurnal frequencies by sums and differences of the various orbital periods. In fact tidal frequencies are often specified by a six digit "Doodson number." The Doodson number is simply a prescription for expressing the frequency as a linear combination of the basic orbital parameters.

Since the orbits of the earth and the moon change slowly with respect to the basic tidal period of one day, the tidal sidebands do not differ greatly from their nominal frequencies of one cycle per day and two cycles per day, and the simple statement that the basic tidal periods are twelve hours and twenty-four hours is not grossly incorrect for short periods of observation.

A more exact theory

The simple model outlined above accounts for the major features of the tidal excitation function. There are, however, higher-order effects that are significant. They are more easily discussed using the tidal potential which is a scalar rather than the tidal force which is a vector. (The tidal potential difference between two points is simply the work done on a unit mass by the tidal force in moving the mass between the two points.)

It is also inconvenient to express the generating function in terms of the distance between the moon (or the sun) and the point of observation. The position of the observer is most easily specified in terms of latitude and longitude; the position of the moon is usually specified in terms of a spherical coordinate system using two angles and a distance. If we use the center of the earth as the origin of these two systems (one to locate the observer and the second to locate the moon), we can express the distance from the moon to the observer as a power series in a/R, where a is the radius of the earth and R is the radial distance to the moon. The coefficients of the terms in the power series are called multipole moments. They are polynomials of sines and cosines of the angular positions of the observer and the moon.

The multipole expansion is convenient because it separates the coordinates of the observer from the coordinates of the moon so that the lunar part of the calculation need be done only once for all observers; the lunar part is relatively easy to calculate using known astronomical constants. Successive terms in the expansion involve higher and higher powers of a/R. This quantity is about 0.02 for the moon so that the series converges quickly and only a few terms are usually needed.

The first term in this expansion is exactly the constant attractive force discussed in the first section. The next term (the "dipole" term in the analogous electrical expansion) is zero since the center of the earth is taken as the center of mass of the system in our model. From the point of view of the earth tides, the first term of interest is the second order term. This term describes a "quadrupolar" distortion of the earth whose shape is exactly the shape deduced above and outlined in Fig. 2. The radial part of this term varies as a^2/R^3. We recognize this term as having the same radial dependence as the differential force we derived above. The next order lunar term is smaller than the quadrupole term by a factor on the order of 0.02 so that the quadrupole term accounts for roughly 98% of the total tide. The next order term is negligible for the sun.

Although the quadrupole term accounts for most of the tide, the higher order terms in the expansion each have a component with a frequency of more than two cycles/day. These higher frequency terms are observable on many instruments. Each of these higher order terms also makes a contribution to the diurnal and semidiurnal bands, and these contributions must be included in accurate computations. Although these terms have almost the same frequencies as the principal diurnal and semidiurnal components, their dependence on latitude and longitude is different; their importance is therefore dependent on the position of the observer.

All of the terms in the multipole expansion also have long-period components. These are tides with periods longer than one day; the next largest component is usually the fortnightly tide. These long-period tides are usually masked by the larger diurnal and semidiurnal tides, but they are important when the shorter period components are very small (at the poles, for example).

The tidal spectrum

We see from the more exact theory that the tidal spectrum is extremely rich. Tables of tidal harmonics have about 400 entries, although many of these terms are quite small and are often combined with nearby larger terms. Tidal analyses usually require approximately 30 frequency bands to deal with data series about one year long. More terms must be used for longer time series.

The response of the earth

The tidal stresses discussed above act on the oceans and on the solid earth. The response of the earth (measured as the distance A − A′ of Fig. 3, for example) is smaller than that of the ocean but, in our current linear approximation, has the same spectrum. In order to calculate the deformation of the earth we must know its elastic properties. Alternatively, we can use the observed deformation to calculate the effective elastic moduli of the earth.

We may characterize the response of the earth in terms of the "equilibrium tide." The tidal forces cannot move the center of mass of the earth-moon system. We have taken the center of mass to be located at the center of the earth, and displacements are measured from that point. *Relative* displacements are measured from the position of a point in the absence of tidal effects.

We imagine a point on the surface of the earth in equilibrium under gravitational forces. As the moon passes

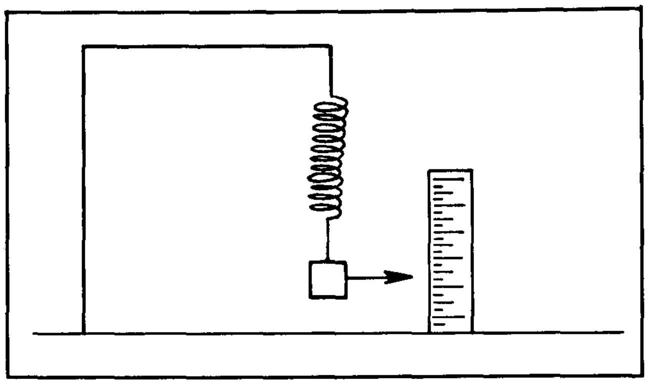

Fig. 4. A schematic diagram of a gravity meter. The local acceleration of gravity can be determined by measuring the displacement produced when a known mass is suspended from a spring of known force constant.

overhead, the point experiences an additional force due to the moon's gravitational attraction. If the particle moves a distance h towards the moon, the work done by the additional force is $m\mathrm{V}$, where m is the mass of the particle and V is the difference in the tidal potential between the two end points. This work increases the gravitational potential energy of the particle by mgh, where g is the local acceleration of gravity. Since the particle is always in equilibrium, these two changes must be equal:

$$mgh = m\mathrm{V}$$

so that the height h is given by

$$h = \mathrm{V}/g$$

This quantity has a magnitude of about 30 cm. (Note that V is the tidal part of the potential – not the entire potential. Apart from constants and angular factors, its mathematical form is a^2/R^3 – a form that we recognize as having the same radial dependence as the differential force.) The vertical displacement due to the earth tides will be smaller by a constant factor derived from the elastic properties of the earth and reflecting the fact that the solid earth is not bound by purely gravitational forces. This constant is usually called Love's number h, and its average magnitude is about 0.62 for the main (i.e., quadrupole) tidal term.

Tidal instrumentation

Although the tidal deformation described above could in principle be measured directly (using astrometric methods, for example), this is not commonly done since the effects are quite small. The tides are usually observed using three types of instruments: gravity meters, tilt meters, and strain meters. Any one of these can be used to study the tides, but each responds in a different way to the tidal potential.

The gravity meter

A gravity meter measures the local acceleration of gravity. We may imagine a prototype gravity meter as a mass on a spring as shown in Fig. 4. The mass must be in equilibrium under the action of two forces: the downward force due to the pull of the earth and the upward force provided by the stretched spring. The pull of the earth has the magnitude mg, where m is the mass of the particle and

g is the local acceleration of gravity. If the spring is stretched by a distance x, the force due to the spring on the mass has the value kx, where k is a constant characterizing the stiffness of the spring. If the mass is to be in equilibrium, the sum of the forces must be zero so that the two forces must be equal in magnitude. Thus

$$kx = mg$$

so that the local acceleration of gravity can be measured in terms of the extension, x, the spring constant, k, and the mass m:

$$g = kx/m$$

When an instrument of this type is placed on the surface of the earth it responds to the tidal potential in three ways:

1. There is a direct attraction between the mass and the tide-generating body (e.g., the moon). When the moon is overhead this effect raises the mass thereby decreasing x and reducing the apparent value of g.

2. The earth on which the instrument is mounted deforms as shown schematically in Fig. 2. The instrument moves further away from the center of the earth. This motion reduces the gravitational attraction of the earth and augments the direct effect described above. This motion is proportional to the Love number h discussed above.

3. The deformation of the earth produced by the tidal potential moves some of the outlying mass of the earth more nearly under the gravity meter. This tends to increase the density of the material under the instrument and therefore to raise the local gravitational potential. This effect is opposite in sign to the first two effects. This effect is usually characterized in terms of a second Love number, k. Its magnitude is about 0.31. (The modification of the tidal response due to the change in the gravitational potential of the earth is not limited to the tides in the solid earth. It occurs in the ocean tides as well.)

We would expect all of these terms to be proportional to the applied potential, and the effects are customarily lumped into a "gravimetric factor" relating the applied potential deduced from the astronomy to the measured change in gravity. The gravimetric factor must be a function of the Love numbers defined above. An exact calculation shows that the gravimetric factor for the quadrupolar tide is given by

$$1 + h - 1.5k$$

which has a magnitude of about 1.16. In the customary definition, the Love numbers are dimensionless. The potential is likewise expressed in a dimensionless way as the equilibrium tide divided by the radius of the earth or V/ga. This quantity is about 200 microgals (a microgal is a unit of acceleration equal to one cm/s^2).

If the reason for measuring the earth tides is to study the elastic response of the earth, then it is clear that a gravity meter dilutes the desired signal with a direct attraction that turns out to be the dominant effect. Thus to achieve the same accuracy in the measurement of the elastic properties of the earth, a gravity meter must have a significantly higher signal-to-noise ratio than a strain meter, for example (to be described later). A detailed calculation shows that a gravity meter must be roughly six times better than a strain meter to achieve the same accuracy in the determination of the earth's contribution to the earth tide signal. This factor of six is not a constant; it depends on the location of the instruments and on the azimuth of the strain meter as discussed below.

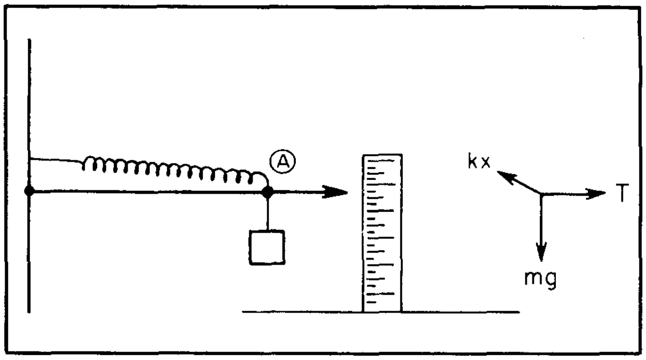

Fig. 5. A gravity meter with increased sensitivity. The point A is in equilibrium under the action of the tension in the bar, the pull of the spring and the weight of the mass as shown on the right. When the acceleration of gravity changes, point A will move up or down so as to keep the sum of the forces equal to zero.

Practical gravity meters are not very different from the prototype instrument shown in Fig. 4. Their design is governed by several constraints:

1. The instrument shown in Fig. 4 is sensitive to changes in temperature, and these changes will change the length of the spring exactly as a change in gravity would. Thus gravity meters using mechanical suspensions are usually operated in controlled-temperature environments.

2. The sensitivity of the instrument is limited by the sensitivity of the device used to measure the change in the equilibrium position. This change is in turn inversely proportional to the stiffness of the spring (the constant k), and high sensitivity implies a very weak spring. Weak springs tend to be fragile and they also tend to be very long since the equilibrium position of the mass is determined by the requirement that the restoring force of the spring (kx, where x is the stretch) must be equal to the downward force of gravity (mg, where m is the mass).

Several ingenious mechanical arrangements have been devised to support a rather large mass on a very weak spring. One such arrangement is shown in Fig. 5. Point A is in equilibrium under the action of the weight of the mass (mg, downward), the compression in the beam (T, horizontal) and the restoring force of the spring (kx, at a slight upward angle). The weight must be supported by the spring since the beam is horizontal. If the weight changes slightly (due to a change in the acceleration of gravity), the new weight must be balanced by a change in the tension in the spring. This in turn requires the spring to change its length. Because of the geometry, the length of the spring changes very slowly as point A moves upward or downward so that small changes in the acceleration of gravity result in large displacements of the point A. The result is a mechanical reduction in the effective stiffness of the spring for *changes* in gravity.

Mechanical gravity meters typically have sensitivities of a few microgals. A good instrument might show a drift of a few microgals per day. The drift is reasonably constant with time, so that it is usually possible to remove it using linear regression analysis.

Several nonmechanical gravity meters have been developed. One type of instrument measures the acceleration of gravity by determining the time it takes an object to fall through a series of known distances. The "object" is actually a part of a laser interferometer. The distances are measured in terms of the wavelength of light; the fall times are measured using precision frequency standards. Devices of this type are able to measure the acceleration of gravity with an uncertainty of a few microgals. Although this corresponds to a fractional uncertainty of only about 0.01 ppm in the value of g, it results in uncertainties of a few percent in the earth tides. Since the *total* elastic contribution (that part of the tides produced by the earth and not the direct attraction on the sensing mass) is on the order of 16%, such instruments cannot tell us much about the elastic properties of the earth.

A second type of nonmechanical gravity meter uses a superconducting ball magnetically suspended in a container of liquid helium. The currents used for the suspension flow in superconductors and are very stable in time. These currents play the same role as the spring in a conventional instrument; the position of the mass must be sensed using some ancillary system. Typical readout systems use the change in the capacitance between the levitated mass and its surroundings as an indicator of the change in the position of the mass.

Instruments of this type show great promise for improving our understanding of the tides. Published results have shown earth tide measurements with uncertainties of 0.01% in the determination of the amplitudes of the largest semidiurnal components.

The tilt meter

A tilt meter measures the angle between the normal to the earth's surface and the acceleration of gravity. We may imagine a tilt meter as a pendulum on a support as shown in Fig. 6. As with a gravity meter, the tilt meter responds to the direct attraction as the tidal body passes overhead. There is also an effect produced by the motion of the support as the earth deforms. These two effects are again lumped into a "gravimetric factor" relating applied potential to measured tilt.

The response of a tilt meter at a given point also depends on the angle between its sensitive axis and the meridian. In our simple model (stationary moon over the equator) for example, a tilt meter on the equator oriented along a north-south axis would see no direct effect, while an east-west instrument would see the maximum direct effect. The tilt along any intermediate azimuth may be found in terms of the north-south and east-west tilts using the rules of vector addition. Thus two orthogonal instruments are necessary to describe fully the tilt vector at any point. A single gravimetric factor may not be adequate to describe the tilt tide at a site located in the real, anisotropic world. Tilt tides vary with latitude and longitude as well as with azimuth; the amplitude of a typical tilt tide is 200 nrad (about 0.4″).

Modern tilt meters tend to fall into two general categories: pendulums and bubbles. A pendulum tilt meter is a precision version of the prototype shown in Fig. 6.

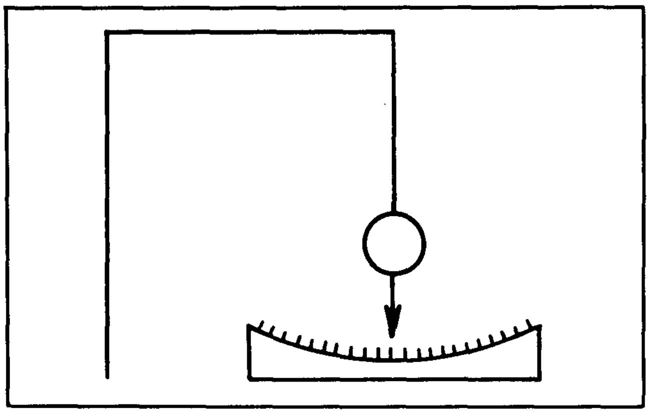

Fig. 6. A schematic diagram of a tilt meter. The tilt angle is read directly from the scale.

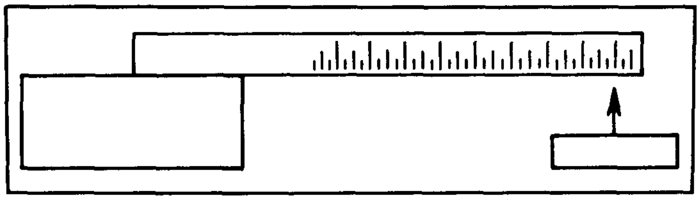

Fig. 7. A schematic diagram of a strain meter. The strain is determined by computing the fractional change in the length of the baseline between the two piers.

Great care must be used in the mechanical construction, and the instrument must usually be operated in a thermally stable environment. The position of the pendulum is usually determined by measuring the distance between the pendulum and two end plates mounted at either end of its swing. A common method is to measure the capacitances between the center plate and the end plates.

A bubble tilt meter is a precision version of the simple bubble level. Great care must be used to be sure that the liquid is pure and that the enclosing glass is smooth. The position of the bubble is often determined optically (the bubble-liquid combination forms a lens that may be used to image a light source on a detector) or electrically (the conductivity of the liquid measured along different secant-line paths changes as the bubble moves).

Several investigators are currently constructing long-baseline levels. These are made by joining two containers with a piece of tubing or pipe. The containers are placed on piers at either end of the baseline to be measured. As the earth tilts, the levels in the containers change. Since the baseline may be quite long, very small tilts may be measured. These tilt meters are under development.

The strain meter

A strain meter measures the change in the length of a baseline on the surface of the earth. We may imagine a strain meter as a ruler and two piers as shown in Fig. 7. Unlike the gravity meter and the tilt meter, the strain meter measures only the deformation of the earth. There is no

Fig. 8. The evacuated path of a 30-m laser strain meter. The instrument is located in a mine approximately 8 km west of Boulder, Colorado. The strain meter is approximately 60 m below ground level.

direct effect. The response of the instrument depends on the azimuth of its sensitive axis, but the relationship is more complicated than for tilt.

The relationship between stress and strain in a three-dimensional solid is complicated since a stress in any direction usually produces strains in all other directions. It turns out that three strain meters are required to determine the strain at any point; the strain along an arbitrary axis cannot be deduced using the laws of vector addition and more complex addition rules must be used.

Although there is a linear relationship between applied potential and measured strain, the "gravimetric factor" is far from simple. It depends on the elastic moduli, on the azimuth of the instrument and on its location. As with tilt, a single gravimetric factor may not adequately describe a site that has anisotropies or crustal inhomogeneities. Strain tides vary with longitude and latitude as well as with the azimuth of the instrument. The amplitude is usually about 2×10^{-8}. (Strain is the ratio of the change in length to the baseline length and is thus dimensionless.)

The earliest strain meters were precision versions of the prototype shown in Fig. 7. Fused silica was usually used as the "ruler" because of its small coefficient of thermal expansion. Both the long-term stability and the thermal stability were poor, however, and precision measurements were not made until the advent of lasers.

Laser strain meters are interferometers that measure the change in the baseline length in units of the laser wavelength. Some means must be found to keep the wavelength stable. Unstabilized lasers show wavelength fluctuations of up to several parts per million – several orders of magnitude too large to be useful for earth-tide measurements. Several stabilization schemes have been proposed; the best systems compare the laser wavelength against a molecular absorption wavelength in a molecule such as iodine or methane and tune the laser to maintain the coincidence between the laser wavelength and the absorption. These instruments have accuracies of about 5×10^{-11} – more than adequate for earth-tide work.

The primary problem with laser strain meters is that they are large, complicated devices. The path over which the measurement is made must be evacuated, and the ancillary vacuum apparatus is expensive and not portable.

In Fig. 8 we show the evacuated path of a 30-m laser strain meter. The instrument is located in a mine approximately 8 km west of Boulder, Colorado. The stabilized laser and the ancillary electronics are not shown.

Measurements

It is only in recent years that the quality of earth-tide measurements has improved to the point where meaningful comparisons between theory and experiment can be made. One of the principal difficulties is the smallness of tidal effects compared to effects due to thermal expansion. Typical materials have coefficients of thermal expansion of several parts per million per Celsius degree; a one degree change in temperature can thus cause distortions many times larger than those produced by the tides. Improvements in instrumentation have only partially solved this problem. The earth is subjected to thermo-elastic stresses, and it is not unreasonable to expect diurnal and semidiurnal motions of a purely thermal origin.

Each of the instruments we have discussed is also sensitive to various spurious effects. Gravity meters respond to changes in the local density. Thus changes in the level of the underground water table may produce spurious changes in *g*. A gravity meter also responds to changes in barometric pressure (the instrument responds to the changing gravitational attraction of the mass of the air above the instrument). Tilt meters and strain meters respond to local inhomogeneities. While this is often an advantage in that they can be used to learn about properties of the site, it is often difficult to invert the data and extract site properties from observed tides. Calculations of this sort are quite complicated and require rather detailed models of the site to be quantitatively useful.

Measurement opportunities

We turn to those problems that can be investigated using earth-tide measurements.

The earth-tide signals are proportional to the elastic parameters of the earth, and precision earth-tide studies are one way of determining these parameters. The globally averaged values of these parameters are modified by local anomalies, and these may be studied with measurements of sufficiently high accuracy. Local anomalies that might be studied in this way include thermal anomalies (e.g., Yellowstone National Park) and dilatancy, a change in elastic properties that may be useful as an earthquake precursor.

Yellowstone National Park is famous as a thermal anomaly, i.e., as a place where very hot material is very close to the surface. This material is so hot that its elastic properties differ quite markedly from normal materials; the earth-tide response will be quite different and it is possible (in theory) to determine the elastic properties of the molten material by measuring the earth tides there.

Dilatancy is a partial failure of a material when it is subjected to a stress that is close to the breaking stress. Although the bulk of the material has not broken, small cracks have opened inside. The material becomes "spongy," and its ability to hold water increases. Its elastic properties change as well; its earth-tide response is different from normal materials. Dilatancy is interesting because it is one of the last stages before failure, and its detection might therefore serve as a useful earthquake precursor. Other techniques have been used to search for dilatancy, with only mixed success; its utility as an earthquake precursor is not yet clear.

In addition to local anomalies, the earth tides at any point are modified by the ocean tides because of the direct attraction of the water and because of the elastic deformation of the earth due to the motion of the water. These ocean-load tides are large along the coasts. They may even contribute several percent to the strain tide at a midcontinent site (e.g., Colorado). Measurements of the earth tides can be used to test models of the ocean tides.

There are many other investigations that require a knowledge of the tidal *displacement*. These include calculations of satellite orbits, radio astronomy, and geodetic techniques such as satellite ranging. In any of these investigations, the position of the observing station must be accurately known. As we have seen, the equilibrium tide is on the order of 30 cm – a large correction for many of these techniques. The motion may be much larger at island or coastal stations because of the ocean load.