14 Other Means for Precision Frequency Control

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14.1 INTRODUCTION

In addition to the quantum electronic devices described in Chapter 10 and the bulk quartz and SAW oscillators discussed in Chapter 8, there are a number of other methods for precision frequency control. The frequency-controlling elements used in these techniques include high-Q tuning forks, high-Q LC circuits, superconducting cavities, and other high-Q resonators.

Techniques for frequency control with these devices have many similarities with those discussed in Chapters 8 and 10. General considerations outlined by Cutler and Searle (1966) can be used to estimate the limits of frequency stability due to wideband additive noise and thermal noise within the bandwidth of the resonator, that is, the frequency-determining element. Specifically,

$$\sigma_y(\tau) = \sqrt{\frac{kT}{2}\left(\frac{FB}{2\pi^2 \nu_0^2 P_0 \tau^2} + \frac{1}{Q_l^2 P_b \tau}\right)}, \tag{14-1}$$

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where k is Boltzmann's constant, T the noise temperature of the resonator, ν_0 the resonance frequency, F the noise figure of the first signal amplifier not filtered by the resonator, B the noise bandwidth of the receiver-plus-measurement system, P_0 the output power received at the first amplifier not filtered by the resonator, Q_l the loaded-resonator quality factor, P_b the total power dissipated in the resonator and its load, and τ the measurement interval. The first term on the right-hand side of Eq. (14-1) is due to additive noise of the receiver–measurement system, whereas the second term is due to noise within the resonator bandwith perturbing the phase of oscillation. These two considerations therefore favor high-Q resonators operating at the highest possible power and frequency. Practical limitations come from systematic shifts or degradation in the resonator and/or degradation of receiver noise performance with increased resonator frequency and/or power levels. The following equation describes the empirical relationship between resonator Q factor and the best frequency stability observed with a given system, which seems to apply to many different techniques for frequency control [see, e.g., Table 3 of Hellwig (1979)]:

$$\Delta\nu/\nu_0 \cong \beta/10^7 Q, \qquad 2 < \beta < 100. \tag{14-2}$$

The empirical constant β is as low as 10 in many systems.

In addition to these random processes, many environmental parameters, such as temperature, excitation power, vibration, shock, radiation, outgassing, chemical processes, etc., affect the frequency and frequency stability of real devices. The performance of these devices indicated in the literature is often directly due to the technology available at the time and the constraints of the specific applications and as such may not apply to a new application due either to changes in technology or the presence of different constraints, such as size, power, weight, complexity, etc.

14.2 LOW-FREQUENCY DEVICES

In this section devices operating at frequencies under about 1 GHz are described.

14.2.1 Quartz Tuning Forks

The first low-frequency quartz resonators used for watch applications were flexure bars at 16 kHz and then at 32 kHz (Forrer, 1969; Musa and Daniels, 1971; Engdahl and Matthey, 1975). However, they were replaced by tuning forks, which typically have better long-term stability, are easier to mount, are more rugged, have less susceptibility to shock, have better frequency stability, and occupy less volume than comparable flexure bars. As a consequence they have been widely used for highly stable, low-frequency oscillators.

The first quartz tuning forks were made in 1928 by Koga (1928). They were of proportions similar to the classical metal tuning forks and were operated at 1 kHz. More recently, the interest has been in obtaining much higher frequencies and much smaller size, but still with "bulky" tuning forks (Yoda *et al.*, 1972; Kanbayashi *et al.*, 1976; Bindal and Singhal, 1976; Schwartz, 1972).

The crucial breakthrough in terms of size and cost originated with Staudte (1973), who used photolithographic techniques to produce "thin" tuning forks with typical dimensions 6 mm long, 1 mm wide, and 0.025 mm thick. These devices can be made with fundamental frequencies from 10 to 300 kHz. Other frequencies can be obtained by using the first overtone (which is about 5.8 times higher than the fundamental) or by frequency multiplication or division. Much of the recent work has been done at $\nu = 2^{15} = 32{,}768$ Hz because of the straightforward division to one cycle per second and the convenient device parameters for the clock industry.

The flexural vibration is obtained by means of two opposite length-extensional vibrations. This requires double electrodes to apply the out-of-phase driving signals on the four faces (bulky tuning fork) or on the two faces (thin tuning fork) of each arm of the tuning fork (Fig. 14-1). As a consequence plating becomes more difficult to perform. The crystal plate can be rotated

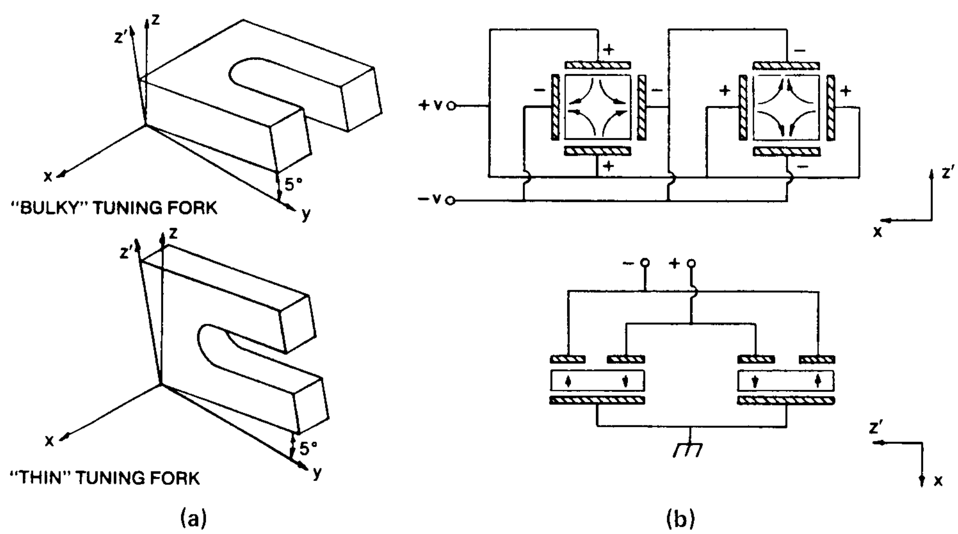

FIG. 14-1 (a) Orientation and (b) electrode configuration of "thin" and "bulky" tuning forks.

TABLE 14-1

Typical Characteristics of 32-kHz Quartz Tuning Forks

Parameter	Approximate value
Approximate size	6 mm × 1 mm × 0.2 mm
Q factor	30,000–120,000
Motional resistance	~10–400 kΩ
Temperature sensitivity	$\Delta\nu/\nu \cong 4 \times 10^{-8}(T - T_0)^2$
Frequency shift vs. power	$\Delta\nu/\nu \cong 10^{-5}/\mu W$
Tuning range	$\Delta\nu/\nu \cong 10^{-4}$
Acceleration sensitivity	$\Delta\nu/\nu \cong 10^{-7}/g$

around the x axis to obtain a 5° X-cut for instance, which would have a better frequency versus temperature characteristic.

In fact, today the choice between bulky and thin tuning forks is not yet clear and the choice is a function of compatibility with the pressure of industrial large-scale production, cost, and reproducibility of the parameters. Table 14-1 shows some approximate parameters for 32 kHz resonators (Engdahl and Matthey, 1975; Kanbayashi *et al.*, 1976; Kusters *et al.*, 1976; Yoda *et al.*, 1972).

Many studies have been made of various parameters versus cut angles and tuning-fork parameters. Temperature sensitivity is a critical point for most applications because these devices exhibit a parabolic frequency versus temperature characteristic rather than a cubic one as for AT-cut thickness-shear crystals. Attempts have been made to reduce this sensitivity, with only moderate success. Kusters *et al.* (1976) investigated first-order temperature-coefficient turnover point and Q factor versus length-to-width ratio at doubly rotated cut angles for 32-MHz tuning forks, and Kanbayashi *et al.* (1976) investigated the effects of asymmetry on frequency.

Limitations on frequency stability due to added noise for the high-series-resistance units characterized by Kusters *et al.* (1976) can be estimated using Eq. (14-1). For the purpose of calculation, let us assume an amplifier noise figure of $F = 2$, a dissipation power of 10^{-8} W, and a measurement bandwidth of 1 kHz. Equation (14-1) then shows that the frequency stability is dominated by the additive noise for times shorter than 10 sec:

$$\sigma_y(\tau) \cong 1.5 \times 10^{-10}\tau^{-1}.$$

The above performance has not, to our knowledge, ever been verified, since virtually all of the tuning forks have been used without protection from environmental effects such as temperature, supply variations, shock, etc. In addition, the CMOS oscillator circuit that is normally used limits the observed frequency stability to about 1×10^{-7} at 1 sec. In a more stable circuit 1-sec frequency stability would be expected to improve by one to three orders of magnitude, limited by the thermal and vibrational environment. Frequency drift or aging of 1×10^{-7} per year or 3×10^{-10} per day has been reported for some types of tuning forks (see, e.g., Fig. 6-2) (Yoda *et al.*, 1972; Kanbayashi *et al.*, 1976).

The relatively low-drift, good short-term stability, and extremely low cost of modern tuning-fork fabrication and divider chips would seem to make this device an excellent candidate for low-cost timing applications and precision low-frequency oscillators.

14.2.2 Other Low-Frequency Devices

Several other techniques are capable of producing precision frequency sources with fractional-frequency stabilities of order 10^{-9}. These techniques can be typified by LC resonators with Q values of order 10^3 and relatively high oscillation powers. An example of this approach is seen in the tunnel-diode oscillators developed by Van Degrift (1975), shown schematically in Fig. 14-2. For an oscillator frequency of 3.7 MHz, resonator Q of $\cong 10^3$, and oscillator power of 10^{-8} W, he obtained the results shown in Fig. 14-3. In principle, even better results could have been obtained for times shorter than $\cong 10$ sec by going to a higher frequency, where the additive noise is less important. Another example is the stripline oscillator developed by D. A. Howe and described by Wineland *et al.* (1977). This oscillator, whose resonator cross section is shown in Fig. 14-4, produces 5 mW at about 500 MHz; however, similar designs can cover the range from $\cong 100$ MHz to several gigahertz. Figure 14-5 shows the frequency stability of a 500-MHz oscillator divided down to 5 MHz. The divider noise masks the stability for very short times. The performance at times longer than a few milliseconds is most likely dominated by fluctuations in the teflon-based laminate used for the stripline. The use of a different substrate material with better dimensional stability and comparable or lower losses, such as alumina, might significantly improve the frequency stability beyond about 10 msec. The improved

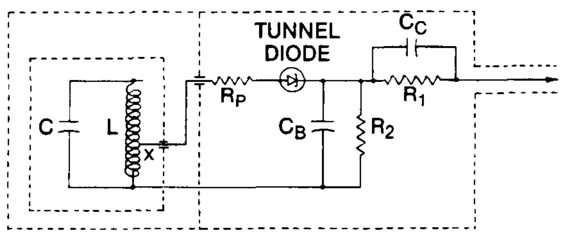

FIG. 14-2 Block diagram of tunnel-diode oscillator. [From Van Degrift (1975).]

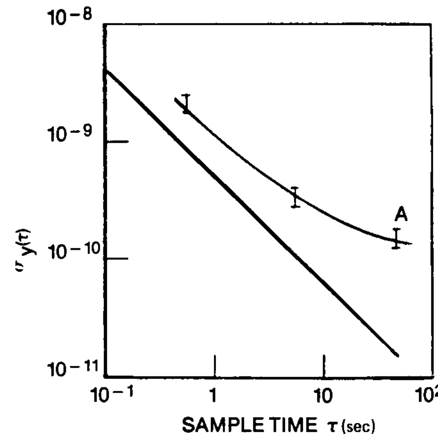

FIG. 14-3 Frequency stability $\sigma_y(\tau)$ versus averaging time for a cryogenic 3.7-MHz tunnel-diode oscillator with $P_0 = 10^{-8}$ W; the measurement bandwidth is 2.5 Hz. The stability at point A is probably limited by the measurement apparatus. [From Van Degrift (1975).]

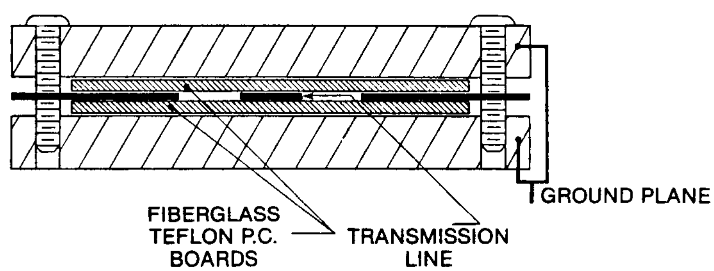

FIG. 14-4 Cross section of a stripline resonator for use at 500 MHz. The unloaded Q factor is about 500. [From Wineland *et al.* (1977).]

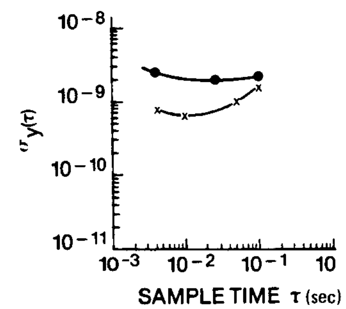

FIG. 14-5 Frequency stability of a 500-MHz stripline oscillator divided down to 5 MHz for a measurement bandwidth of 40 kHz (upper curve) and 400 Hz (lower curve). [From Wineland *et al.* (1977).]

stability from 1 to 10 msec, as compared to the tunnel-diode circuit, is due to the higher operating frequency and oscillation power. The limitations on fractional-frequency stability at 1 to 100 msec due to added thermal noise is totally negligible ($< 10^{-14}$) in this case.

In principle, this oscillator design can be made with a much lower sensitivity to vibration than a quartz-crystal-controlled oscillator. The unit shown in Fig. 14-4 achieved a sensitivity of approximately 5×10^{-10}/g for vibration at 40 Hz. The use of a more rigid substrate material and chip-bonding techniques should improve performance considerably. Such an oscillator might prove to be significantly superior to quartz crystal oscillators for use in systems with very high vibration and shock. A hybrid solution might be to use an *LC* oscillator locked to a quartz-crystal-controlled oscillator in order to reduce some of the vibration-induced sidebands and still retain the better long-term stability of the quartz-crystal-controlled oscillator.

14.3 MICROWAVE DEVICES

In this section devices operating at frequencies of about 1 GHz are described. Microwave cavities can be used in all the traditional ways low-frequency resonators have been used to stabilize the frequency of an oscillator. [See, for example, Stein (1975) and Jimenez and Septier (1973).] Superconductive cavities have the distinction of having the best combination of practically achievable power, Q factor, and operating frequency of all known microwave devices.

Recently, wide interest has developed in the use of dielectrically loaded microwave cavities to stabilize microwave oscillators and for various filtering applications. The term "dielectrically loaded" designates that a solid dielectric material is used instead of vacuum or air. Although the Q factor is not as high as in superconducting cavities, these devices handle higher power, operate over a very large temperature range, and are much cheaper.

14.3.1 Superconducting Cavities

In order to achieve the high Q values characteristic of superconductors, it is generally necessary to use cavity modes and assembly techniques that minimize surface imperfections and eliminate the need for rf current through the assembly joints. In the case of niobium, electron-beam welding techniques have been developed to a sufficient degree that the assembly joint can carry rf current without seriously degrading the Q factor. Figure 14-6 shows the measured Q values for X-band cavities fabricated from lead and niobium, the two most promising superconductors for microwave cavities. These two

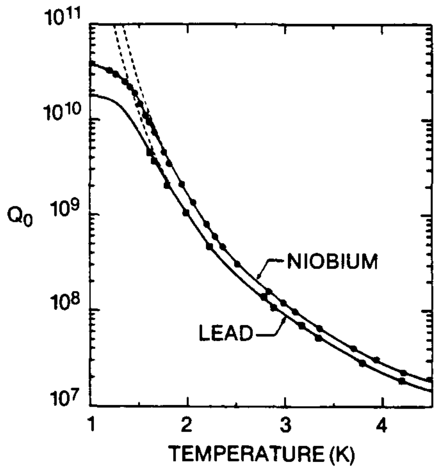

FIG. 14-6 Unloaded Q value of typical lead and niobium X-band cavities with a measurement bandwidth of 10^4 Hz. The dashed curves represent the theoretical behavior. [From Pierce (1974).]

metals are available commercially in sufficient purity and the fabrication techniques are well known (Turneaure and Weissman, 1968; Pierce, 1974). The superconducting transition temperature for lead is 7.19 K, whereas that for niobium is 9.25 K. Other superconductors with higher transition temperatures are known; however, they are generally brittle and difficult to fabricate. Lead cavities, on the other hand, are particularly easy to fabricate by electroplating. At Stanford niobium cavities have been fabricated using electron-beam welding followed by firing at 1900°C and chemical polishing (Turneaure and Weissman, 1968). Another technique developed at Siemens (Diepers *et al.*, 1971) uses chemical polishing followed by anodizing. The Q value produced by this latter technique may degrade in time, however (Pierce, 1974). The rf magnetic fields achieved without serious Q-factor degradation are approximately 60% higher in niobium than in lead cavities. From Eq. (14-1) it is seen that this yields somewhat better noise performance and is the primary reason for the predominant concentration on niobium cavities in the early work. Experimental results, however, show that in every practical application the environmental sensitivities dominate the frequency stability. The white-frequency level from Eq. (14-1) for a loaded Q value of 5×10^9, $P_b = 10^{-6}$ W, $T = 1$ K, and $\nu = 10^{10}$ Hz corresponds to $\sigma_y(\tau) \cong 10^{-20}\tau^{1/2}$, while the additive noise only contributes $\sigma_y(\tau) = 4 \times 10^{-20}\tau^{-1}$ for a 300-K sustaining circuit. Figure 14-7 shows the frequency stability of 8.4-GHz niobium cavities, whose characteristics are given in Table 14-2 (Stein, 1975). The environment was carefully controlled and yet the vibration–acceleration

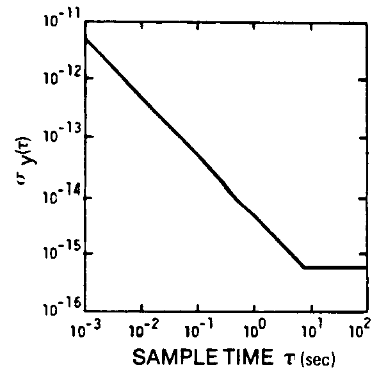

FIG. 14-7 Fractional-frequency fluctuations of an X-band superconducting-cavity-stabilized voltage-controlled oscillator. [From Stein (1975).]

sensitivity of approximately 10^{-9}/g limits the short-term stability. The long-term drift was of order 2×10^{-13}/day.

One approach having greater potential for better short-term stability is the all-cryogenic parametric oscillator stabilized by a superconducting-cavity transmission filter proposed by Stein (1975). This all-cryogenic-oscillator approach eliminates many of the long leads associated with earlier work. Frequency stabilities of order 1×10^{-16} at 1 sec appear feasible with this scheme, which also uses the superconducting cavity as a transmission filter. Superconducting cavities can also be used as transmission filters for conventional oscillators. The filtering process greatly improves the spectral purity (i.e., the phase noise for Fourier frequencies outside the bandpass of the cavity) and improves the short-term frequency stability for times long compared to the inverse linewidth of the cavity.

As indicated above, the primary disadvantages of superconducting cavities are their environmental sensitivities. In order to achieve a stability of 10^{-16} in the present TM_{010} X-band niobium cavities, the power must be fractionally

TABLE 14-2

Typical Characteristics of X-Band Niobium Superconducting Cavities

Parameter	Approximate value
Q factor	10^{10}–10^{11}
Operating power	10–100 μW
Temperature coefficient	$\Delta\nu/\nu \cong 4 \times 10^{-10}$/K at 1.4 K
Acceleration sensitivity	$\Delta\nu/\nu \cong 10^{-9}$/g
Energy shift	$\Delta\nu/\nu = 10^{-11}/10^{-7}$ J stored energy

stable to 10^{-5}, the acceleration constant to 10^{-7} g, and the temperature constant to 10^{-7} K at 1.3 K. Of these effects the vibration–acceleration sensitivity is presently the most serious limit on the short-term stability.

A new approach that might yield a major reduction in the vibration sensitivity is to plate lead on the inside of a ceramic cavity. The mechanical strength of several ceramics exceeds that of niobium by an order of magnitude. The small loss in Q factor should have an insignificant effect on the noise performance. Also, new cavity designs that are inherently less sensitive to mechanical deformation need to be studied. One such approach is being pursued by Dick and Strayer (1983). In their design the cavity is a sapphire sphere coated on the outside. The gain element is a ruby maser at about 3 GHz. This approach sacrifices cavity Q due to the losses within the sapphire dielectric; however, it has the nice features of parametric pumping and compactness suggested by Stein (1975).

14.3.2 Dielectrically Loaded Cavities

The use of dielectrically loaded cavities dates from the work of Richtmeyer (1939), but the early dielectrics suffered from large temperature coefficients. Recent materials have dielectric constants from 30 to about 100, temperature coefficients of order 10 to -10×10^{-6}/°C, and loss tangents less than 10^{-3}.

TABLE 14-3

Characteristics of Temperature-Compensated Dielectric-Resonator Materials at 7 GHz

	Material			
Parameter	Ba_2TiO_6–$Ba_2Ti_6O_{20}$	R–04C; series TiO_2–ZrO_2–SnO_2–$(ZrSn)TiO_2$; monoclinic	R-09C; series $(BaPb)TiO_3$	$Ba(NiTa)O_3$–$Ba(ZrZnTa)O_3$
Loss tangent	0.5×10^{-3}	1.6×10^{-4} from −55 to 100°C	10^{-3}	6.8×10^{-5}
Q_{unloaded}	2000 (minimum)	6300 (minimum)	1000	14500
Relative dielectric constant	$37 \pm 5\%$	36.8–38.9, selectable	90	30
Temperature coefficient of frequency from −50 to 100°C	−25 ppm/°C	0 possible selectable from −4 to 12 ppm/°C	0 possible selectable from −10 to 10 ppm/°C	0 possible selectable from 18 to −10 ppm/°C

The availability of temperature coefficients from 10 to -10×10^{-6}/°C allows one to compensate the entire resonator–gain system to $<10^{-6}$/°C. The high dielectric constants greatly reduce the size of the resonator as compared to traditional vacuum dielectric cavities.

Table 14-3, from the review article of Stiglitz (1981) and a paper by Wakimo *et al.* (1983), compares several of the most interesting dielectric families for application in frequency metrology. Clearly there is great opportunity to make extremely stable miniature microwave resonators and oscillators with high signal-to-noise ratios and hence frequency stability. As an example, Wakimo *et al.* (1983) describes a 5-g, 11-GHz oscillator having an uncompensated frequency stability of ± 1 MHz from -40 to 60°C. The size is 2.03 cm $\times$ 1.26 cm $\times$ 0.88 cm.