WEIGHTING AND SMOOTHING OF DATA IN GPS COMMON VIEW TIME TRANSFER

Marc A. Weiss, Time and Frequency Division,
National Bureau of Standards, Boulder, Colorado

ABSTRACT

It is now possible to compare a clock with UTC(NBS) anywhere in common view of a GPS satellite with Boulder, Colorado at the full level of accuracy and stability of the NBS atomic time scale for integration times of about four days and longer via the NBS Global Time Service. This availability includes Japan, Europe, and the entire United States. The service includes a dial-up service for current estimates of the user's clock performance and a monthly report with improved estimates after the fact. We discuss here the method by which the common view time transfer values in the monthly reports are computed. Measurements are taken using a satellite in common view of NBS and a second location. These measurements are repeated each sidereal day so that the geometry at measurement time remains fixed. The data are carefully examined for bad points, and these are removed by interpolation or extrapolation. A measurement geometry which repeats each sidereal day defines a time series. The measurement noise of each time series is determined using a decomposition of variance or N-corner hat technique which in turn defines weights used to compute a weighted average. Finally, a Kalman smoothed estimate of time and fractional frequency offset is computed for each time series separately using the measurement noise estimates, and these are also combined with the weights to define optimal estimates of time and frequency offsets. Using this technique we have been able to transfer time with time stability of less than 10 ns, time accuracies of the order of 10 ns, and a frequency stability of 1 part in 10^{14} and better for measurement times of four days and longer. In addition to using this method for the above-mentioned Global Time Service, it is used in computing the data sent to the BIH for the generation of UTC and TAI. These data include comparisons of the time and frequency of UTC(NBS) with other principal timing centers: NRC in Ottawa, PTB in Braunschweig, RRL in Tokyo, and USNO in Washington DC.

Data Selection and Rejection

Locations interested in comparing a clock with UTC(NBS) via GPS should measure their clock against GPS time via satellites according to an NBS tracking schedule. A satellite is tracked for intervals up to 13 minutes, and the data taken during that time is reduced to a value of GPS minus reference clock and a rate offset. It has been shown elsewhere[1] that tracking longer than about 10 minutes is of little value since the fluctuations appear to be flicker noise limited for Fourier frequencies smaller than about one cycle per 10 minutes. One gains significantly by averaging the white phase noise at the higher Fourier frequencies. Tracks are extended to 13 minutes to ensure use of the most recent ionospheric correction, since that parameter is transmitted every 12.5 minutes. The tracking schedule tells which satellites to track at what time on a certain modified Julian day (MJD) for all locations in a given large area of earth. Each track in the tracking schedule is assigned to at least two areas and is chosen to maximize the elevation of a GPS satellite as seen from those areas. The elevation of a satellite changes little over a large area of Earth since the satellite orbits are 4.2 Earth radii. The track times decrement by 4 minutes a day to preserve the geometric relationship between the satellites and the ground location at each measurement. This follows since the satellites are in 12 hour sidereal orbits. A sidereal day is not exactly 4 minutes less than a solar day, but this is close enough since the satellites deviate somewhat from exact sidereal orbits. The tracking schedule needs to be recomputed from time to time (about once or twice a year).

GPS minus reference measurements are gathered together at NBS in Boulder from many locations in the United States and around the world. UTC(NBS) can be transferred to any location having common view data with NBS. A track is in common view between two locations if both locations have received the same signals from a satellite, i.e. both locations have tracked the same satellite at the same time. In this way many sources of noise cancel or nearly cancel upon subtraction of the GPS minus reference measurements.[2] If there is some discrepancy between the times of tracking at the two locations the noise of the difference measurement may grow rapidly. A common view track repeats every sidereal day and in this way defines a time series comparing the clocks at the two locations. Each time series represents a different path from one location to the satellite to the other location repeated every sidereal day and used to compare the two reference clocks. Each satellite in common view can be used for such a time series. Indeed a satellite can give rise to two time series if there are two different optimal common view paths each day: one when the satellite is above the two locations, one when they look over the pole at it. Time transfer between two locations is accomplished by determining measurement noise and weights for each path, using this to smooth each time series separately and combine them into a weighted average.

We often find that the entire time series of common view measurements via one satellite is biased from the data via another satellite. This is not entirely understood, but it must be due either to consistent error in the transmitted ephemeris or ionospheric model, consistent error in the tropospheric model, coordinate errors at the local receiver or a frequency offset between the reference clocks. Because of these biases, we work with the time series separately before they are combined into a weighted average. Also, the presence of the biases makes choosing the weights very important since the resultant average can change significantly with different weights.

First, each time series is studied for bad points. This can be a difficult task because deviations in a time series can come from several places. A reference clock may have a time or frequency step, in which case points in the time series may seem bad but are actual measurements of the clock. If this happens when there are missing data from several satellites it can be difficult to interpret. Bad measurements are caused by either troubles in the data transmitted from the satellite or problems in the receiver. When these are found in a given time series they are replaced by a value either linearly interpolated from neighboring good points, or, if it is an end point, by a value extrapolated from the entire time series by a quadratic curve fit. In this way a bad measurement is replaced with a value which maintains the bias of the time series when it is included in the weighted average.

Measurement Noise and Weights

The measurement noise and the weight of each time series is estimated using a decomposition of variance or N-corner hat technique with the modified Allan variance. The N-corner hat technique is a generalization of the three-corner hat, where the variance of the stability of a particular clock is estimated using variances of the stability of difference measurements among three clocks. The generalization is that we have N clocks instead of three. We apply this technique to a differences of the time differences of our GPS data, i.e., differencing the common view measurements across satellites. The equations are as follows.

A time difference is a difference of measurements at two locations via satellite i:

$$(\mathrm{Ref}_2 - \mathrm{Ref}_1 + \text{C-V noise})_i = (\mathrm{GPS} - \mathrm{Ref}_1 + \mathrm{noise})_i - (\mathrm{GPS} - \mathrm{Ref}_2 + \mathrm{noise})_i,$$

where "C-V noise" denotes the common view measurement noise for that path.

The difference of the differences is:

$$\mathrm{Noise}_i - \mathrm{Noise}_j = (\mathrm{Ref}_2 - \mathrm{Ref}_1 + \mathrm{Noise})_i - (\mathrm{Ref}_2 - \mathrm{Ref}_1 + \mathrm{Noise})_j.$$

Thus we see that the set of variances of the difference of differences is the set of variances of noise differences. We may apply N-corner hat to this to find the variance of a particular process just as we apply N-corner hat to the set of variances of clock stability differences to find the stability of a particular clock. Let us consider the equations for the decomposition of variances.[3] We want to find

$$\sigma_i^2 = \text{estimate of the variance of process } i;\ i = 1,2,\ \dots\ ,\ N$$

given

s_{ij}^2 = measurement of the variance of i-j difference.

We choose the σ_i^2 to minimize

$$A = \sum_{j=2}^{N} \sum_{i=1}^{j-1} (s_{ij}^2 - \sigma_i^2 - \sigma_j^2)^2.$$

The result after solving $\partial A/\partial \sigma_i^2 = 0$ is

$$\sigma_i^2 = \frac{1}{N-2} \left(\sum_{k=1}^{N} s_{ij}^2 - B \right),$$

where

$$B = \frac{1}{2(N-2)} \left(\sum_{k=1}^{N} \sum_{j=1}^{N} s_{kj}^2 \right), \text{ with } s_{jj}^2 = 0.$$

If we use the modified Allan variance in these equations we see that the common view noise has a spectrum consistent with the hypothesis of white noise phase modulation.[4] This means that the square root of the variance as a function of time interval, τ, should be proportional to $\tau^{-3/2}$. Because of this we may multiply $\sigma_i^2(N \cdot \tau_o)$ by N^3 and take the mean over the number, M, of variance computations. Thus the common view noise squared, n_i^2, for path i is proportional to

$$n_i^2 \approx \frac{1}{M} \sum_{k=0}^{M-1} [\ \sigma_i^2(2^k \tau_o) * (2^k)^3].$$

The constant of proportionality is

$\tau_o^2/3p_i$, where p_i is the percentage of good points.

The factor of $\tau_o^2/3$ comes from the relationship between time and frequency stability with the modified Allan variance under the assumption of white phase noise.[4,5] The percentage of good points comes in because the confidence of the estimate gets worse with fewer points.[6] The weight of path i, w_i, is the reciprocal of the normalized noise estimate

$$w_i = (1/n_i^2)/(\sum_j 1/n_j^2).$$

These are the weights which are used to combine the time series for the different satellite paths into a single weighted average. The result is that the more stable the series the heavier it is weighted. This makes sense for unbiased data, and we assume here that the bias of a path is proportional to its instability. If a bias is due to local coordinate errors this assumption will fail. If the bias is due to error in transmitted data it is possible the bias would be unstable from attempts to correct the error. More study needs to be done to understand these biases.

Kalman Estimates

In addition to the unsmoothed weighted average, a forward-backward Kalman smoother is used to remove the noise from the time transfer. Each path is smoothed separately using the estimates of the noise for each path, as well as estimates for the noise characteristics of the two clocks, as input to the Kalman smoother. The state vector consists of two elements: the time, x, and frequency, y, of a clock offset from UTC(NBS). Interpolated or extrapolated values are not used by the Kalman, rather it replaces these with its own optimal estimates. The x and y values from the different paths are combined using the weights to generate a smoothed estimate of the time and frequency offset of a reference clock from UTC(NBS).

Results

Results of approximately 10 ns accuracy and 1 part in 10^{14} stability for integration times of four days have been reported elsewhere[8]. Here we simply note results on more recent data. We consider time comparisons between NBS in Boulder, Colorado, USA, and PTB in Braunschweig, West Germany during a fifty day period from MJD 46300 to 46350, August 23 to October 12, 1985. We use measurements via paths for five satellite vehicles (SV's), SV 8, SV 9, SV 11, SV 12, and SV 13. The raw data can be seen in Figures 1a and 1b via each of these paths. The biases between the paths can be seen here. To reveal the biases more clearly we plot some of the second differences of satellites measured against SV 13 in Figures 2a and 2b. The second differences are the input to the 5-corner hat. The mod $\sigma_y(\tau)$ values for SV's 11, 12, 13 are plotted in figure 3a, along with the -3/2 slope line for white phase modulation. We see an excellent agreement. The values for SV's 8 and 9 are very similar and were not plotted simply because they would confuse the graph. If we compare this with the $\sigma_y(\tau)$ plots for the time transfer via each of SV's 11, 12, and 13 in figure 3b we see that the measurement noise may yet be present at one or two days of integration, but quickly drops below the noise of the clocks for periods of four days or greater. Table 1 below shows the average bias against SV 13 and the computed measurement noise for each of the satellite paths.

Table 1

SV #	Average Bias vs. SV# 13 (ns)	Meas Noise (ns)
8	-23.1	9.1
9	-11.9	6.1
11	-12.0	5.7
12	-10.1	7.2
13	0.	7.0

The biases of SV's 9, 11 and 12 are grouped together about 11 ns below SV 13 and the SV 8 has a bias approximately 12 ns below that. The measurement noise estimate is our only key for weighting the different biases. While it tends to give the bias of SV 8 a somewhat lower weight than the majority opinion of SV's 9, 11, and 12, the bias of SV 13 is weighted similarly to each of them. We simply do not know the right answer here. The standard deviation of the different SV paths averaged over the fifty days is 10.8 ns. This is due primarily to the biases, since the composite measurement noise is only 3.0 ns. This latter is an indication of the measurement noise remaining in the weighted average. The weighted average for the time transfer across the satellites is in figure 1c. The $\sigma_y(\tau)$ for these data is plotted in figure 3c. Knowing something about the clocks involved we see there can be little remaining measurement noise. We attempt to remove this with a Kalman Smoother. Figure 1d shows the time residuals for PTB - UTC(NBS) after the smoother, and figure 3d shows the associated Allan variance. Here, the $\sigma_y(\tau)$ values seem a little too low for integration periods of 1 and 2 days. Finally, we give the frequency estimates from the weighted average and the Kalman in figures 4a and 4b.

Conclusions

Time transfer via GPS satellites is possible at the level of accuracy of state of the art time standards for periods of 4 days or more, depending on the baseline if done with care. Care is needed in making strictly simultaneous measurements at two locations repeated every sidereal day to maintain a common-view measurement with a constant geometry. Care is needed in removing bad points from each of the time series. Weights for each path are very important since, due to biases in the system, a change in weights significantly changes the weighted mean. Finally, a Kalman smoother may be employed to remove measurement noise from the weighted average, but its results must be interpreted carefully. Understanding the biases in the system remains an important unsolved problem.

References

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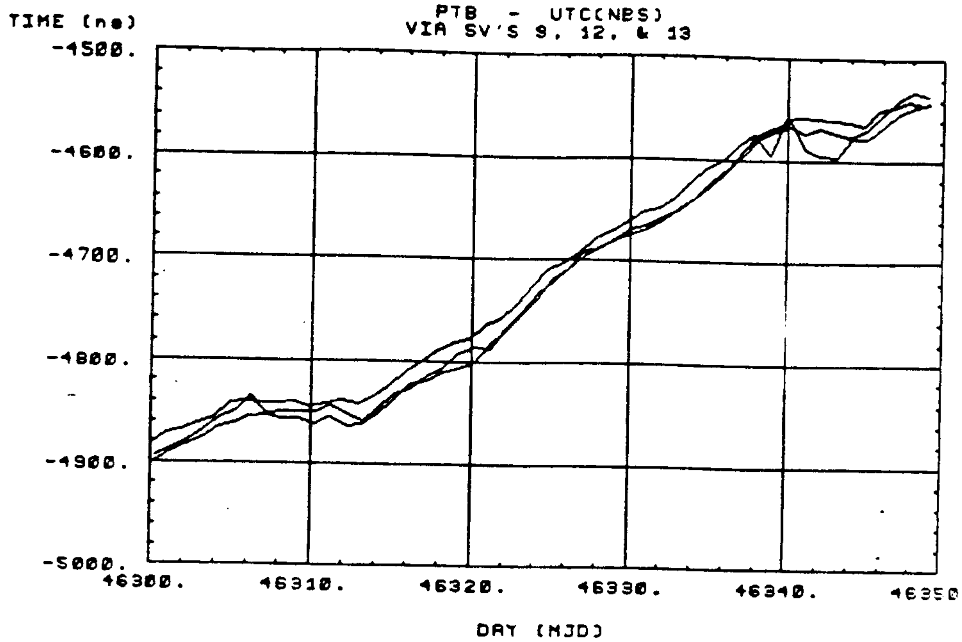


Figure 1a

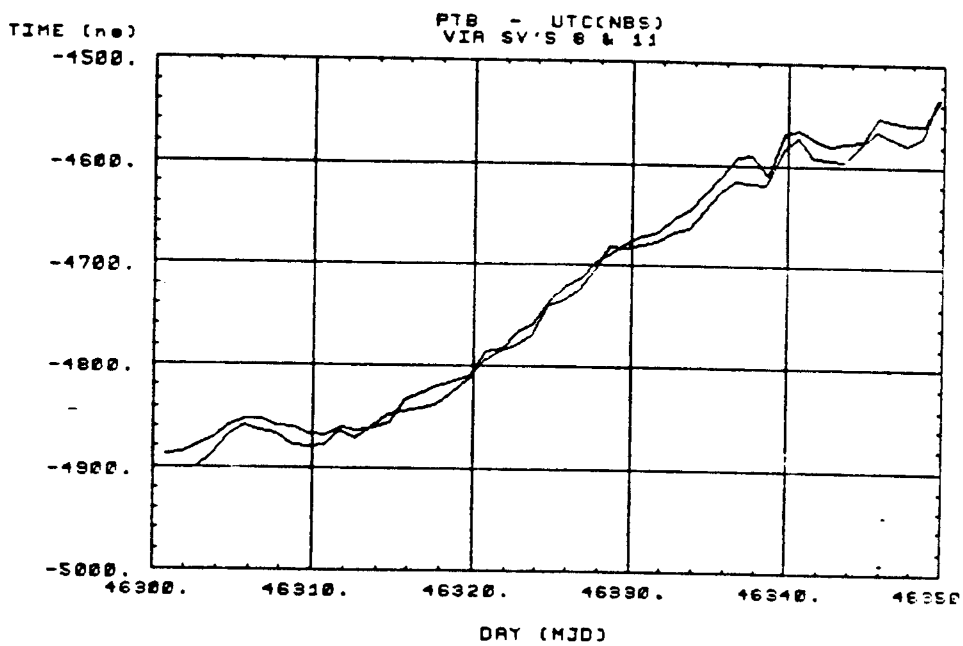


Figure 1b

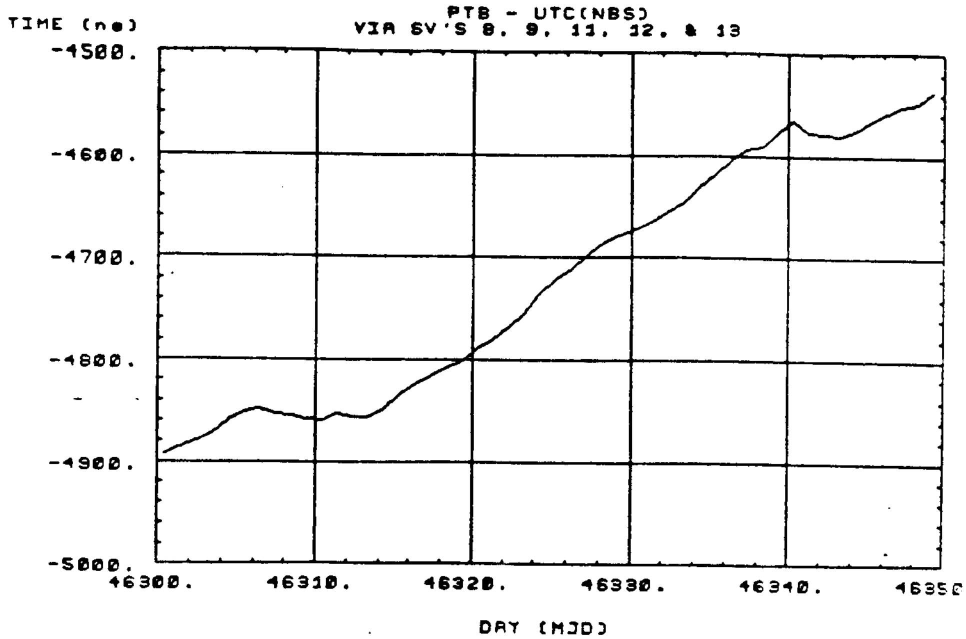


Figure 1c

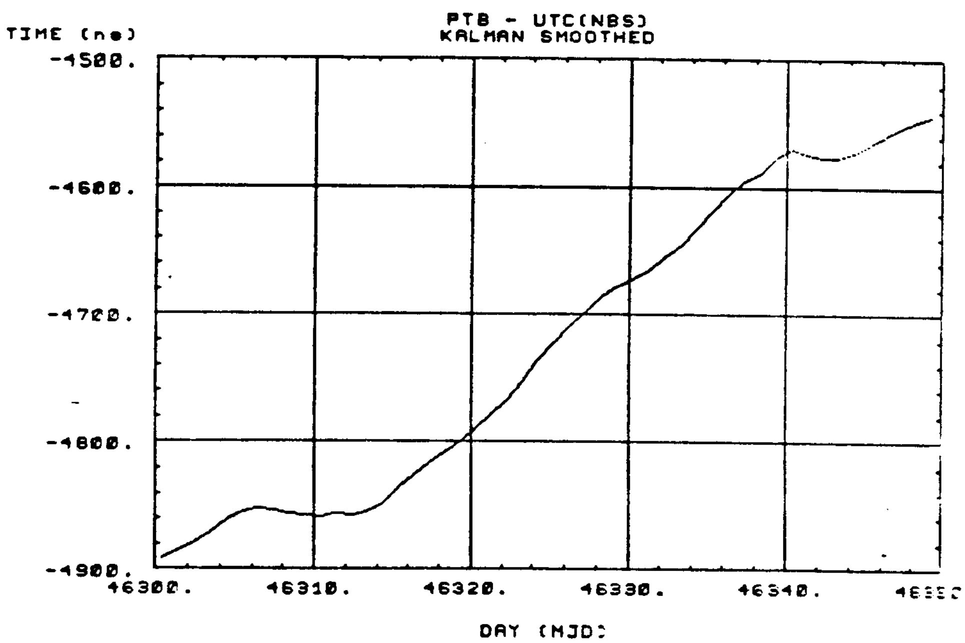


Figure 1d

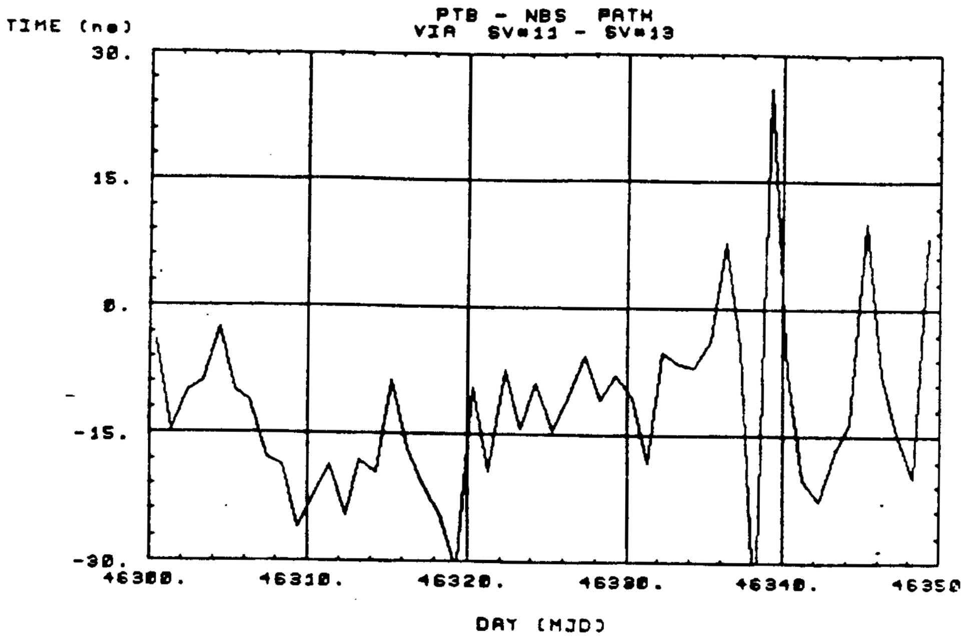


Figure 2a

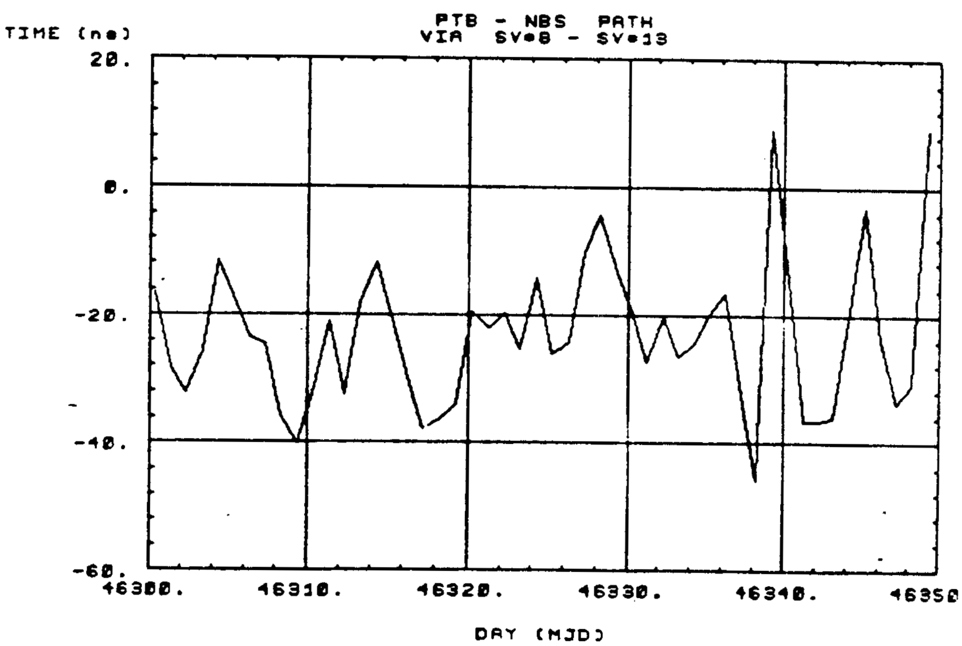


Figure 2b

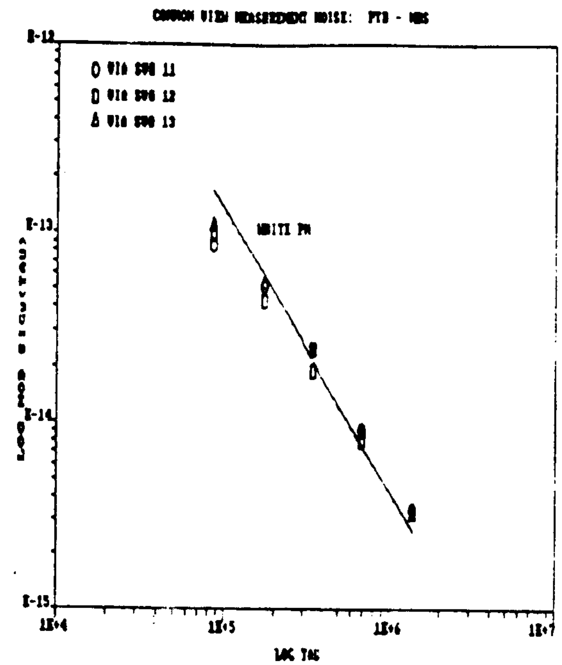


Figure 3a

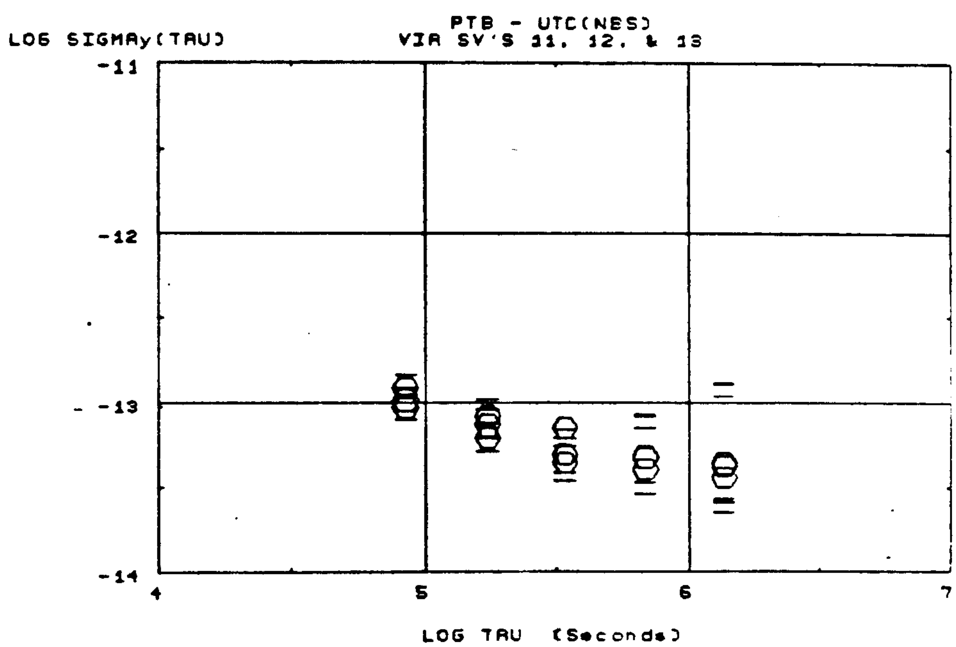
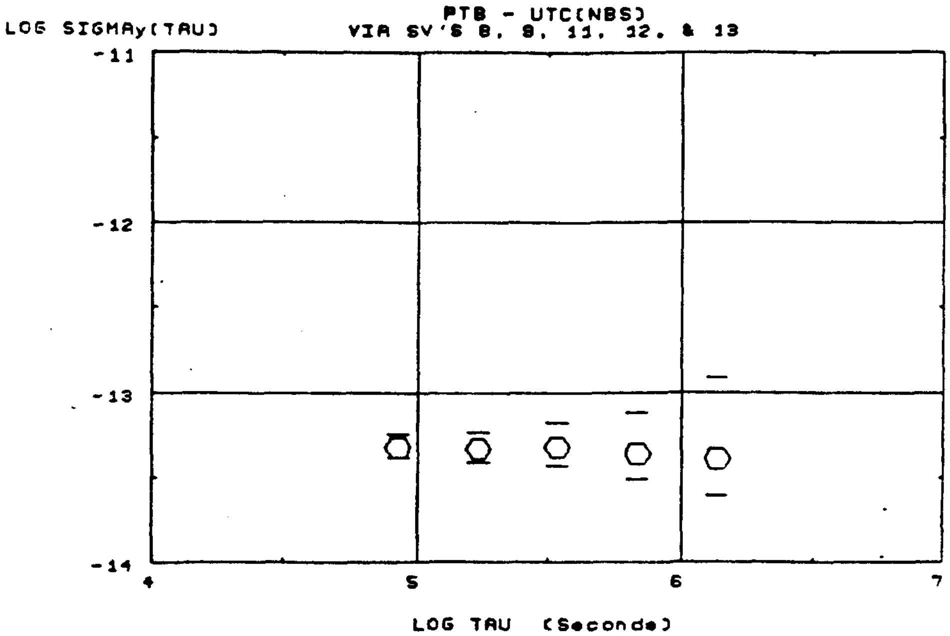


Figure 3b



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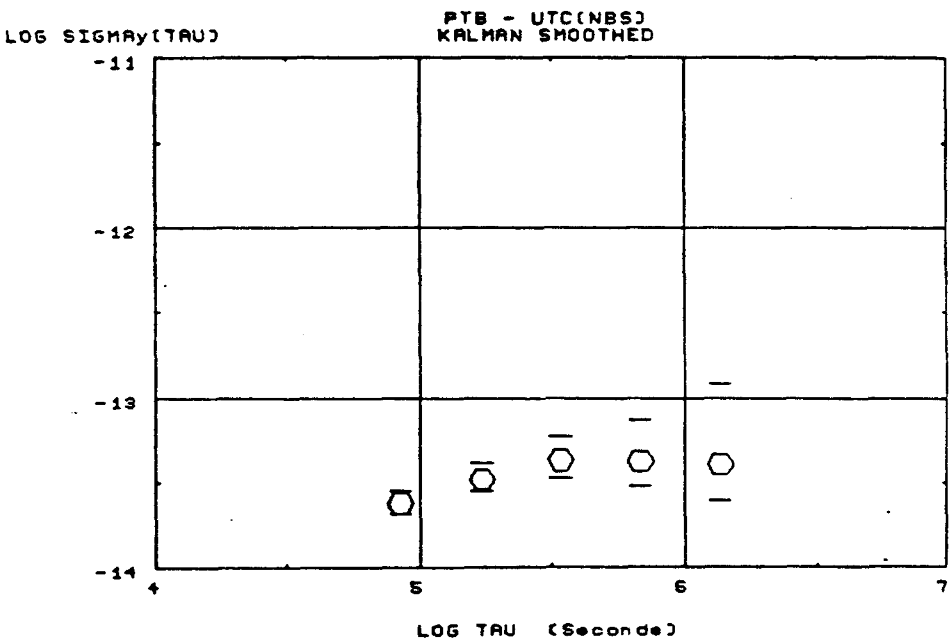

Figure 3d

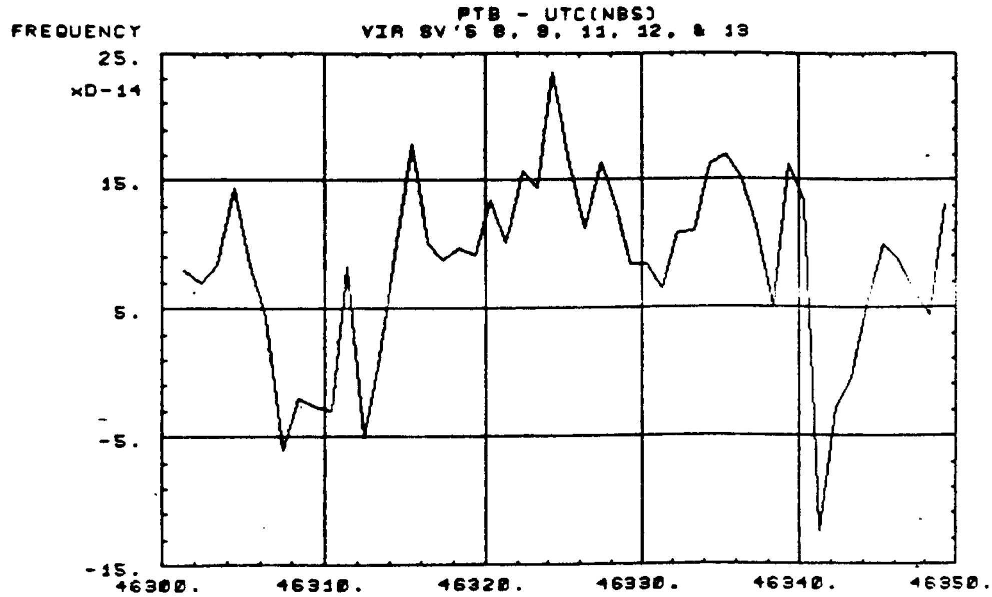

Figure 4a

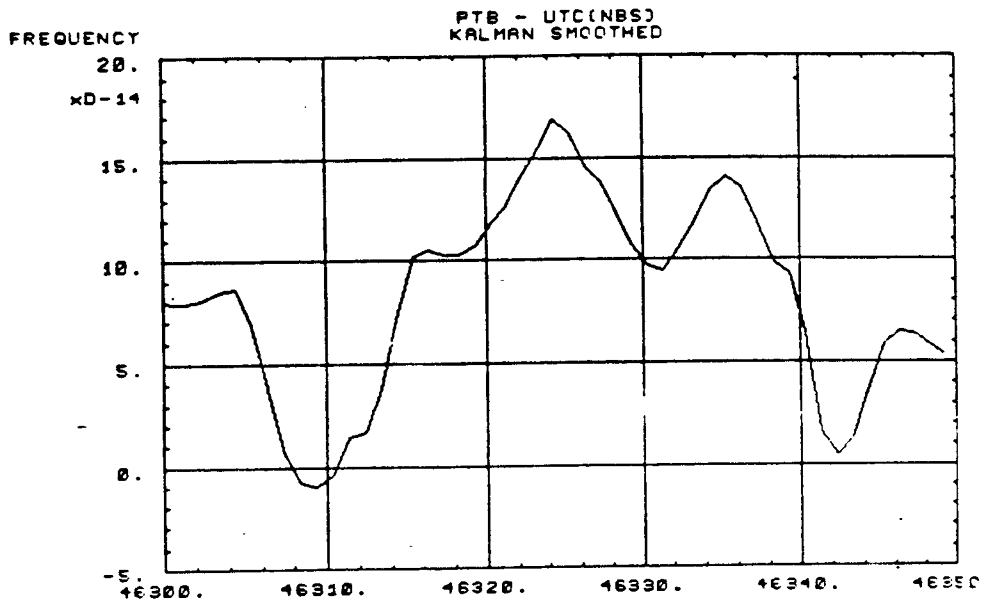

Figure 4b