

ULTRA-ACCURATE INTERNATIONAL TIME AND FREQUENCY COMPARISON VIA AN ORBITING HYDROGEN-MASER CLOCK

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Abstract. - Hydrogen maser clocks have exhibited fractional frequency stabilities of better than 1×10^{-15} for averaging times as large as 20,000 seconds [1]. This represents an rms time deviation of about 20 ps for $\frac{1}{4}$ day prediction times [2]. S-band Doppler cancellation frequency comparison techniques have been developed with phase stabilities of a few picoseconds [2,3]. Laser ranging systems have been developed with accuracies of a few cm [4]. Combining the virtues of these developments and choosing a satellite with an appropriate orbit would allow worldwide time comparisons at the subnanosecond level, and frequency comparison uncertainties of the order of 1×10^{-16} . Such a capability would open up new horizons to the frequency standards laboratories, to the VLBI community, to the Deep Space Tracking Network, and to fundamental time and frequency (T/F) metrology on a worldwide basis, as well as greatly assisting the BIH in the generation of UTC and TAI.

Introduction. - For the ultra-accurate frequency metrologist, it would be useful both to have a perfect reference and also to have that reference available to other metrologists so that comparisons could be made with respect to the same reference. Though the experiments outlined in this paper do not perfectly achieve these goals, they allow giant steps to be taken from where we are now in the comparison of remote T/F standards. The culmination of these experiments would result in a system which would be the marriage of five very successful methods that have been developed in the recent past and whose individual accuracies and capabilities are documented in the literature. These are: 1) the clock flyover experiment, originally investigated and conducted by Besson, et al [5], and later by others [4]; 2) the subnanosecond accuracies available using reciprocity of laser pulses (propagation delay times in both directions are equal)

off of retroreflectors [4,6]; 3) the three-way microwave Doppler cancellation system outlined herein results in picosecond phase stability between remote line-of-sight clocks [3]; 4) the unparalleled frequency stability of hydrogen masers for sample times in the vicinity of ~ 100 s to several hours [1]; and 5) the great advantage gained by using signals in simultaneous common-view from two sites, leading to large amounts of common-mode error cancellation in many instances [7,8,9,10]. The accuracy and characteristics of these methods will be reviewed; we shall then discuss how they reasonably can be married together to give an ultra-accurate worldwide T/F metrology system.

The proposed Shuttle time and frequency transfer (STIFT) concept is illustrated in figure 1. STIFT can be viewed as an extension of the transportable clock, or clock flyover, method with a hydrogen maser clock in a space vehicle. Time and frequency transfer between the space clock and a clock on the ground is accomplished in one of two ways. The first method is by two-way microwave transmission involving three carrier frequencies. The CW microwave system will permit a direct frequency comparison between the space clock and the ground clock with an accuracy of 10^{-14} for sample times of the order 100 s and longer. This is a unique feature of the STIFT system. A time code modulation can be applied on the microwave carrier to perform time transfer with an accuracy of 1 ns or better. The user of the system will require a microwave ground terminal located next to the clock to receive signals from and to transmit back to the spacecraft. Further, a method of identifying unambiguously a cycle of the microwave downlink from the spaceborne clock from orbit to orbit adds great leverage to the microwave system. The few picosecond phase stability of the microwave system combined with the long integration times will yield frequency measurements accurate to the order of a part in 10^{16} .

Second, the STIFT system can accomplish time transfer with subnanosecond accuracy using the short-pulse laser technique in conjunction with existing laser ground stations. The laser technique can also be used to calibrate the microwave system. The short-pulse laser method is the most accurate documented technique of time transfer available [4]. The equipment onboard the space vehicle consists of a hydrogen maser clock, a microwave transponder system with antenna, a corner reflector array with photodetectors, and an event timer.

A first step implementation of the STIFT system would be a demonstration flight on the Space Shuttle. The experiment would be returned and could be reflown on a later Shuttle mission. Initially, Shuttle missions are limited to altitudes of about 370 km (200 nmi), with a 57° inclination orbit. Such an orbit will provide many contacts with existing ground stations, and sufficient global coverage to demonstrate the time and frequency transfer capability of the STIFT method. The number of participating laboratories and users will depend on the limited number of ground terminals which can be made available for the demonstration experiment. Redistribution of ground terminals for reflights of the experiment would permit wider participation in the experiment.

SHUTTLE TIME AND FREQUENCY TRANSFER ACTIVITY

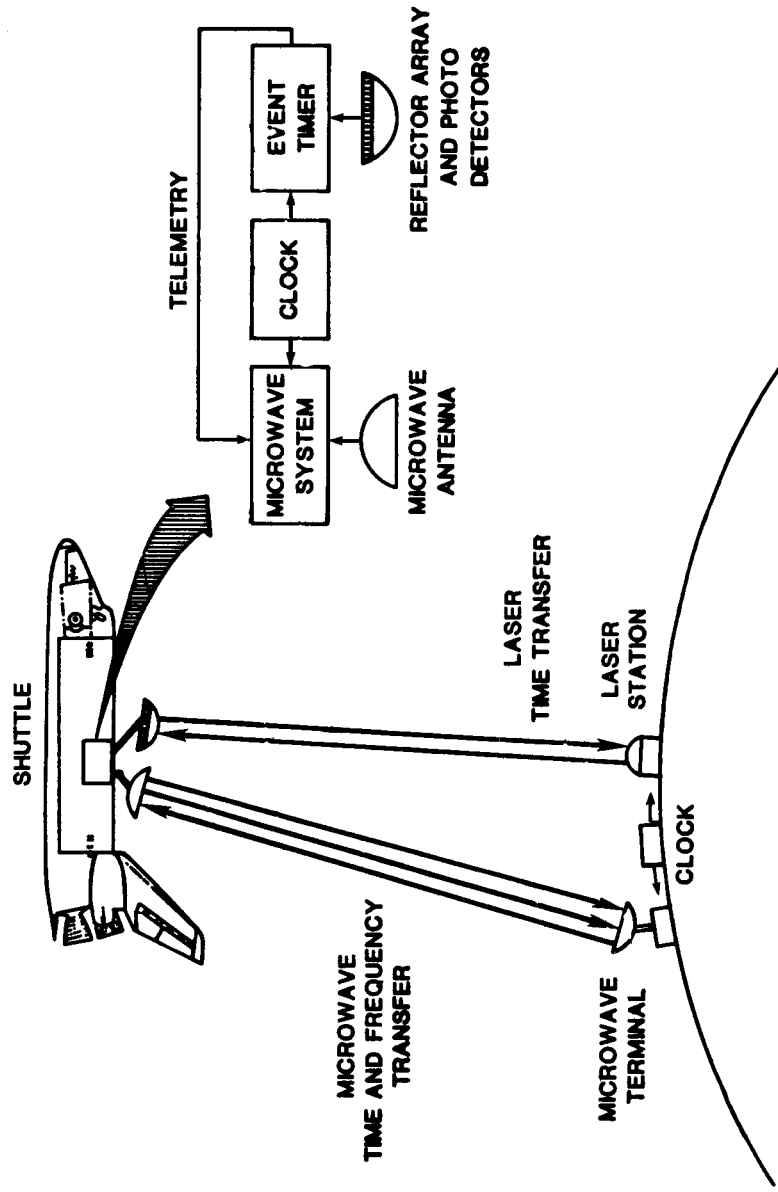


Figure 1. - A sketch of the STIFT concept showing the microwave Doppler cancellation system and the laser ranging system to the Shuttle.

The next phase of the implementation program could be the deployment of STIFT on the Power Systems Platform, a small, unmanned space station being planned by the National Aeronautic and Space Administration (NASA). The orbit would be very similar to the Space Shuttle orbit because the platform will be serviced by the Shuttle. The main advantage would be the extended (in principle, unlimited) operation time which would make the STIFT system available in a quasi-operational mode.

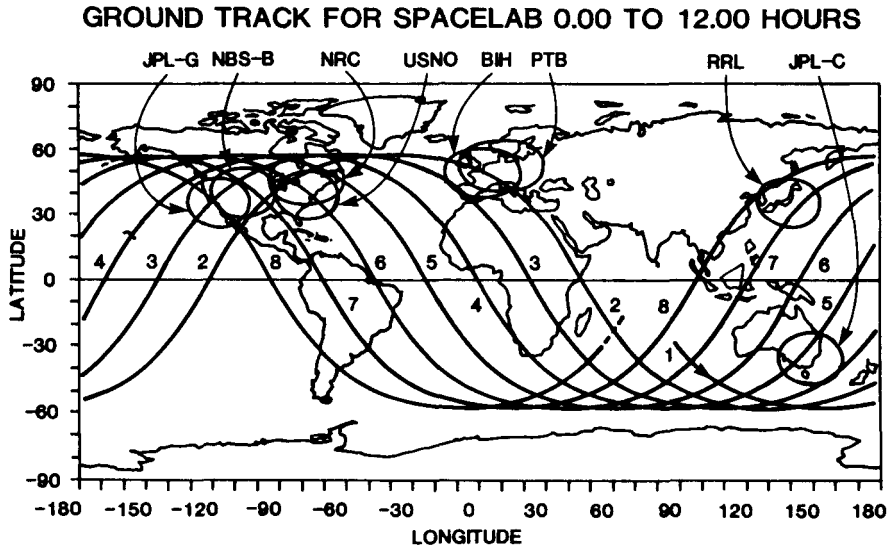
The ultimate goal is a free-flying satellite in an orbit optimized for a global operational system. Since the basic technology required for STIFT is available, one could directly implement the STIFT satellite system without demonstration flights. However, making use of available Shuttle missions offers several advantages. The program could start at a lower initial cost and the reflight possibility would permit optimization of the STIFT design based on practical results from demonstration flights.

The spaceborne part of the STIFT system consists of the hydrogen maser clock, a microwave transponder, microwave antenna, corner reflector array with photodetectors, and associated electronics. For Shuttle missions, the experiment package would be mounted on a pallet in the Shuttle bay. A deployable antenna may be required to obtain a clean radiation pattern.

The ground tracks of a 57° , 370 km Shuttle orbit are shown in figures 2a and 2b for the first 16 orbital revolutions. Also shown are the visibility circles for selected ground terminal locations for 10° elevation limits. The following station locations have been included to illustrate the current study, but could clearly be broadened to include any ground station within a latitude of $\pm 57^\circ$: GSFC/Washington, DC; PTB/Braunschweig, FRG; JPL/Goldstone, CA; JPL/Canberra, Australia; NBS/Boulder, CO; NRC/Ottawa, Canada; BIH/Paris, France; and NRL/Tokyo, Japan. The GSFC would be very accurately tied in time to UTC(USNO).

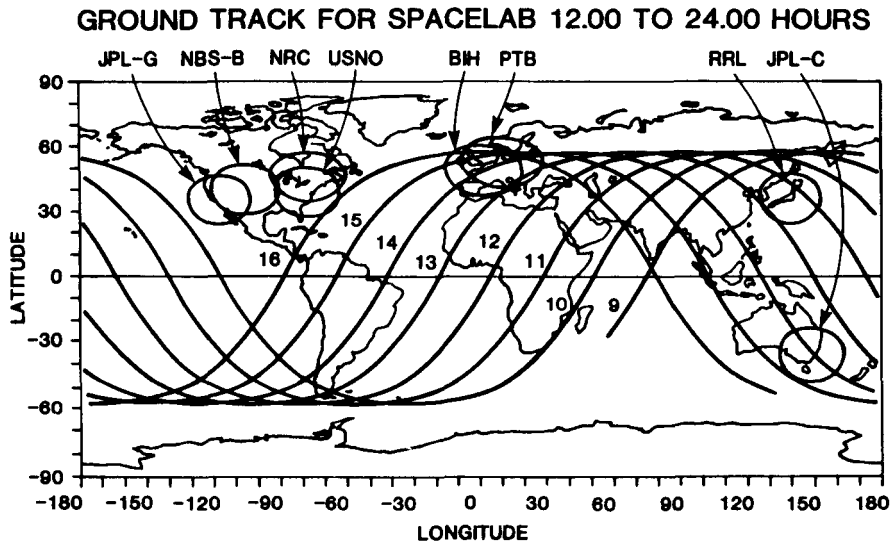
The techniques to be employed have been used successfully in earlier experiments. The microwave portion of the system is similar to a system used with the Gravitational Redshift Probe (GPA) flown in 1976 [3]. The short-pulse laser technique was successfully used in airborne clock experiments in 1975 which measured general relativistic effects on time [4]. Because of this experience with previous experiments, basic new technology development is not required for the Shuttle experiment. The main new contribution of this paper is to show that the orbital elements of the Shuttle, or of a satellite, can be determined well enough so that the uncertainties of the relativistic effects on the spaceborne clock can be kept sufficiently small to allow an individual cycle of the microwave signal to be unambiguously identified. As will be shown, the implications of accomplishing this are very far-reaching.

The current operational mode for comparison of primary standards in the U.S. (NBS), Canada (NRC), and West Germany (PTB) utilizes Loran-C, which suffers from limited global coverage and fluctuations of the ground-wave propagation delay,



TRACKING CIRCLES - ALTITUDE 370.00 KM ELEVATION ANGLE 10.00 DEG

Figure 2a.



TRACKING CIRCLES - ALTITUDE 370.00 KM ELEVATION ANGLE 10.00 DEG

Figure 2b.

Figures 2a and 2b. - Sixteen successive ground tracks of a Shuttle flight and how they intercept various key timing centers. It is interesting to not that PTB has six successive passes, e.g., a sinewave oscillator spends most of its time at the extremes.

INTERNATIONAL TIME AND FREQUENCY COMPARISON (<< 1 μs)

<u>Method</u>	<u>Inaccuracy</u>	<u>Stability</u>	<u>Cost Factor</u>	<u>24-Hour Frequency Accuracy</u>	<u>Comments</u>
GPS (Common-view)	10 ns	1 ns	0.25	≤ 8 x 10 ⁻¹⁴	Potential for global coverage—test started May 1981
Shuttle (STIFT)	1 ns	0.001 ns	0.25	≤ 10 ⁻¹⁴	Coverage ± 57° latitude—earliest flight 1986
VLBI and GPS (common-view)	0.3 ns	0.1 ns	0.1	3 x 10 ⁻¹⁵	Only available when VLBI systems are operating and accurate GPS orbits are known
TDRSS	10 ns	1 ns	1.0	< 10 ⁻¹³	Limited global coverage—planned for 1982
LASSO	1 ns	0.1 ns	1.0	~ 10 ⁻¹⁴	Limited global coverage—planned for 1982, experimental system
GPS	40 ns*	10 ns	1.0	~ 3 x 10 ⁻¹³	Global coverage—operational 1985, under test since 1978
Communication Satellite (2-way)	50 ns	≤ 1 ns	5.0**	≤ 10 ⁻¹³	Almost global coverage except near poles—experimental
Portable Clock	100 ns	N/A	6.0	~ 10 ⁻¹³	Global coverage—best accuracy within reasonable driving vicinity of airports
Loran-C	500 ns	≤ 40 ns	3.0	≤ 10 ⁻¹²	Coverage excludes most of Asia and Southern Hemisphere—new stations planned

Cost Factor = M\$.ns (user cost)
 *Inaccuracy may increase to 600 ns if C/A code is deteriorated for strategic reasons
 **Cost includes estimate for annual rent

TABLE I.

making the system practically incompatible with requirements of high-precision standards laboratories and time services. Other laboratories are interested in the accuracy capabilities of primary frequency standards, either directly or indirectly, but adequate means for international frequency comparison do not exist at the state-of-the-art accuracy of the standards themselves. Other techniques of accurate time transfer have been tested experimentally or are planned for implementation in the future. A performance comparison of available and planned methods, including the proposed Shuttle experiment (Shuttle Time and Frequency Transfer Experiment, STIFT) is shown in table I. The following definitions apply to the table. Inaccuracy is expressed relative to a perfect portable clock. Stability is the measure of time variations over the course of the measurement (i.e., related to the phase stability of the measurement system with sampling intervals and length of data determined by the method). Cost-effectiveness is the product of inaccuracy and user cost dollars (in mega-dollar nanoseconds), of course, the smaller the number the better. The 24-hour frequency accuracy is derived from time stability over 24 hours, which determines the accuracy of absolute remote frequency comparison for that sample time. Though many of the numbers represent anticipated performance, it is believed that they are well within an order of magnitude of what is feasible. Where applicable, the figures in the table I are rms values. As can be seen, the STIFT experiment looks extremely attractive, when compared with other techniques.

Microwave System. - A block diagram of the microwave system is shown in figure 3. The key feature of the system is, for time durations longer than the signal round-trip time, that we obtain significant cancellation of several effects including the first-order Doppler effect in the frequency comparison loop. The ground clock signal is first transmitted to the spacecraft and transponded back to the ground terminal to obtain the two-way Doppler shift, which is divided by two to generate the one-way Doppler shift, which in turn is subtracted from the one-way space clock downlink signal. The resulting beat signal at the output of the mixer is the frequency difference between the two clocks with the first-order Doppler shift removed. This process cancels propagation variations in the ionosphere. In addition, this process cancels tropospheric delay, assuming reciprocity. The S-band transmission frequencies shown are those used with the Gravitational Probe A. The three frequencies were selected to compensate for ionospheric dispersion. A single antenna is used at the spacecraft and at the ground station for transmission of the three microwave links.

The frequency transfer uses the phase information of the CW phase-coherent carrier signals. Time transfer is accomplished by modulation of the carrier signals. The time code generated by the space clock is modulated on the clock downlink and compared with the time code of the ground clock. The one-way propagation delay, needed as correction for time transfer, is obtained from the range

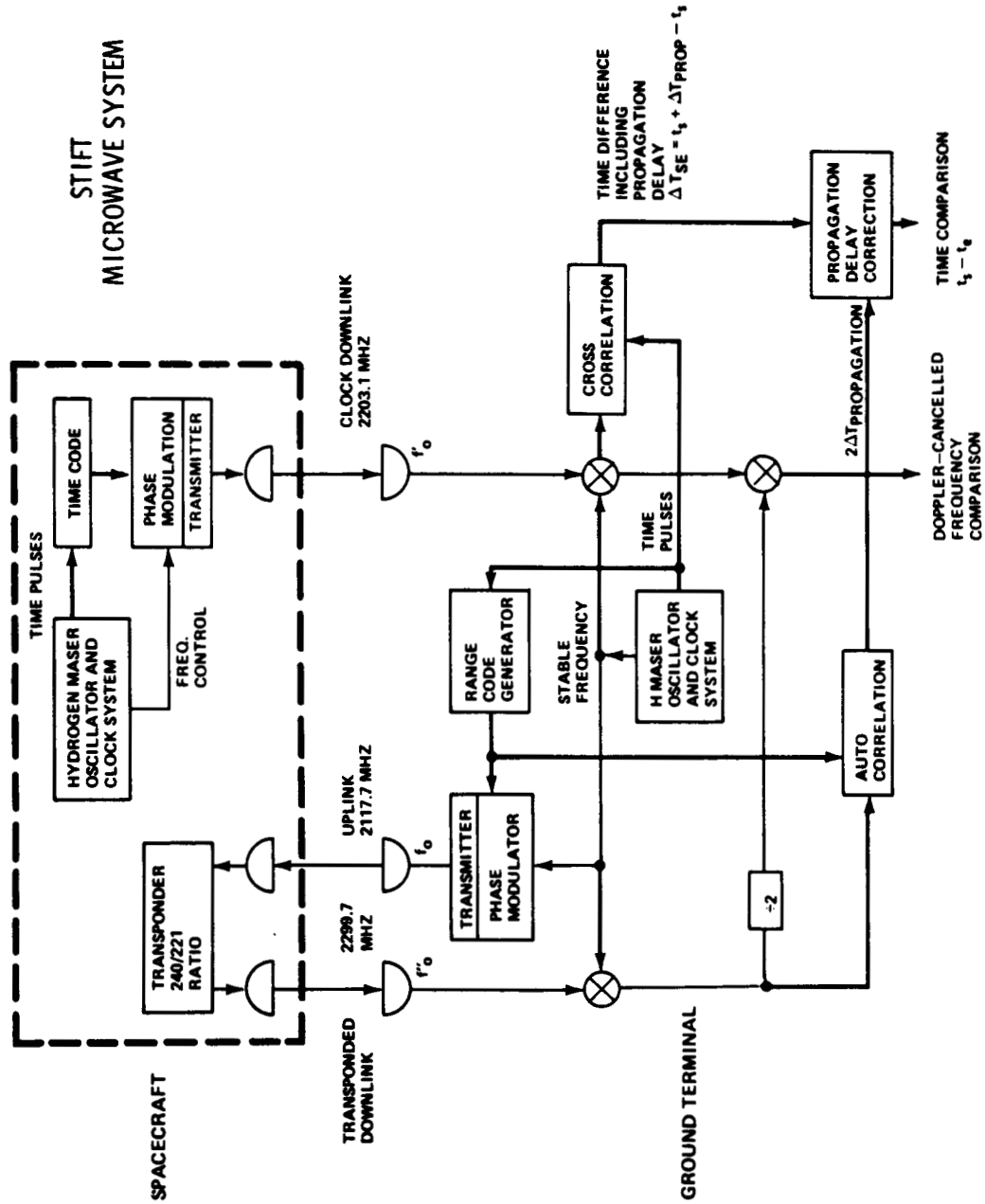


Figure 3. - The STIFT microwave system showing the appropriately chosen frequencies for ionospheric delay correction and also the modulation system for all weather ns time transfer.

measurement accomplished by PRN phase modulation of the transponded links. The time difference between the two clocks is derived from the delay of the two time codes.

One important objective is the development of a low-cost, simple ground terminal that can be afforded by a large number of users of an operational STIFT system.

Laser System. - The short-pulse laser technique is the most accurate documented line-of-sight method of time transfer [2]. Existing laser ground stations can use STIFT for subnanosecond time transfer if they are equipped with an atomic clock and an event timer. Because of the higher accuracy, the short-pulse laser technique can serve as a calibration tool for the microwave time transfer. It should also provide interesting data on microwave versus laser propagation through the atmosphere. Using this technique, neither the relative velocity nor the distance between space vehicle and ground stations enters into the clock comparison.

The laser ground station transmits short laser pulses which are returned by the corner reflector array on the space vehicle. The reflector array is equipped with photodiodes to detect the arrival of the laser pulses. The arrival time t'_2 of the pulses is measured in the time frame of the onboard clock by the event timer. This information is telemetered to the laser ground station which measures the time of transmission, t_1 , and reception, t_3 of the laser pulse. From this data, the time difference between the space clock and the ground clock can be determined with an accuracy of 1 ns or better.

Picosecond Timing from Microwave Cycle Identification. - There is no convenient way to divide down from the S-band signal and retain an unambiguous identity of a particular cycle of the microwave signal. However, picosecond phase measurements can be made if the clocks, the medium, and the ephemeris errors are known with sufficiently small uncertainties. In this case, it is possible to resolve a particular cycle from one orbit of the Shuttle to the next or from one site to another. Once a particular cycle has been identified, we can use its phase or zero crossing for timing on a reproducible basis. When this is accomplished, the STIFT experiment becomes the most precise of any of the international methods of T/F comparisons being considered. The basic limit of this method is the reproducibility and stability of the microwave system, which has been previously documented to be at the 1 to 10 ps level for integration times of the order of the Shuttle orbit period [3]. If the 10 ps level of stability can be maintained over a day or longer—and there is good evidence that this can be done—then one can make frequency comparisons of 1 part in 10^{16} or better at one day or longer with this method.

The period of the downlink microwave signal is 454 ps. Therefore, one would like to keep rms errors well below 1 radian at that frequency, which would be 72 ps, in order to resolve a particular cycle. The contributions to that uncertainty are represented as the root-sum-square (rss) of terms in the following equation where the first term is due the relativistic uncertainties from the errors in the ephemeris determination of the orbiting clock. The second term is due to the random uncertainties of time predictability of the onboard clock itself and the last term results from uncertainties in the overall delay from the space clock to the ground clock.

$$\delta t_{\text{rss}} = \left[\overline{\delta t_{\text{rtv}}^2} + \overline{\delta t_{\text{clock}}^2} + \overline{\delta t_{\text{delay}}^2} \right]^{\frac{1}{2}} \quad (1)$$

We have investigated the uncertainties in the ephemeris determination given the ideal case of a satellite in freefall and being ranged with the laser system and/or alternatively with the microwave-integrated Doppler technique. The uncertainties in orbit determination for the freefall condition compared favorably with those for GEOS-3 [11,12]. The errors for the Shuttle experiment were estimated to be about 15 ps (see below). To account for Shuttle drag, solar pressure, and space maneuvers, it will be assumed that an onboard inertial guidance system is available to monitor non-gravitational accelerations. Such accelerations can be used to compute perturbations in the free-fall orbit. Peak uncertainties due to these perturbations are estimated to be about 35 meters for the 1½ hour Shuttle orbits [13]. Clearly, all of the terms will be dependent on the integration time, which may range from 1½ hours to 24 hours for various experiments. Taking the case of the Shuttle period of 1½ hours, a 35 meter peak ephemeris error would lead to about 13 ps rms uncertainty in the gravitational red shift effect. The velocity determination uncertainty is of the order of about 2 cm/s, which would yield an uncertainty in the second-order Doppler effect of less than 1 ps over that same integration time. Figure 4 shows the rms time dispersion time characteristics of the hydrogen maser and of the microwave system as transformed from the stability data shown in figure 5; one observes about 9 ps rms error for a hydrogen maser clock after 1½ hour integration time.

The absolute delay from the last term in equation (1) cannot be known with an accuracy of less than 72 ps (except perhaps with the laser ranging system), but the stability of the microwave measurement system makes reproducibility possible at an accuracy level of about 10 ps over the 1½ hour integration time (see figure 4). Hence, one knows that the total path delay is an integral number of cycles of the microwave signal and, using modular arithmetic, one can deduce rather than measure the delay, and hence identify a particular cycle of the microwave signal. In other words, we can predict the Shuttle clock time to an rms level of about 18 ps from the previous pass to the current one with respect to the ground clock time using our estimates of the Shuttle ephemeris. We obtain

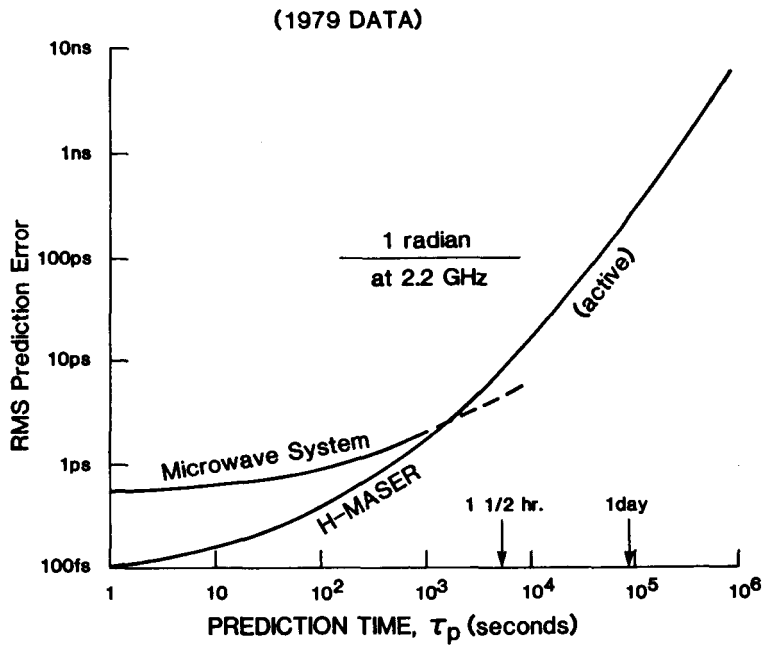


Figure 4. - An estimate of the rms prediction error of the microwave system and of the hydrogen maser calculated from the frequency stability data in Fig. 5.

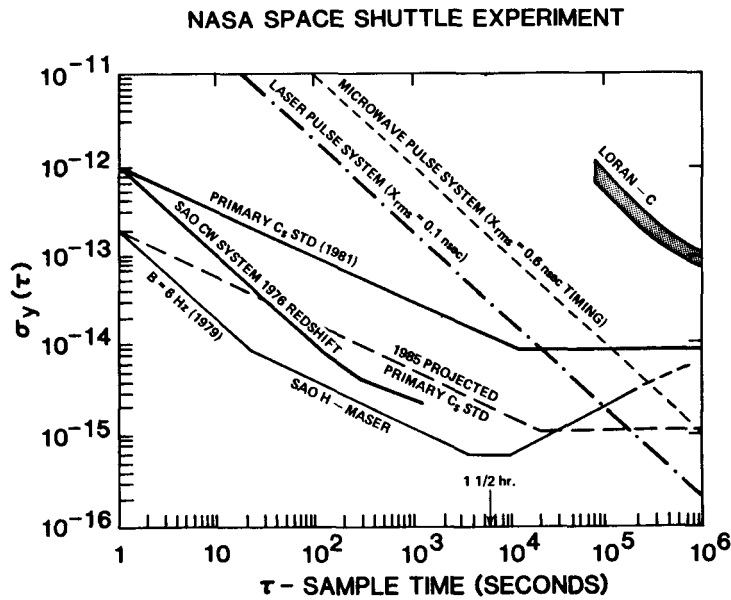


Figure 5. - A comparison of the fractional frequency stability (square root of the Allan variance) for a variety of sampling times. The dates given by each curve indicate when the stabilities were determined or when their determination is anticipated. The Loran-C data is for ground-wave transmissions. The microwave pulse and laser pulse systems are projected stabilities from previously obtained results. It is interesting that the maser has its best stability for the orbit period of the Shuttle.

from the previous measurement, a time difference and a delay factor. Using these measures, we calculate the current predicted time difference and delay value and compare it with what is measured. This comparison difference is an integer multiple of the microwave period plus an error in prediction plus the instabilities in the microwave system, which this paper shows can be kept well below the 72 ps level—thus allowing us to calculate the above integer. Knowing this integer allows us to remeasure the time difference of the Shuttle clock with respect to the ground clock limited only by the instabilities in the microwave system. The tremendous leverage gained by this technique is obvious and would allow international time and frequency comparisons at super-accurate levels. The ground station would require both a clock with similar low time deviation for the integration time of interest, as well as a microwave station. We have plans to develop such a station so that it is economically feasible for several of these to exist at key locations around the globe.

Figure 5 shows an overall fractional frequency stability comparison of some standards and comparison methods relevant to the STIFT activity. The net effect of microwave cycle identification on this plot is to continue "SAO CW System 1976 Redshift" curve going down nominally as τ^{-1} from where it ends on the right.

Freefall Orbit Determination. - We next discuss the accuracy with which the Shuttle ephemerides can be determined. Assuming that nongravitational accelerations of the Shuttle can be monitored, we are here primarily interested in gravitational and other systematic effects. The gravity model GEM 10 [11,12] incorporates surface gravity data and represents one of the best available models for the gravity field experienced by the Shuttle, which is in a relatively low orbit. Model GEM 10 is characterized by the presence of a large number of higher harmonics (of degrees 10 to 20) in the potential, whose coefficients have fractional uncertainties of approximately 0.1 to 0.3.

Evaluations of the GEM 10 model for predicting the orbit of GEOS-3, a spacecraft with an altitude of about 840 km, showed that the spacecraft position could be predicted to within about two or three meters. However, this required refined modeling which included lunar and solar tidal gravitational perturbations, solar radiation pressure on the satellite, polar motion, UT1 data, ocean tides, and solid earth tides; the prediction uncertainty reflects numerous errors besides ephemeris errors.

For a Shuttle in an orbit of 370 km altitude, uncertainties in higher harmonics of the gravitational field of the earth could be expected to contribute two or three times the uncertainties they contribute to the GEOS-3 orbit prediction. Thus, using GEM 10, one would not expect predicted orbit positions to be in error by more than about 10 meters; if high-accuracy tracking data is available from many stations distributed about the earth's surface, this figure could be significantly reduced. The 10 meters transform into a clock error of only 8 ps

for a Shuttle orbit. We have taken a conservative 2σ limit of 16 ps for the purposes of this paper.

Relativistic Corrections to Satellite Clock Time. - We adopt the point of view that the center of mass of the earth, as it falls freely in the gravitational field of the sun, provides an origin for a local, nonrotating, inertial frame, and that a worldwide network of coordinate clocks synchronized in such a reference frame is our objective [14]. The presence of the earth itself and the location of standard time and frequency labs on the surface of the rotating earth require that clocks in motion in the local frame must have relativistic corrections applied to them in order to obtain readings of "coordinate time" in the local inertial frame. The corrections for the satellite clock relative to a clock on the geoid have been derived elsewhere (e.g., Ashby and Allan, 1979) [14], and are given by

$$\Delta t_{\text{rltv}} = \int_{\text{path}} ds \left[\frac{v^2}{2c^2} - \frac{V}{c^2} + \frac{V_0}{c^2} \right] \quad (2)$$

where ds is the increment of proper time elapsed on the standard clock, v is the velocity of the satellite with respect to the local inertial frame, V is the gravitational potential due to the earth's mass, and V_0 is the effective gravitational potential of a point at rest on the geoid, including the effects due to rotation. The expression for V is

$$V = \frac{GM_e}{r} \left[1 - \left(\frac{a_1}{r} \right)^2 J_2 P_2(\cos \theta) \right], \quad (3)$$

where r is the distance of the satellite from the earth's center, a_1 is the earth's equatorial radius, θ is the colatitude, and J_2 is the quadrupole moment coefficient of the earth. Also,

$$V_0 = \frac{GM_e}{a_1} \left[1 + \frac{1}{2} J_2 \right] - \frac{1}{2} \omega_e^2 a_1^2 \quad (4)$$

where ω_e is the angular rotational speed of the earth. Values of the constants GM_e , a_1 , J_2 , and ω_e are given in table II.

Table II. - Values of Useful Constants

ω_e , rotational angular velocity of the earth	$= 7.292115 \times 10^{-5}$ rad/s
J_2 , quadrupole moment coefficient for earth	$= 1.08263 \times 10^{-3}$
GM_e , earth's mass times gravitational constant	$= 3.9860045 \pm 0.0000002 \times 10^{14}$ m ³ /s ² [15]
a_1 , equatorial radius of the earth	$= 6.378140 \times 10^6$ m.

The value of V_0/c^2 obtained from these constants is

$$V_0/c^2 = -6.9692815 \times 10^{-10} .$$

If the quadrupole term were neglected, an error of about 4×10^{-13} would be introduced.

The relativistic corrections can then, with the neglect of the quadrupole correction terms, be expressed exactly in terms of the orbital elements as follows:

$$\Delta t_{rltv} = \frac{2}{c^2} (GM_e a)^{1/2} \Delta E - \frac{GM_e}{2c^2 a} \frac{\Delta t}{1-e^2} + \frac{V_0 \Delta t}{c^2} \quad (5)$$

where ΔE is the change in eccentric anomaly, and Δt is the change in the coordinate time.

Using the relation between the eccentric anomaly and coordinate time, the change in eccentric anomaly is

$$\Delta E = e\Delta(\sin E) + n(\Delta t) \quad (6)$$

where $\Delta(\sin E) = \sin E_{\text{final}} - \sin E_{\text{initial}}$ is the change in the sine of the eccentric anomaly. Then the relativistic correction can also be expressed as

$$\Delta t_{rltv} = \frac{2e}{c^2} (GM_e a)^{1/2} \Delta(\sin E) + \frac{GM_e}{c^2 a} \left[2 - \frac{1}{2(1-e^2)} \right] \Delta t + \frac{V_0 \Delta t}{c^2} . \quad (7)$$

We now examine the sensitivity of this correction, to errors in each of the parameters on which it depends. If the elapsed coordinate time, Δt , can be

determined to ± 1 ms, then the uncertainty in the last term, $V_0 \Delta t / c^2$, is less than a picosecond. We shall assume that GM_e is a known quantity having the value given in table II. The effect of an error, $\delta(GM_e)$, on the calculation of the relativistic correction, $\Delta t_{r|tv}$, may be estimated by calculating the following quantity:

$$\begin{aligned} \delta(\Delta t_{r|tv}) &= \frac{\partial(\Delta t)}{\partial(GM_e)} \cdot \delta(GM_e) \\ &= \delta(GM_e) \left[\frac{1}{c^2 a} \left[\frac{1}{2} \Delta t + \Delta \left(\frac{t}{1-e \cos E} \right) \right] + \frac{e}{c^2} \sqrt{\frac{a}{GM_e}} \Delta(\sin E) \right]. \end{aligned} \quad (8)$$

The uncertainty in GM_e is

$$\delta GM_e \cong 2 \times 10^7 \text{ m}^2/\text{s}^3. \quad (9)$$

The orbit eccentricity is typically very small; using $e = 0.003$, orbit altitude = 360 km, $\Delta t = 5500$ s for one orbit, the error in the correction becomes approximately

$$\begin{aligned} \delta(\Delta t_{r|tv}) &= \delta(GM_e) \left[\frac{3}{2} \frac{\Delta t}{c^2 a} + \frac{e}{c^2} \sqrt{\frac{a}{GM_e}} \Delta(\sin E) \right] \\ &\cong 4.94 \times 10^{-17} \Delta t + 8.66 \times 10^{-17} \Delta(\sin E) \\ &\cong 0.27 \text{ ps} \end{aligned} \quad (10)$$

Such a small uncertainty may safely be neglected.

The effect of errors in the knowledge of the other parameters appearing on the right side of equation (7) may similarly be estimated by differentiating with respect to the appropriate quantity. We then find, for a Shuttle orbit of altitude 370 km and eccentricity 0.003, that an uncertainty of 10 m in semi-major axis contributes an uncertainty of about 8 ps per revolution; an uncertainty of 10^{-6} in e contributes about 2.4 ps, and an uncertainty in Δt of 1 ms contributes 1.6 ps.

There are two additional small effects which merit some discussion. First, the slightly non-spherical mass distribution of the earth gives rise to higher multipole contributions to the gravitational potential. The largest of these is the earth's quadrupole moment, which gives a potential approximately a factor of 10^{-3} smaller than the principal monopole term. The gravitational redshift on a satellite clock due to this term depends on the orbit inclination, position of perigee, and initial and final longitudes. An estimate of the quadrupole potential's effect on an equatorial orbit at an altitude of 360 km is sufficient to show that the net effect is quite small. For one orbital revolution, the required

correction due to the quadrupole potential is about 2 ns. This is a systematic effect which can be modeled quite accurately. Even if only the quadrupole term is retained, the error in estimation of this effect is only about 5 parts in 10^4 of the effect itself, and hence, is about 1 ps or less.

Second, the process of synchronization of the satellite clock with a ground station by means of transmission of electromagnetic signals cannot be thought of simply in terms of Einstein synchronization and, since the earth is rotating with respect to the local inertial frame, a relativistic correction must be included. If t_A is the coordinate time of emission of a synchronizing signal from the ground station at position \vec{r}_A , t'_A is the time of arrival of the signal at the satellite at position \vec{r}'_A , $\vec{\omega}_e$ is the angular velocity of the earth, and τ is the total round trip time of the signal as measured on the ground station clock, then (Ashby and Allan, 1979) [14]

$$t'_A = t_A + \frac{1}{2} \tau + \frac{2\vec{\omega}_e}{c^2} \cdot \left[\frac{1}{2} \vec{r}_A \times \vec{r}'_A \right]. \quad (11)$$

The correction term depends on the angles between the angular velocity vector, $\vec{\omega}_e$, and the vector area, $\vec{A} = \frac{1}{2} \vec{r}_A \times \vec{r}'_A$, of the triangle enclosed by the radius vectors to the positions of the ground station and the satellite. For observation of the satellite within an acceptance cone of 10° or greater elevation, and for a satellite altitude of $h = 360$ km, the magnitude of the area, A , is at most $\frac{1}{2} r_A (r_A + h) \sin 11.2^\circ = 4.17 \times 10^6 / \text{km}^2$, while $2 \omega_e / c^2 = 1.6227 \times 10^{-6} \text{ ns}/\text{km}^2$. Thus, if the ground station is at a latitude of 40° , the correction is of maximum magnitude, 5.2 ns. By choosing the instant of synchronization close to that during which the satellite and the ground station lie in the same north-south plane, so that the angle between \vec{A} and $\vec{\omega}_e$ is close to 90° , the correction can be made very much smaller. This correction is also systematic and can be computed to a high accuracy in a specific application. In this example, an error of 100 m in the knowledge of the altitude of the satellite would introduce an error into the computation of this correction to Einstein synchronization of 0.2 ps at most. Other time transfer processes, such as by means of signals from the satellite to the ground and back, involve errors of comparable magnitude.

For a satellite in a high orbit, the uncertainties in predicted position arising from uncertainties in the gravity model should be significantly reduced. Figure 6 shows the orbit of a satellite of 12-hour period projected on the earth's surface. The orbit is nearly equatorial; the orbit parameters are: semi-major axis, $a = 26,610$ km; inclination, $I = 0.5^\circ$. Intersection with observation cones, of 10° elevation angle, from NBS and USNO are shown. The shaded area is the area within which the satellite may be received simultaneously from NBS and USNO. We have carried out a worst-case systematic error analysis of the satellite orbit determination problem for this satellite, assuming that the gravity field can be

modeled accurately by a single term in the potential. Assuming laser retroreflector tracking data of 2 cm accuracy are available from both NBS and USNO, the resulting errors in $\Delta t_{r|tv}$ are less than 1 ps. One may also use integrated Doppler tracking with the microwave system; in this case, errors in the determination of $\Delta t_{r|tv}$ are even less.

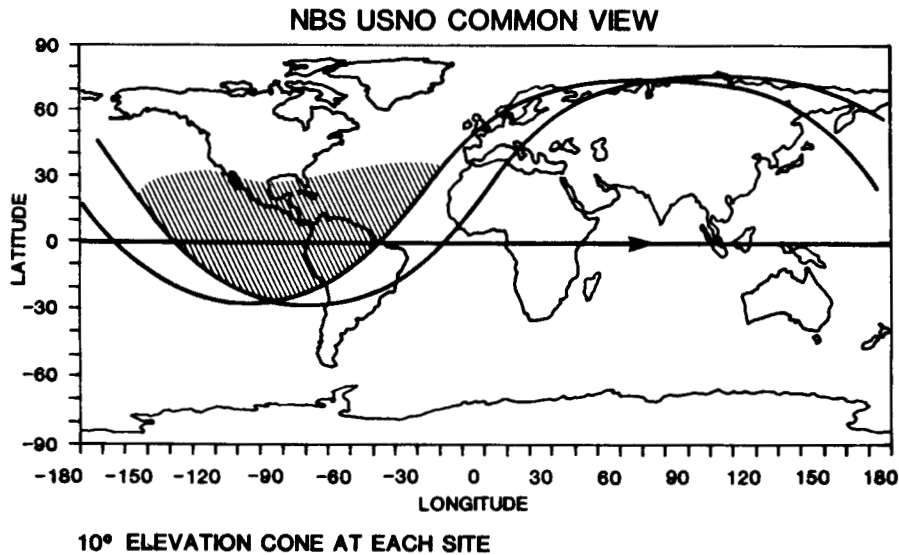


Figure 6. - Projection of satellite orbit on the earth's surface for an equatorial orbit of 12-hour period, $i = 0.5^\circ$, and intersection with observation cones, of 10° elevation angle, from NBS and USNO. The shaded area is the area within which the satellite may be received simultaneously from NBS and USNO. The projection of the orbit lies practically on top of the equator.

Plans and Conclusions. - The STIFT concept will allow worldwide time and frequency comparisons and/or measurements at or below 1 ns and 1×10^{-14} , respectively. In addition, if unique identification of a cycle of the 2.2 GHz downlink frequency can be made, then its phase can be used for timing. Nominally, this would yield 10 ps timing precision and 1×10^{-16} frequency measurements and comparisons. As has been shown in the text, this can be accomplished if the rms error of the Shuttle or satellite clock time can be kept below the limit of about 72 ps (1 radian). There are three sources contributing to this rms error: 1) for the $1\frac{1}{2}$ hour Shuttle orbit period, the random uncertainty due to the Shuttle clock itself is about 9 ps; 2) the relativistic uncertainties arising from uncertainties in the free-fall Shuttle orbit determination are worst case about 16 ps rms; and 3) the same uncertainties arising from orbital perturbations due to solar pressure, drag, maneuvers, etc. as would be estimated from an onboard inertial guidance system are about 13 ps. The root-sum-square of the above leads to an overall Shuttle clock time uncertainty of 22 ps, which is nicely below the 72 ps limit.

Knowing the Shuttle clock time with this level of uncertainty allows us to calculate the delay from pass to pass, since it is modulo one cycle of the carrier (15 cm or 454 ps) to a level which is only limited by the instabilities in the microwave system—approximately 10 ps or 3 mm.

With the ability to use the microwave system for timing at the few picosecond level on a world wide basis, a whole set of new experimental opportunities open up both in terms of experiments involving accurate time metrology as well as in accurate frequency metrology on an international basis. In addition, it relieves the need to have a laser ranging system to do subnanosecond timing except as may initially be needed at some sites, as a calibration check and for special propagation studies. If the time stability of the STIFT microwave measurement system persists at about the 10 ps level, and there is good indication that it will, then the STIFT approach would allow one to make worldwide frequency comparisons with an uncertainty of about 1×10^{-16} for one day, 1×10^{-17} for 10 days, etc. This opens up opportunities for studying with much greater accuracy: special and general relativistic effects, changes in some of the fundamental constants, etc. In addition, VLBI stations could reference the same frequency standard and thus have the same coherent source to within a few picoseconds at each station, which could greatly enhance radio astronomy data reduction.

We plan to do more work on the analysis of errors in the determination of relativistic effects on coordinate time at the picosecond level. In particular, we intend to examine more carefully the effects of non-gravitational accelerations due to atmospheric drag and space maneuvers, which perturbs the free-fall satellite orbit, as well as contributions to uncertainties in the satellite ephemeris due to uncertainties in modeling the gravity field. Versatile software packages have been developed which will allow us to consider any variety of sites, orbit inclinations, eccentricities, and altitudes. We anticipate that the STIFT experiments would be flown in the '85 or '86 time frame and we have received indications from several principal timing centers that they would like to participate in the experiments.

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