Spectrum Analysis of Extremely Low Frequency Variations of Quartz Oscillators*

The rather numerous discussions of the spectrum of oscillator frequency fluctuations prompted the application of the Tukey[1] technique of spectrum analysis to five years of daily frequency comparisons between an atomic frequency standard and free-running quartz crystal oscillators at the National Bureau of Standards. Three of the four spectra shown in Fig. 1 were computed from observations made on oscillators 5a and 35 which are 100-kc quartz oscillators utilizing vacuum tubes in amplifiers and automatic level control circuits. The aging rate of these oscillators had stabilized several years before these measurements were begun.

The fourth spectrum of Fig. 1 is an estimate of the contribution to the oscillator spectra due to instrumentation noise. From a series of random numbers this estimate was computed assuming that the daily frequency measurements were in error by as much as ± 1 part in 10^{10} and that the measurement error on any given day was independent of the errors made on neighboring days. Before undertaking the spectrum analyses a detrending procedure described by Norton, *et al.*,[2] was used to remove frequency drifts and variations at frequencies lower than the lowest frequency for which the spectrum was obtained. The spectrum of $\Delta f/f$, the relative or fractional frequency deviations of the oscillator, is shown with normalization such that integration from 0 to ∞ with respect to the Fourier frequency F in cycles per second would formally yield the total

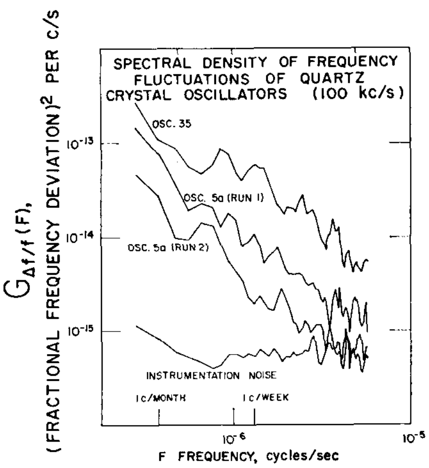

Fig. 1.

relative mean-square fluctuation. Fig. 1 shows spectral densities $G_{\Delta f/f}(F)$ which appear to increase without limit as one considers lower and lower Fourier frequencies. This behavior is similar to that of klystron and LC oscillators at Fourier frequencies about 10^8 times greater than those of Fig. 1.[3,4] A least-squares-type fit to the lowest oscillator curve of Fig. 1 is $G(F)=3.6 \times 10^{-23}/F^{1.4}$ (fractional frequency deviation)^2 per cycles per second.

Other spectrum measurements were made in a higher fluctuation, or Fourier, frequency range on more recently constructed transistorized 5-Mc quartz oscillators. The equipment consisted of an NH_3 maser, frequency multipliers, a mixer, an electronic frequency meter, and a tunable mean-square indicator. Analysis of these data yielded $G(F)=(5\times 10^{-23})/0.9$ (fractional frequency deviation)^2 per cycles per second in the range $0.02<F<2$ cps. This result is comparable to that for the lower frequency range. Above $F=2$ cps the spectral density was observed to increase with increasing Fourier frequency F.

For both types of crystal oscillators studied, the spectral behavior observed below $F=2$ cps is similar to that of current fluctuations in current carrying semiconductors. Current noise possessing a spectrum of this type occurs widely in nature and has been referred to as excess noise, flicker noise, or simply $1F$ noise.[5,6] Excess noise has been observed at frequencies as low as 6×10^{-5} cps.[7] In many theoretical treatments[8–10] of oscillator stability it has been customary to consider only the direct effects of tube shot noise and thermal noise on frequency stability with resulting theoretical spectra which disagree with the actually observed spectra reported here. The effect of flicker noise on oscillator stability has been treated theoretically by Troitsky.[11] His analysis resulted in a spectrum similar to the oscillator spectra of Fig. 1; however, experiments of Driagin[4] failed to verify certain aspects of this theory.

A difficulty associated with the oscillator spectra of the type shown in Fig. 1 is in determining how to extrapolate curves to lower frequencies in a meaningful way. It is desirable to compute for an oscillator such quantities as the power spectrum width, the mean-square frequency variation, and the expected time error of a clock connected to the oscillator. Formal relationships may be easily derived expressing these quantities as integrals over frequency of the frequency fluctuation spectrum times some appropriate filter function which weights the spectrum in accordance with the application under consideration. However, these integrals sometimes diverge at the lower limit if the spectrum is assumed to behave at arbitrarily low frequencies as a straight line extrapolation of Fig. 1 might suggest. The convergence difficulty was discussed by Malakhov[6] with reference to the expressions that lead to the spectral distribution of energy of an oscillator randomly frequency modulated with flicker noise. After disposing of the problem associated with the upper limit, Malakhov proposed proceeding according to the customary formalism that is valid for stationary processes, with the exception that the lower limit $F=0$ was to be replaced by $F=1/T$ where T is characteristic of the duration of an experiment which would determine the width of the power spectrum. This replacement seems reasonable and could be applied in the computation of the fluctuational aspects of various experiments involving oscillators. However, the exact relationship between T and the duration of an observation or experiment is somewhat vague, though this in itself would be of little consequence were it not for the fact that in some cases examined, the pertinent integral was found to be rather dependent on the exact value of the lower limit. Closer analysis of the whole problem might show that the weighting function, rather than the lower limit, could, and should, incorporate the duration of the experiment in such a way as to avoid the difficulty

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