PERFORMANCE, MODELING, AND SIMULATION OF SOME CESIUM BEAM CLOCKS

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Summary

With the availability of a new primary frequency standard, NBS-5 at the National Bureau of Standards, we have been able to evaluate with greater confidence than in the past the performance characteristics of the commercial cesium beam clocks used in the AT(NBS) atomic time scale. Two other techniques have also been employed to evaluate a clock's performance, viz., interclock comparisons and comparisons with other national laboratories.

Utilizing the above performance data we have constructed models for the behavior of cesium beam atomic clocks. Based on these models and appropriate optimization procedures, algorithms have been developed to generate an atomic time scale, AT(NBS), from the ensemble of standards available to us. The model is shown to well fit both individual clocks as well as clock ensembles. This modeling provides a direct opportunity for clock data simulation. Simulation techniques are developed and applied in the testing of some diagnostic tests for frequency and/or time steps. The results are very encouraging as a new effort for even better clock modeling.

Rate calibrations of AT(NBS), UTC(NBS), TAI, and other national time scales are given with reference to NBS-5, and these are compared with other past primary cesium beam frequency standards. TAI was measured as too high in rate by 12 ± 5 parts in 10^{13}.

Key words: Atomic Clock, Atomic Clock Modeling, Atomic Clock Noise, Atomic Clock Performance, Atomic Time Scale Accuracy, Comparison of Atomic Time Scales, Comparison of Frequency Standards, Frequency Calibration, Frequency Drift, Simulation of Clock Performance.

I. Introduction

A mathematical model describing the systematic and random behavior of atomic clocks is useful in the design of algorithms for constructing time scales, and we have developed such a model for cesium beam clocks [1]. The validity of our model is supported by three types of evidence. First, it is consistent with time-domain measurements of stability obtained by comparison of several independent clocks. Second, it includes the frequency drift which can be observed with respect to our new frequency standard, NBS-5 [2, 3]. Third, it is consistent with long-term comparisons of our clocks with other timing centers via the International Atomic Time Scale (TAI) [4].

In principle, if one had a perfect clock, one could set its time and frequency in agreement with some perfect reference, and there would be no time dispersion. In practice, of course, there is always time dispersion, and the modeling of the dispersion characteristics permits the design of time scale algorithms which minimize its effects. The extent of this dispersion can be seen by comparing some of the best atomic time scales in the world with TAI, which is probably the best reference in the world.

The rate origin of TAI was determined as follows: During 1968 B. Guinot, Director of the Bureau International de l'Heure (BIH) where TAI is generated [4], made very careful measurements via Loran-C of the time and frequency differences between the three atomic time scales AT(F), AT(PTB), and AT(USNO)[1]. Weights of 1, 1, and 2 were given respectively to the frequencies of these three scales to determine a weighted frequency to be given to the TAI scale (see Table 1) beginning 1 January 1969. Four other atomic time scales were added during 1969: Royal Greenwich Observatory (RGO), 8 April 1969; National Research Council (NRC), 18 May 1969; NBS, 27 June 1963; and Observatoire de Neuchatel (ON), 4 December 1969, but in each case a rate or frequency compensation was applied so as to not perturb the rate of TAI [4]. In Fig. 1, we have plotted the time dispersion of the three time scales which determined the rate origin of TAI with appropriate time and rate compensations (as determined by B. Guinot) as of 1 January 1969 (i.e., there were no initial time or rate errors in the compensated scales). In addition we have plotted with similar compensation the time dispersion of AT(NRC) since it is frequently calibrated with the NRC Cs III primary frequency standard, and of AT(NBS) for comparison purposes. It is interesting to note that the peak-to-peak dispersion among these atomic time scales is almost 100 μs in a little over four years' time.

TABLE 1 - TAI Rate
Determined 1 January 1969

Time Scale	Weight	Rate
AT(F)	25%	+ 65 ns/day
AT(PTB)	25%	+ 15 ns/day
AT(USNO)	50%	- 40 ns/day

[1] AT(F) is the atomic time scale for France and has consisted of an ensemble of from 5 to 7 commercial cesium beam clocks located at various of the key laboratories in France. AT(PTB) is the atomic time scale of the Physikalisch-Technische Bundesanstalt, Braunschweig, Federal Republic of Germany. This scale has consisted of an ensemble of 4 to 6 commercial cesium beam clocks. AT(USNO) is the atomic time scale of the U. S. Naval Observatory denoted by its staff as MEAN(USNO) or A.1, and has consisted of 14 to 16 commercial cesium beam clocks [5].

II. Clock Characterization, and Simulation

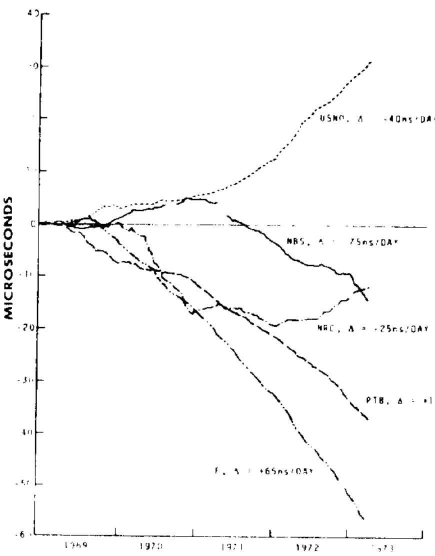

Fig. 1 - Time dispersion with respect to TAI of some of the independent atomic time scales contributing to make up TAI. Time and rate compensations were made to bring each scale in agreement with TAI at the origin of each scale as plotted.

Each of these atomic time scales is composed of an ensemble of commercial cesium beam clocks whose times are combined through some algorithm to generate one atomic time scale. One would expect that the dispersion of any one of these ensembles would be less on the average than that of an individual clock. The dispersion shown in Fig. 1 is large compared to the needs for much smaller synchronization tolerances and to the theoretical dispersion due to pure shot noise (idealistic limit) of an individual cesium beam clock (about 1 μs over the same 4-year period). We will investigate this dispersion in more detail.

The need for a primary frequency standard of high accuracy becomes apparent if either a frequency drift or a frequency offset exists in a clock--both of which appear to be present in the ensembles considered above. The time dispersion is quadratic or linear respectively if such a problem exists. These and other kinds of dispersion are examined using the new accuracy and stability available with NBS-5.

Once a tractable and well-fitting model has been established to describe a clock's performance, then one may employ optimization techniques based on the model to objectively detect clock problems and to minimize time dispersion. Some of these ideas are tested on a pair of clocks in the AT(NBS) ensemble for measuring the particular parameters for modeling this pair of clocks.

A Model for the Time Dispersion in a Cesium Beam Clock

Let the ideal time be represented by t, and the time of clock i by t_i. Then time difference of clock i from ideal time is:

$$T_i(t) = t_i - t \tag{1}$$

The proposed model for characterizing why Eq. (1) is non-zero is represented by the four terms in Eq. (2).

$$T_i(t) = T_i(t_o) + R_i(t_o) \times \left[t - t_o\right] + \frac{1}{2} D_i \times \left[t - t_o\right]^2 + x_i(t) \tag{2}$$

The first term on the right of Eq. (2) represents the error in synchronization originally (at $t = t_o$). The second term represents the error in syntonization originally; i.e., an error in the calibration of the rate or the frequency of clock i. The third term represents the time dispersion due to linear frequency drift in the i-th clock. The fourth term is due to the random fluctuations of the time of clock i.

The first three terms of Eq. (2) are categorically deterministic and non-random. The last term being random is best characterized statistically. The time dispersion due to contributions from the first three terms is constant, linear, and quadratic respectively. The time dispersion due to contributions from the last term depends on the noise spectrum of the random fluctuations in the i-th clock. Typically, we have found that the random fluctuations in a cesium beam clock are well represented by power law spectral densities with the predominant terms as follows:

$$S_{y_i}(f) = h_{o_i} + \frac{h_{-1_i}}{f}, \quad f \leq 1 \text{ Hz} \tag{3}$$

where $S_{y_i}(f)$ denotes the spectral density of the fractional frequency fluctuations, $y_i = \frac{dx_i}{dt}$, at Fourier frequency f for the i-th clock. The coefficients h_{o_i} and h_{-1_i} are the intensities of the white noise frequency modulation (FM) and of the flicker noise FM respectively. The value of h_{o_i} may be calculated from physical measurements (i.e., the signal-to-noise ratio at the detector of the cesium beam tube, resonance linewidth, limitations in the excitation electronics, etc.). The cause of the flicker noise FM is not known and should be measured for each cesium beam clock. For Eq. (3) the time domain stability using an Allan variance measurement method is given by [6, 7]:

$$\sigma_{y_i}^2(\tau) = \frac{h_{o_i}}{2\tau} + 2h_{-1_i} \ln 2, \quad \tau \gtrsim 1 \text{ s} \tag{4}$$

The time dispersion contributed by these two kinds of noise depends upon the particular algorithm used in generating time [8]. If we assume optimal procedures (optimum in the sense of minimum squared error), the time dispersion for white noise FM is $\tau\sigma_y(\tau)$, and for flicker noise FM is $1.3\,\tau\sigma_y(\tau)$. For the two noise processes in Eq. (3) the minimum squared time dispersion utilizing optimum processing is:

$$\left\langle \epsilon_{opt}^{2}(\tau) \right\rangle = \frac{h_o \tau}{2} + 2.4 h_{-1} \tau^2, \quad \tau \gtrsim 1\,\mathrm{s}. \tag{5}$$

One observes that the time dispersion is proportional to the square root of time for white noise FM and linear with time for flicker noise FM (square root of Eq. (5)).

If a significant level of linear frequency drift is present and is not removed before analyzing the random contributions, it is easy to show that this will cause a contribution to the instability as follows:

$$\sigma_y(\tau) = \frac{|D_i|}{\sqrt{2}} \tau. \tag{6}$$

Simulation of the Performance of a Cesium Beam Clock

Assuming that the model stated in the previous subsection is a good one for a cesium beam clock, it becomes obviously desirable to be able to simulate this model. One of the main advantages of this simulated data is the freedom gained in testing timekeeping algorithms. It is beyond the scope of this paper to fully exploit this advantage. We will simply point out some of the insights gleaned from simulation, show how to simulate, and later in the paper do some simulation and perform some tests on some particular algorithms.

A model representative of Eq. (2) is:

$$z(t) = a + bt + ct^2 + x(t), \tag{7}$$

where the a, b, and c can be chosen appropriate to the time error, frequency error, and frequency drift rate respectively of the clock being modeled. The random variable, $x(t)$, as has been shown [9] can be generated from a set of random uncorrelated numbers (RUNs) (i.e., zero mean), which are often available as a computer function.

Specifically, if a plot of Eq. (4) is made with $\log \sigma_y(\tau)$ as the ordinate and $\log \tau$ as the abscissa, there will be an intercept between the two kinds of noise, i.e., the point where the intensity of the white noise FM is equal to the intensity of the flicker noise FM. Associated with this point there will be a particular value of τ--call it τ_I ($\tau_I = h_o/(4h_{-1} \ln 2)$). We wish to generate a discrete set of data y_n with spacing τ_0 and which are statistically modeled by Eq. (4) where now y becomes discrete and is given by $y_n = (x_{n+1} - x_n)/\tau_0$. Also for equally spaced discrete data, an estimate of $\sigma_y^2(\tau)$ from a finite set of such data takes on a very simple form:

$$\sigma_y^{\ 2}(\tau) = \frac{1}{2(M-1)} \sum_{n=1}^{M-1} (y_{n+1} - y_n)^2, \tag{8}$$

where M is the number of data points. A set y_n can be generated using the following recursive equations [9]:

$$y_{n+1}^{(1)} = (1 - \gamma^{(1)})\, y_n^{(1)} + \frac{1}{2} y_{n+1}^{(0)} - (\frac{1}{2} - \gamma^{(1)})\, y_n^{(0)}, \tag{9a}$$

$$y_{n+1}^{(2)} = (1 - \gamma^{(2)})\, y_n^{(2)} + \frac{1}{2} y_{n+1}^{(1)} - (\frac{1}{2} - \gamma^{(2)})\, y_n^{(1)}, \tag{9b}$$

and

$$y_{n+1}^{(3)} = (1 - \gamma^{(3)})\, y_n^{(3)} + \frac{1}{2} y_{n+1}^{(2)} - (\frac{1}{2} - \gamma^{(3)})\, y_n^{(2)}, \tag{9c}$$

where

$$\gamma^{(1)} = 0.777\, \tau_0/\tau_I,$$

$$\gamma^{(2)} = \gamma^{(1)}/4, \text{ and } \gamma^{(3)} = \gamma^{(2)}/4.$$

The superscript on y is the order in the recursive sequence--the final output for each entry in the discrete data set being $y_{n+1}^{(3)}$. Initially, set $y_0^{(1)} = y_0^{(2)} = y_0^{(3)} = 0$, and set $y_1^{(0)}$ equal to the first random uncorrelated number (RUN), which then allows one to generate $y_1^{(3)}$--the first discrete data point. Next make the replacements $y_{n+1}^{(j)}$ from the last recursive sequence into $y_n^{(j)}$ for the new recursive sequence, and then call a new RUN for $y_{n+1}^{(0)}$, etc. If the RUNs are normally distributed with unit variance (and zero mean since they are uncorrelated), then the $y_n^{(3)}$ will be normally distributed with $\sigma_y^2(\tau_0) = 1/64$. The factor 1/64 is due to the filter response given in Eq. (9). Since time is the integral of the frequency, the final simulated random time dispersion output is, therefore,

$$x_m = 4\sqrt{2h_o\tau_0} \sum_{n=1}^{m} y_n^{(3)}, \quad \tau_I \gg \tau_0, \tag{10}$$

where m is the m-th discrete random time fluctuation; this equation completes the model for the discrete case given by Eq. (7). The above recursive equations simulate a random process whose statistical properties are within about 5% of those given by Eq. (4) over at least the range $\tau_0 \le \tau \le 1000\,\tau_0$.

One very interesting observational result from our studies of simulated data is that for both white noise FM and flicker noise FM the time fluctuations as a function of time appear to have definite changes in slope from time to time. One is tempted to assume that the clock suffered a discrete change in frequency, even though no such change occurred, and, furthermore, these changes are totally explainable by the amount of energy in the low frequencies of the random process. A more serious caution, however, is that if the data are optimally processed, based on a well-fitting model, any other algorithm which manipulates the data further can only make the time dispersion worse. Fig. 2 is an example of simulated flicker noise FM with lines drawn per temptation. Flicker noise FM has another interesting property that, as was shown by Mandelbrot [10], it is self similar--i.e., an expansion or contraction of the abscissa and ordinate equally does not change the statistical character of the data. Or, in other words, one could continue to draw straight lines as illustrated down to the finest detail as long as the flicker noise FM process predominated in that region of Fourier frequencies. An interesting result of this argument is that one's temptation to draw straight lines is strongly influenced by the size of the graph paper he uses. It is apparent then that one may be prone to overmanipulate the data. We will develop an objective test for recognizing changes in frequency in Section IV.

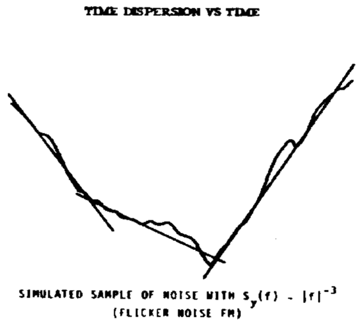

Fig. 2 - A simulated sample of flicker noise FM with arbitrary straight lines drawn in to nominally fit the data. The ordinate and abscissa are arbitrary.

III. Some Characteristics of the AT(NBS) Scale and its Constituents and Other Clock Ensembles

Characterization via Internal Comparisons

The physical arrangement of the clocks in the AT(NBS) ensemble is designed so that they are independent to a very high degree [11], and the assumption of statistical independence seems to be valid. Thus, it is straightforward to show that the individual clock's stabilities may be sorted out using the following equation:

$$\sigma_i^2 = \frac{1}{2}\left[\left(\sigma_{ij}^2 - \sigma_{n_{ij}}^2\right) + \left(\sigma_{ik}^2 - \sigma_{n_{ik}}^2\right) - \left(\sigma_{ij}^2 - \sigma_{n_{jk}}^2\right)\right], \quad (11)$$

where σ_{ij}^2 is some kind of variance measure between clocks i and j, etc.; $\sigma_{n_{ij}}^2$ is the measurement system noise using the same variance measure between clocks i and j, etc.; and σ_i^2 is an estimate of the same variance measure for the i-th clock. Given three clocks one can permute i, j, and k to obtain estimates for j and k as well. Given L clocks in an ensemble there will be (L-1) ! / (2 ! (L-3) !) different (not all independent) estimates of σ_i^2. By taking an appropriate weighted combination--recognizing that some estimates will have much better confidence than others--one can get a best estimate along with the confidence of the estimate for this particular variance measure. If the measure is $\sigma_y^2(\tau)$, then one can repeat the above routine for different τ values, and thus estimate the stability characteristics for each member of the clock ensemble. Using the above technique we show the stability characteristics of Cs 323 clock in the AT(NBS) scale in Fig. 3. Note the characteristics of white noise FM, flicker noise FM, and of linear frequency drift.

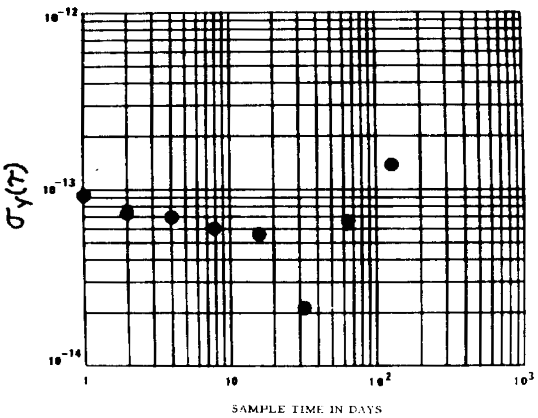

Fig. 3 - Estimated stability of Cs 323 as deduced from independent interclock comparisons.

In order to get an idea of the deterministic characteristics, such as the frequency drift, one needs a reference clock which is better than any one of the clocks being considered. The current algorithm used to generate AT(NBS) is designed so that the stability of AT(NBS) is better than the best clock in the ensemble. In principle this can be used as a reference clock, but has the disadvantage that it is not independent, and hence gives an optimistically biased estimate of the performance of any clock which is a member of the ensemble.

Fig. 4 is a plot of the frequencies of some of the clocks in the AT(NBS) ensemble over about the past 3 years with respect to AT(NBS). A rate compensation has been applied to each to normalize it nominally at its origin. As will be shown later, and as is evident from the plot, there is a significant relative frequency drift in the Cs 323 clock. This plot gives clear evidence for the need of a primary frequency standard with an accuracy of about 1 part in 10^{13} in order to reduce significantly the quadratic time dispersion that is apparently present in some cesium beam clocks.

Utilization of NBS-5 to Calibrate the AT(NBS) Ensemble

NBS-5, a new primary cesium beam frequency standard of the National Bureau of Standards, became operational at the end of November 1972, after which a multitude of experiments were conducted. NBS-5 was evaluated using the "power shift" and the "pulse" methods [2, 3]. Optimum microwave power is that setting which gives a maximum signal at the cesium detector. In order to evaluate the reproducibility of NBS-5 and to determine if there was any significant deterministic frequency drift in AT(NBS) or in some of the main clocks of which it is composed, we examined the frequency difference between NBS-5 and these clocks on each occasion when NBS-5 was operating at optimum power. These frequency differences are plotted in Fig. 5. Experimentally, we estimated that the microwave power could be set at optimum, reproducible to within a fractional frequency of 1×10^{-13}. In Fig. 5 the peak variations are about 1×10^{-13} of NBS-5 vs. AT(NBS) giving credence to this estimated uncertainty. The cesium beam intensity changed some over the course of these experiments because the cesium oven was often turned off and on between experiments. The last run, April 1973, was just prior to a recharge procedure of the cesium oven in NBS-5.

As can be seen, within the uncertainty of NBS-5's reproducibility at optimum power (1 part in 10^{13}) there is no significant frequency drift measurable in AT(NBS) or in the other clocks shown from 6 December 1972 to 6 April 1973. Note that there is a relative drift indicated in Fig. 5 between Cs 167 and Cs 323 consistent with that shown in Fig. 4 for these same clocks.

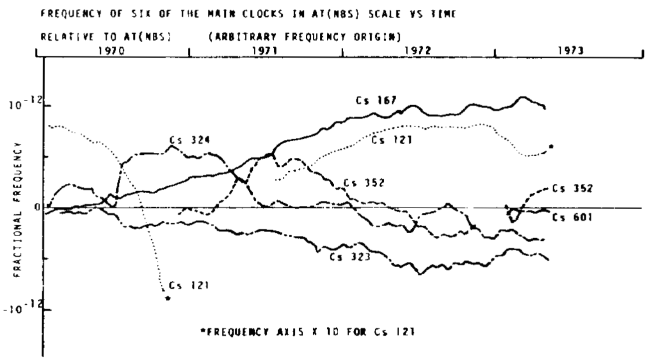

Fig. 4 - Relative fractional frequencies of 6 selected clocks in the AT(NBS) scale as a function of time with the AT(NBS) scale used as the reference.

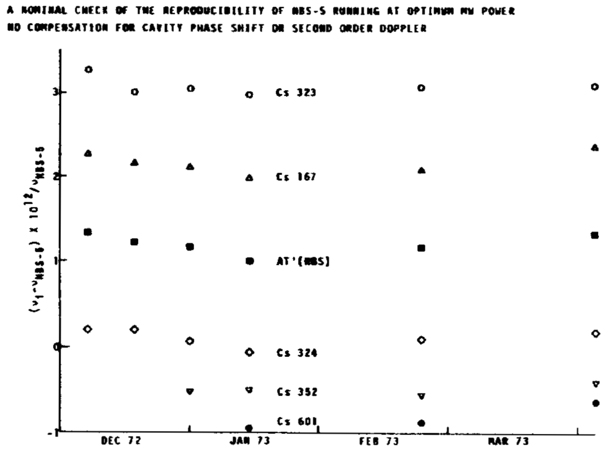

Fig. 5 - A check of the reproducibility of the frequency of NBS-5 using 6 selected clocks in the AT(NBS) scale as comparison standards. AT(NBS) is also plotted with respect to NBS-5 to determine if any systematic frequency drift is present in AT(NBS).

The UTC)NBS) scale, which is based on the same ensemble of cesium beam clocks, is coordinated to be in time and frequency agreement with the UTC scale maintained by the BIH (UTC-TAI = integral number of seconds, currently 12 s and the rates of UTC and TAI are the same). The rate of UTC(NBS) has been held constant within the capabilities of the NBS clock ensemble and the particular algorithm employed since 1 January 1973, and it is estimated to be within about 1 part in 10^{13} of the rate of UTC and within about 2 μs of time agreement. The frequency bias parameters for NBS-5 were evaluated sufficiently to give preliminary calibrations of the absolute rate of UTC(NBS), and hence, in essence, of UTC and TAI during January, February, March, and April of 1973. These are plotted in Fig. 6 along with the internal estimate of the accuracy associated with each preliminary calibration. The best estimate shown in Fig. 6 is based on some methods for optimal filtering [1], and would indicate that UTC(NBS) is too high in fractional frequency by $(10.4 \pm 5) \times 10^{-13}$. The uncertainty of 5×10^{-13} is larger than the internal estimates for most of the individual preliminary calibrations because we have included some accounting for aspects of NBS-5's performance which remain to be investigated more thoroughly [1, 2, 3][2]. Taking into account the gravitational red shift due to the elevation difference between the BIH and Boulder, Colorado, we would estimate that the rate of UTC or TAI is too high by about 12 ± 6 parts in 10^{13}.

[2] In [2] an uncertainty of 2×10^{-13} is quoted. However, this is a preliminary, tentative accuracy subject to further experimental verification.

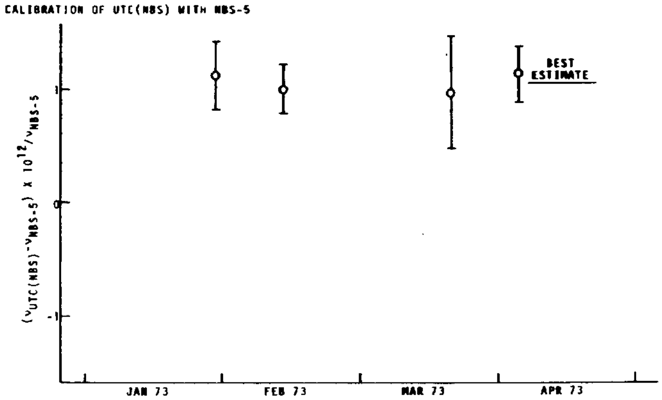

Fig. 6 - Relative fractional frequencies of six selected clocks in the AT(NBS) scale as a function with the AT(NBS) scale used as the reference.

During the dates of 13 to 22 January 1973 we measured several of the cesium beam clocks in the NBS clock ensemble with NBS-5 running at optimum power. This measurement was in an effort to observe the stability characteristics of the clocks as well as to calibrate their absolute frequency. Fig. 7 is an example of a $\sigma_y(\tau)$ vs. τ stability plot. The sample time τ is the time interval over which each of an adjacent pair of fractional frequency differences was averaged per Eq. (8). Cs 601 is a high-performance commercial cesium beam tube. For Cs 601 vs. NBS-5 we obtained a value for $\sqrt{h_o/2}$ of 7.6×10^{-12}. The contribution due to NBS-5 was about 4×10^{-12} as determined from short-term stability data vs. a quartz crystal oscillator (see Fig. 8). From these data one would conclude that Cs 601 had a white noise FM level of about 5×10^{-12} for $\sqrt{h_o/2}$.

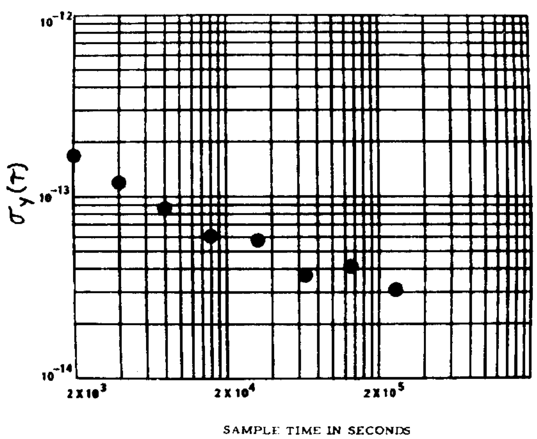

Fig. 7 - Fractional frequency stability of Cs 601 (a member of the AT(NBS) ensemble) as measured by NBS-5.

The apparent flattening at the right of Figs. 7 and 8 is characteristic of flicker noise FM, but the data were of insufficient length to confirm this characterization with confidence. It is not known whether this apparent flattening is due to Cs 601 or NBS-5, or both; more data are needed. Note the stability of $\sigma_y(\tau \geq 64,000\ \mathrm{s}) \leq 4 \times 10^{-14}$. Using Eq. (5), however, one calculates a time dispersion of 4.7 ns for Cs 601 at $\tau = 1$ day and with $\sqrt{2h_{-1}\ \ln 2} = 4 \times 10^{-14}$. A measured estimate of the time dispersion using the current AT(NBS) algorithm is 5 ns for Cs 601--quite good agreement in support of the proposed model. The corresponding random time fluctuations x(t) for Cs 601 vs. NBS-5 from the same measurement is shown in Fig. 9.

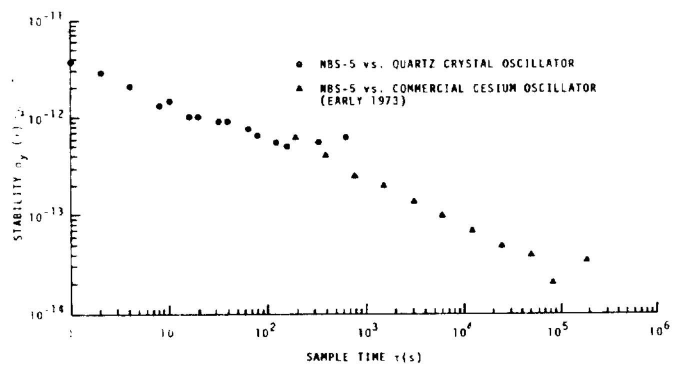

Fig. 8 - Fractional frequency stability comparison of NBS-5 vs. a quartz crystal oscillator in short term, and Cs 601 for longer sample times.

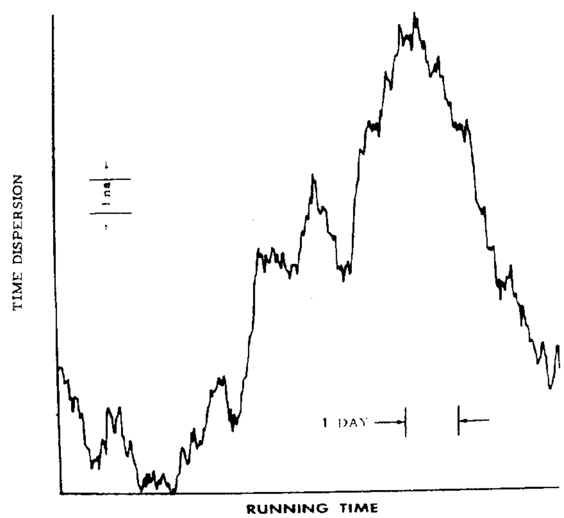

Fig. 9 - Random time fluctuations of Cs 601 (a member of the AT(NBS) ensemble) as measured by NBS-5.

Comparison of AT(NBS) with National and International Standards

Aside from NBS-5 the past three evaluated primary cesium beam frequency standards have been the NRC CsIII [12, 13], the PTB Cs 1 [14], and NBS-III [15]. We have plotted the fractional frequencies of the five time scales AT(F), AT(NBS), AT(NRC), AT(PTB), and AT(USNO), whose time dispersions were plotted in Fig. 1, plus that of TAI with reference to the four primary frequency standards mentioned above, and at the time each was evaluated. These results are shown in Fig. 10. The accuracies (1σ) quoted for these four standards taken chronologically are 5×10^{-13} for NBS-III, 4.5×10^{-13} for PTB Cs 1, 15×10^{-13} ($\sim 2\sigma$) for NRC Cs III, and 5×10^{-13} for NBS-5. The calibration periods for each were May 1969, March-July 1970, 2 July - 9 November 1970, and January-April 1973, respectively[3]. There are several very interesting observations one can make about the data shown in this figure. Some of them are: Most of the time scales are too high in frequency; AT(F) appears to have a negative frequency drift, while AT(NRC) and AT(USNO) appear to have a positive frequency drift; the fractional frequency of PTB Cs 1 appears to be lower than that of NRC Cs III by about 4×10^{-13} as observed from the correlation (this, of course, is well within the accuracies quoted); and the fractional frequency of AT(NBS) appears to have changed less than 1×10^{-13} over about 4 years according to the NBS-III and NBS-5 calibrations (this is much better than either the accuracies of NBS-III or NBS-5 or of the estimated frequency dispersion in AT(NBS)).

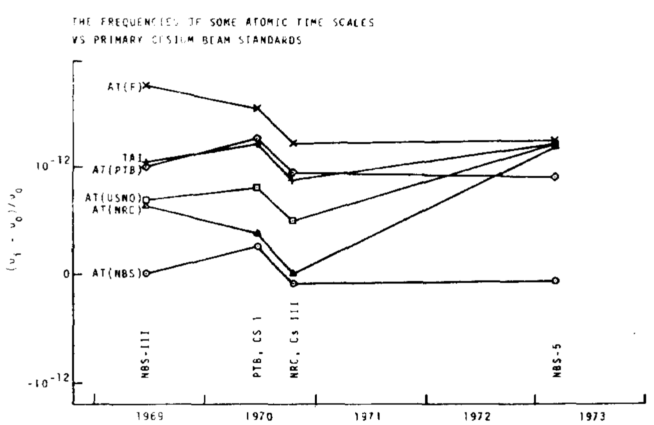

Fig. 10 - The frequency offsets of some AT scales as measured by different primary laboratory cesium beam frequency standards (via Loran-C).

[3] Data were presented at 12-14 June 1973 Frequency Control Symposium on NRC Cs V. The NRC staff have measured the fractional frequency of TAI at $(+10 \pm 2) \times 10^{-13}$ with respect to this new NRC frequency standard [16] in excellent agreement with NBS-5.

In Fig. 11 we have plotted the fractional frequencies of the same five scales during the same calibration periods but with respect to TAI instead of the primary standards. In this case both AT(F) and AT(NBS) appear to have negative frequency drifts. The uncertainty in each of the points plotted is about 1 or 2 parts in 10^{13} due to the uncertainty of frequency transfer via the Loran-C system [17].

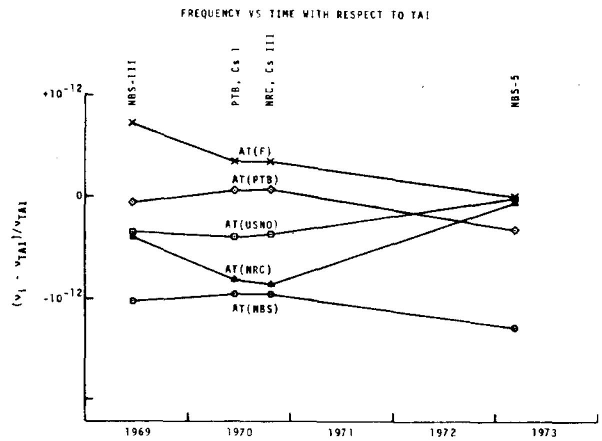

Fig. 11 - The frequencies of some atomic time scales as compared with TAI (via Loran-C) as measured during the same evaluation periods as for the data in Fig. 10.

Analyzing the stability characteristics of each of these five national time scales with respect to TAI using the data from the BIH annual report [4] and from their monthly Circular D Bulletin yields further confirmation of the validity of the model proposed in Section II. (Note: One must realize that the following results with TAI as the reference will be optimistically biased since each of these five time scales contributes to TAI.) In Fig. 12 we have plotted $\sigma_y(\tau)$ vs. τ for AT(F) as a typical example. The stability plotted on the left of each figure where $\sigma_y(\tau)$ decreases with increasing τ is the noise caused by the Loran-C system including its propagation fluctuations [17]. The flat portion is an optimistically biased estimate of the flicker noise FM of the particular clock ensemble. The $\sigma_y(\tau)$ proportional to τ^{+1} at the right may be caused by linear frequency drift. This seems to be confirmed when looking at the quadratic behavior of the time dispersion shown in Fig. 13, but one should be cautious about such appearances (e.g., step changes in frequency as mentioned earlier).

One can estimate linear frequency drift in several ways, e.g., by fitting a least-squares quadratic to the time fluctuations, a linear least-squares fit to the frequency fluctuations, or by using Eq. (6) which is a linear estimator. It is apparent from other considerations [8] that the last two examples are better estimators than the first but we have not determined which is better of the last two. Taking a linear least-squares fit to the frequency yields the results shown in Table 2. If we use Eq. (6) we obtain results in reasonable agreement with Table 2.

TABLE 2 - Determined from a Least-Squares-Fit to the Frequency with Respect to TAI (8 Jan 69-28 Mar 73)

AT(i)	Frequency Drift (Parts in 10^{13}/year)
F	- 1.6
NBS	- 1.2
NRC	+ 1.4
PTB	- 0.7
USNO	+ 1.3

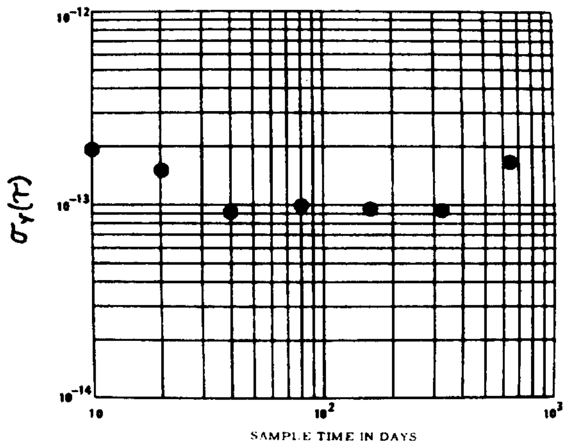

Fig. 12. - Fractional frequency stability of AT(F) with respect to TAI (no frequency drift removed). Typical of other AT scales.

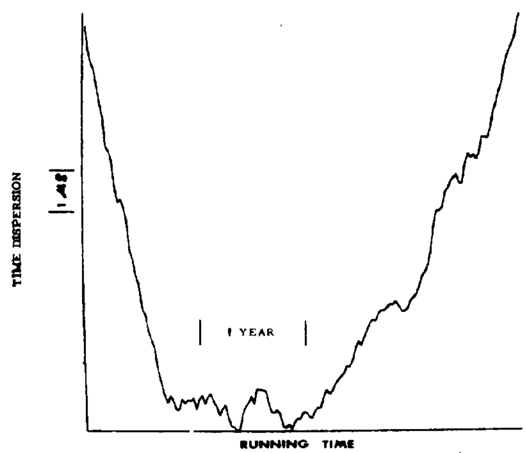

Fig. 13 - Time fluctuations of AT(F) with respect to TAI (no frequency drift removed). Typical of other AT scales.

After we removed a linear least-squares fit to the frequency we analyzed the remaining random fluctuations and the stability of AT(F) as an example is plotted in Fig. 14. It is interesting to note that most of the clock ensembles were characterized by flicker noise FM in the range 40 days $\leq \tau \leq$ 200 days. The least-squares fit applied to the data filters the very low frequencies [8] so that $\sigma_y(\tau)$ for τ = 320 days and τ = 640 days are decreased significantly. A typical example of the residual time fluctuations is shown for AT(F) in Fig. 15. Note how similar this plot is in character to the simulated flicker noise FM shown in Fig. 2.

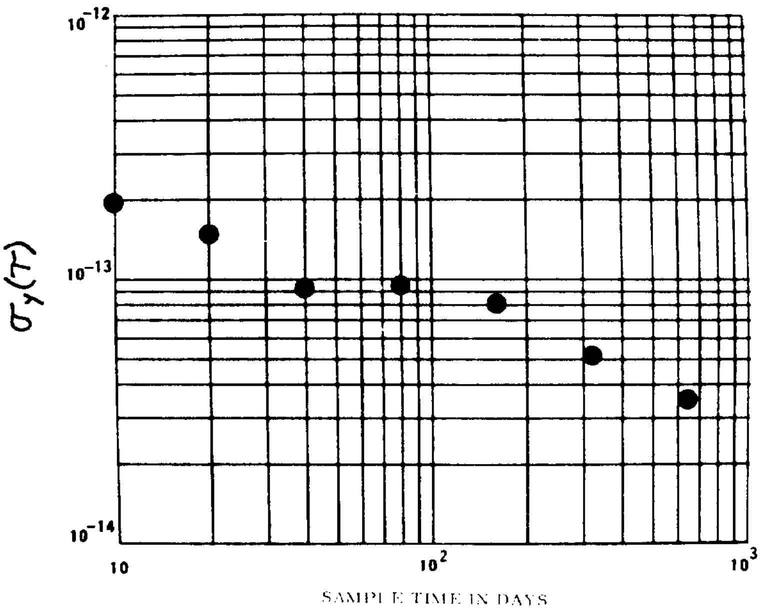

Fig. 14 - Fractional frequency stability of AT(F) with respect to TAI (frequency drift removed by least-squares fit to the frequency). Typical of other AT scales.

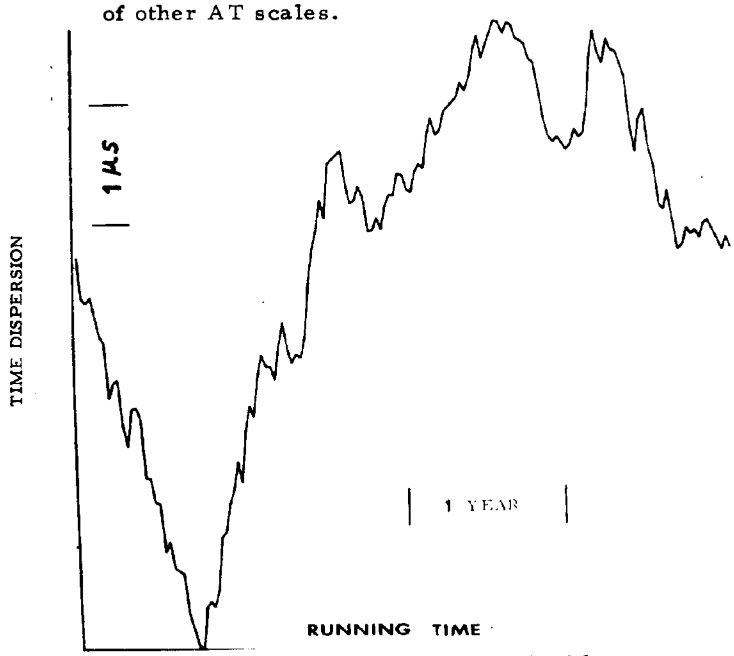

Fig. 15 - Residual time fluctuations of AT(F) with respect to TAI (frequency drift removed by least-squares fit to the frequency). Typical of other AT scales.

IV. Objective Filtering of Time and Frequency Steps in a Clock

It is commonly recognized that discrete steps in time and in frequency occasionally occur in cesium beam clocks [5]. In such cases diagnostic routines should be implemented to objectively ascertain if a change in frequency is statistically significant. If such steps occur the clock model given by Eq. (2) is not complete, but needs a reevaluation of $T_i(t_o)$ or $R_i(t_o)$ if a time or frequency step occurs respectively (for convenience call these T-steps or F-steps respectively).

An F-step may be detected as follows: First, take the first derivative of the frequency $\frac{dy(t)}{dt}$. Define the signal s to be an F-step, which will be a δ-function after the derivative is taken. The spectral density of the signal $S_s(f)$ is a constant from zero hertz up to the Nyquist frequency of the assumed discrete data. Define the noise as that given by the proposed model in Eq. (2) via the above time derivative, i.e., $\frac{d^2}{dt^2}(T_i(t)) = \ddot{z}(t)$. Using Eq. (3) we find the spectral density of the noise to be:

$$S_{\ddot{z}}(f) = (2\pi)^2 h_o f^2 + (2\pi) h_{-1} f. \tag{12}$$

Taking the second time derivative removes the first two terms of Eq. (2), viz., time and frequency offsets; however, the frequency drift term causes $\ddot{z}$ to have a non-zero average. With signal and noise defined, the problem becomes a classic one in optimum signal detection in the presence of noise. We desire to determine if the signal is present or not, and therefore need to design for a maximum signal-to-noise ratio. It is clear by examining the spectral densities for the signal and noise above that this can be accomplished with an appropriate low-pass filter. Since the size of the F-steps is not known a priori, it generally may be desirable to use filters with different time constants so that big steps may be recognized quickly while smaller ones take more time. Once an F-step is recognized one can return to quantify it with an optimum filter time constant. Compromises, also, often have to be made due to finite data lengths, i.e., as the filter time constant gets longer the degrees of freedom reduce. Further, one may not be willing to wait too long before correcting and updating a time scale's performance since its time readings are typically periodically published as an ongoing process (backtracking is difficult).

Fig. 16a shows the results of processing 608 days of the time difference $T_{Cs\ 167}(t) - T_{Cs\ 323}(t)$ through an F-step detector consisting of a cascade of three R-C low-pass filters--each with a time constant of 10 days. The time constant of the cascade is $3\tau = 17$ days; hence, the plot represents $608/17 \simeq 36$ degrees of freedom. Thus for normally distributed noise, the probability of one excursion past a 3σ limit is less than 10%. We conclude that with reasonable probability, two F-steps were detected between the clocks during the 608 days. Notice also that the average is biased above zero due to the relative frequency drift between the two cesium clocks. This bias amounts to 0.18 ns/day^2 or 7.4×10^{-13}/year.

The same data--after removing the frequency drift, the two F-steps, and the one T-step--were processed through the same step detector and the output of the processor is shown in Fig. 16b. (The T-step detector is explained below.) A plot of the fractional frequency of both the raw data and the data corrected for the two F-steps and the one T-step is shown in Fig. 17. Each data point is a 40-day average. A time fluctuation plot of the same corrected data is also shown in Fig. 18, and a time-domain stability plot is shown in Fig. 19 as the circles "o".

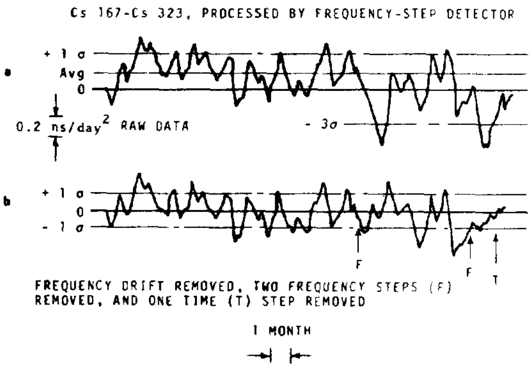

Fig. 16a - Second finite difference of the time differences (~ second derivative of the time differences) between Cs 167 - Cs 323 processed through an F-step detector to ascertain if frequency steps have occurred.

Fig. 16b- Same as 16a but with frequency drift, 2 F-steps, and 1 T-step removed.

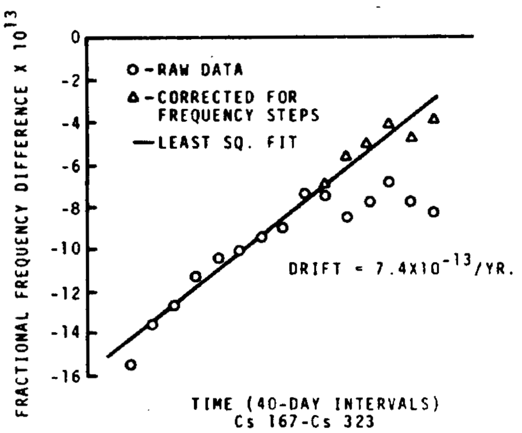

Fig. 17 - Fractional frequency of Cs 167 - Cs 323 with and without corrections applied.

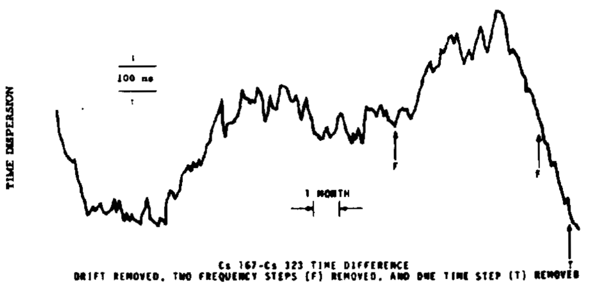

Fig. 18 - Random time fluctuations of Cs 167 - Cs 323 after removal of frequency drift, 2 F-steps, and 1 T-step.

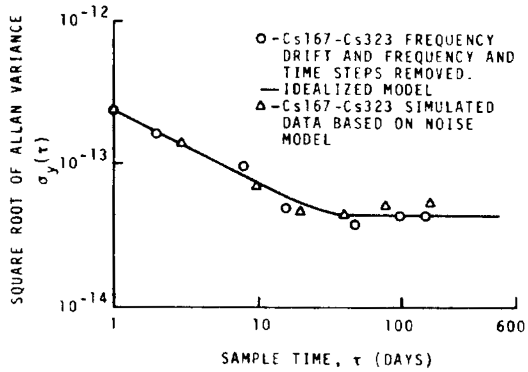

Fig. 19 - Fractional frequency stability of Cs 167 - Cs 323 after removal of frequency drift, 2 F-steps, and 1 T-step.

Once the stability characteristics are known, these data can be simulated using Eq. (9). This was done and the time-domain stability plot of these simulated data are shown as the triangles "Δ" in Fig. 19; also shown as the solid line in Fig. 19 is the theoretical stability given by Eq. (4). Note the apparent flicker noise FM level of about 4×10^{-14}. The simulated data were also processed through the same step detector and the resultant output is shown in Fig. 20--the appearance being very similar to that shown in Fig. 16b for the real clocks after corrections. Fig. 21 shows the simulated time fluctuations for the same clock pair, and again the characteristic appearance is very similar to the real clock data after corrections shown in Fig. 18. A least-squares fit to the frequency of the simulated data was also determined where such a drift was known not to exist. We measured 0.67×10^{-13}/year more than a factor of 10 smaller than was measured for the real clock data.

To detect optimally T-steps, invert Eq. (9) as follows:

$$y_{n+1}^{(2)} = 2\left[y_{n+1}^{(3)} - (1 - \gamma^{(3)})\, y_n^{(3)} + (\tfrac{1}{2} - \gamma^{(3)})\, y_n^{(2)}\right], \quad (13a)$$

$$y_{n+1}^{(1)} = 2\left[y_{n+1}^{(2)} - (1 - \gamma^{(2)})\, y_n^{(2)} + (\tfrac{1}{2} - \gamma^{(2)})\, y_n^{(1)}\right], \quad (13b)$$

and

$$y_{n+1}^{(0)} = 2\left[y_{n+1}^{(1)} - (1 - \gamma^{(1)})\, y_n^{(1)} + (\tfrac{1}{2} - \gamma^{(1)})\, y_n^{(0)}\right]. \quad (13c)$$

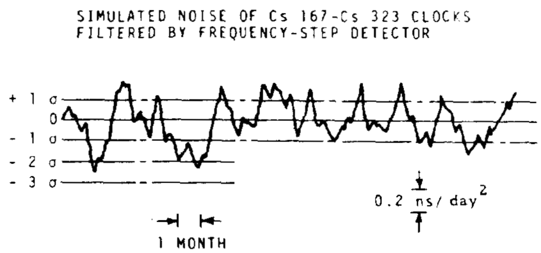

Fig. 20 - Computer simulated data for Cs 167 - Cs 323 processed through same F-step detector as in 16 above.

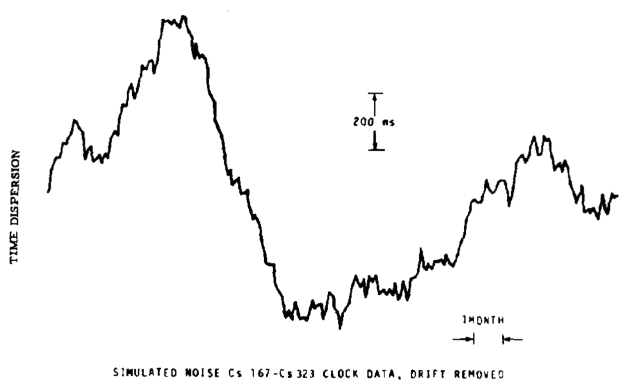

Fig. 21 - Random time fluctuations of computer simulated model of Cs 167 - Cs 323 (to be compared with 18 above).

Using the stability characteristics as determined from Fig. 19, calculate $\gamma^{(1)}$, $\gamma^{(2)}$, and $\gamma^{(3)}$ as in Eq. (9). (A T-step has very little effect on the stability characterization and $\sigma_y(\tau)$ was actually calculated prior to the determination of the T-step, even though the stability plotted in Fig. 19 was made *after the* determination and removal of the T-step.) In inverse order of Eq. (9) the input to the filter (recursive sequence) is the real clock data corrected for frequency drift and F-steps; these data are entered at $y_{n+1}^{(3)}$. And, similar to Eq. (9), the output is the $y_{n+1}^{(0)}$ which in theory will be random uncorrelated numbers, RUNs, with unit variance after similar normalization as in Eq. (9). Further, if the real clock data are normally distributed then the $y_{n+1}^{(0)}$ output will be also. If a T-step occurs, once this sequence is initiated, it will generate through the recursion sequence a perturbed point which, with some probability, will exceed the probable excursion for the sample size. For the particular clock pair used above a step in the time difference of - 86 ns ± 14 ns was detected at the point indicated in Figs. 16b and 18. For the forward pass on the data set the T-step was out by 4.2 σ. The confidence of the estimate was improved after the fact to 14 ns by processing the data backward to where the T-step occurred.

Finally in Fig. 22 the inverse filtered data were plotted to test for distribution and variance after the T-step was removed. The data are obviously in very good agreement with the theoretical model hypothesized.

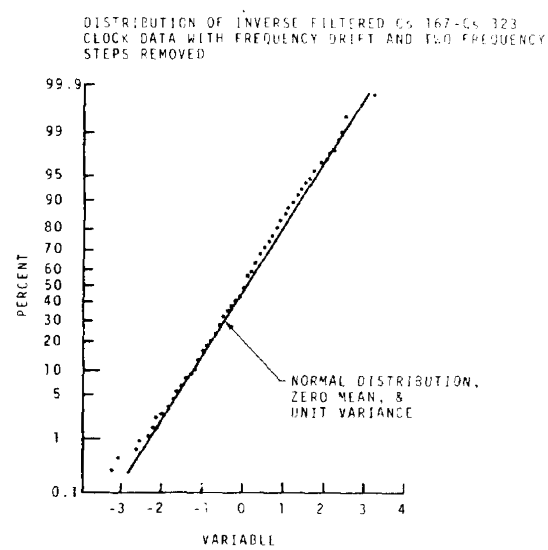

Fig. 22 - The corrected data for Cs 167 - Cs 323, inverse filtered and plotted to test for normal distribution and unit variance.

V. Conclusions

A tractable well-fitting model for the time dispersion in cesium beam atomic clocks has been developed. The greatest time disperion--which is quadratic with time--may result from a linear frequency drift which has been observed in individual clocks and in clock ensembles. The size of this drift varies from unit to unit and ensemble to ensemble, and values have been measured from less than 1 part in 10^{13} per year up to several parts in 10^{13} per year. The above dispersion characteristic alone points out the need for an evaluable primary frequency standard with an accuracy of about 1 part in 10^{13}. The operation of NBS-5 has helped considerably in ascertaining the degree of some of these frequency drift problems. *It is apparent* that subsequent calibrations with NBS-5 and with primary standards from other national laboratories will be very meaningful.

Specifically, NBS-5 was used to ascertain if there were any measurable drift in the AT(NBS) scale, and within an uncertainty of 1 part in 10^{13} no observable drift was measured between 6 December 1972 and 6 April 1973. In addition, we calibrated the rate of AT(NBS) and the rate of UTC(NBS) using NBS-5. The current best estimate of the rate for UTC(NBS) is $+10.4 \pm 5$ parts in 10^{-13}. The last calibration of AT(NBS) prior to NBS-5 was with NBS-III in May 1969. Using AT(NBS) as a memory, NBS-III and NBS-5 agree in frequency to within 1 part in 10^{13}.

Linear rms time dispersion may result from either an error in the frequency calibration of a clock or from the clock being perturbed by flicker noise FM. The rms time dispersion due to the various levels of flicker noise FM measured in individual clocks and ensembles varied from about 1 to 8 μs per year. In contrast, the four last calibrations with evaluable primary cesium beam frequency standards would indicate that most of the major atomic time scales in the world, including UTC and TAI, are currently too high in frequency by about 1 part in 10^{12}; and, hence, their time scales run too fast by about 30 μs per year compared to a clock more nearly in agreement with the above primary standards. Specifically, NBS-5 measures TAI to be too high in frequency by about 1.2 parts in 10^{12}.

Diagnostic routines have been developed to detect optimally steps in either the time or the frequency of a clock. These diagnostics were tested on a pair of clocks in the AT(NBS) ensemble, and detected two frequency steps and one time step in 608 days of data. A method for simulating a cesium beam clock's performance is developed and utilized to test the diagnostics. An obvious opportunity is available to use simulated clock data to test the performance of a variety of time scale algorithms that are currently being used throughout the world.

With the availability of NBS-5 we were also able to measure the white noise FM, which is another important performance characteristic of cesium beam clocks, to high precisions and over many sample time decades. The time dispersion due to this kind of noise is proportional to the square root of time. Using NBS-5 as the reference a root mean square time dispersion for a high performance commercial cesium beam clock was estimated to be:

$$\left\langle \epsilon_{opt}^{2}(\tau) \right\rangle_{Cs\ 601}^{1/2} \simeq \left(2.5 \times 10^{-24} s \cdot \tau + 27 \times 10^{-28} \tau^{2}\right)^{1/2} \qquad (14)$$

over the range of $1\ s \leq \tau \leq 2 \times 10^{5}$ s. For $\tau = 1$ day Eq. (14) calculates to be 4.7 ns. The AT(NBS) scale algorithm is designed to fully utilize this performance.

Acknowledgments

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