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Frequency Stability Specification and Measurement: High Frequency and Microwave Signals

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Frequency Stability Specification and Measurement: High Frequency and Microwave Signals

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FREQUENCY STABILITY SPECIFICATION AND MEASUREMENT: HIGH FREQUENCY AND MICROWAVE SIGNALS*

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This report gives concise definitions for specifying frequency stability for measurements in the frequency domain and time domain. Standards of terminology and of measurement techniques are recommended. Measurement systems in the high frequency and microwave regions are described in adequate detail so that the systems may be duplicated.

Key Words: Allan variance; Frequency stability measurements; Measurement system description; Phase noise; Spectral density; Stability definitions; Terminology standards.

I. INTRODUCTION AND BACKGROUND

At the beginning of FY-71 the Department of Defense Joint Services Calibration Coordinating Group (DoD/CCG), J. L. Hayes (Chairman), Metrology Engineering Center, Pomona, California, contracted with the National Bureau of Standards (NBS) to write a paper pertaining to the specification and measurement of frequency stability. The project was under the jurisdiction of the DoD/CCG Time and Frequency Working Group. The current members of this group are Peter Strucker (Chairman), Metrology Engineering Center, Pomona, California; J. M. Rivamonte, U. S. Army Metrology and Calibration Center, Redstone Arsenal, Alabama; and E. L. Kirkpatrick, Aerospace Guidance and Metrology Center, Newark Air Force Station, Newark, Ohio.

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The first year's work (FY-71) culminated as NBS Report 9794 [1]. The second year's work (FY-72) extended the frequency range to include a working measurement system at X-band. The results of work done in both FY-71 and FY-72 are documented in this paper.

The purpose of the project is to establish recommended standards of terminology and measurement techniques for frequency stability. Emphasis is placed on details of useful working systems (apparatus) that could be duplicated by others in the field of frequency stability measurements. Uniformity of data presentation is stressed in order to facilitate interpretation of stability specifications and to enable one to communicate and compare experimental results more readily. An authoritative paper [2] by the Institute of Electrical and Electronics Engineers (IEEE) Subcommittee on Frequency Stability is used extensively as the prime reference in the preparation of this report. This report presents the description and performance of frequency stability measurement systems capable of precise measurements on state-of-the-art sources. Sufficient theory of the measurement systems is given to enable a person to readily understand the principle of operation.

2. TERMINOLOGY FOR SPECIFICATION OF FREQUENCY STABILITY

The term frequency stability encompasses the concepts of random noise, intended and incidental modulation, and any other fluctuations of the output frequency of a device. In this report we are mainly (but not totally) concerned with random fluctuations corresponding to Fourier frequencies in the 10^0 to 10^6 hertz range. The measurement of frequency stability can be accomplished in both the frequency domain (e.g., spectrum analysis) and the time domain (e.g., gated frequency counter).

In the aforementioned manuscripts [1], [2] the authors chose to use two independent definitions, each related to different useful methods of measurement. (See Appendix A for a Glossary of Symbols.)

The frequency domain definition of frequency stability is the one-sided spectral density of the fractional frequency fluctuations, $S_y(f)$, where $y \equiv \delta\nu/\nu_o$. The fractional frequency fluctuation spectral density $S_y(f)$ is not to be confused with the radio frequency power spectral density $S_{\sqrt{RFP}}(\nu)$, nor with $S_{\delta V}(\nu)$, which are not good primary measures of frequency stability [2]. (There is some discussion of this in Appendix D.) Phase noise spectral density plots [i.e., $S_{\delta\phi}(f)$ versus f] are a common alternative method of data presentation. The spectral density of phase fluctuations is related to $S_y(f)$ by

$$S_{\delta\phi}(f) = (\nu_o^2/f^2)\, S_y(f) \quad . \tag{1}$$

The time domain definition of frequency stability uses the type of sample variance called the Allan variance [3] of y:

$$\langle \sigma_y^2(N, T, \tau, f_h) \rangle \equiv \left\langle \frac{1}{N-1} \sum_{n=1}^{N} \left(\bar{y}_n - \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k\right)^2 \right\rangle \quad . \tag{2}$$

The particular Allan variance with N = 2 and $T = \tau$ is found to be especially useful in practice. It is denoted by:

$$\sigma_y^2(\tau) \equiv \langle \sigma_y^2(N=2, T=\tau, \tau, f_h) \rangle = \left\langle \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2} \right\rangle \quad . \tag{3}$$

The bar over the y indicates that y has been averaged over a specified time interval τ. The angular brackets indicate an average of the quantity over time. (See example in Appendix F.)

In the time domain we are concerned with the measurement of Allan variances at different time intervals. Plots of $[\sigma_y^2(\tau)]^{\frac{1}{2}}$ versus τ ("sigma versus tau") on a log-log scale are commonly used for data presentation. A convenient chart which enables one to translate from frequency domain measures to time domain measures (and often conversely) is found in Appendix B. An example of this translation is given in Appendix C.

Script $\mathscr{L}(f)$ is a frequency domain measure of phase fluctuations (noise, instability, modulation) used at NBS. Script $\mathscr{L}(f)$ is defined as the ratio of the power in one phase noise sideband, referred to the input carrier frequency, on a per hertz of bandwidth spectral density basis, to the total signal power, at Fourier frequency f from the carrier, per one device. (See Appendix D.)

$$\mathscr{L}(f) \equiv \frac{\text{Power density (one phase modulation sideband)}}{\text{Power (total signal)}} . \tag{4}$$

For small $\delta\phi$,

$$S_{\delta\phi}(f) = 2\mathscr{L}(f) . \tag{5}$$

A practical system for the measurement of Script $\mathscr{L}(f)$ or $S_{\delta\phi}(f)$ will be described in detail later.

It seems appropriate here to discuss briefly the types of noise that affect the output frequency of all known signal sources. The noises can be characterized by their frequency dependence.

Common Name	$S_{\delta\phi}(f)$	$S_{\delta\nu}(f) = f^2 S_{\delta\phi}(f)$
White Phase Noise	f^0	f^{+2}
Flicker Phase Noise	f^{-1}	f^{+1}
White Frequency Noise (Random walk of phase)	f^{-2}	f^0
Flicker Frequency Noise	f^{-3}	f^{-1}

3. OPERATIONAL SYSTEMS FOR MEASUREMENT OF FREQUENCY STABILITY AT NBS (HIGH FREQUENCY REGION)

John H. Shoaf and Donald Halford

Most conventional systems for measurement of frequency stability until recently have utilized time domain techniques primarily. As indicated previously, in order to have a comprehensive and sufficient measure of frequency stability, it is preferred that the measurements involve both frequency and time domain techniques.

Fortunately, frequency domain and time domain methods for measuring frequency stability require similar apparatus except that: to make measurements in the frequency domain you must have a frequency window (spectrum analyzer) following the detector; for the time domain you must have a time window (gated counter) following the detector.

It was the introduction of good double-balanced mixers that permitted measurement of frequency stability by improved techniques [4]-[7], [11]. The double-balanced mixer, considered as a phase sensitive detector, makes possible meaningful frequency stability measurements of high-quality signal sources in both the frequency domain and the time domain. The results are quantitative and may be obtained from a measurement system which is reasonable in cost.

The frequency stability measurement systems described below have been used at NBS since 1967. The functional block diagrams in figures 1, 7, 9, and 10 are referred to in the detailed descriptions of the particular systems. The carrier frequency range 10^3 Hz to 10^9 Hz is easily covered with these techniques.

3.1. Frequency Domain Measurements

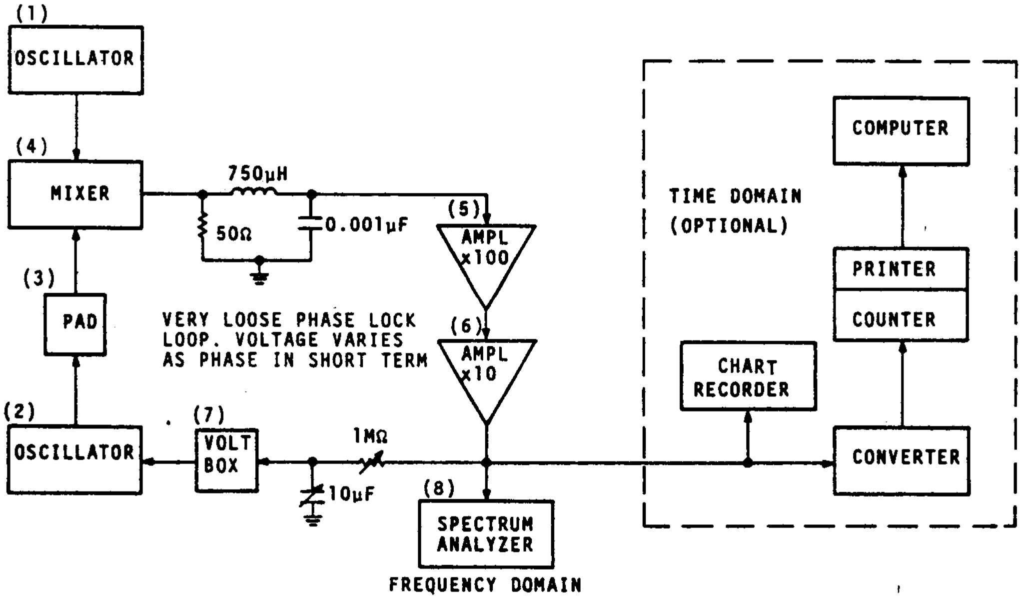


FIGURE 1: TYPICAL FREQUENCY DOMAIN MEASUREMENT OF FREQUENCY STABILITY (PHASE SENSITIVE MODE)

(1) OSCILLATOR UNDER TEST
(2) REFERENCE OSCILLATOR OF HIGH SPECTRAL PURITY (VARACTOR TUNING)
(3) ADJUSTABLE ATTENUATOR (TYPICALLY 10dB)
(4) DOUBLE-BALANCED MIXER (SCHOTTKY BARRIER DIODES)
(5) NBS LOW-NOISE dc AMPLIFIER (SEE FIGURE 3)
(6) OPERATIONAL AMPLIFIER (SEE FIGURE 2)
(7) BATTERY BIAS BOX (SEE FIGURE 5)
(8) SPECTRUM ANALYZER (LOW FREQUENCY, HIGH RESOLUTION)

(Figures 1, 7, and 10 show circuit values appropriate for measuring high quality 5-MHz quartz crystal oscillators.)

Figure 1 illustrates the measurement system typically used at NBS for <u>frequency domain</u> measurements. It should be noted that time domain data can also be obtained simultaneously, although usually this system is used only for frequency domain measurements. (For time domain measurements it is often more convenient to use a slightly modified measurement setup to be described later.) In this frequency domain setup the oscillator under test is fed into one side of a low-noise double-balanced mixer which utilizes Schottky barrier diodes. The reference oscillator is fed into the other side of the mixer through an attenuator, typically 10 dB. The mixer acts as a phase sensitive detector so that when the two signals are identical in frequency and are in phase quadrature the output is approximately zero volts dc. When this output is sent back to the reference oscillator via the varactor tuning, phase lock is achieved. As with any feedback system, care must be observed to avoid instabilities, e.g., servo loop oscillations. The phase-lock loop contains proper termination at the output of the mixer followed by operational amplifiers with adjustable gain. The time constant of the loop may be adjusted as needed by varying the amplifier gain within the loop and by use of the RC filter (fig. 4) indicated in the diagram. Finally, a battery bias box is included at the varactor input in order to operate in a suitably linear portion of the varactor's frequency versus voltage curve.

A very loose phase-lock loop is indicated inasmuch as the voltage varies as phase (in short term), and in this frequency domain measurement we are observing the small phase variations directly. By the phrase very loose phase-lock loop, we mean that the bandwidth of the servo response is small compared to the lowest frequency f at which we wish to measure (i.e., the response time is very slow). Operational amplifiers are arranged in a circuit as shown in figure 2 for convenience of adjusting the gain and for self-contained battery supply voltage. Special NBS low-noise dc amplifiers used in certain precision measurements are shown in figure 3. At NBS we have arranged in a small chassis the adjustable RC or CR filters utilizing low-noise components, with rotary switches for various combinations of R and C (see fig. 4).

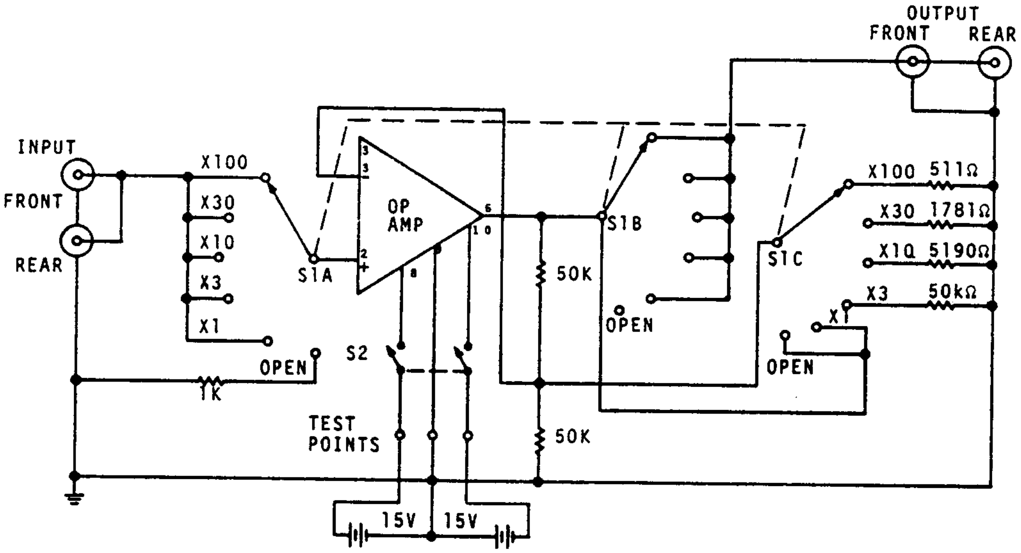
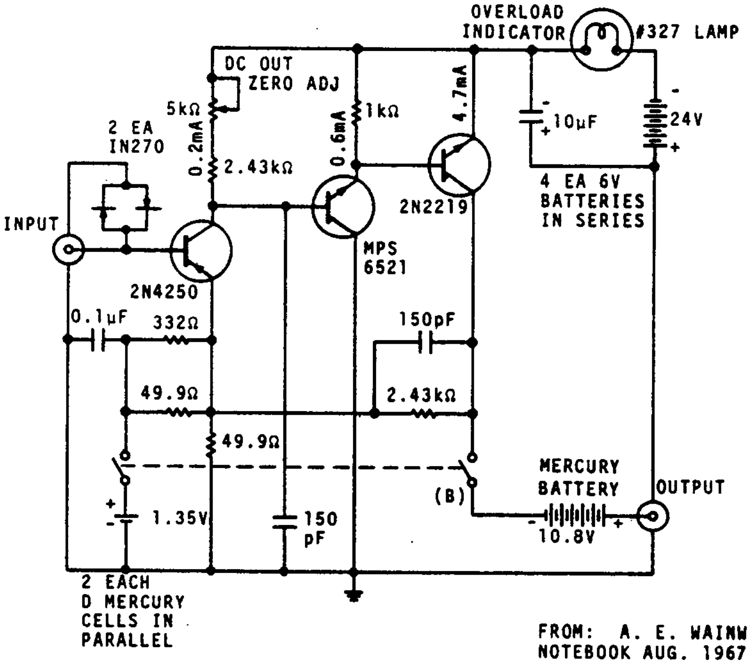


FIGURE 2: STEPPED GAIN OPERATIONAL AMPLIFIER



FROM: A. E. WAINWRIGHT
NOTEBOOK AUG. 1967, p. 111

FIGURE 3: LOW NOISE AMPLIFIER

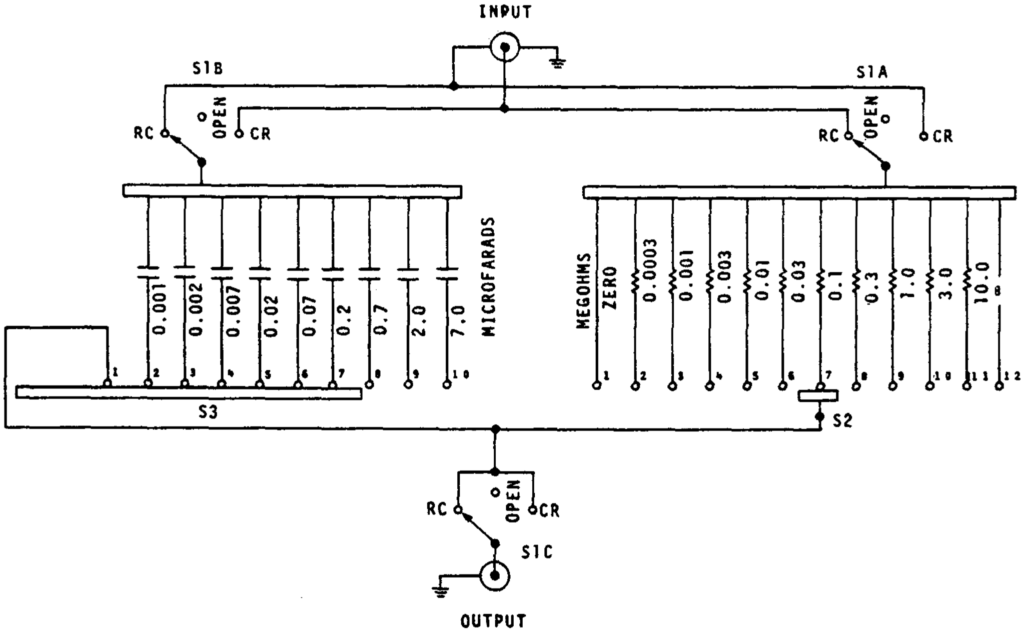


FIGURE 4: ADJUSTABLE RC FILTER

S1: FILTER MODE SWITCH (ROTARY, 3 WAFER)
S2: RESISTOR SWITCH (ROTARY, SHORTING TYPE)
S3: CAPACITOR SWITCH (ROTARY, PROGRESSIVE SHORTING)

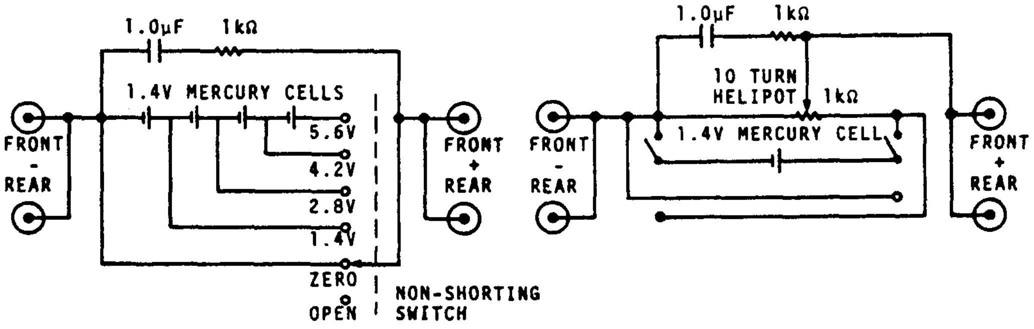
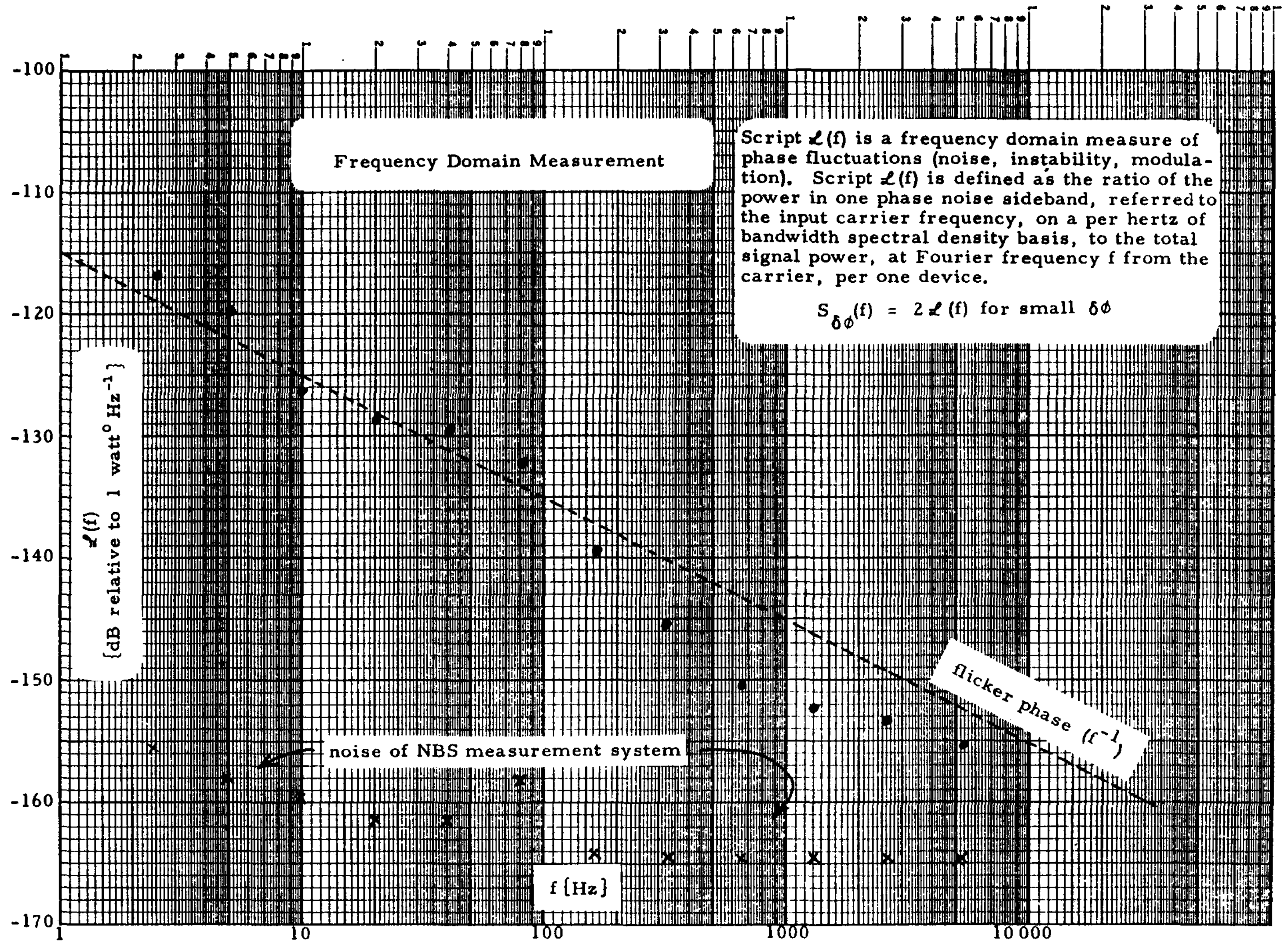


FIGURE 5: BATTERY BIAS BOX

THE UNITS ARE BUILT ON SEPARATE CHASSIS AND CONNECTED IN SERIES TO FACILITATE FINE ADJUSTMENT BETWEEN STEP VOLTAGES

The battery bias box is arranged with vernier as shown in figure 5, facilitating fine frequency adjustments via the varactor frequency adjustment in one oscillator. A wave analyzer is used to obtain the noise plot information relevant to stability (frequency domain). The phase noise sideband levels are read out in rms volts on the analyzer set to certain chosen values of frequency, f. For typical high quality signal sources this corresponds to measuring only those phase noise sidebands which are separated from the carrier by the various f intervals chosen. Script $\mathcal{L}(f)$ may be calculated with the assumption that both sources contribute equally; however, if one source were the major contributor, then the noise of that source would be no worse than 3 dB greater than the value of Script $\mathcal{L}(f)$ so calculated. A typical plot is shown in figure 6. A sample calculation may be found in Appendix E.



Script ℒ(f) is a frequency domain measure of phase fluctuations (noise, instability, modulation). Script ℒ(f) is defined as the ratio of the power in one phase noise sideband, referred to the input carrier frequency, on a per hertz of bandwidth spectral density basis, to the total signal power, at Fourier frequency f from the carrier, per one device.

$$S_{\delta\phi}(f) = 2\mathcal{L}(f) \text{ for small } \delta\phi$$

FIGURE 6: SCRIPT ℒ(f) VERSUS FREQUENCY f

3.2. Time Domain Measurements

In figure 7 a measurement system typically used at NBS for stability measurements in the <u>time domain</u> is shown. It will be noted

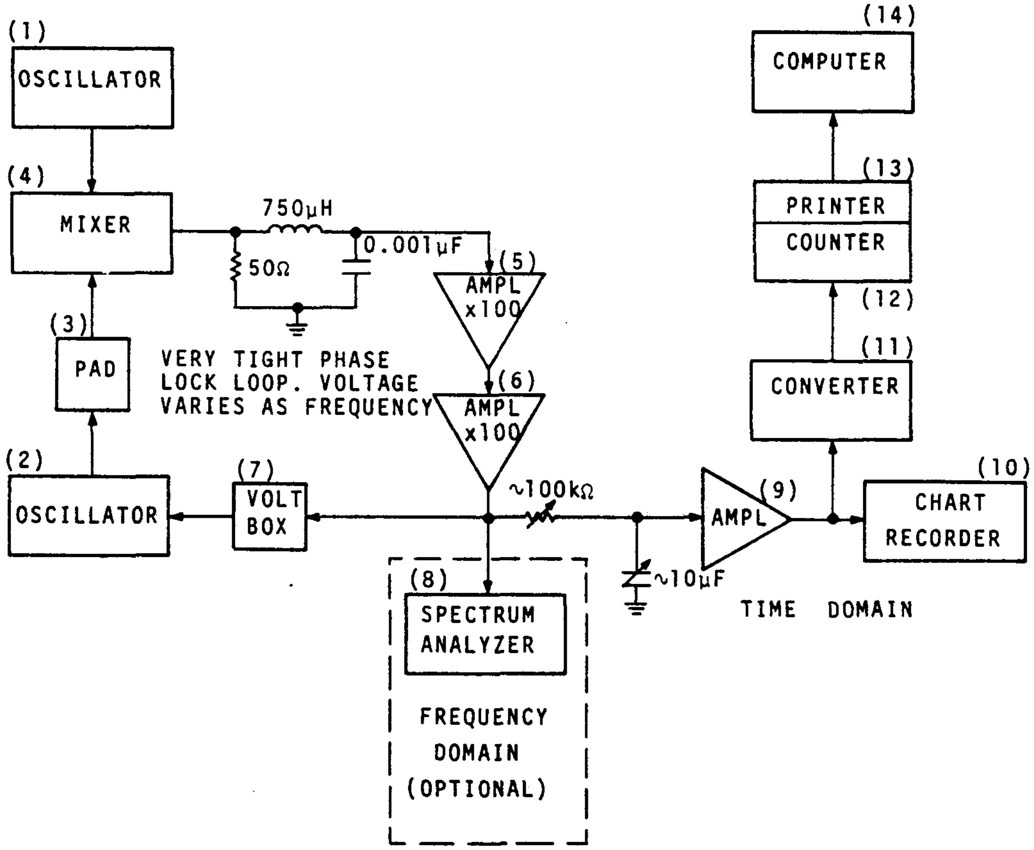


FIGURE 7: TYPICAL TIME DOMAIN MEASUREMENT OF FREQUENCY STABILITY (FREQUENCY SENSITIVE MODE)

ITEMS (1) THROUGH (8) SAME AS FIGURE 1
(9) OPERATIONAL AMPLIFIER
(10) STRIP CHART RECORDER FOR QUALITATIVE OBSERVATION
(11) VOLTAGE-TO-FREQUENCY CONVERTER
(12) FREQUENCY COUNTER WITH LOW DEAD TIME
(13) DIGITAL RECORDER WITH FAST RECORDING SPEED
(INHIBIT TIME COMPATIBLE WITH COUNTER DEAD TIME)
(14) COMPUTER (OPTIONAL METHOD OF DATA ANALYSIS)

that the principle of operation is similar to that used in the frequency domain measurement wherein the reference oscillator is locked to the test oscillator. However, for the time domain measurement we use a *very tight phase-lock loop* and the correction voltage at the oscillator varies as frequency. By the phrase *very tight phase-lock loop*, we mean that the bandwidth of the servo response is relatively large (i.e., the response time is much smaller than the smallest time interval τ at which we wish to measure). Caution--there are problems which are potentially present in tight phase-lock loop systems. This is a very convenient setup for observing frequency fluctuations in longer term. However, with the time constant appropriately adjusted and the means for taking sufficiently fast samples the system is readily used for short term measurements, as well as for the longer term measurements, in the time domain. For qualitative observations any suitable oscilloscope or strip chart recorder may be used. For quantitative measurements the system at NBS utilizes a voltage-to-frequency converter, a frequency counter, and a printer capable of recording rapid samples of data with very short dead time. The data are analyzed typically by computer via a program designed to compute the appropriate Allan variance [2], [3]. In our computer program log σ versus log τ along with the associated confidence in the σ are automatically plotted on microfilm. For small batches of data a desk calculator could be used and the computer analysis would not be necessary. An example of a specific Allan variance computation is shown in Appendix F. A typical plot of log σ versus log τ is shown in figure 8. The dashed lines indicate the *slopes* which are characteristic of the types of noise indicated.

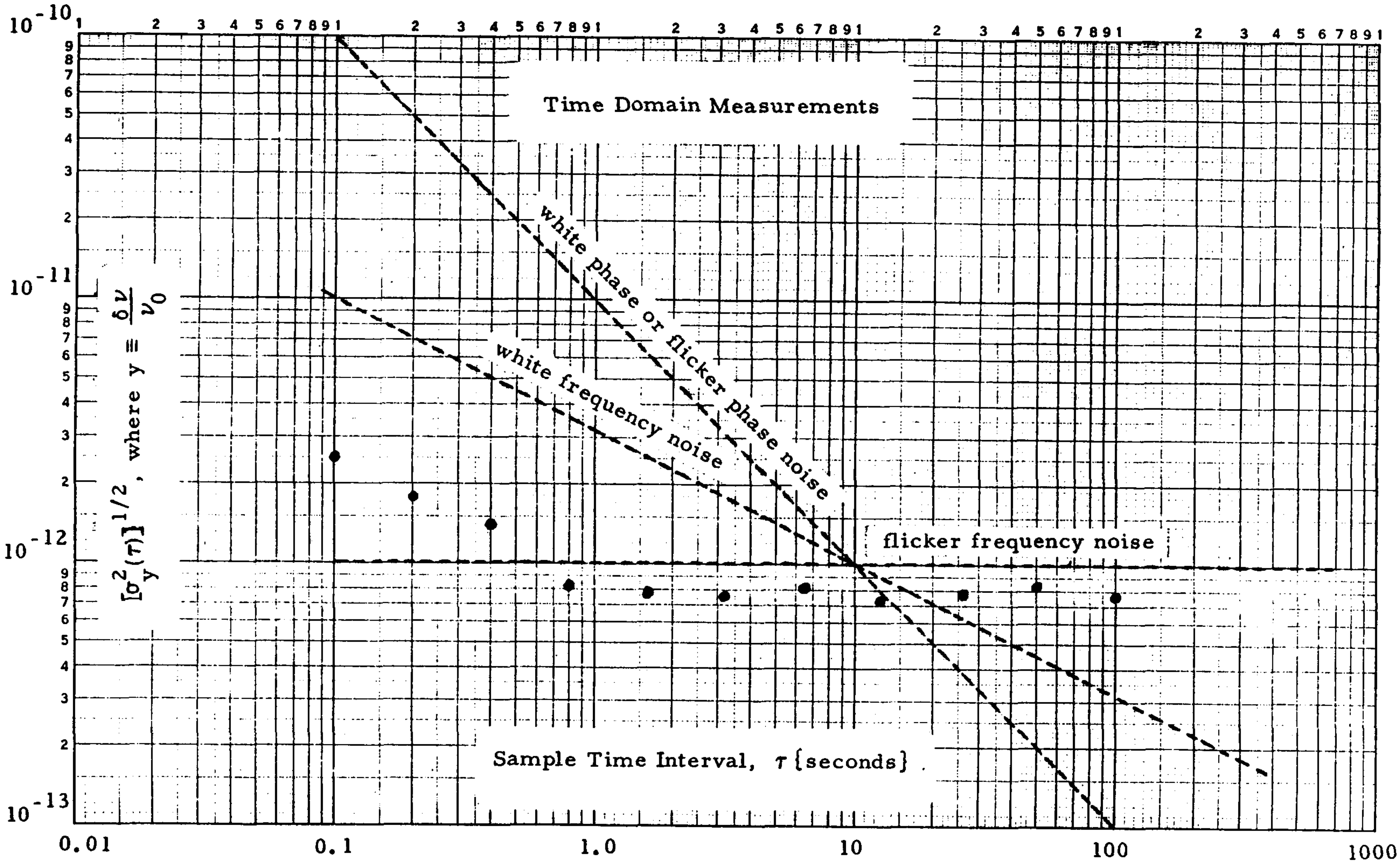


FIGURE 8: SIGMA VERSUS TAU

The convenience of obtaining time domain data has been greatly enhanced by utilizing recently developed counters [8] which are programable to automatically compute σ versus τ. A block diagram (fig. 9) shows the measurement setup using a computing counter programed for $\sigma_y(\tau)$. The program which was used is given in Appendix G. Certain limitations of deadtime are inherent in the use of this time domain method. However, in general (except for very short τ), frequency stability in the time domain may be measured quickly and accurately using a computing counter.

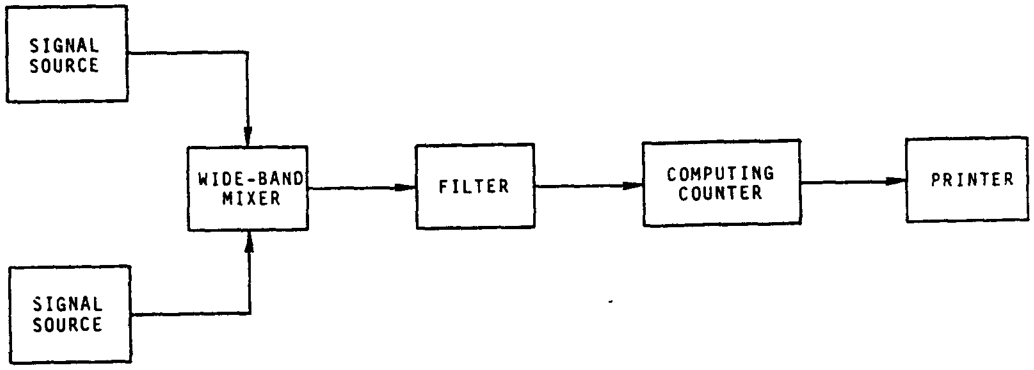


FIGURE 9: FREQUENCY STABILITY MEASUREMENT UTILIZING A COMPUTER COUNTER

3.3. Differential Phase Noise Measurements

An additional useful system illustrated in figure 10 is used for differential phase noise measurements of various discrete components which are frequently used in stability measurement systems. In this system only one frequency source is used. Its output is split so that part of the signal passes through the component to be tested.

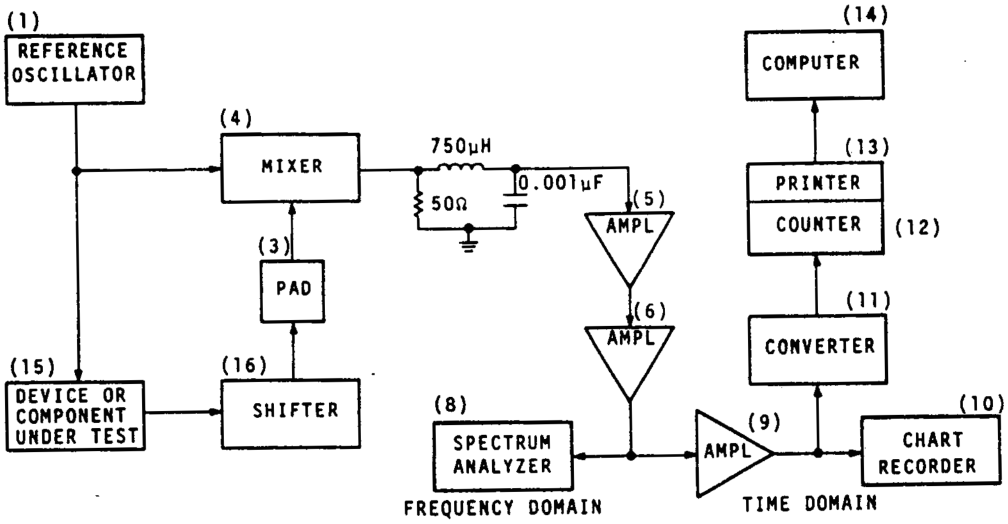


FIGURE 10: DIFFERENTIAL PHASE NOISE MEASUREMENT

ITEMS (1) THROUGH (14) SAME AS FIGURES 1 AND 7
(15) ANY DEVICE OR COMPONENT UPON WHICH NOISE MEASUREMENTS ARE DESIRED (AMPLIFIERS, FILTERS, CAPACITORS, CABLES, PADS, ETC.)
(16) NBS ADJUSTABLE PHASE SHIFTER, 5MHz (SEE FIGURE 11)

The signal is adjusted via a phase shifter (fig. 11) so that it is in phase quadrature with the other part of the original signal and is down-converted in the Schottky barrier diode mixer as described in the other

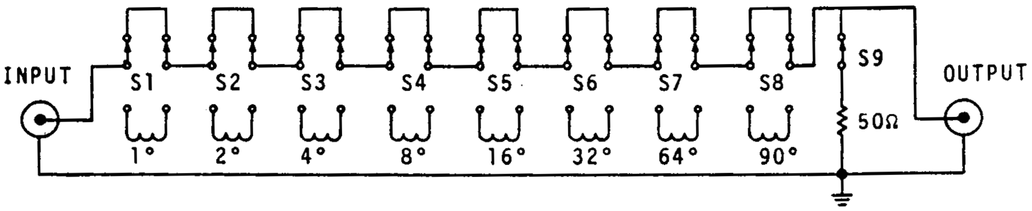


FIGURE 11: ADJUSTABLE PHASE SHIFTER (5MHz DELAY LINE)
RG174/U CABLE WAS USED FOR EACH SEGMENT OF PHASE SHIFT CALCULATED AT ~10 cm PER DEGREE AT 5 MHz.

systems. The switchable 50Ω load in figure 11 is not essential but is included for convenience. A low-pass filter is included before the signal is amplified in special low-noise, low-level dc amplifiers and observed on the spectrum analyzer. Script $\mathcal{L}(f)$ values are calculated at various frequency values, f, and plotted. A sample calculation is shown in Appendix E.

The measurement system noise level (e.g., see fig. 6) is easily evaluated. Using the differential phase noise measurement system shown in figure 10, let the "device under test" be a short length of coaxial cable (which is itself not a source of noise). The small amount of noise observed on the spectrum analyzer represents the system noise, mainly due to the mixer (4) or the first amplifier (5). The calculation of the system noise is then the procedure given in Appendix E.

4. OPERATIONAL SYSTEMS FOR MEASUREMENT OF FREQUENCY STABILITY AT NBS (MICROWAVE REGION)

John H. Shoaf and A. S. Risley

Thorough investigation of stability measurement techniques in the X-band region revealed that a method different from that described above for HF measurements was desirable in most cases. The recommended measurement system is described here in detail. Other techniques of stability measurements in X-band will be discussed in less detail later. The recommended system* is a single-oscillator system as shown in the photograph (fig. 12) and in the block diagram (fig. 13). Prime references are papers by Ashley et al. [9] and Ondria [10].

4.1. Discussion of the Measurement System

The single-oscillator frequency stability measurement system is basically a frequency modulation (FM) demodulator. That is, it can retrieve from the modulated carrier the signal with which the carrier was originally frequency modulated.

An important consideration when making measurements is that of maintaining the quadrature condition--a 90° average phase difference between the signals in the reference channel and the signal channel as seen at the mixer. Unfortunately, there is a fairly high probability that during the course of a measurement the average phase difference will fluctuate a few degrees about the desired 90° setting. Therefore, it is recommended that an occasional check of the quadrature condition be

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* The recommended system is discussed here as a frequency domain measurement. However, time domain measurements can also be made.

made. (In a two-oscillator system of measurement discussed later the quadrature condition--in long term--is established and maintained by phase-locking one source to the other. A similar procedure could be used here, but we consider it to be unnecessary in practice.)

In practice, there is a low frequency limit to the usefulness of this method for the measurement of FM noise. We have seen limiting values of f ranging from as high as 500 Hz to as low as 2.5 Hz.

The single-oscillator system and the two-oscillator system each has an upper frequency limit; i.e., a value of f above which frequency stability measurements cannot meaningfully be made. For the single-oscillator system this upper limit is $f \approx W_c$, where W_c is the 3-dB resonance linewidth of the loaded discriminator cavity. In the NBS single-oscillator system, measurements were made at values of f as high as 100 kHz.

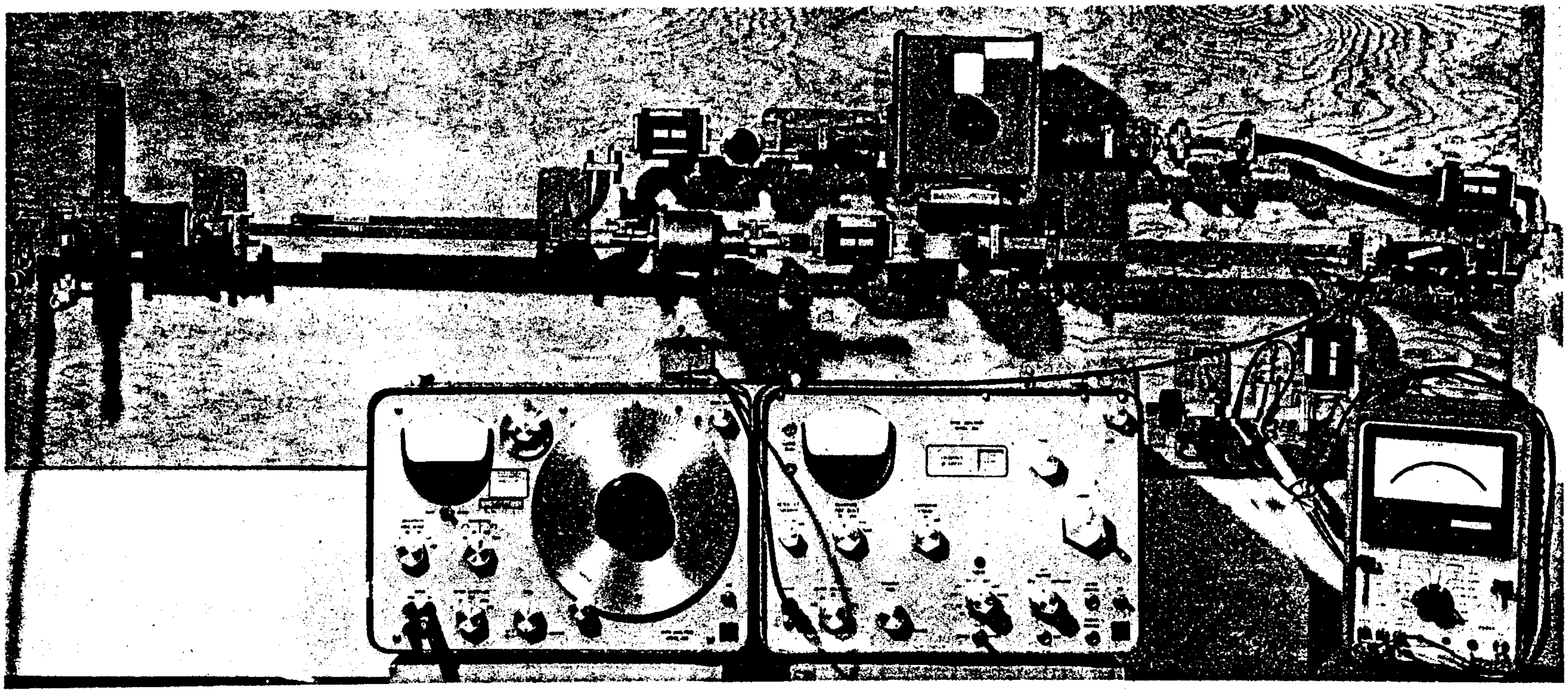

FIGURE 12. SINGLE OSCILLATOR FREQUENCY STABILITY MEASUREMENT SYSTEM

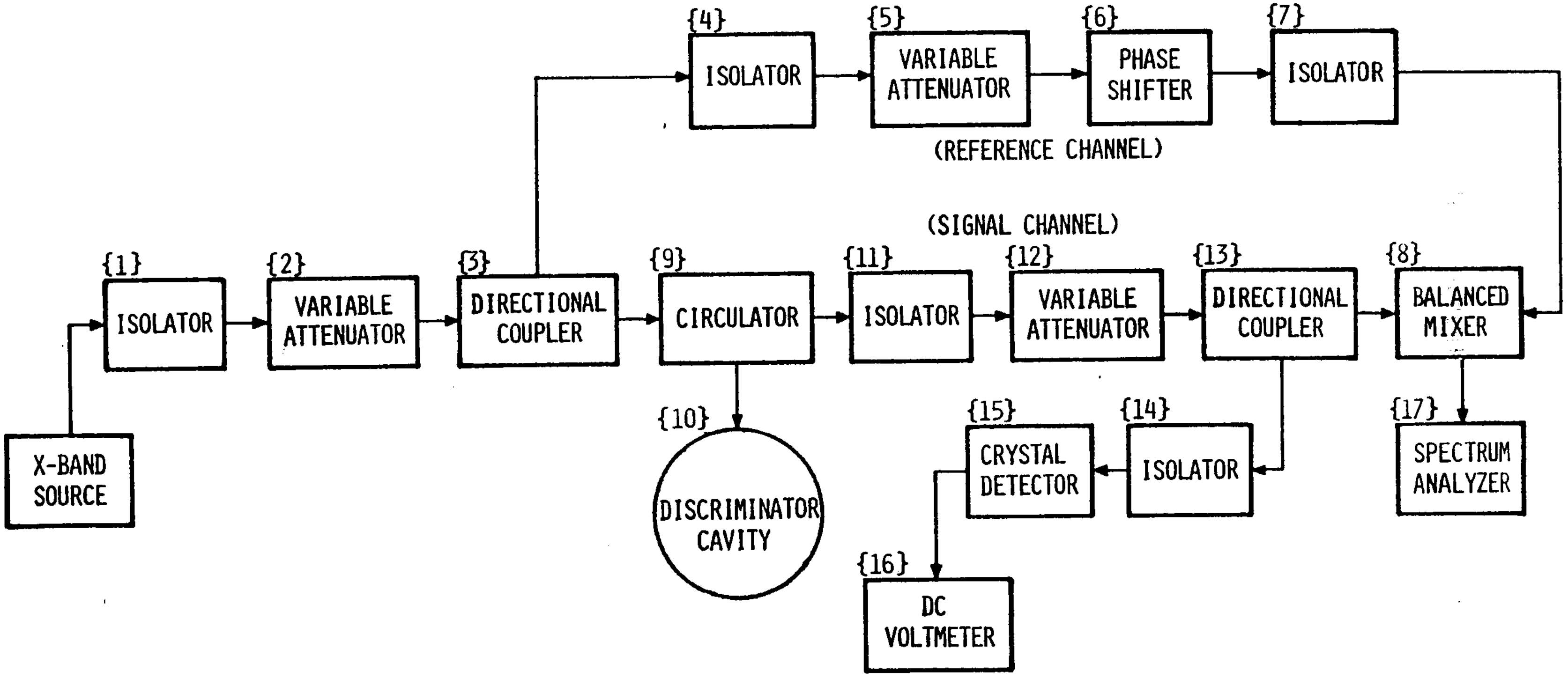

FIGURE 13: SINGLE OSCILLATOR FREQUENCY STABILITY MEASUREMENT SYSTEM

4.2. Description of the Measurement System

A description of the single-oscillator measuring system may readily be followed by referring to the block diagram (fig. 13). The X-band source under test is connected at the left-hand side of the system. The signal passes through an isolator {1} and variable attenuator {2} before it is split via a 3-dB directional coupler {3}. (It should be noted that isolators {1}, {4}, {7}, {11}, and {14} are used at several points throughout the system as a means of preventing any serious reflections which might otherwise exist.) Part of the signal enters the reference channel (upper arm), passing through a phase shifter {6} via a variable attenuator {5} and eventually through a 90° twist into the balanced mixer {8}. The other part of the signal enters the signal channel (lower arm), passing through a three-port circulator {9} connected to a discriminator cavity {10} at one port. A variable attenuator {12} is also in the signal channel before the signal reaches the balanced mixer. The output of the mixer goes to either of two spectrum analyzers {17}. A 10-dB directional coupler {13} is utilized in the signal channel to facilitate detection of resonance tuning of the cavity. This is observed via a detector {15} with a dc voltmeter readout {16}.

The only component in the system which is not readily available commercially as a stock item is the discriminator cavity {10}. It is a TE_{011} right circular one-port cavity. The coarse tuning is accomplished by means of a movable end wall. The fine tuner is a small diameter rod which can be moved coaxially in the cavity. The diameter of the rod should be such that the cavity frequency changes no more than 1.5 MHz for 0.05 inch (~ 1 millimeter) change in depth of insertion. At any desired frequency the coupling of the cavity should be such that the absorption is very nearly complete. For further discussion of coupling see reference [4]. The cavity Q needs to be high enough for good sensitivity but adequately low for sufficient bandwidth. The cavity used in the frequency stability measurement system described here has an unloaded Q of approximately 20,000. Additional details are available upon request from the authors.

4.3. Calibration Procedure

Initial calibration of the measurement system is necessary in order to assign an absolute scale to the stability measurements. To facilitate calibration, a <u>sinusoidally-modulated</u> X-band source is used to drive the system. The frequency-modulated signal is observed on an RF power spectral density analyzer and the modulation level is adjusted to a value sufficient to completely suppress the X-band carrier. For sinusoidal modulation, the first carrier null corresponds to a modulation index of 2.4. Modulation at 5 kHz was found to be convenient to use because of the particular dispersion and bandwidth settings which were available on the particular spectrum analyzer used to display the carrier suppression. The detailed procedure for obtaining the calibration factor follows.

(a) With the discriminator cavity {10} far off resonance, set the level of the X-band signal (as determined with a dc voltmeter {16} at the detector {15}) to a convenient value and <u>record</u> the value. The first variable attenuator {2} should be used for this adjustment. Any convenient level may be chosen provided an equal amount of power also will be available from the signal source which is to be evaluated.

(b) Adjust the cavity to resonance. The dc voltmeter at the detector is used to determine resonance. Place the dc voltmeter at the output of the mixer {8} and adjust the phase shifter {6} in the reference channel until the dc output at the mixer is zero (phase quadrature). Remove the dc voltmeter and connect the output of the mixer to a spectrum analyzer {17} tuned to the modulation frequency, 5 kHz. Record the rms voltage reading (V_{rms}) of the spectrum analyzer.

(c) It is now possible to calculate the calibration factor K.

$$K = \frac{(\Delta \nu)_{\text{rms}}}{V_{\text{rms}}} , \tag{6}$$

where $(\Delta \nu)_{\text{rms}}$ is the rms frequency deviation of the carrier due to intentional frequency modulation. This deviation is calculated using the equation

$$(\Delta \nu)_{\text{rms}} = 0.707 \, (\Delta \nu)_{\text{peak}}, \tag{7}$$

where $(\Delta \nu)_{\text{peak}}$ is the product of the modulation index with the frequency of sinusoidal modulation, i.e., 2.4 × 5 kHz = 12.025 kHz. Therefore the calibration factor in our case is

$$K = \frac{0.707(12.025 \text{ kHz})}{V_{\text{rms}}} = \frac{8.51 \text{ kHz}}{V_{\text{rms}}} . \tag{8}$$

4.4 Measurement Procedure

The procedure for obtaining data for the spectral density plot is quite similar to the calibration procedure except that the X-band carrier is <u>not</u> subjected to intentional modulation.

(a) With the cavity far off resonance, set the level at the detector to the same value obtained during calibration. Use the variable attenuator {2} for this adjustment. The other variable attenuators {5}, {12} are set to zero.

(b) Adjust the cavity to the resonant frequency of the X-band source. Adjust the phase shifter so that the dc output at the mixer is zero. Attach the spectrum analyzer to the output of the mixer and record rms voltage readings (v'_{rms}) for various frequency settings of the spectrum analyzer. A low noise amplifier may be necessary to obtain useful readings at large Fourier frequencies. A second reading

(v''_{rms}) should be taken at each value of f with the signal strongly attenuated in the signal channel. This is to record the residual additive background noise not attributable to actual phase noise on the carrier. This attenuation is accomplished by inserting all (> 20 dB) of the attenuation in the variable attenuator {12}.

(c) In order to calculate values of $S_{\delta\phi}(f)$ for plotting at various Fourier frequencies it is convenient to make a tabulation of results. An example of some typical results is given below in Table 1. The following relations are used:

$$v_{rms} = \sqrt{(v'_{rms})^2 - (v''_{rms})^2} \tag{9}$$

$$\delta\nu_{rms} = v_{rms} \times K \tag{10}$$

$$S_{\delta\nu}(f) = \frac{(\delta\nu_{rms})^2}{B} \tag{11}$$

where B is the bandwidth at which the readings were made on the spectrum analyzer, and

$$S_{\delta\phi}(f) = \frac{S_{\delta\nu}(f)}{f^2} \ . \tag{12}$$

Values of $S_{\delta\phi}(f)$ which were calculated this way (Table 1) are plotted in figure 14.

TABLE 1

Worksheet for Calculation of $S_{\delta\phi}(f)$

Calibration factor $K = \dfrac{8.51 \times 10^3 \text{ Hz}}{0.88 \text{ V}} = 9.67 \times 10^3 \text{ Hz/V}$

f(Hz)	f^2 (Hz^2)	B (Hz)	v_{rms} (μV)	$\delta\nu_{rms}$ (Hz)	$(\delta\nu_{rms})^2$ (Hz^2)	$S_{\delta\nu}(f)$ (Hz)	$S_{\delta\phi}(f)$ (dB)*
5×10^3	2.5×10^7 (74 dB)†	100	660	6.38	40.7	0.41 (-3.9 dB)‡	-77.9
2.5×10^3	6.2×10^6 (68 dB)	100	780	7.54	56.9	0.57 (-2.4 dB)	-70.4
1×10^3	1.0×10^6 (60 dB)	100	1000	9.67	93.5	0.94 (-0.3 dB)	-60.3
640	4.1×10^5 (56 dB)	10	1200	11.6	135.0	1.4 (1.5 dB)	-54.5
320	1.0×10^5 (50 dB)	10	460	4.45	19.8	2.0 (3.0 dB)	-47.0
210	4.4×10^4 (46 dB)	10	580	5.61	31.5	3.2 (5.1 dB)	-40.9
150	2.2×10^4 (44 dB)	10	680	6.58	43.3	4.3 (6.3 dB)	-37.7
90	8.1×10^3 (39 dB)	1	230	2.22	4.93	4.9 (6.9 dB)	-32.1
40	1.6×10^3 (32 dB)	1	300	2.90	8.41	8.4 (9.3 dB)	-22.7
20	400 (26 dB)	1	450	4.35	18.9	19.0 (12.8 dB)	-13.2
10	100 (20 dB)	1	700	6.77	45.8	46.0 (16.6 dB)	- 3.4

*$S_{\delta\phi}(f)$ is tabulated in decibels relative to 1 $\text{radian}^2\ \text{Hz}^{-1}$

† dB relative to 1 Hz^2

‡ dB relative to 1 $\text{Hz}^2\ \text{Hz}^{-1}$

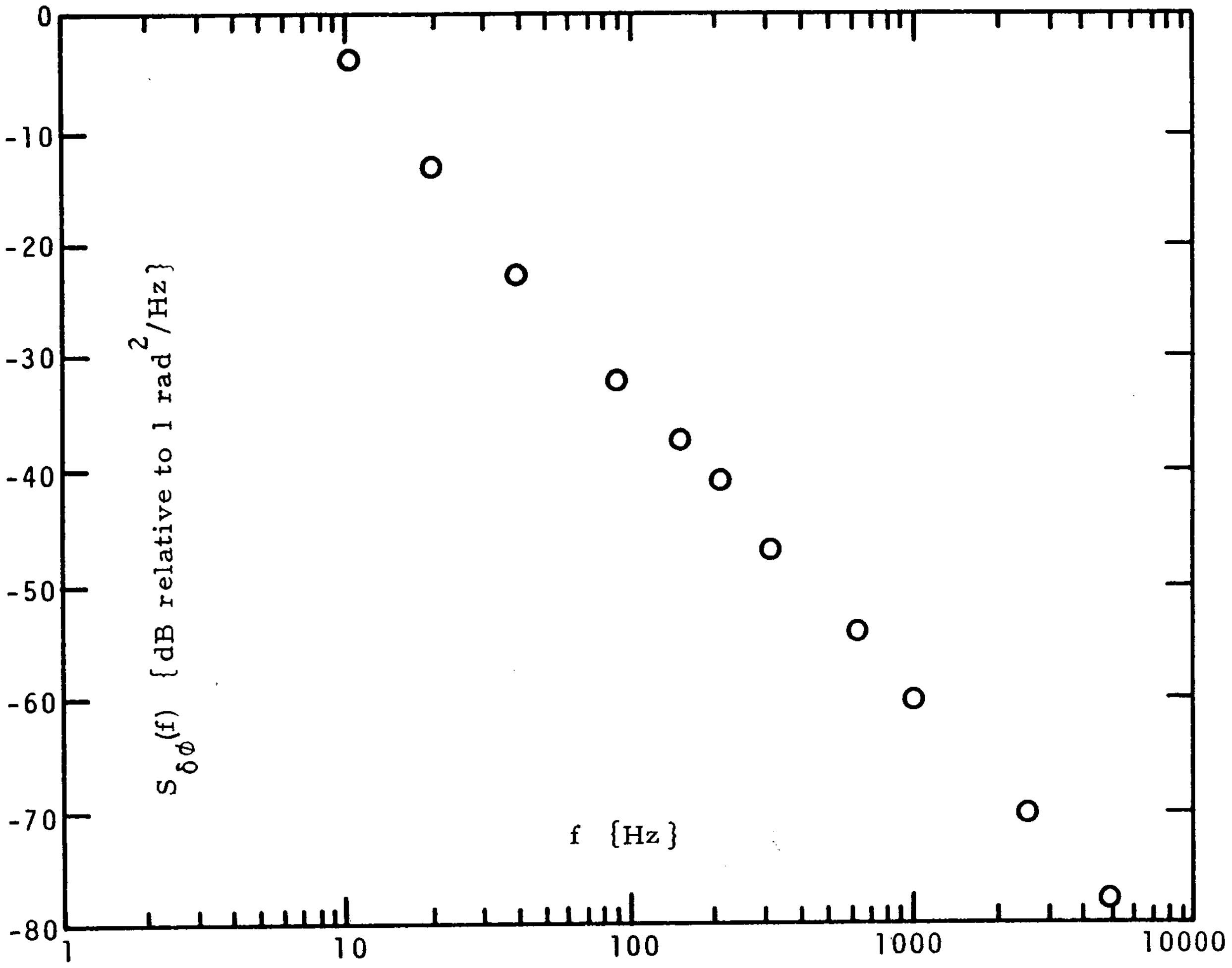

FIGURE 14. FREQUENCY DOMAIN PLOT OF X-BAND GUNN DIODE OSCILLATOR SIGNAL SOURCE (SINGLE OSCILLATOR METHOD)

4.5. Additional Techniques for Frequency Stability Measurements at X-Band

It has been found convenient and desirable, under certain circumstances, to use other techniques for measuring frequency stability at X-band. Where two X-band sources are available, phase- or frequency-locking techniques similar to those used at HF can be used. (See figs. 15, 16, and 17.) Good wide-band double-balanced mixers with coaxial connectors are available [11] which permit many of the measurements to be performed without use of waveguide components.

The measurement setup as shown in the block diagram of figure 9 can also be used at microwave frequencies utilizing a computing counter for time domain measurements. Extensive measurements of frequency stability have been made on stabilized X-band sources [12]. Time domain data obtained via the computing counter have been compared with frequency domain data obtained via several methods.

An example shown in Appendix C translates the frequency domain data of figure 14 into estimated time domain performance shown in figure 18. In the same figure we have plotted time domain data taken directly via a computing counter (see fig. 9).

5. SUMMARY

Terminology and concise definitions for the specification of frequency stability have been given. Recommended techniques and measurement systems were described in detail for both high frequency and X-band signals. Experimental results are compared using various systems. Examples of computations are also included.

A. E. Wainwright, Howard E. Bell, and David W. Allan have assisted in the development of the measurement systems reported here. Their contributions are gratefully acknowledged. The authors appreciate the assistance of Mrs. E. Helfrich in the preparation of the manuscript.

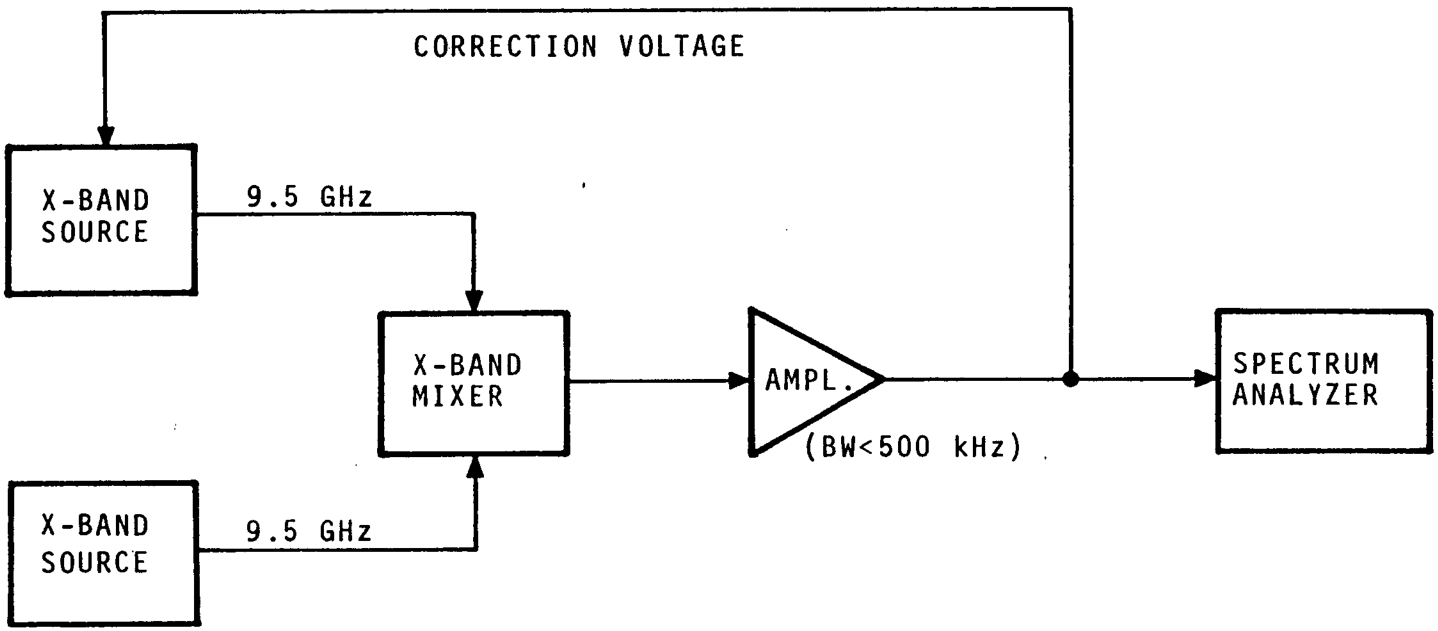

FIGURE 15: FREQUENCY STABILITY MEASUREMENT SYSTEM (PHASE-LOCK SERVO LOOP)

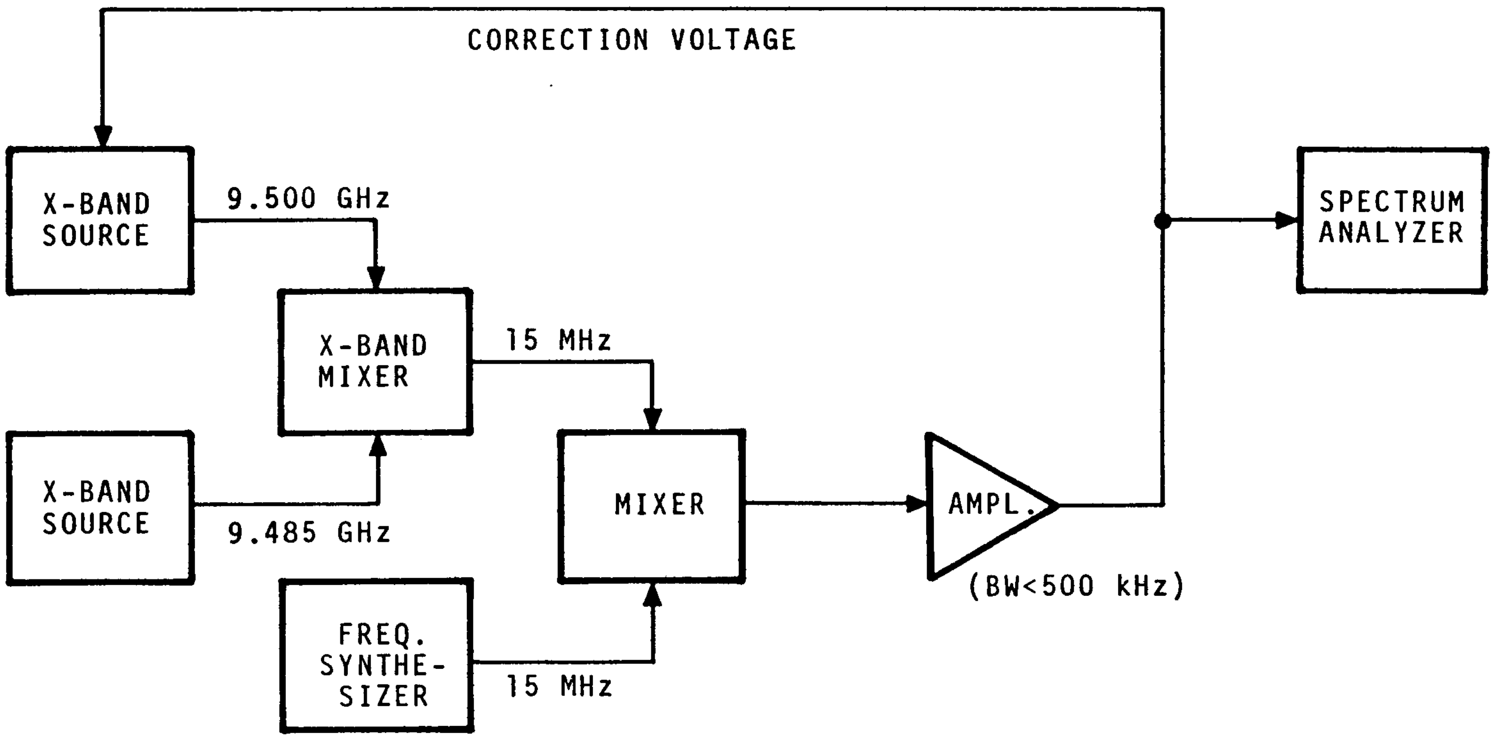

FIGURE 16: FREQUENCY STABILITY MEASUREMENT SYSTEM (OFFSET-FREQUENCY PHASE-LOCK SERVO LOOP)

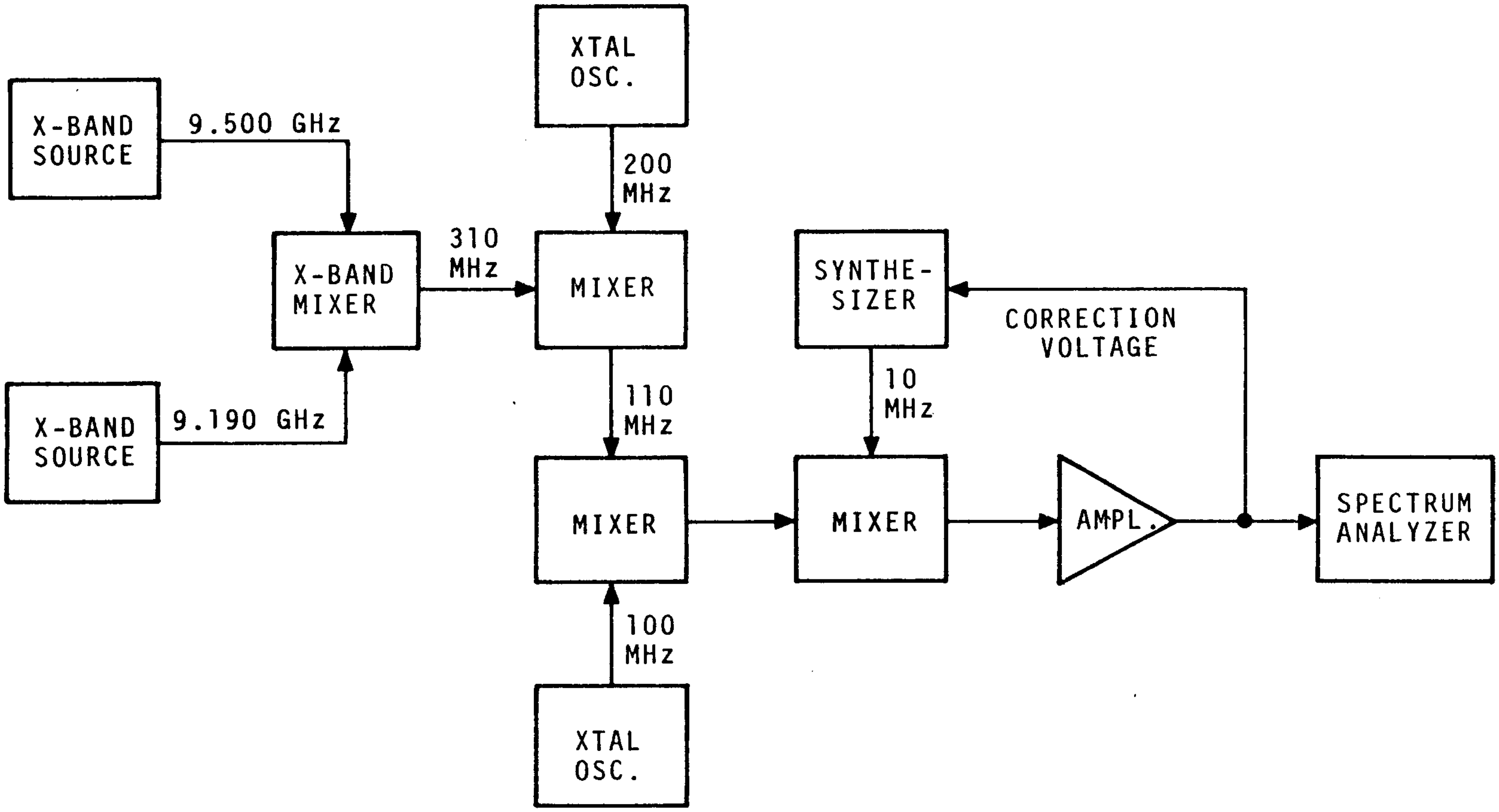

FIGURE 17: FREQUENCY STABILITY MEASUREMENT SYSTEM
(LARGE FREQUENCY-OFFSET PHASE-LOCK SERVO LOOP)

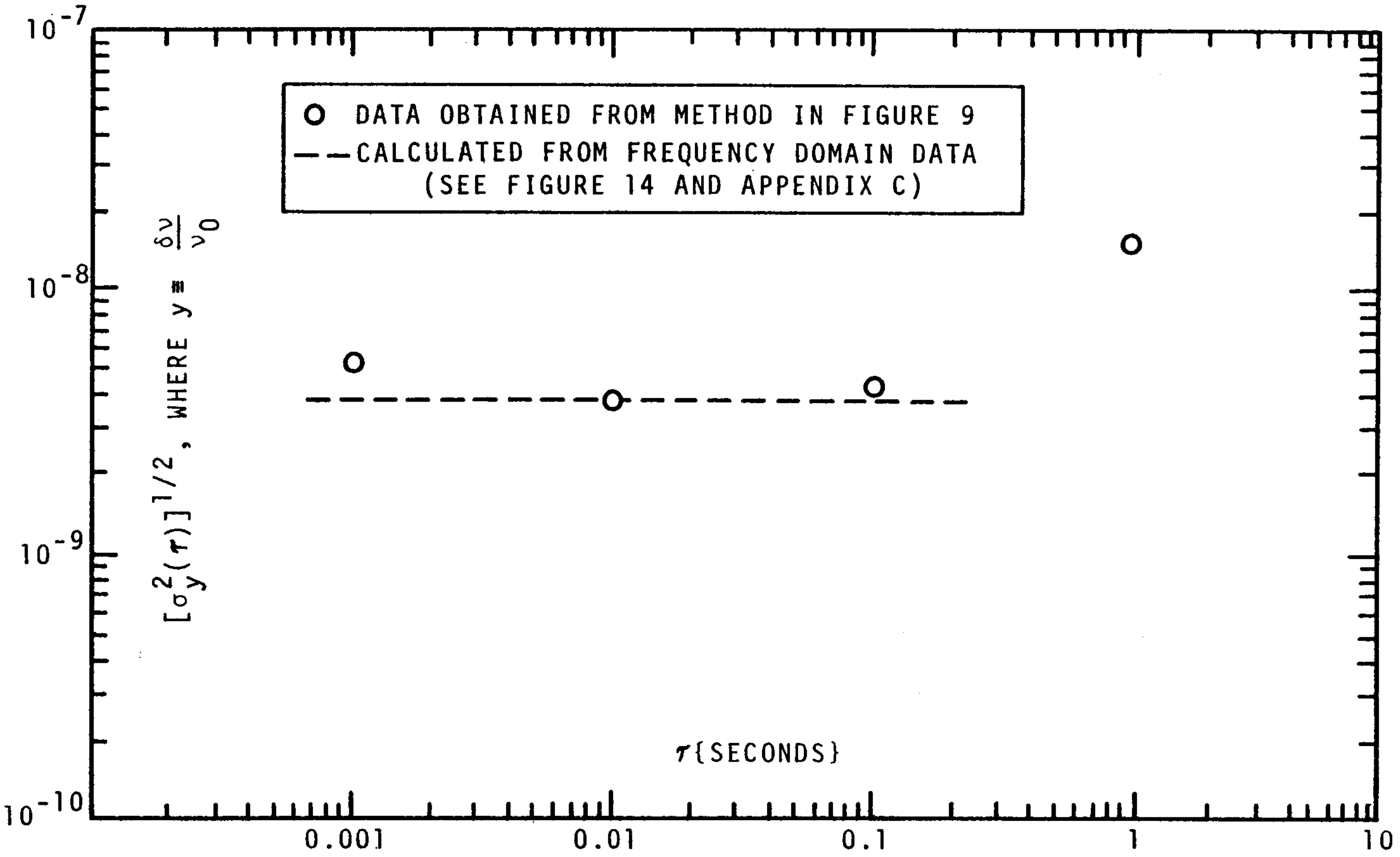

FIGURE 18. TIME DOMAIN PLOT OF X-BAND GUNN DIODE OSCILLATOR SIGNAL SOURCE

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APPENDIX A

Glossary of Symbols

A_{ptp}	Peak-to-peak voltage of a beat frequency at output of mixer
B	High frequency cutoff f_h (bandwidth)
B_a	Analysis bandwidth (frequency window) of the spectrum analyzer
f	Fourier frequency of fluctuations
f_h	Defined as B, high frequency cutoff (bandwidth)
h_α	Coefficient of f^α in spectral density representation
K	Calibration factor used in the single oscillator stability measurement system for microwave frequencies, $K = (\Delta\nu)_{rms}/V_{rms}$
k, n	Integers (used as index of summation)
$\mathscr{L}(f)$	Frequency domain measure of phase fluctuations; Script $\mathscr{L}(f)$ is defined as the ratio of $\frac{\text{Power density (one phase modulation sideband)}}{\text{Power (total signal)}}$. For small $\delta\phi$, $S_{\delta\phi}(f) \approx 2\mathscr{L}(f)$
M	Total number of data values available (usually $M >> N$)
N	Number of data values used in obtaining a sample variance
P_{total}	Total power of signal
r	Parameter related to dead time; $r \equiv T/\tau$
$S_{\delta\nu}(f)$	Spectral density of frequency fluctuations
$S_{\delta V}(f)$	Spectral density of voltage fluctuations
$S_{\delta\phi}(f)$	Spectral density of phase fluctuations; $S_{\delta\phi}(f) = \frac{S_{\delta\nu}(f)}{f^2} = S_y(f)\frac{\nu_o^2}{f^2}$

APPENDIX A cont.

$S_{\sqrt{RFP}}(\nu)$	Spectral density of the (square root of the) radio frequency power
$S_y(f)$	Spectral density of y (Spectral density of fractional frequency fluctuations)
T	Time interval between the beginnings of two successive measurements
t	Time variable
v	Root-mean-square (noise) voltage at output of mixer as measured by a spectrum analyzer
V_{rms}	Root-mean-square voltage of the output of an FM demodulator due to intentional modulation
W_c	The 3-dB resonance linewidth of the loaded discriminator cavity
x	Time interval fluctuations; $\frac{dx}{dt} \equiv y$, hence $x = \delta\tau$
y	Fractional frequency fluctuations, $y \equiv \frac{\delta\nu}{\nu_o}$
$\bar{y}$	Average of y over a specified time interval τ
$\langle\ \rangle$	Time average operator (usually over a large but finite time interval τ)
Δ	Difference operator
δ	Fluctuation operator
$\delta\nu$	Frequency fluctuations
$\delta\phi$	Phase fluctuations
δV, v	Voltage fluctuations
ν	Signal frequency (carrier frequency) variable
ν_o	Average frequency of source (nominal frequency)

APPENDIX A cont.

σ	Square root of a variance
$\sigma_y^2(\tau)$	Specific Allan variance where $N = 2$, $T = \tau$
$\sigma_y^2(N, T, \tau, f_h)$	Sample variance of N averages of $y(t)$, each of duration τ and repeated every T units of time (Allan variance) measured in a post-detection noise bandwidth of f_h
τ	Sampling time interval
τ_a	Post-detection averaging time of the spectrum analyzer
Ω	Signal angular frequency (carrier angular frequency), $\Omega \equiv 2\pi\nu$
ω	Fourier angular frequency of fluctuations, $\omega \equiv 2\pi f$

APPENDIX B

Stability Measure Conversion Chart*

(Frequency Domain - Time Domain)

$S_y(f) \equiv$ one-sided spectral density of y (dimensions are y^2/f), $0 \le f \le f_h$, $f_h \equiv B$, $2\pi f_h \tau >> 1$; $S_y(f \ge f_h) = 0$

General Definition: $\langle \sigma_y^2(N, T, \tau, f_h) \rangle \equiv \left\langle \frac{1}{N-1} \sum_{n=1}^{N} \left(\bar{y}_n - \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k\right)^2 \right\rangle$, $\frac{dx}{dt} \equiv y \equiv \frac{\delta\nu}{\nu_o}$, $r \equiv \frac{T}{\tau}$

Special Case: $\sigma_y^2(\tau) \equiv \langle \sigma_y^2(N=2, T=\tau, \tau, f_h) \rangle = \left\langle \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2} \right\rangle$

Useful Relationships:

$(2\pi)^2 = 39.48$

$\ln 2 = 0.693$

$2 \ln 2 = 1.386$

$\ln 10 = 2.303$

Time Domain (Allan variances, . . .) Frequency Domain (Power law spectral densities)	$\sigma_y^2(\tau)$ $[N = 2, r = 1]$	$\langle \sigma_y^2(N, T = \tau, \tau, f_h) \rangle$ $[r = 1]$	$\langle \sigma_y^2(N, T, \tau, f_h) \rangle$
WHITE x $S_y(f) = h_2 f^2 \quad \left(S_x(f) = \frac{h_2}{(2\pi)^2}\right)$ $2\pi f_h \tau >> 1$	$h_2 \cdot \frac{3 f_h}{(2\pi)^2 \tau^2}$	$h_2 \cdot \frac{N+1}{N(2\pi)^2} \cdot \frac{2 f_h}{\tau^2}$	$h_2 \cdot \frac{N + \delta_k(r-1)}{N(2\pi)^2} \cdot \frac{2 f_h}{\tau^2}$ $\delta_k(r-1) \equiv \begin{cases} 1 \text{ if } r = 1, \\ 0 \text{ otherwise} \end{cases}$
FLICKER x $S_y(f) = h_1 f \quad \left(S_x(f) = \frac{h_1}{(2\pi)^2 f}\right)$ $2\pi f_h \tau >> 1$, $2\pi f_h T >> 1$	$h_1 \cdot \frac{1}{\tau^2 (2\pi)^2} \left[\frac{9}{2} + 3 \ln(2\pi f_h \tau) - \ln 2\right]$	$h_1 \cdot \frac{2(N+1)}{N \tau^2 (2\pi)^2} \left[\frac{3}{2} + \ln(2\pi f_h \tau) - \frac{\ln N}{N^2 - 1}\right]$	$h_1 \cdot \frac{2}{(2\pi\tau)^2} \left\{\frac{3}{2} + \ln(2\pi f_h \tau)\right.$ $\left. + \frac{1}{N(N-1)} \sum_{n=1}^{N-1} (N-n) \cdot \ln\left[\frac{n^2 r^2}{n^2 r^2 - 1}\right]\right\}$, for $r >> 1$
WHITE y (Random Walk x) $S_y(f) = h_0 \quad \left(S_x(f) = \frac{h_0}{(2\pi)^2 f^2}\right)$	$h_0 \cdot \frac{1}{2} \tau^{-1}$	$h_0 \cdot \frac{1}{2} \tau^{-1}$	$h_0 \cdot \frac{1}{2} \tau^{-1}$, for $r \ge 1$ $h_0 \cdot \frac{1}{6} r(N+1) \tau^{-1}$, for $Nr \le 1$
FLICKER y $S_y(f) = \frac{h_{-1}}{f} \quad \left(S_x(f) = \frac{h_{-1}}{(2\pi)^2 f^3}\right)$	$h_{-1} \cdot 2 \ln 2$	$h_{-1} \cdot \frac{N \ln N}{N-1}$	$h_{-1} \cdot \frac{1}{N(N-1)} \sum_{n=1}^{N-1} (N-n) \left\{-2(nr)^2 \ln(nr)\right.$ $\left. + (nr+1)^2 \ln(nr+1) + (nr-1)^2 \ln\lvert nr-1 \rvert\right\}$
RANDOM WALK y $S_y(f) = \frac{h_{-2}}{f^2} \quad \left(S_x(f) = \frac{h_{-2}}{(2\pi)^2 f^4}\right)$	$h_{-2} \cdot \frac{(2\pi)^2 \tau}{6}$	$h_{-2} \cdot \frac{(2\pi)^2 \tau}{12} \cdot N$	$h_{-2} \cdot \frac{(2\pi)^2 \tau}{12} \quad [r(N+1) - 1]$, $r \ge 1$

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National Bureau of Standards
February 1973

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*Adapted from J. A. Barnes et al., "Characterization of Frequency Stability," NBS Technical Note 394 (October 1970); also published in IEEE Trans. on Instrumentation and Measurement IM-20, No. 2, pp. 105-120 (May 1971).

APPENDIX C

Translation of Data from Frequency Domain Into Time Domain Performance Using the Conversion Chart

Referring to the frequency domain plot in figure 14 it is determined that $S_{\delta\phi}(f)$ indicates approximate f^{-3} behavior over the total range plotted. Therefore $S_{\delta\nu}(f)$ is nearly proportional to f^{-1} (i.e., flicker frequency noise). At f = 1000 Hz, $S_{\delta\nu}(f)$ is equal to -0.3 dB relative to 1 Hz (see Table 1). The carrier frequency, ν_o, is 9.5 GHz.

$$S_y(f) = \frac{S_{\delta\nu}(f)}{\nu_o^2} = \frac{(10^{-0.03}\,\text{Hz})}{(9.5 \times 10^9\,\text{Hz})^2} = 1.04 \times 10^{-20}\,\text{Hz}^{-1}, \tag{C1}$$

for f = 1000 Hz.

$$S_y(f) = \frac{h_{-1}}{f} \quad \text{(see conversion chart)} \tag{C2}$$

$$h_{-1} = S_y(f) \times f = (1.04 \times 10^{-20}\,\text{Hz}^{-1}) \times (10^3\,\text{Hz}) = 10.4 \times 10^{-18} \tag{C3}$$

$$\sigma_y^2(\tau) = h_{-1} \cdot 2 \ln 2 = 10.4 \times 10^{-18} \times 1.39 = 14.5 \times 10^{-18} \tag{C4}$$

$$\sigma_y(\tau) = 3.8 \times 10^{-9}\,. \tag{C5}$$

For a flicker frequency noise there is no τ dependence. A dashed line at this calculated value is plotted on the same graph as data taken directly in the time domain (fig. 18).

APPENDIX D

Spectral Densities: Frequency Domain Measures of Stability

Donald Halford

Stabilities in the frequency domain are commonly specified as spectral densities. We have used the concept of spectral density extensively in the preparation of this report. The spectral density concept is simple and very useful, but care must be exercised in its use. There are at least four different, but related, types of spectral densities which are used in this paper. In this Appendix, we state and explain some of the simple, often-needed relations among these four often-used types of spectral densities.

The four types which we have used, and which are most relevant to frequency and phase fluctuations, are

$S_y(f)$	Spectral density of fractional frequency fluctuations (noise, instability, modulation). The dimensionality is Hz^{-1}. The range of f is from zero to infinity.
$S_{\delta\nu}(f)$	Spectral density of frequency fluctuations (noise, instability, modulation). The dimensionality is Hz^2 per Hz. The range of f is from zero to infinity.
$S_{\delta\phi}(f)$	Spectral density of phase fluctuations (noise, instability, modulation). The dimensionality is rad^2 per Hz. The range of f is from zero to infinity.
$\mathscr{L}(f)$	Script $\mathscr{L}(f)$ is a frequency domain measure of phase fluctuations (noise, instability, modulation). Script $\mathscr{L}(f)$ is defined as the ratio of the power in one phase noise sideband, referred to the input carrier frequency, on a per hertz of bandwidth spectral density basis, to the total signal power, at Fourier frequency f from the signal's average frequency ν_o, per one

device. The dimensionality is Hz^{-1}. The range of f is from minus ν_o to plus infinity.

Each of these spectral densities is one-sided and is on a per hertz of bandwidth density basis. This means that the total mean-square fluctuation (the total variance) of frequency, for example, is given mathematically by

$$\int_o^{\infty} S_{\delta\nu}(f)\, df ,$$

and, as another example, since Script $\mathcal{L}(f)$ is a normalized density, that

$$\int_{-\nu_o}^{+\infty} \mathcal{L}(f)\, df$$

is equal to unity.

Two-sided spectral densities are defined such that the range of integration is from minus infinity to plus infinity. For specification of noise as treated in this paper, our one-sided spectral density is twice as large as the corresponding two-sided spectral density. That is,

$$\int_{-\infty}^{+\infty} [S^{\text{Two-Sided}}]\, df = 2\int_{o}^{+\infty} [S^{\text{Two-Sided}}]\, df = \int_{o}^{+\infty} [S^{\text{One-Sided}}]\, df. \tag{D1}$$

Two-sided spectral densities are useful mainly in pure mathematical analysis. We recommend and use one-sided spectral densities for experimental work. References [1] and [2] also use one-sided spectral densities.

We use the definition

$$y \equiv \frac{\delta \nu}{\nu_o}, \qquad \text{(D2)}$$

and it follows that

$$S_y(f) \equiv S_{\frac{\delta\nu}{\nu_o}}(f) = \left(\frac{1}{\nu_o}\right)^2 S_{\delta\nu}(f) \, . \qquad \text{(D3)}$$

To relate frequency, angular frequency, and phase we use

$$2\pi[\nu(t)] = \Omega(t) = \frac{d\,\phi(t)}{dt} \, . \qquad \text{(D4)}$$

This may be regarded as a definition of instantaneous frequency $\nu(t)$. From equation (D4), a direct result of transform theory is

$$S_{\delta\phi}(f) = \left(\frac{1}{\omega}\right)^2 S_{\delta\Omega}(f) = \left(\frac{1}{f}\right)^2 S_{\delta\nu}(f). \qquad \text{(D5)}$$

Script $\mathscr{L}(f)$ can be related in a simple way to $S_{\delta\phi}(f)$, but only for the condition that the phase fluctuations occurring at rates f and faster are small compared to one radian. Otherwise Bessel function algebra must be used to relate Script $\mathscr{L}(f)$ to $S_{\delta\phi}(f)$. Fortunately, the "small angle condition" is often met in random noise problems. Specifically we use

$$\mathscr{L}(f) \approx \left(\frac{1}{2\ \text{rad}^2}\right) S_{\delta\phi}(f) \, , \qquad \text{(D6)}$$

provided that

$$\int_{f}^{+\infty} S_{\delta\phi}(f')\,df' \ll 1 \text{ rad}^2 . \qquad \text{(D7)}$$

For the types of signals under discussion and for $|f| < \nu_o$, we use as a good approximation

$$\mathcal{L}(-f) \approx \mathcal{L}(f) . \qquad \text{(D8)}$$

Script $\mathcal{L}(f)$ is the normalized version of $S_{\sqrt{RFP}}(\nu)$, with its frequency parameter f referenced to the signal's average frequency ν_o as the origin such that f equals $\nu - \nu_o$.

Some Mathematics of Phase Sideband Power as Related to Phase Fluctuations: A simple derivation of equation (D6) is possible. We combine the derivation with an example which illustrates the operation of a double-balanced mixer as a phase detector. Consider two sinusoidal 5-MHz signals (having negligible amplitude modulation) feeding the two input ports of a double-balanced mixer. When the two signals are slightly out of zero beat, a slow sinusoidal beat with a period of several seconds at the output of the mixer is measured to have a peak-to-peak swing of A_{ptp}.

Without changing their amplitudes, the two signals are retuned to be at zero beat and in phase quadrature (that is, $\pi/2$ out of phase with each other), and the output of the mixer is a small fluctuating voltage centered on zero volts. Provided this fluctuating voltage is small compared to $A_{ptp}/2$, the phase quadrature condition is being closely maintained, and the "small angle condition" is being met.

Phase fluctuations $\delta\phi$ between the two signals of phases ϕ_2 and ϕ_1, respectively, where

$$\delta\phi \equiv \delta(\phi_2 - \phi_1) , \tag{D9}$$

will give rise to voltage fluctuations δV at the output of the mixer

$$\delta V \approx \frac{A_{ptp}}{2} \delta\phi, \tag{D10}$$

where we have used radian measure for phase angles, and we have used

$$\sin \delta\phi \approx \delta\phi \tag{D11}$$

for small $\delta\phi$ ($\delta\phi << 1$ rad). We solve equation (D10) for $\delta\phi$, square both sides and take a time average

$$\langle(\delta\phi)^2\rangle \approx 4 \frac{\langle(\delta V)^2\rangle}{(A_{ptp})^2} . \tag{D12}$$

If we interpret the mean-square fluctuations of $\delta\phi$ and of δV, respectively, in equation (D12) in a spectral density fashion, we may write

$$S_{\delta\phi}(f) \approx \frac{S_{\delta V}(f)}{2(A_{rms})^2} \;, \qquad \text{(D13)}$$

where we have used

$$(A_{ptp})^2 = 8(A_{rms})^2 \;, \qquad \text{(D14)}$$

which is valid for the sinusoidal beat signal.

For the types of signals under consideration, by definition the two phase noise sidebands (lower sideband and upper sideband, at -f and +f from ν_o, respectively) of a signal are coherent with each other. As already expressed in equation (D8), they are of equal intensity also. The operation of the mixer when it is driven at quadrature is such that the amplitudes of the two phase sidebands add linearly in the output of the mixer, resulting in four times as much power in the output as would be present if only one of the phase sidebands were allowed to contribute to the output of the mixer. Hence for $|f| < \nu_o$ we obtain

$$\frac{S_{\delta V}(|f|)}{(A_{rms})^2} \approx 4\,\frac{S_{\sqrt{RFP}}(\nu_o + f)}{P_{total}} \;, \qquad \text{(D15)}$$

and, using the definition of Script $\mathcal{L}(f)$,

$$\mathcal{L}(f) \equiv \frac{S_{\sqrt{RFP}}(\nu_o + f)}{P_{total}} \approx \frac{1}{2} S_{\delta\phi}(|f|) \;, \qquad \text{(D16)}$$

provided the phase quadrature condition is approximately valid.

The phase quadrature condition will be met for a time interval at least τ long, provided

$$\int_{(2\pi\tau)^{-1}}^{\infty} S_{\delta\phi}(f')\,df' << 1 \text{ rad}^2, \qquad \text{(D17)}$$

and hence equation (D16) is useful for values of f at least as low as $(2\pi\tau)^{-1}$. Equations (D16) and (D17) correspond to equations (D6) and (D7) respectively.

APPENDIX E

A Sample Calculation of Script $\mathcal{L}$

Script $\mathcal{L}(f)$ is easily measured using a double-balanced mixer as a phase sensitive dctector, together with a spectrum analyzer which can measure at the frequency f. Equation (E1) is valid for the case where the reference signal has negligible phase noise compared to the test signal.

$$\mathcal{L}(f) = \frac{2}{n^2}\left(\frac{v_{\text{one unit}}}{A_{\text{ptp}}}\right)^2 . \tag{E1}$$

However, eq (E2) is the valid equation when we have two equally noisy signals (test and reference) driving the mixer.

$$\mathcal{L}(f) = \frac{1}{n^2}\left(\frac{v_{\text{two units}}}{A_{\text{ptp}}}\right)^2 . \tag{E2}$$

In case the device being measured has amplification of times n (i. e., frequency multiplication or frequency synthesis), the definition of Script $\mathcal{L}(f)$ requires that the factor n^2 appear in eqs (E1) and (E2). For example, a frequency synthesizer with output at 45.55 MHz, and with its input driven at 5 MHz, is characterized by n equal 9.11 corresponding to a phase fluctuations increase expressed in decibels of 19.2 dB.

$$10 \log n^2 = 20 \log n = 20 \log 9.11 = 19.2 \text{ dB}. \tag{E3}$$

Both (E1) and (E2) require that the beat signal out of the mixer be sinusoidal when the two input signals are slightly out of zero beat. The peak-to-peak amplitude of the beat is A_{ptp}.

For convenience of computation and plotting it often is advantageous to set the beat frequency voltage (before locking) to some special voltage such as $\frac{1}{\sqrt{10}}$ volts (0.316 V) peak-to-peak at the mixer output. Then (after lock) with the output of the phase detector expressed in rms nanovolts per root hertz, direct plotting is facilitated for Script $\mathcal{L}(f)$ in decibels versus frequency in hertz. In this case 1000, 100, and 10 nanovolts per root hertz correspond to -110, -130, and -150 dB respectively. A sample calculation demonstrating this convenience is shown below.

Given:

$$A_{ptp} = 0.316\,\mathrm{V}\ (\text{i.e., } \frac{1}{\sqrt{10}}\ \mathrm{V}), \tag{E4}$$

$$v = 100\ \mathrm{nV \cdot Hz^{-1/2}\ rms} \tag{E5}$$

at f equal to 20 Hz for a pair of equally noisy devices having no frequency multiplication (n = 1):

$$\mathcal{L}(f) = \left(\frac{v_{\text{two units}}}{A_{ptp}}\right)^2 \tag{E6}$$

$$= \left(\frac{100\ \mathrm{nV\,Hz^{-1/2}}}{0.316\ \mathrm{V}}\right)^2 = \left(\frac{10^{-7}}{\sqrt{10^{-1}}}\right)^2 \mathrm{Hz^{-1}} = \frac{10^{-14}}{10^{-1}}\mathrm{Hz^{-1}} \tag{E7}$$

$$= 10^{-13}\ \mathrm{Hz^{-1}} = -130\ \mathrm{dB} \tag{E8}$$

at f equal to 20 Hz. Or, using logarithms:

$$\mathcal{L}(f) = 20 \log_{10}\left(\frac{v_{\text{two units}}}{A_{\text{ptp}}}\right) \qquad \text{(E9)}$$

$$= 20 \log_{10} \frac{(10^{-7}\ \mathrm{V} \cdot \mathrm{Hz}^{-1/2})}{(10^{-1/2}\ \mathrm{V})} = 20(-7 + 0.5) = -130\ \mathrm{dB} \qquad \text{(E10)}$$

at f equal to 20 Hz.

If the phase noise were to follow flicker law (1/f), at f equal to 1 Hz the mean square noise would be 20 times worse (13 dB greater). That is

$$\mathcal{L}(1\ \mathrm{Hz}) = -130\ \mathrm{dB} + 13\ \mathrm{dB} = -117\ \mathrm{dB}. \qquad \text{(E11)}$$

APPENDIX F

A Sample Calculation of Allan Variance, $\sigma_y^2(\tau)$

$$\sigma_y^2(\tau) \equiv \langle \sigma_y^2(N = 2, T = \tau, \tau) \rangle = \left\langle \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2} \right\rangle \approx \frac{1}{2(M-1)} \sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 \qquad \text{(F1)}$$

in the example below:

Number of data values available, $M = 9$

Number of differences averaged, $M - 1 = 8$

Sampling time interval $\tau = 1\text{s}$

Data values $(\bar{y})$	First differences $(\bar{y}_{k+1} - \bar{y}_k)$	First differences squared $(\bar{y}_{k+1} - \bar{y}_k)^2$
892		
809	- 83	6889
823	14	196
798	- 25	625
671	-127	16129
644	- 27	729
883	239	57121
903	20	400
677	-226	51076

$$\sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 = 133165$$

Based on these data:

$$\sigma_y^2(\tau) = \frac{133165}{2(8)} = 8322.81, \qquad \text{(F2)}$$

$$[\sigma_y^2(\tau)]^{1/2} = \sqrt{8322.81} = 91.23, \; N = 2, \; T = \tau = 1\text{ s}. \qquad \text{(F3)}$$

In this example, the data values may be understood to be expressed in parts in 10^{12}.

APPENDIX F cont.

Using the same data as in the above example it is possible to calculate the Allan variance for $\tau = 2$ s by averaging pairs of adjacent data values and using these averaged values as new data values to proceed with the calculation as before. Allan variance values may be obtained for $\tau = 3$ s by averaging three adjacent data values in a similar manner, etc., for larger values of τ.

Ideally the calculation is done via a computer and a large number, M, of data values should be used. (Typically $M = 256$ data values are used in the NBS computer program.) The statistical confidence of the calculated Allan variance improves nominally as the square root of the number, M, of data values used. For $M = 256$, the confidence of the Allan variance is not expected to be better than approximately $\frac{1}{\sqrt{256}} \times 100\% \approx 7\%$ in the rms sense. The use of $M \gg 1$ is logically similar to the use of $B_a \cdot \tau_a \gg 1$ in spectrum analysis measurements, where B_a is the analysis bandwidth (frequency window) of the spectrum analyzer, and τ_a is the post-detection averaging time of the spectrum analyzer.

APPENDIX G

COMPUTING COUNTER $\sigma_y(\tau)$ PROGRAM USING AN EFFICIENT OVERLAPPING ESTIMATOR

David W. Allan

(1) MANUAL
(2) Enter carrier or basic frequency
(3) $c \rightleftarrows x$
[skip to (33) if program is already in]
(4) LEARN
(5) CLEAR x
(6) $b \rightleftarrows x$
(7) MODULE or PLUG-IN
(8) $a \rightleftarrows x$
(9) X FER PROGRAM
(10) MODULE or PLUG-IN
(11) $a \rightleftarrows x$
(12) $\overrightarrow{axy}$
(13) - (subtract)
(14) $\overrightarrow{xy}$
(15) × (multiply)
(16) $\overrightarrow{bxy}$
(17) + (add)
(18) $b \rightleftarrows x$
(19) REPEAT
(20) X FER PROGRAM
(21) $\overrightarrow{Nxy}$
(22) $\overrightarrow{Nxy}$
(23) + (add)
(24) $a \rightleftarrows x$
(25) $\overrightarrow{bxy}$
(26) $\overrightarrow{axy}$
(27) ÷ (divide)
(28) $\sqrt{x}$
(29) $\overrightarrow{cxy}$
(30) ÷ (divide)
(31) DISPLAY x
(32) PAUSE
(33) RUN
(34) START

Program will automatically repeat unless righthand PAUSE switch is in HALT position

$\tau \equiv$ Sample time (computing counter "measurement time")
$T - \tau \simeq 0.003$ seconds (compute + cycle time)
$N = 2$

Number set on repeat loop corresponds to the number of estimates of the variance. For good confidence levels 100 or more estimates usually are required.

APPENDIX H

Some Important References for Measurement and Specification of Frequency Stability

General References

1. November or December of even-numbered years IEEE Transactions on Instrumentation and Measurement (Conference on Precision Electromagnetic Measurements, held every two years).

2. February 1966 Proceedings of the IEEE, Special Issue on Frequency Stability (IEEE-NASA Symposium). Very useful.

3. Proceedings of the IEEE-NASA Symposium on the Definition and Measurement of Short-Term Frequency Stability at Goddard Space Flight Center, Greenbelt, Maryland, November 23-24, 1964. Prepared by Goddard Space Flight Center (Scientific & Technical Information Division, National Aeronautics and Space Administration, Washington, D.C., 1965). Copies available for $1.75 from the U. S. Government Printing Office, Washington, D.C. 20402.

4. The annual Proceedings of the Symposium on Frequency Control (Fort Monmouth). The Proceedings are not edited nor reviewed.

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6. September 1968 IEEE Transactions on Microwave Theory and Techniques, Special Issue on Noise.

7. Frequency and Time Standards, Application Note 52, Hewlett-Packard Company (November 1965).

APPENDIX H cont.

Some Specific Papers

8. D. W. Allan, "Statistics of Atomic Frequency Standards," Proc. IEEE, vol. 54, pp. 221-230, February 1966. A thorough understanding of this paper is important for everyone who wishes to measure and quote performance of frequency standards in the time domain, e.g., σ versus τ plots. The data analysis must take into account the number of samples taken and how they are used.

9. David W. Allan, B. E. Blair, D. D. Davis, and H. E. Machlan, "Precision and Accuracy of Remote Synchronization via Portable Clocks, Loran C, and Network Television Broadcasts," Proc. 25th Annual Symposium on Frequency Control, Fort Monmouth, N.J., pp. 195-208, April 1971.

10. J. R. Ashley, C. B. Searles, and F. M. Palka, "The Measurement of Oscillator Noise at Microwave Frequencies," IEEE Trans. on Microwave Theory and Techniques MTT-16, No. 9, pp. 753-760, September 1968.

11. E. J. Baghdady, R. N. Lincoln, and B. D. Nelin, "Short-Term Frequency Stability: Characterization, Theory, and Measurement," Proc. IEEE, vol. 53, pp. 704-722, July 1965. Among many other topics, the possible problem of AM noise is discussed.

12. Helmut Brandenberger, Frederic Hadorn, Donald Halford, and John H. Shoaf, "High Quality Quartz Crystal Oscillators: Frequency Domain and Time Domain Stability," Proc. 25th Annual Symposium on Frequency Control, Fort Monmouth, N.J., pp. 226-230 April 1971. An example of 1970 "state-of-the-art" measurements.

13. L. S. Cutler, "Present Status in Short Term Frequency Stability," Frequency, vol. 5, pp. 13-15, September-October 1967. This is a well-written, concise progress report with indications for future effort. Caution: His examples I and II were not in fact state-of-the-art, while example IV has some factor-of-ten typographical errors as given.

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14. L. S. Cutler and C. L. Searle, "Some Aspects of the Theory and Measurement of Frequency Fluctuations in Frequency Standards," Proc. IEEE, vol. 54, pp. 136-154, February 1966. This is a useful treatment of some of the theory, mathematics, and measurement techniques--with physical insight into the noise processes of practical concern.

15. W. A. Edson, "Noise in Oscillators," Proc. IRE, vol. 48, pp. 1452-1466, August 1960. This paper is an "old classic."

16. C. Finnie, R. Sydnor, and A. Sward, "Hydrogen Maser Frequency Standard," Proc. 25th Annual Symposium on Frequency Control, Fort Monmouth, N. J., pp. 348-351, April 1971. State-of-the-art H maser stability.

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19. J. A. Mullen, "Background Noise in Nonlinear Oscillators," Proc. IRE, vol. 48, pp. 1467-1473, August 1960. An "old classic."

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21. H. P. Stratemeyer, "The Stability of Standard-Frequency Oscillators," General Radio Experimenter, vol. 38, pp. 1-16, June 1964. Easy reading.

22. V. Van Duzer, "Short-Term Stability Measurements," IEEE-NASA Symposium on Short-Term Frequency Stability, Washington, D. C.: U. S. Government Printing Office, pp. 269-272, NASA-SP80. The basic method described by Van Duzer is simple, elegant, easily instrumented, easily modified, extremely versatile, and capable of the best resolution which is attained at today's state-of-the-art.

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