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BASIC LABORATORY METHODS FOR MEASUREMENT OR  
COMPARISON OF FREQUENCIES AND TIME INTERVALS

J. T. Stanley and J. B. Milton



**U. S. DEPARTMENT OF COMMERCE**  
**NATIONAL BUREAU OF STANDARDS**

Institute for Basic Standards  
Boulder, Colorado 80302

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# NATIONAL BUREAU OF STANDARDS REPORT

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### BASIC LABORATORY METHODS FOR MEASUREMENT OR COMPARISON OF FREQUENCIES AND TIME INTERVALS

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U.S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS

## FOREWORD

The purpose of this report is to bring together under one cover a comprehensive, up-to-date digest of basic laboratory methods for the measurement or comparison of time intervals and frequency. The text is written in an informal conversational tone with very little emphasis on mathematics. This approach was chosen to meet the needs of the laboratory technician whose field of specialization entails the use of electronic equipment for the measurement of frequencies and/or time intervals.

This is the third in a series of background information documents provided to the Air Force Communications Service by the National Bureau of Standards under contract EIIIM-6.

The purpose of the series is to provide a compilation of information on time and frequency subjects for use by the Air Force Communications Service in conceiving, designing, and operating advanced communications systems. The documents are intended to serve systems engineers, communications facilities administrators, and operating personnel. They are also intended for use in connection with training activities, both as background information for benefit of course designers and as reference material for students.

Work under this contract is being conducted under the sponsorship and guidance of the Directorate of Communications Engineering, COMSEC/Data Systems Division, Hq. AFCS, Richards-Gebaur Air Force Base, Missouri.

L. E. Gatterer, Project Leader  
Air Force Time and Frequency Studies  
National Bureau of Standards

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# BASIC LABORATORY METHODS FOR MEASUREMENT OR COMPARISON OF FREQUENCIES AND TIME INTERVALS

J. T. Stanley and J. B. Milton

## Abstract

A discussion of the basic laboratory methods for the measurement of frequencies and time intervals is presented. Included are the major techniques of frequency and time measurement by means of cathode ray oscilloscopes, wavemeters, bridges, frequency meters, comparators, and electronic digital counters. Also included is a brief discussion of some auxiliary devices which are quite useful for extending the accuracy, versatility, and range of instrumentation. Relative merits of the various instruments are discussed along with their fundamental principles of operation.

Key Words: Cathode ray oscilloscope; Digital counter; Frequency bridge; Frequency comparator; Frequency measurement; Frequency meter; Phase comparison; Time interval measurement; Wavemeter

## 1. INTRODUCTION

Time and frequency determination dates from the very dawn of the physical sciences. Simple devices for the measurement of electrical frequency were invented during the 1800's, yet modern instruments for such measurements were not developed until the middle of the twentieth century. Since World War II the technology of time and frequency measurement has advanced at an extremely rapid rate. Measurement of time intervals as small as  $10^{-10}$  second and of frequencies as high as  $10^{14}$  hertz are now possible.

During the past decade large-scale integrated circuits have brought about a prodigious change in the capabilities of electronic instruments. At least one commercial firm is now marketing a 1-GHz digital counter, the development of which was not feasible just ten years ago.

Such refinements, of course, are in keeping with the need for higher precision and accuracy in today's computerized, space age. Not long ago the finest precision required for timing measurements in most fields of science or engineering was on the order of a millisecond. Today, however, the need for precision of plus or minus one microsecond is commonplace. For synchronization of high-speed computers, for improved missile tracking, for control of space missions, and for a variety of laboratory measurements, timing accuracies to the nearest nanosecond will soon be essential.

Time and its reciprocal, frequency, can each be intercompared with greater precision than any other known physical quantity. Merely by allowing two oscillators to run for a sufficiently long time, we can compare their average frequencies to any desired degree of precision. Conversely, we can measure time intervals with the same precision by counting the individual cycles in the output waveform of an oscillator, provided its frequency remains constant. But a serious problem arises in attempting to measure time accurately.

Unlike two standard meter bars which can be left side by side indefinitely while their lengths are compared, time doesn't stand still for measurement. No two time intervals can be placed alongside each other in a stationary state, so we must rely upon the stability of an oscillator, a clock, a counter, or a similar measuring device to measure or compare time intervals accurately.

In following the practices set forth in this report, one should keep in mind that the end result of a measurement can be no better than the performance of the equipment with which the measurement is made. Frequent and careful calibration of the measuring instruments against reliable standards is absolutely necessary if maximum accuracy is to be achieved.

## 2. OSCILLOSCOPE TECHNIQUES

The cathode-ray oscilloscope is probably the single most useful laboratory tool available to the electronics technician, engineer or scientist. Whole volumes<sup>1</sup> have been devoted entirely to its many uses.

The basic physical principles of the oscilloscope have been known for years. An early oscillograph (as it was called then) was in operation during the 1890's. This device used wires, mirrors and light beams instead of electronic circuits, but it could display waveforms of low-frequency alternating current none the less.

The modern cathode-ray tube was developed at Bell Telephone Laboratories in the early 1920's. Since then the electronic oscilloscope has evolved into an instrument that can display waveforms in the microwave region and store images for days at a time. The principles of the modern oscilloscope are at the heart of such devices as television sets and radar displays.

Measurements or comparisons of frequencies and time intervals are among the major uses of the oscilloscope. Let us begin by examining the internal workings of this remarkable instrument.

### 2.1 Basic Sections of the Oscilloscope

There are four main parts to the oscilloscope: (1) the cathode ray tube and its power supply, (2) the vertical deflection amplifier, (3) the horizontal deflection amplifier, and (4) the time-base generator. A block diagram is shown in figure 1.

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<sup>1</sup>See Bibliography, section 6.

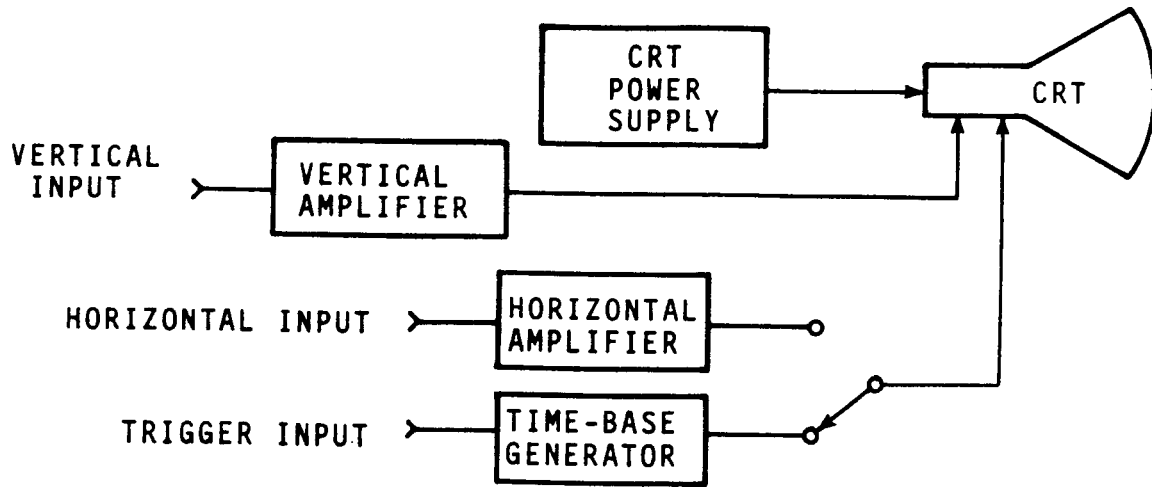


Figure 1 Simplified Block Diagram of a Cathode Ray Oscilloscope

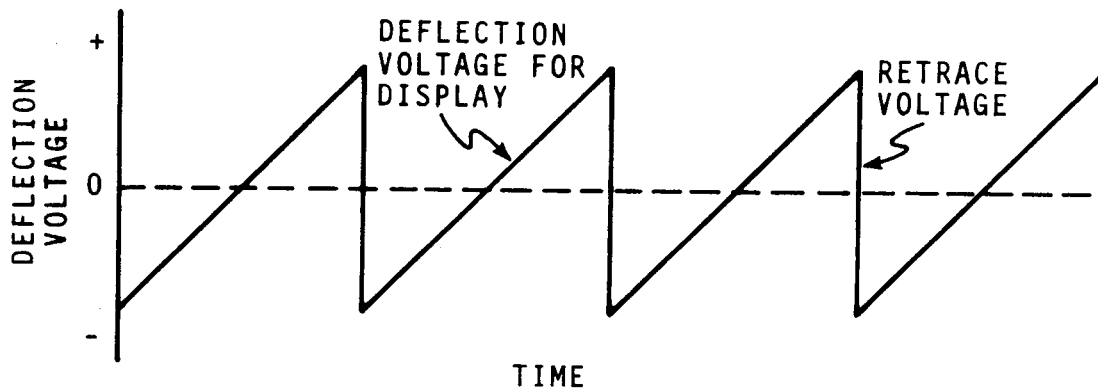


Figure 2 Waveform of Horizontal Deflection Voltage from Time Base Generator

The cathode-ray tube (CRT) is used to display a spot of light created when electrons in a stream generated by the CRT and its power supply strike the phosphorous coating on the back of the CRT face. The spot is made to move up or down by the action of a dc or ac signal applied to the vertical deflection plates via the vertical deflection amplifier. Likewise, the spot is caused to move from side to side by a signal from the horizontal deflection amplifier.

The role of the time-base unit is slightly more involved. In order to display faithfully a signal that is a function of time, the spot of light must traverse the CRT screen at a uniform rate from one side to the other and then return to the original side as quickly as possible. The voltage that causes this horizontal sweep is generated by the time-base unit. The waveform is a ramp or "saw-tooth" as shown in figure 2.

The frequency of the ramp voltage is variable so that waveforms of any frequency within the limits of the oscilloscope can be displayed. The time-base generator has a control to change the frequency of the ramp. What is of interest to the operator is how fast the spot moves across the screen; therefore the frequency control is usually labeled in seconds (or fractions of seconds) per centimeter.

When the frequency of the traverse is high enough, the persistence of the phosphorous and the retention capability of the human eye combine to make the moving spot appear as a solid line across the screen. If a periodic wave is applied to the input of the vertical amplifier and the time-base unit is properly adjusted, the waveform will be displayed as a stationary pattern on the CRT screen.

Certain sections of the oscilloscope, such as deflection amplifiers and time-base unit, may be disregarded if the instrument is to be used exclusively for frequency comparisons at high signal levels. The CRT with its power supply, but minus the amplifiers and time-base unit, is sometimes called a basic oscilloscope.

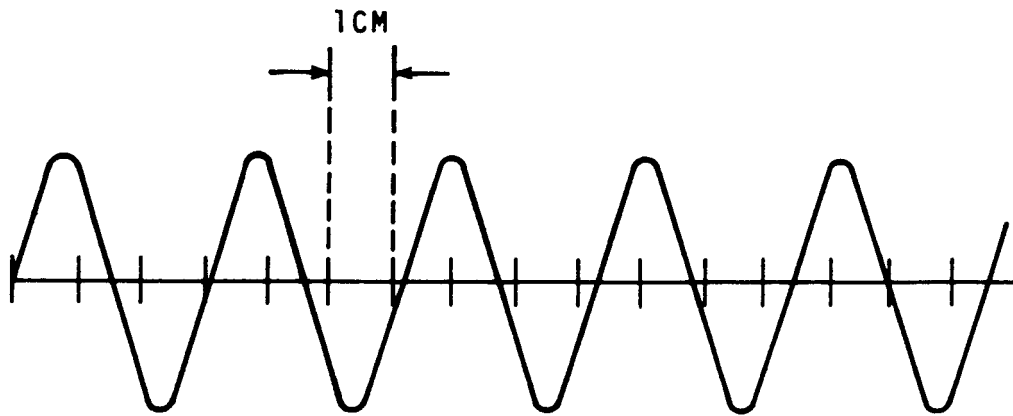


Figure 3 Sine Wave Display as Viewed on Oscilloscope

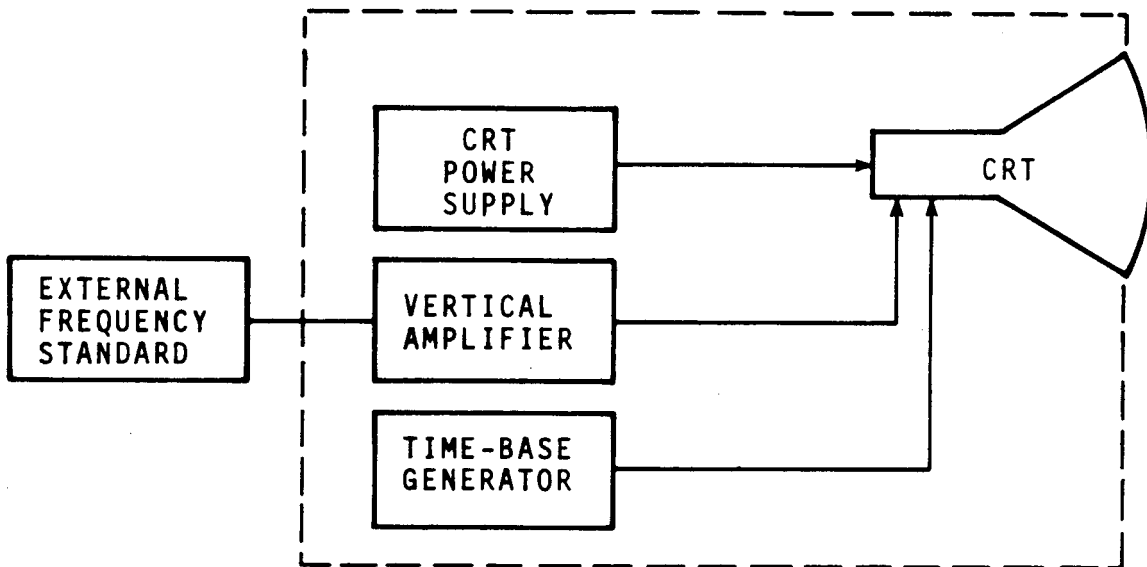


Figure 4 Time Base Calibration Hookup Using an External Frequency Standard



## 2.2 Direct Measurement of Frequency

Suppose now that we want to measure the frequency of a sine wave when it appears as shown in figure 3. By counting the number of cycles and the number of scale divisions (centimeters), we see that five cycles occur in 15.5 centimeters. Now we observe the time-base generator dial, first making sure it is in the calibrated mode. If the dial reads 0.1 millisecond per centimeter, for instance, the frequency computation would be:

$$\frac{5 \text{ cycles}}{15.5 \text{ cm}} \times \frac{1 \text{ cm}}{0.1 \text{ ms}} \times \frac{1000 \text{ ms}}{1 \text{ s}} \approx 3230 \text{ Hz} .$$

Alternatively, we may divide the display (in cycles per centimeter) by the time-base rate (in seconds per centimeter):

$$\frac{\text{cycles}}{\text{cm}} \div \frac{\text{s}}{\text{cm}} = \frac{\text{cycles}}{\text{cm}} \times \frac{\text{cm}}{\text{s}} = \text{Hz} .$$

Accuracy of the measurement is limited by the ability of the operator to read the screen and by the accuracy of the time-base and its linearity. In general, one should expect to do no better than about 0.5 percent. Still, if you want to know whether you are looking at 60 Hz or 120 Hz, or if you are checking a radio carrier frequency for harmonics, the accuracy is good enough.

If there is much doubt as to the accuracy of the time-base generator, a standard should be used for calibration. In this case, the standard can be any signal that is more accurate than the signal being measured.

The calibration is done in much the same way as the frequency measurement but in reverse. If you have a crystal oscillator or other standard source that operates in the same frequency neighborhood as the one being measured, proceed as follows.

Display the standard signal on the screen, and adjust the time-base switch to the setting that should give one cycle per centimeter. If the standard is 100 kHz, for example, the time-base step switch should be set to  $10\mu\text{s}/\text{cm}$ . Compute this setting just as you would if the standard were an unknown frequency. If the time-base generator is off frequency, there will be more or less than one cycle per centimeter displayed on the screen. Adjust the variable frequency control (not the step switch) of the time-base unit until the display is correct.

The adjustment may or may not hold for other positions of the step switch, so it is best to recalibrate each position. Many laboratory oscilloscopes have built-in calibration oscillators which can be used in the same way as an external frequency standard.

### 2.3 Frequency Comparison

As stated previously, direct frequency measurements with the oscilloscope have an accuracy limit of about 0.5 percent. Frequency comparisons with the oscilloscope, however, can easily be made with a precision of one part per million. A precision of one part per 100 million or even one part per billion can be achieved by using appropriate auxiliary equipment described later. The power and versatility of the oscilloscope as a frequency comparison device are unexcelled, especially when one remembers that comparisons can be made without any other equipment except possibly a few capacitors and resistors. The most popular comparison technique involves the interpretation of complex displays known as Lissajous patterns.<sup>2</sup>

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<sup>2</sup>The Lissajous patterns in this section are reproduced from Encyclopedia on Cathode Ray Oscilloscopes and Their Uses by John F. Ryder and Seymour D. Uslan (courtesy of Seymour D. Uslan). The publication is no longer in print.

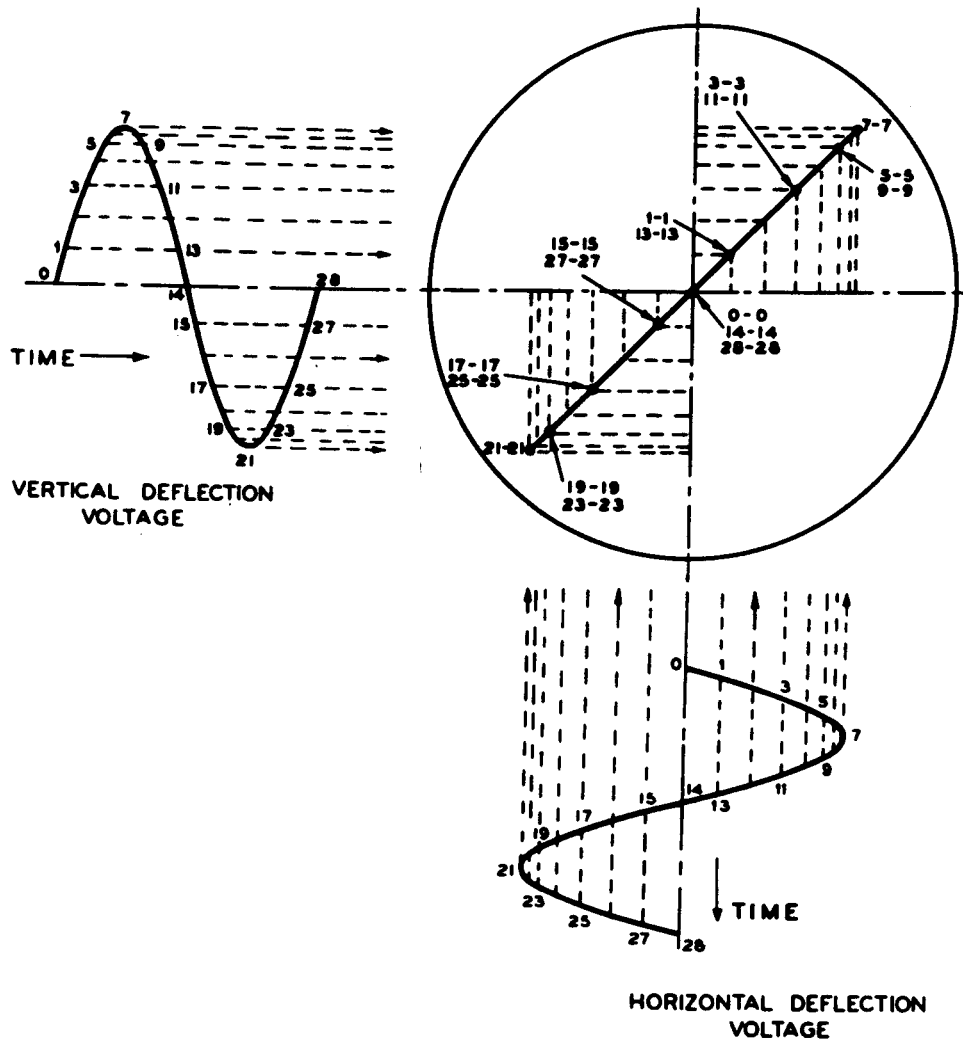
### a. Lissajous Patterns

The optical-mechanical oscillograph was developed in 1891, but the idea of a Lissajous pattern is even older--having been demonstrated by a French professor of that name in 1855. Professor Lissajous' patterns were developed using the same types of optical-mechanical devices (mirrors and light beams) that constituted the original oscillograph.

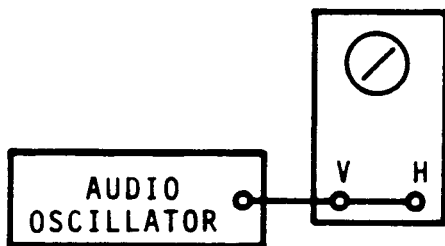
We have observed that if a periodic waveform is applied to the vertical deflection plates, either directly or through the vertical deflection amplifier, the spot on the screen will move up and down in such manner that the deflection from center screen is a measure of the instantaneous applied voltage. The same spot movement occurs from side to side if we apply this voltage to the horizontal deflection plates. If we apply a sine wave simultaneously to the horizontal deflection plates and the vertical deflection plates, the spot will move in some pattern that is a composite function of the instantaneous voltages on each set of deflection plates.

The simplest Lissajous pattern occurs if we connect the same sine wave to both sets of plates. The equipment hookup and the development of the pattern are shown in figure 5. The resulting pattern, as you can see, is a straight line. Furthermore if both signals are of the same amplitude, the line will be inclined at  $45^{\circ}$  to the horizontal. If instead of being in phase the two input signals were oppositely phased, the line would be perpendicular to the one shown.

If the input signals are equal in amplitude and  $90^{\circ}$  out of phase, a circular pattern results. Other phase relationships produce an elliptical pattern. The phase difference in degrees between the horizontal sine wave and the vertical sine wave can be determined as shown in figure 6.



b. Pattern Development



a. Equipment Hookup

Figure 5 The 1:1 Lissajous Pattern for Two Identical Frequencies in Phase

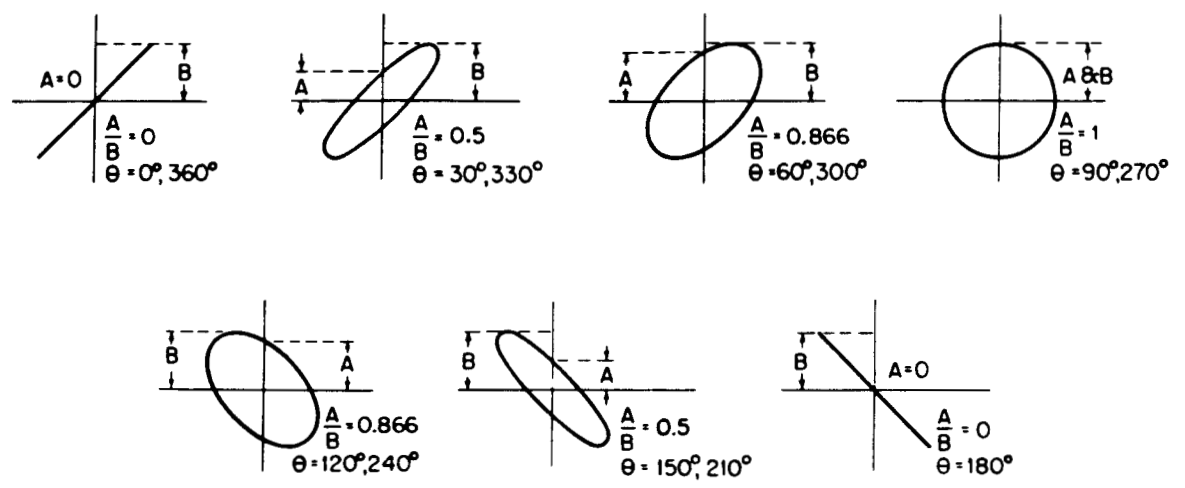
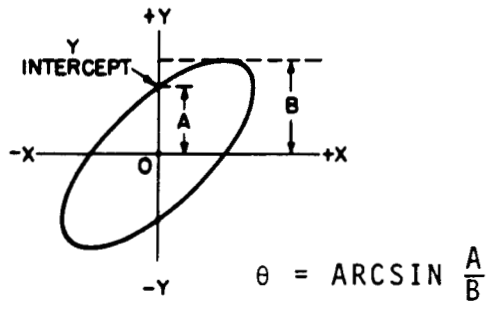


Figure 6 Elliptical Lissajous Patterns for Two Identical Frequencies of Different Phase

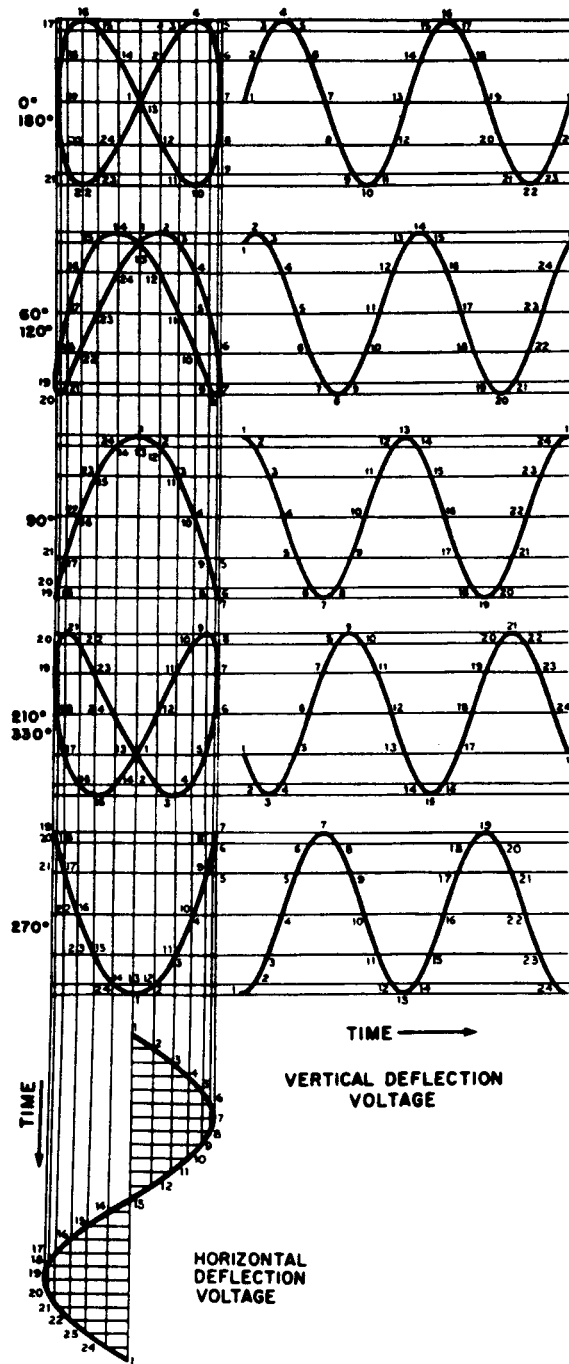


Figure 7 Development of 2:1 Lissajous Patterns

The phase relation between signals that generate linear, circular, or elliptical patterns is given by

$$\sin \theta = \frac{A}{B}$$

where  $\theta$  is the phase angle between the two sine waves, and A and B are relative distances measured along the vertical axis of the trace.

If sine waves of different frequency are applied to the deflection plates, more complex patterns result. Shown in figure 7 are several 2:1 patterns. Figures 6 and 7 reveal that the shape of the Lissajous pattern depends upon the frequency ratio as well as the initial phase of the two signals.

The actual determination of frequency ratio is done in a straightforward manner. The number of loops touching, or tangent to, a horizontal line across the top of the pattern is counted. Likewise the number of loops tangent to either the right or left side is counted. The frequency ratio is then computed from the formula

$$\frac{f_v}{f_h} = \frac{T_h}{T_v}$$

where  $f_v$  is the frequency of the vertical deflection voltage,  
 $f_h$  is the frequency of the horizontal deflection voltage,  
 $T_h$  is the number of loops tangent to the horizontal line, and  
 $T_v$  is the number of loops tangent to the vertical line.

Figure 8 shows the graphical construction of a 3:1 pattern. Note that the ratio of  $T_h/T_v$  is 3/1. Lissajous patterns with  $f_v = 3f_h$  are shown in figure 9; those with  $f_h = 3f_v$  are shown in figure 10.

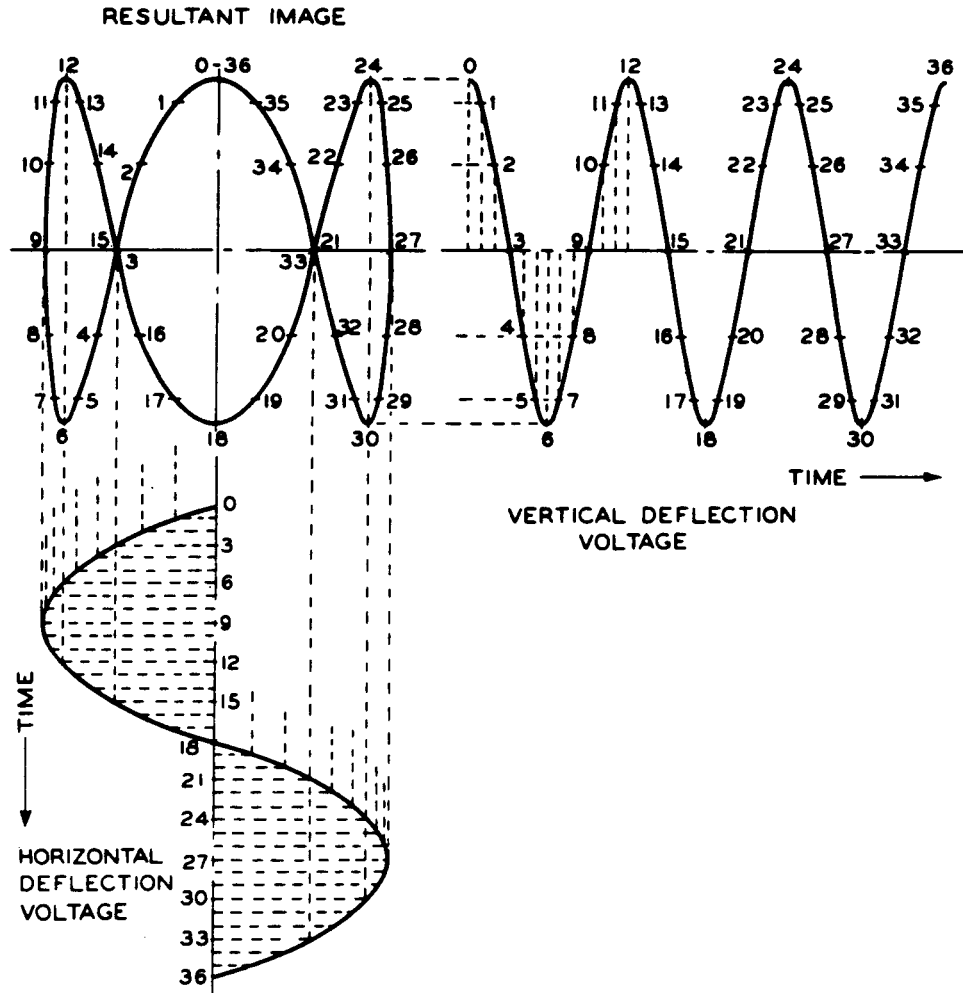


Figure 8 Development of a 3 : 1 Lissajous Pattern



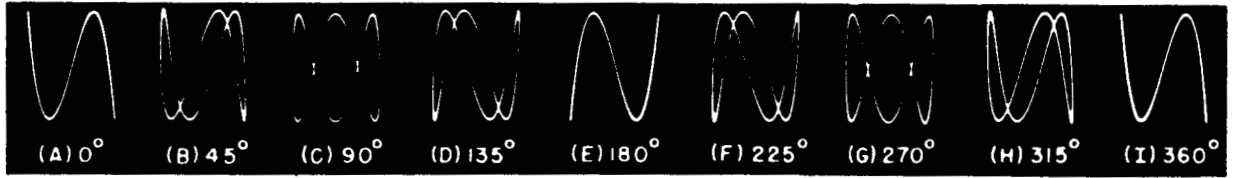


Figure 9 Lissajous Patterns with  $f_v = 3f_h$

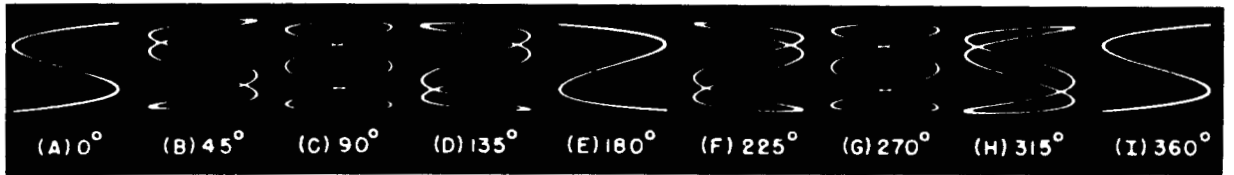
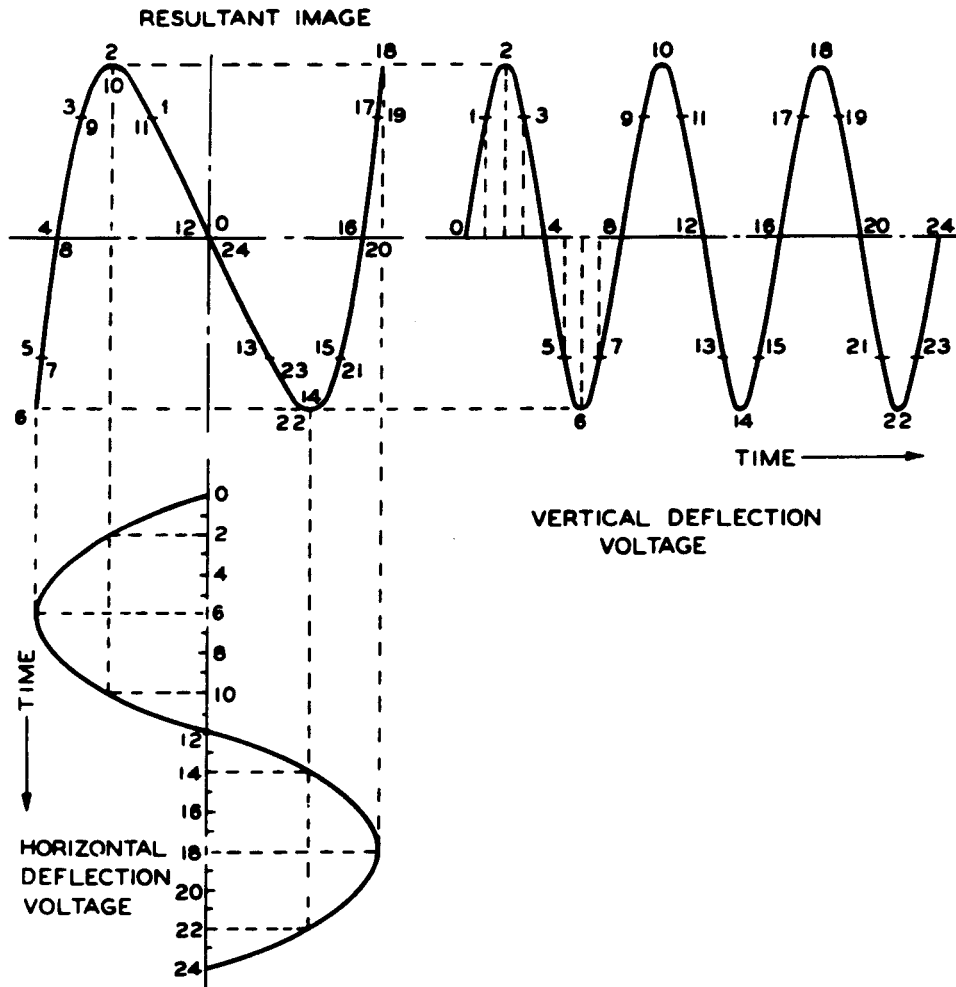
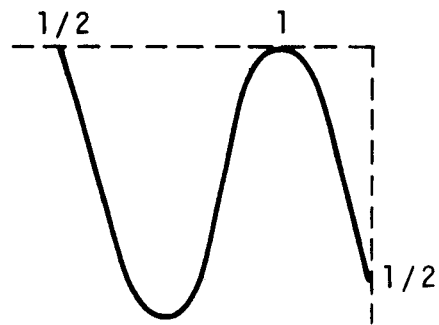


Figure 10 Lissajous Patterns with  $f_h = 3f_v$



a.  $180^\circ$  Initial Phasing



b.  $0^\circ$  Initial Phasing

Figure 11 Folded Lissajous Patterns of 3:1 Ratio

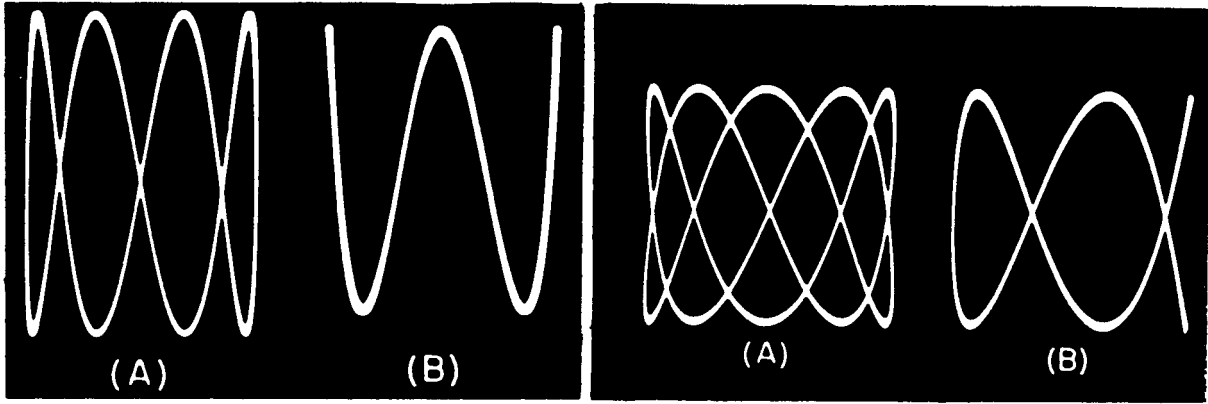
Figure 11a depicts a 3:1 pattern with 180° initial phasing. If the initial phase is shifted by 90° in either direction, the three-lobed pattern of figure 8 occurs. A further shift of 90° in the same direction produces another folded pattern, or double image, as shown in figure 11b.

The double images are somewhat tricky to interpret for, strictly speaking, a vertical line cannot be drawn tangent to either the right or left edge. In such cases each crossover point along the margin is counted as one-half while each point of tangency is given a full count of one. Thus for the pattern of figure 11b,

$$\frac{f_v}{f_h} = \frac{T_h}{T_v} = \frac{1 + \frac{1}{2}}{\frac{1}{2}} = \frac{3}{1} .$$

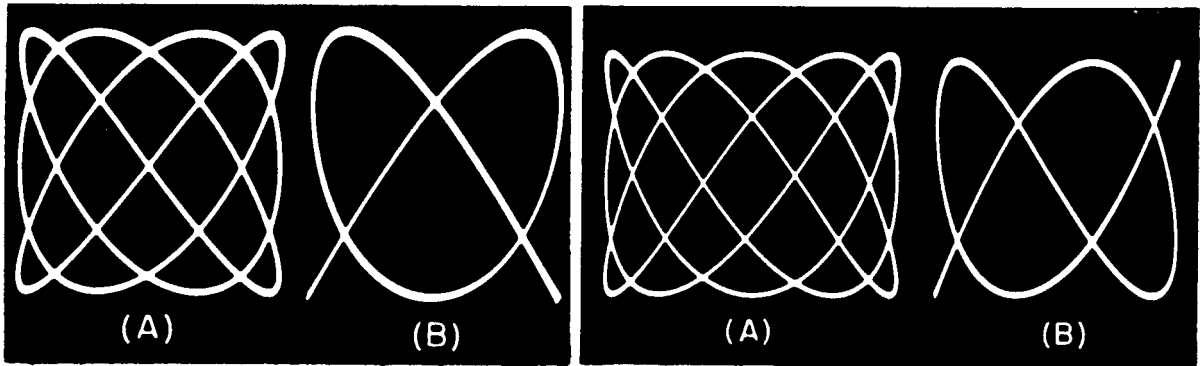
As an alternate procedure one may determine the frequency ratio by drawing imaginary lines both vertically and horizontally through the Lissajous pattern at any convenient point. The ratio is then obtained by counting the number of places where the vertical line and the horizontal line intersect the pattern. When using this technique it is best to avoid drawing either line through those points where the pattern crosses itself. If some of the crossovers cannot be avoided, however, each crossover point should be counted twice where it falls on either the vertical or horizontal axis chosen.

Lissajous patterns become quite complicated when the constituent frequencies are widely separated. Some oscillograms for higher ratios are shown on the following pages. Two versions of each pattern are depicted. Version (A) is the typical closed-lobe pattern, whereas version (B) is the double image that results if phase relations cause the pattern to be folded and exactly superimposed upon itself.



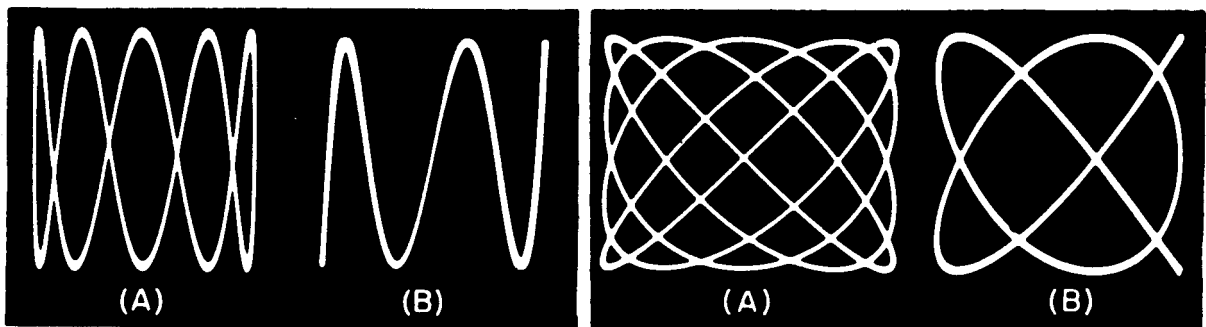
4:1 Patterns

5:2 Patterns



4:3 Patterns

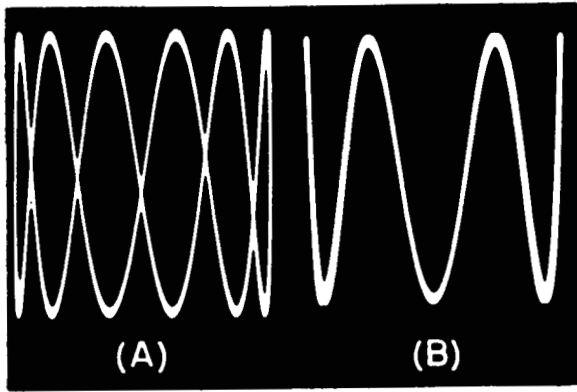
5:3 Patterns



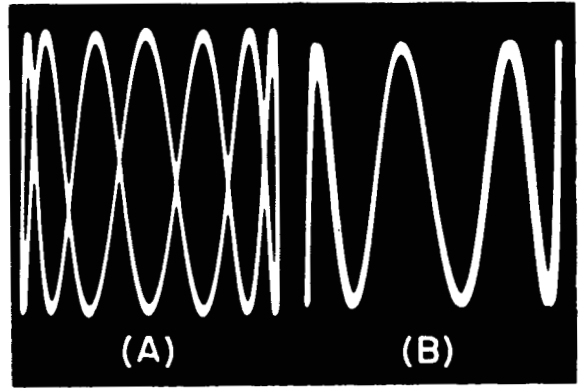
5:1 Patterns

5:4 Patterns

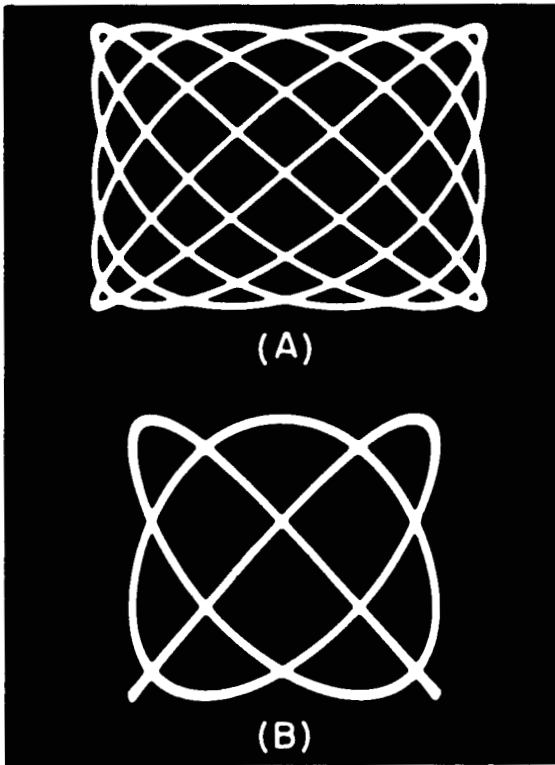
Figure 12 Lissajous Patterns, 4:1 Through 5:4



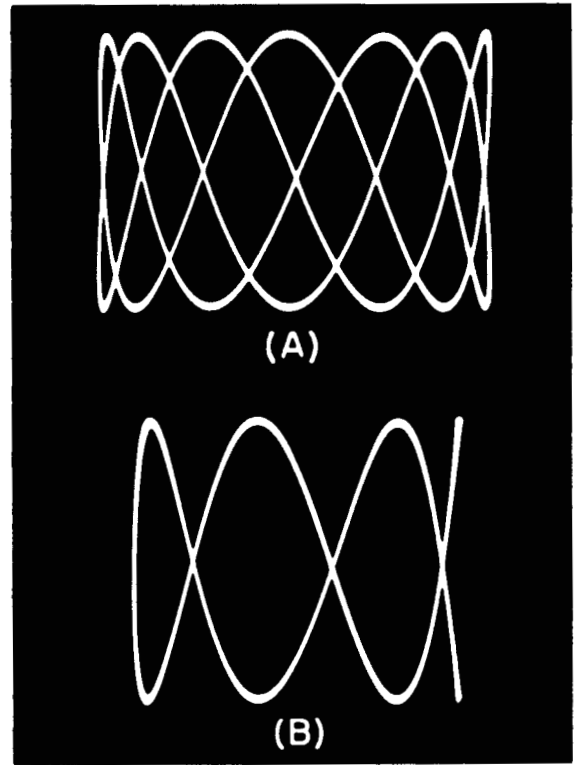
6:1 Patterns



7:1 Patterns

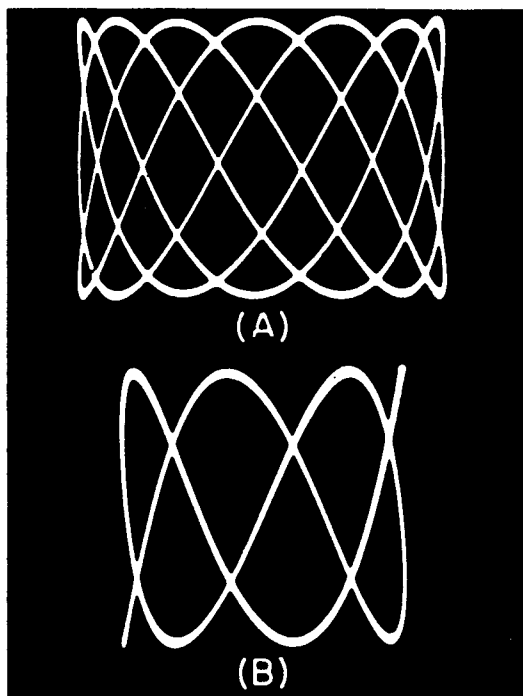


6:5 Patterns

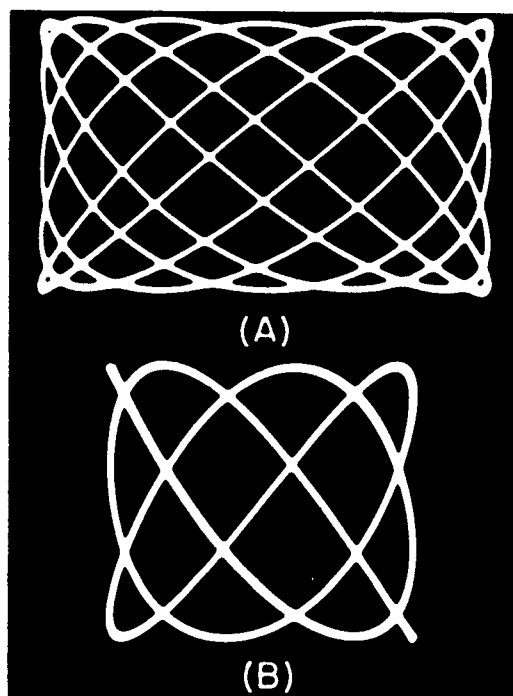


7:2 Patterns

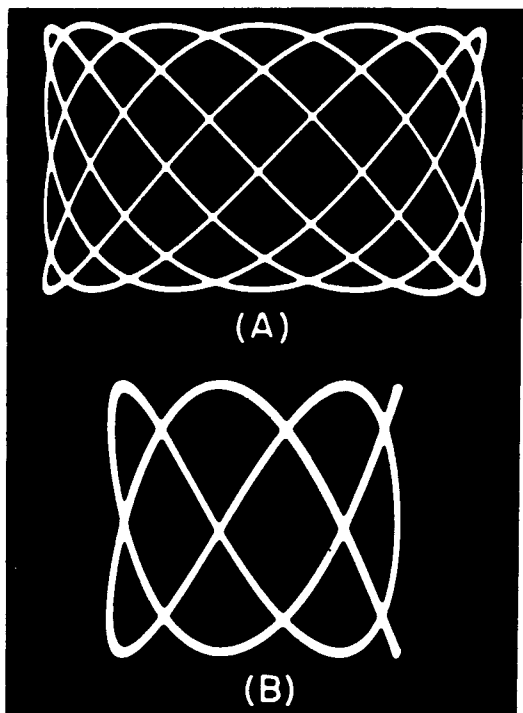
Figure 13 Lissajous Patterns, 6:1 Through 7:2



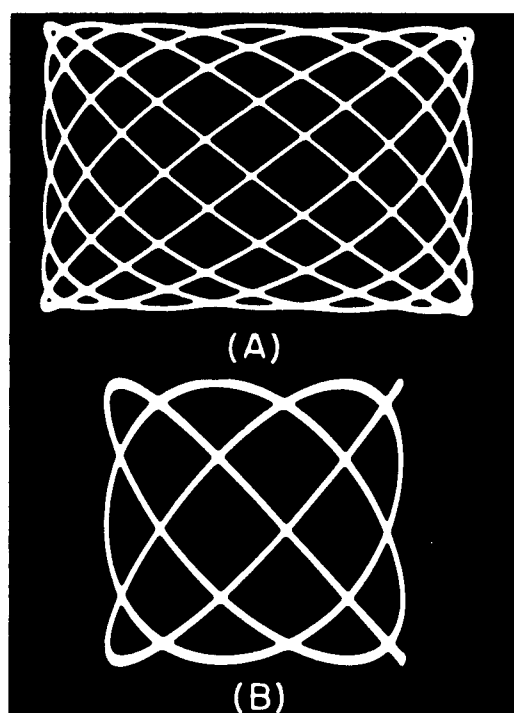
7:3 Patterns



7:5 Patterns

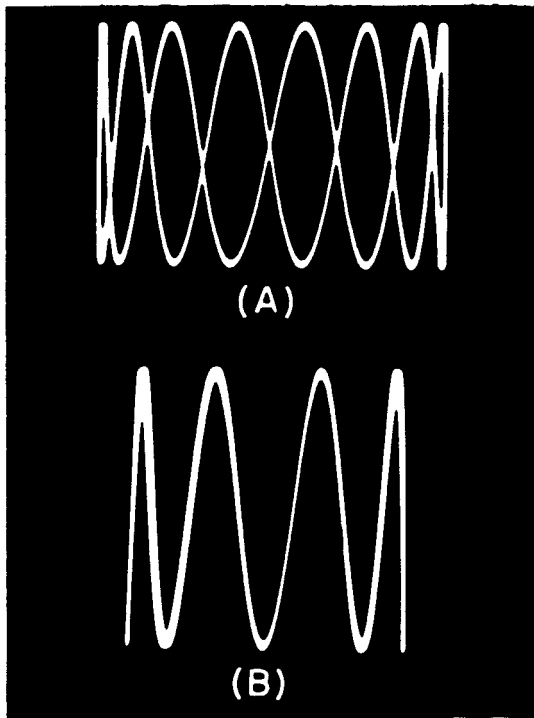


7:4 Patterns

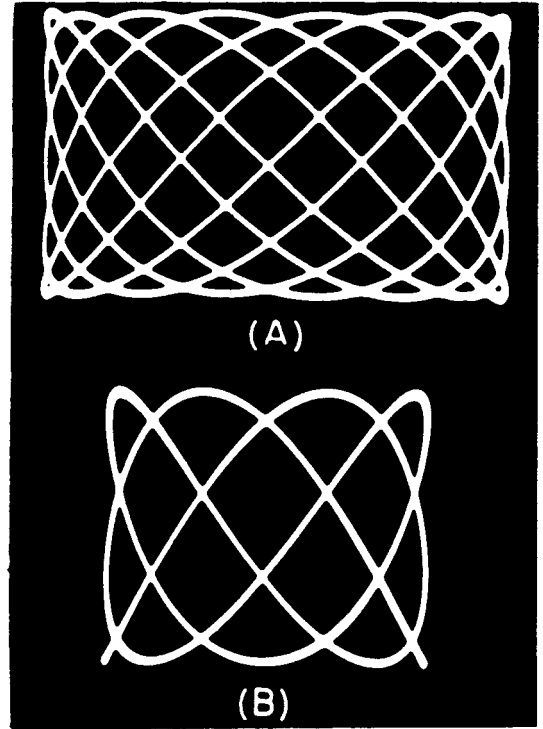


7:6 Patterns

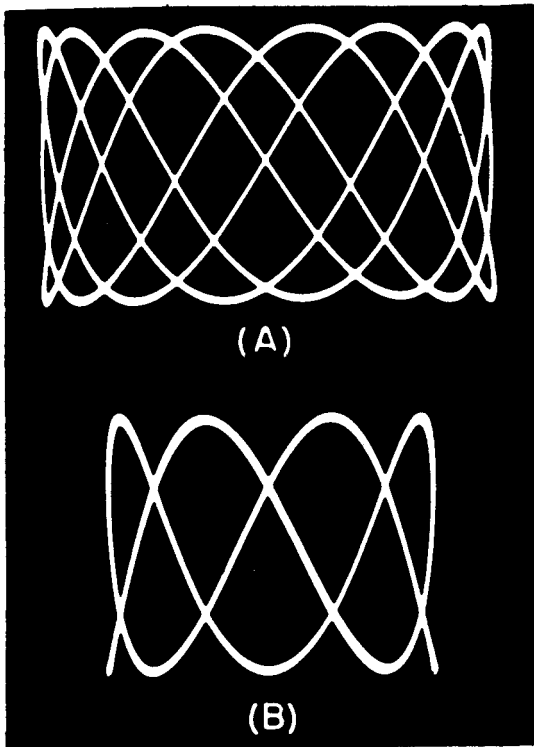
Figure 14 Lissajous Patterns, 7:3 Through 7:6



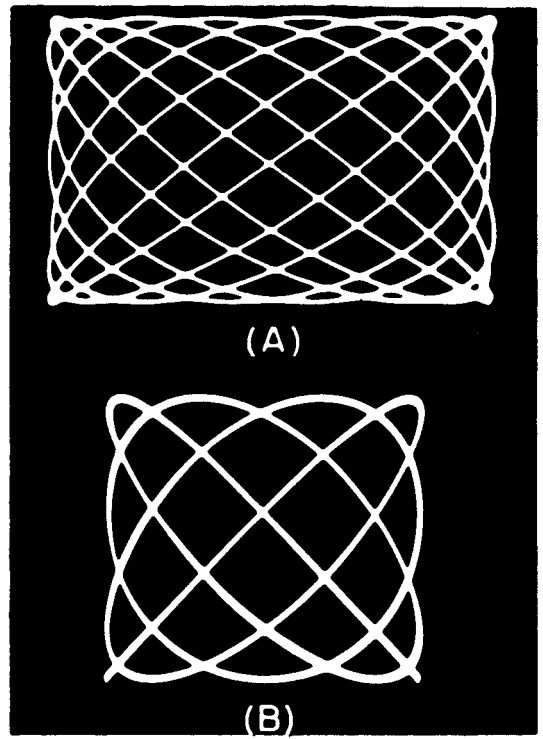
8:4 Patterns



8:5 Patterns

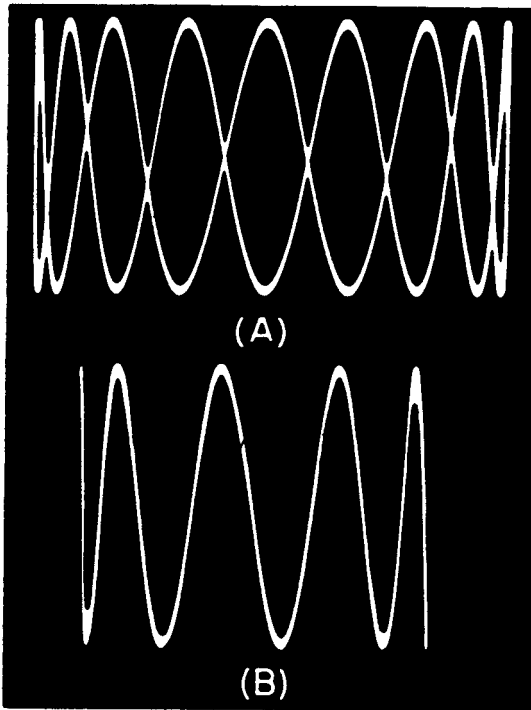


8:3 Patterns

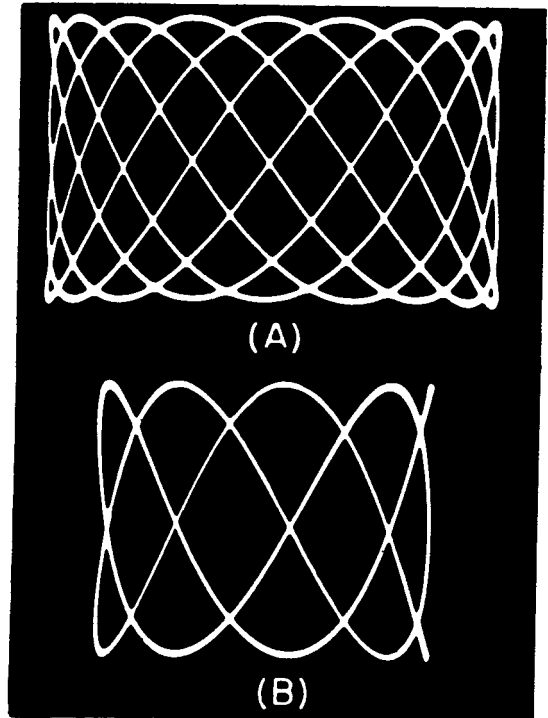


8:7 Patterns

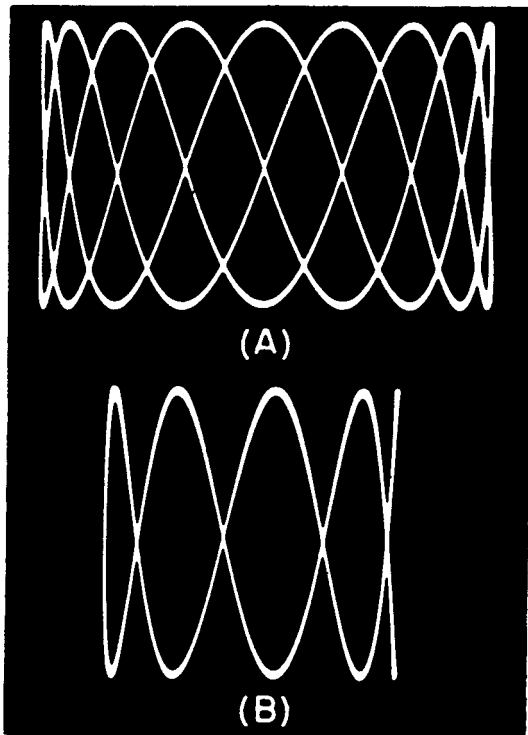
Figure 15 Lissajous Patterns, 8:1 Through 8:7



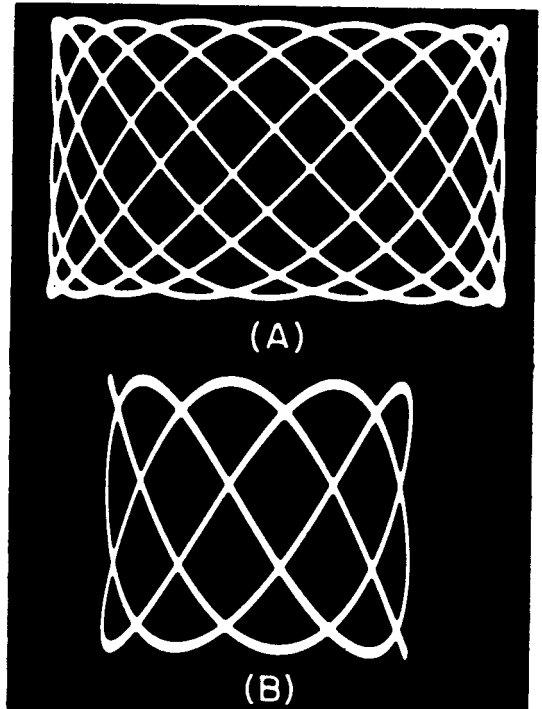
9:1 Patterns



9:4 Patterns



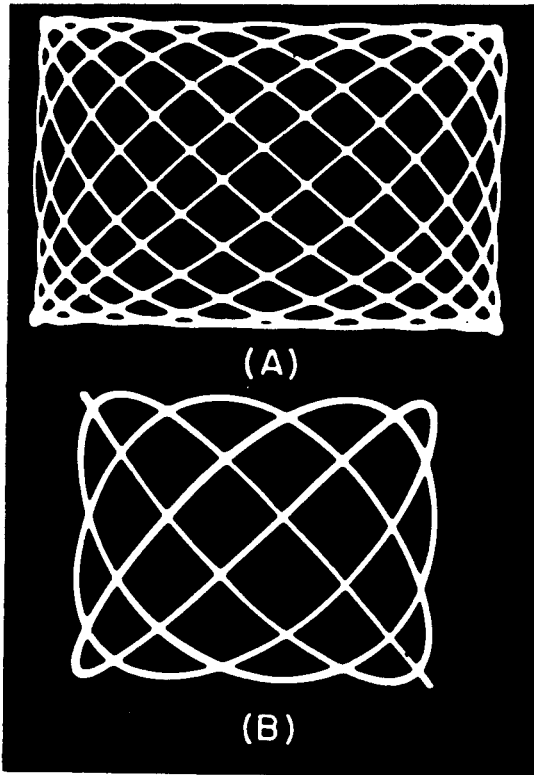
9:2 Patterns



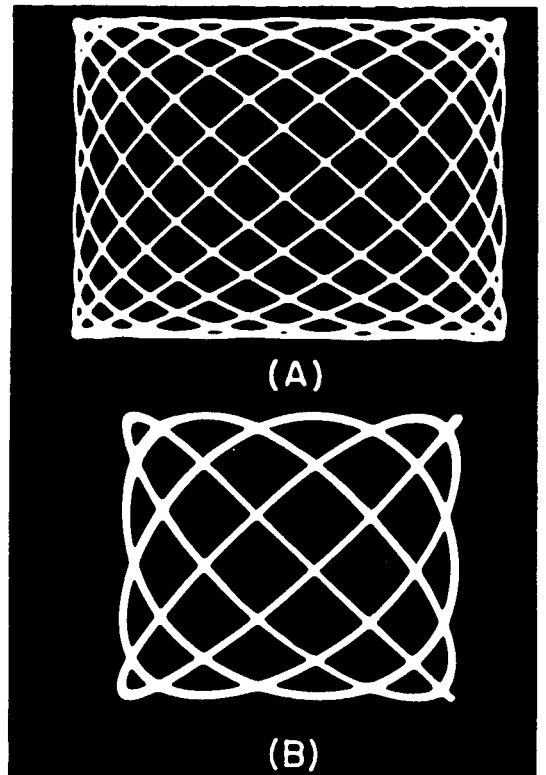
9:5 Patterns

Figure 16 Lissajous Patterns, 9:1 Through 9:5

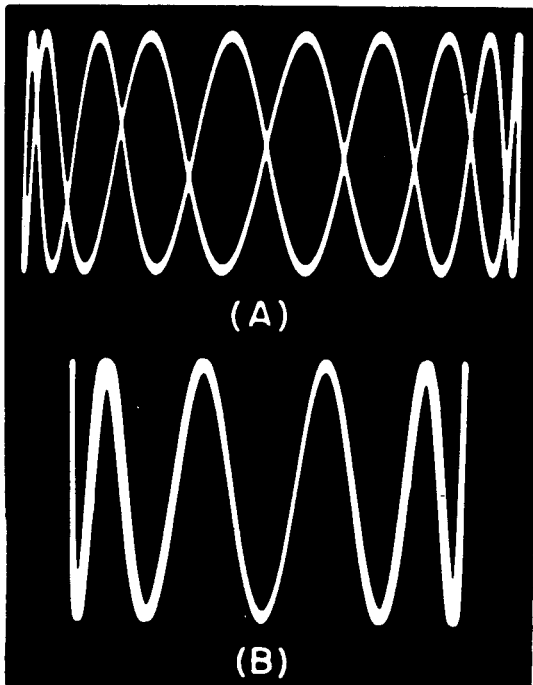




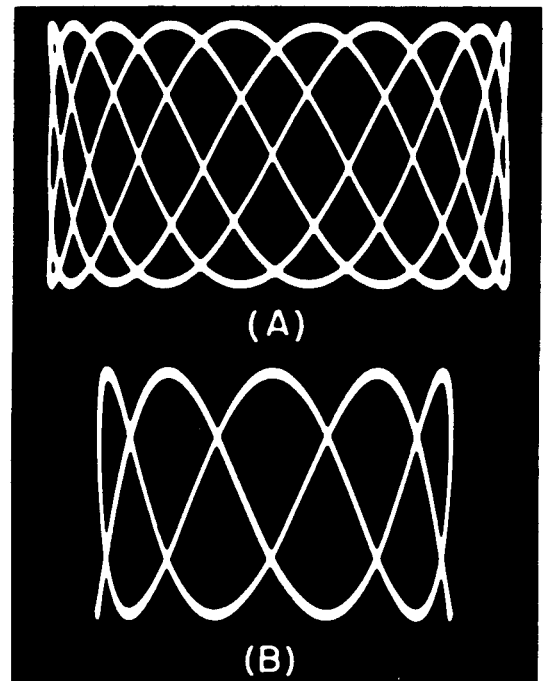
9:7 Patterns



9:8 Patterns



10:1 Patterns



10:3 Patterns

Figure 17 Lissajous Patterns, 9:7 Through 10:3

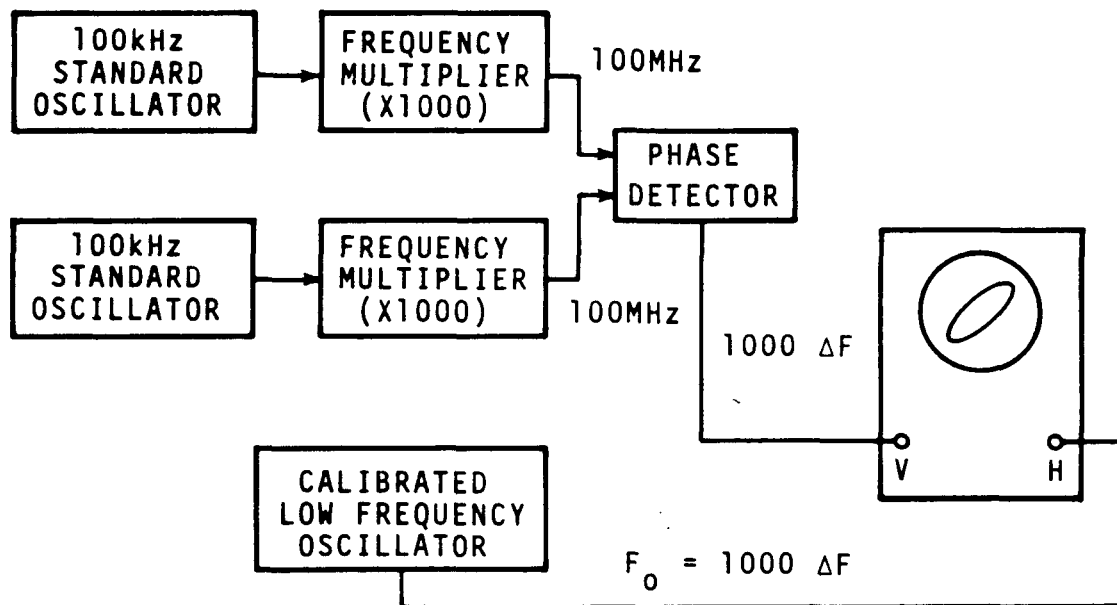


Figure 18 Arrangement for Measuring Small Frequency Differences with Oscilloscope and Frequency Multipliers

The precision of frequency determination by Lissajous patterns depends upon a number of factors. For the moment let us consider only 1:1 patterns. If we are trying to synchronize two power generators at 60 Hz and our pattern is changing through one complete sequence every second, we have achieved synchronization to 1 Hz or about 1.7 percent--not very good. If our deflection voltages however have a frequency of 100 kHz, the same change of one revolution per second in our pattern would give a precision of  $1 \times 10^{-5}$ . This demonstrates that the higher the frequencies we are working with, the easier the job of synchronization becomes for a given degree of precision.

In comparing or synchronizing precision oscillators, a frequency multiplier may be useful. Every time our signals are multiplied in frequency by a factor of ten, the precision of measurement improves by a factor of ten. If we multiply to the extent that we exceed the frequency range of the oscilloscope, however, we may be obliged to return to our bag of tricks and pull out a phase detector. (A detailed discussion of frequency multipliers and phase detectors appears in section 4.)

When the two signals are multiplied and then fed to a phase detector, the phase detector acts as a frequency-difference generator; i.e., it produces a low-frequency signal equal to the difference in hertz between our two multiplied signals. This difference signal may then be compared via Lissajous pattern against the sine wave from a precision low-frequency oscillator.

If we again use two 100-kHz signals as an example but multiply their frequencies to 100 MHz before comparing them, we achieve a measurement precision of  $1 \times 10^{-8}$  for our previously assumed resolution of one hertz. An equipment arrangement for this measurement is shown in figure 18.

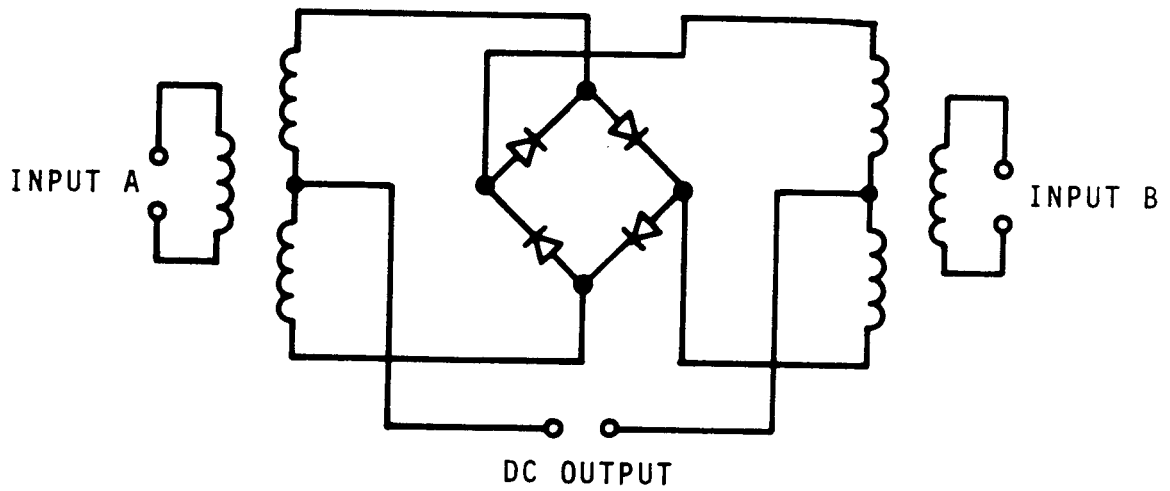


Figure 19 Double Balanced Mixer

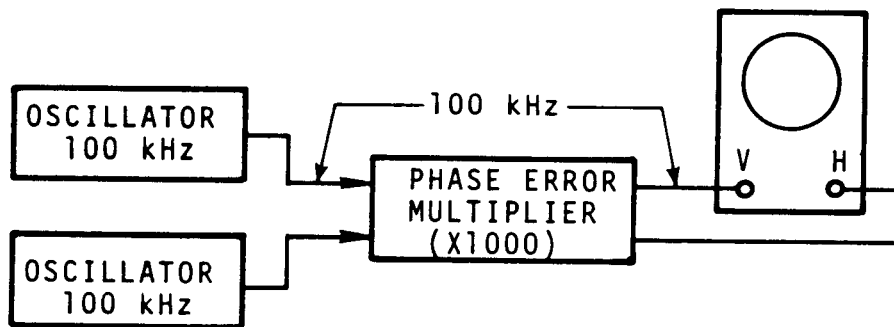


Figure 20 Arrangement for Frequency Comparison Using Oscilloscope and Phase Error Multiplier

The phase detector may be a double-balanced mixer of the type shown in figure 19. These mixers are available commercially, but they may also be constructed rather easily in the laboratory. High quality vhf components should be used, especially for the diode bridge.

Another system for comparing oscillators is indicated in figure 20. A phase-error multiplier increases the phase or frequency difference between the oscillators by some factor, say 1000, without entailing frequency multiplication into the vhf region. (See section 4 for details of how this is accomplished.) The two output signals from the phase-error multiplier can be fed directly to the oscilloscope. The resulting Lissajous pattern will allow oscillator synchronization with the same precision as provided by the frequency multiplier technique. If care is taken, synchronization of  $1 \times 10^{-9}$  to  $1 \times 10^{-10}$  can be accomplished.

A principal use of the 1:1 Lissajous pattern is for synchronization of oscillator frequencies. If the pattern is drifting, no matter how slowly, it means that a frequency difference exists between the oscillators. Neither the oscilloscope nor the operator can tell which oscillator is at the higher frequency. The Lissajous pattern is the same regardless of which frequency is higher. What can be done though is to slightly change the frequency of one of the oscillators. If the rate of pattern drift increases when the frequency of oscillator #1 is raised, then oscillator #1 is at a higher frequency than oscillator #2. The opposite relation exists if the pattern drift slows down when the frequency of oscillator #1 is increased.

Calibration of variable oscillators such as rf signal generators, audio oscillators, etc. is easily accomplished using Lissajous patterns of higher than 1:1 ratio. Figure 21 shows a method of calibrating an rf signal generator using a fixed 100-Hz frequency standard.

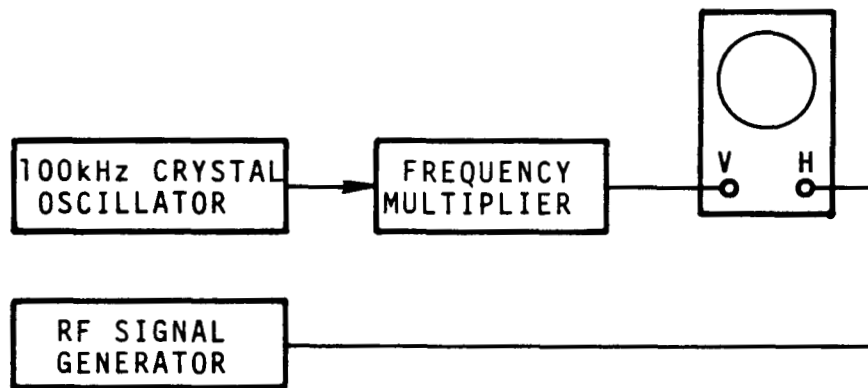


Figure 21 Arrangement for Calibrating a Signal Generator

Most modern crystal oscillators are of higher frequency than 100 kHz. If this is true of the standard oscillator available, the multiplier shown in figure 21 will not be necessary. Starting with a 1:1 Lissajous pattern between the standard oscillator and the signal generator, mark the calibration points on the generator dial each time the pattern becomes stationary as the dial is turned. Each stationary pattern represents a definite frequency ratio which can be evaluated by one of the methods explained previously. Patterns of ratio up to 20:1 can be recognized if the signal generator is stable enough.

If the standard oscillator has a base frequency of 1 MHz with internal dividers to 100 kHz, calibration points can be found on the signal generator dial every 100 kHz from 0.1 MHz to 2 MHz and every 1000 kHz from 2 MHz to 20 MHz. This range is adequate for most rf signal generators. Multipliers can be used to extend the markers into the vhf range if necessary, but the calibration points will be farther apart.

Not much can be determined from a drifting pattern of high ratio, for even the stationary pattern may be so complex as to defy interpretation. A complete rotation of the pattern in one second corresponds to a 1-Hz deviation by the higher frequency no matter what the ratio, but for ratios other than 1:1 the possible extent of deviation by the lower frequency is reduced proportionately.

With a 10:1 pattern, for instance, a drift of one revolution per second could mean that the higher frequency is off by 1 Hz, or it could mean that the lower frequency is off by 0.1 Hz. Unless one of the two frequencies is taken to be the standard, there is no way of telling from the Lissajous pattern which frequency is actually in error. Only their relative difference can be deduced by this method.

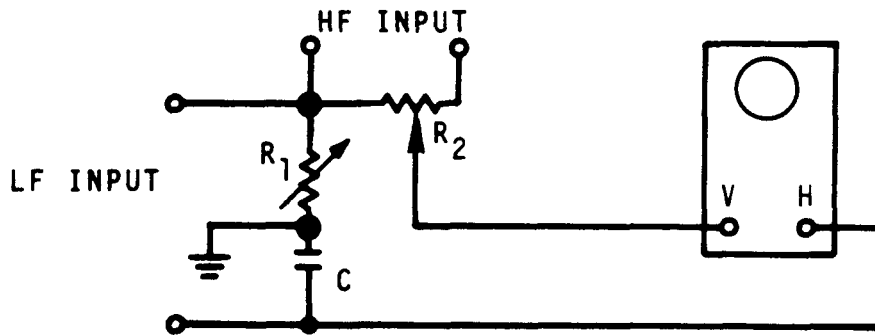
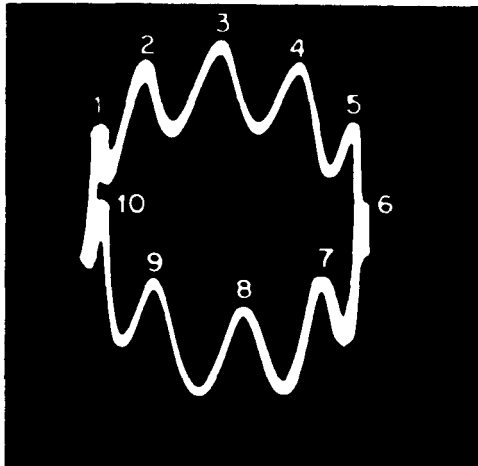
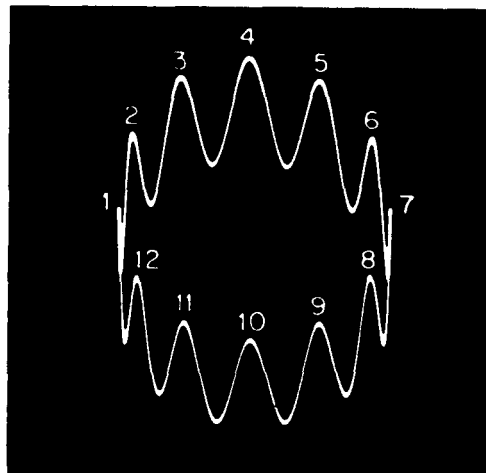


Figure 22 Phase Shifter for Modulated Lissajous Patterns



(a)



(b)

Figure 23 Modulated Lissajous Patterns, 10:1 and 12:1



### b. Modulated Lissajous Patterns

Normal Lissajous patterns are difficult to interpret if the ratio is much higher than 10:1, but ratios up to 20:1 or higher can be recognized from modulated patterns. The method of generating such patterns is shown in figure 22.

In the discussion of 1:1 Lissajous ratios it was pointed out that a circle is formed if the initial phase relation of the two signals is a  $90^\circ$  or  $270^\circ$  difference. Components  $R_1$  and C of figure 22 form a simple phase shifting network that allows a circle to be displayed on the oscilloscope when a low-frequency signal is applied to the LF input. This frequency is low only in the sense that it is lower than the high-frequency signal which is applied to the HF input. The conditions for  $90^\circ$  phase shift are

$$R_1 = X_C = \frac{1}{2\pi f_{\text{low}} C}$$

If a low-amplitude signal of higher frequency is applied to the HF input, the Lissajous circle will become modulated as shown in figure 23.

A Lissajous figure can be thought of as a wave traveling around a glass cylinder in such manner that all portions of the wave are visible at one time or another. The modulated Lissajous figure can be thought of the same way. It has the added advantage, however, that regardless of the initial phase relationship no part of the waveform will be doubled or covered over by another part. If the positive peaks of the pattern in figure 23 are counted, this will be the numerator of the frequency ratio. If there are no intersections in the pattern, the denominator is one. Thus figure 23a is the pattern for a 10:1 ratio, whereas figure 23b is the pattern for a 12:1 ratio.

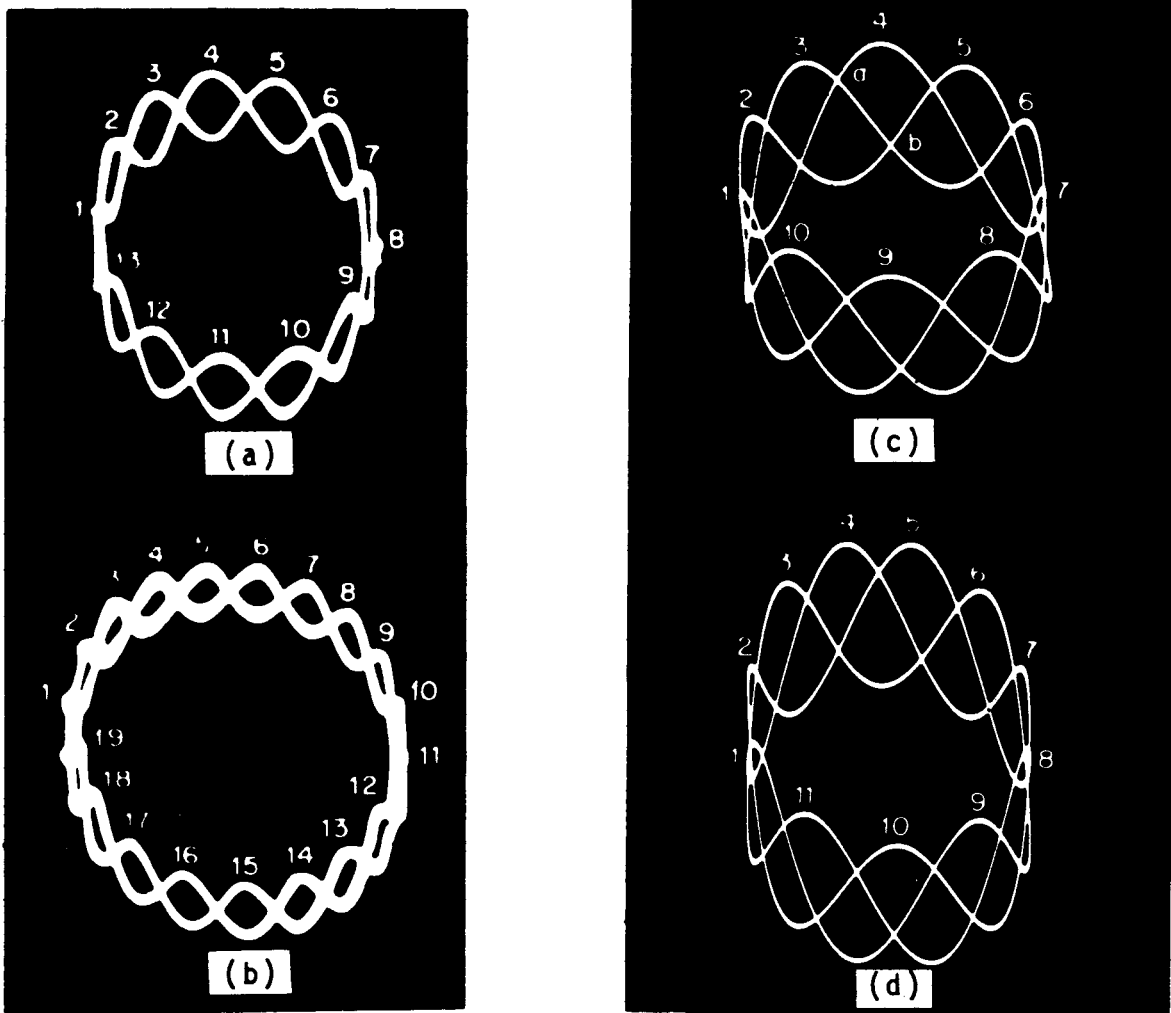


Figure 24 Modulated Lissajous Patterns, 10:3 Through 19:2

In figure 24 the peaks of the patterns are numbered. Pattern (a) has 13 peaks; therefore the numerator of the ratio is 13. The denominator is determined by counting the number of intersections between a positive peak and a negative peak and then adding one to the result. Pattern (a) has one intersection, so the denominator is  $1 + 1 = 2$  and the ratio is 13:2. Pattern (b) has 19 positive peaks and again one intersection between a positive peak and a negative peak. Its ratio is 19:2. Pattern (c) has 10 positive peaks and two intersections, giving a ratio of 10:3. The ratio for pattern (d) is 11:3.

### c. Recorded Slit Method

Most frequency comparisons made in a standards laboratory are between oscillators that have very small frequency differences. For instance a 1:1 Lissajous pattern between oscillators having a fractional difference of  $1 \times 10^{-11}$  would take longer than a day to complete one revolution if the comparison were made at 1 MHz. There are faster ways, however, to measure the rate of change on the oscilloscope. These methods allow the difference frequency to be determined in much shorter time than is possible with a slow moving Lissajous pattern.

If the output wave of a standard oscillator is displayed on the oscilloscope, the sweep frequency can be calibrated by adjusting the variable sweep rate until the display shows the correct number of cycles for a particular setting of the sweep rate step switch. Now if the oscilloscope is triggered by another standard oscillator, the pattern or waveform will appear to be stationary provided the frequencies of the two oscillators are quite close. Actually the pattern will drift unless the oscillators are locked together as sometimes happens. The rate of drift in microseconds per day is directly convertible to the frequency offset between the oscillators. The direction of drift tells us which oscillator is high or low in frequency with respect to the other.

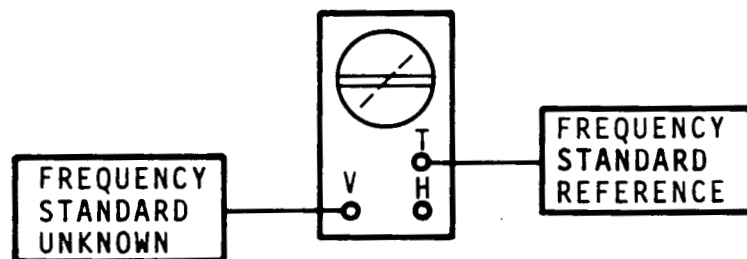


Figure 25 Arrangement for Recorded-Slit Photographic Method

For many years laboratory workers used a recorded-slit photographic technique to compare oscillators. This method is no longer extensively used, but it still bears mentioning. The oscilloscope screen in figure 25 has been taped over except for a narrow horizontal slit across the face at the zero axis. When a sine wave of unknown frequency is displayed and the oscilloscope is triggered by a reference oscillator, all that shows through the slit is a dot of light. This dot will move to the left if the unknown frequency is high and to the right if low with respect to the frequency of the reference oscillator. The question becomes how fast is the dot moving? How many microseconds is the spot moving in how many hours or days? Remember the sweep is calibrated in microseconds per centimeter or in milliseconds per centimeter.

One could make a measurement of the dot position at, say, 10:05 a. m. then come back at 2:30 p. m. for another look. Suppose in the meantime the dot moves exactly three centimeters and the sweep speed is set at one microsecond per centimeter. What does this tell us?

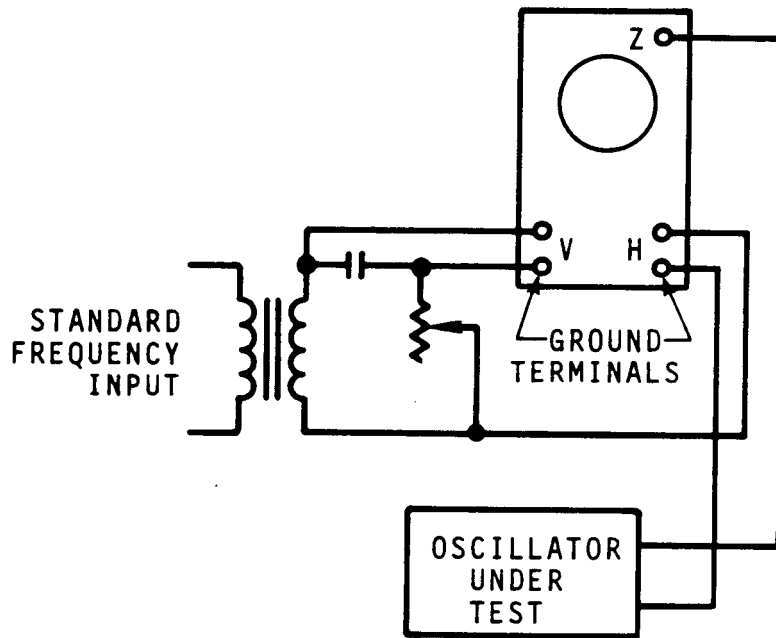
$$\text{elapsed time} = 2:30 \text{ p. m.} - 10:05 \text{ a. m.} = 4\text{h } 25\text{min} = 0.184 \text{ day}$$

$$\text{phase drift rate} = \frac{3.00 \text{ cm} \times 1.00 \mu\text{s/cm}}{0.184 \text{ day}} = 16.3 \mu\text{s/day}$$

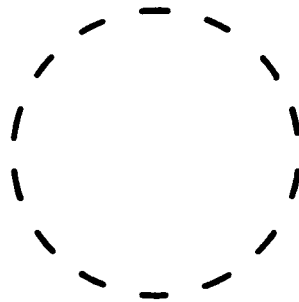
Thus the relative phase of the two oscillators is changing at a rate of 16.3 microseconds per day.

To convert the phase drift rate into a fractional frequency offset we use the fact that there are 86,400 seconds (or  $86,400 \times 10^6$  microseconds) in a day. Hence

$$\frac{16.3 \mu\text{s/day}}{86,400 \times 10^6 \mu\text{s/day}} = \frac{16.3 \mu\text{s/day}}{8.64 \times 10^{10} \mu\text{s/day}} = 1.89 \times 10^{-10}$$



a. Arrangement for Producing a Spotwheel Pattern



b. Spotwheel Pattern for 14:1 Ratio

Figure 26 Calibration by Spotwheel Method

Observe that the actual comparison frequency does not enter into the computation for frequency offset. Also it was assumed in the foregoing example that the oscillators are running at a constant frequency. If this assumption is valid, the computed rate of 16.3 microseconds per day is accurate even though the measurement was confined to an interval of only a few hours. But what if the spot had moved three centimeters in one hour and remained there for the duration of the measurement?

The average frequency offset during the interval from 10:05 a.m. to 2:30 p. m. would still be  $1.89 \times 10^{-10}$ , yet one could not safely extrapolate such behaviour to a period of one full day. Nor is it practical to sit in front of the oscilloscope all day and watch the motion of the spot. The only alternative is to record the spot movement automatically.

One way to accomplish the recording is to use a 35 mm camera with a clock motor drive attached to the film advance shaft. A convenient rate for advancing the film is about two centimeters per hour. A shortcoming of this system is that there are no time calibration marks on the film, and furthermore one doesn't know what is happening until the film is developed. With the advent of modern linear phase detectors, the clock-driven camera has been superseded by other recording devices; but the photographic method still may have application in some instances.

A variation of the recorded-slit arrangement is shown in figure 26. Here the unknown frequency is applied to the z-axis input of the oscilloscope and used to intensity modulate the horizontal trace generated by the internal sweep oscillator. Triggering is initiated by the external reference oscillator. Lateral movement of the intensified spot is recorded and analyzed as described before.

## 2.4 Time Comparison

Time comparisons, or more properly, time-interval measurements are quite easy to perform with the oscilloscope. Usually one time pulse is displayed on the screen while a different time pulse is used to trigger the oscilloscope. Another method is to display both time pulses on a dual-trace scope. Triggering can be achieved from either of the two displayed pulses or from a third time pulse. A slewable divider (see section 4.4) is handy for this purpose.

A point worth remembering is that the accuracy of the time interval measurement is inversely proportional to the length of the interval. If the time pulses are 10 milliseconds apart, for instance, the measurement can be performed to probably no better than  $\pm 50$  microseconds. But if the pulses are only 100 microseconds apart, the measurement can be performed with an accuracy of  $\pm 0.5$  microsecond. In both cases, however, the fractional uncertainty is the same ( $\pm 5$  parts in  $10^3$ ).

For time pulses spaced close together the pulse width may occupy an appreciable percent of the time interval; therefore large errors can be introduced by having the wrong trigger slope or improper trigger amplitude. Unless care is taken by the operator, such errors can nullify the accuracy that is otherwise attainable in the measurement of brief time intervals.

The main advantage in using an oscilloscope for time interval measurements is that one can observe the pulse waveform. For this reason time intervals between odd-shaped or noisy pulses are better measured with the oscilloscope than with other types of instruments. Time intervals in the submicrosecond range can be measured easily and economically with high-frequency oscilloscopes, but long intervals are more accurately measured with an electronic counter, as described in section 5.



## 2.5 Signal Averaging

Signal averaging is a method widely used to extract time signals or other periodic events from a noisy background. Two criteria must be met if signal averaging is to be successful: (1) the periodic event must be relatively stable in time, and (2) the noise must be random. For our purposes random noise is considered to be that type of noise for which the average amplitude at any given frequency is zero.

There are three methods of signal averaging which employ the oscilloscope as a display device. In the following discussion we shall assume some periodic event such as a standard seconds pulse. If the seconds pulse were displayed on an oscilloscope that is triggered in synchronism with the pulse, the same point on each succeeding pulse would be displayed at the very same spot on the oscilloscope face. If noise is present, the amplitude of the pulse at that point will move up and down depending upon the amplitude of the noise there. As long as the noise is random, the average amplitude at that spot will be the same as the amplitude of the pure signal. Now if we could increase the persistence of the oscilloscope so that many seconds pulses could be observed at once, the brightest part of the image would coincide with the average of all the individual traces.

Storage oscilloscopes have the ability to increase their screen persistence so the image is retained for long periods of time. Each successive trace is stored on top of the others so the average signal can be discerned easily by eye. Another method uses camera and film as the storage device. The scope-mounted camera is adjusted so its shutter is open for a considerable length of time. Whenever a seconds pulse occurs, the image is stored on film. As many pulses as desired may be photographed. The required number depends upon the noise level present. The third method uses a signal averager, described fully in section 4.4.f.

### 3. WAVEMETERS AND BRIDGES

Frequency can be measured conveniently and with moderate precision by means of wavemeters and bridges. The operation of both instruments is similar insofar as the controls must be manipulated to bring about a balance point or other resonance indication.

#### 3.1 Basic Wavemeters

A wavemeter is an adjustable circuit that can be made to resonate over a definite range of frequencies as indicated on a calibrated dial. The dial may be marked directly in units of frequency or indirectly in terms of wavelength. A typical wavemeter covers a frequency range on the order of one octave unless special wide-range tuners or band-changing features are employed. Resonance is detected by observing the response of a meter or other device incorporated into the measuring instrument or by noting a disturbance in the behavior of the system under test.

Wavemeters are sometimes classified as transmission types or as absorption types depending upon their manner of connection to the frequency source being checked. A transmission-type wavemeter is fitted with suitable connectors at both the input and output ends so it can be inserted directly into the circuit under test. An absorption-type (or reaction-type) meter on the other hand is designed for loose coupling via the electromagnetic field of the frequency source.

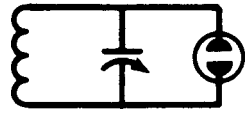
##### a. LC Absorption Wavemeter

The simplest of all instruments for measuring radio frequencies consists of a coil and a variable capacitor in series. A calibrated dial is affixed to the capacitor shaft so the resonant frequency of the LC circuit can be determined as the capacitor is varied from its minimum to its maximum value.

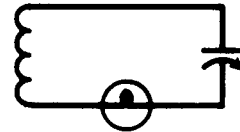
The simple wavemeter is operated by holding it near the frequency source being measured, tuning it for a sharp reaction by the source, and then reading the unknown frequency from the calibrated dial. The reaction is brought about by the transfer of a small amount of rf energy from the source to the wavemeter when the two are resonant at the same frequency. Resonance between the wavemeter and a low-power oscillator, for example, might be indicated by an abrupt jump in the emitter current of the oscillator. In determining the output frequency of a small transmitter one might look for a flicker in the PA plate current as the wavemeter is tuned through resonance. Because of the loose coupling and the miniscule power consumed by the wavemeter, the energy absorbed from high-power sources might be unnoticeable unless a suitable indicator is incorporated into the wavemeter itself.

A small lamp is sometimes used in conjunction with the wavemeter for coarse measurements of frequencies at high power levels. A neon lamp in parallel with the wavemeter's tuned circuit or an incandescent lamp in series with the tuned circuit will glow visibly if sufficient power is absorbed from the frequency source. More refined measurements can be made, of course, if the lamp is replaced by a sensitive meter. Figure 27 depicts some basic indicator arrangements commonly employed with absorption wavemeters.

In operation the wavemeter is tuned for maximum brilliance of the lamp or for peak deflection of the meter. A resonance indicator in series with the tuned circuit must have very low impedance so as not to degrade the  $Q$ , and hence the resolution, of the wavemeter. An indicator connected across the tuned circuit must have a high impedance for the same reason. The resonant elements themselves should exhibit the lowest losses possible, since the accuracy with which frequency can be measured by these means is mainly dependent upon the  $Q$  of the tuned circuit.



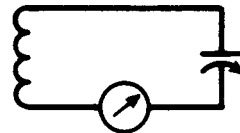
(a) Neon Lamp



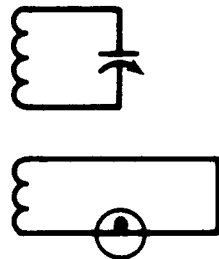
(b) Incandescent Lamp



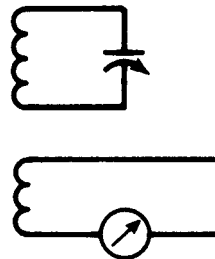
(c) RF Voltmeter



(d) Thermocouple Milli-ammeter



(e) Secondary Winding with Lamp



(f) Secondary Winding with Meter

Figure 27 Wavemeter Resonance Indicators

The inductor and capacitor should be very stable under changes of temperature, humidity, pressure, age, and conditions of handling. Where fine precision is needed, a thermometer may be attached to the wavemeter so corrections can be applied for temperature.

Wavemeters comprised of lumped LC elements are usable at frequencies from about 100 kHz to perhaps as high as 1200 MHz. Different bands of operation are usually provided by plug-in coils, each covering a tuning range of roughly 2.5 to 1. Wider ranges are sometimes achieved at vhf through the use of butterfly resonators or other arrangements whereby the inductance and capacitance are varied simultaneously.

For use above the hf spectrum, wavemeters of the lumped-constant type are somewhat inferior to those having both inductance and capacitance distributed uniformly. Although its accuracy is rather poor, the LC absorption wavemeter is none the less handy as an inexpensive portable instrument.

#### b. Lecher Frame

One of the early contrivances for vhf measurements was a specialized form of wavemeter called a Lecher frame, or sometimes Lecher wires. It consists of a hairpin-shaped section of transmission line constructed of rigid wire or metal tubing. The tuning element is a short-circuiting bar which slides along the parallel portion of the line.

The Lecher frame makes use of a characteristic feature of standing waves, viz. that successive nodes in the standing wave pattern are separated by one-half wavelength. The hairpin bend serves as a pickup loop for loose inductive coupling to the frequency source. A schematic diagram of the Lecher frame is shown in figure 28.



Figure 28 The Basic Lecher Frame

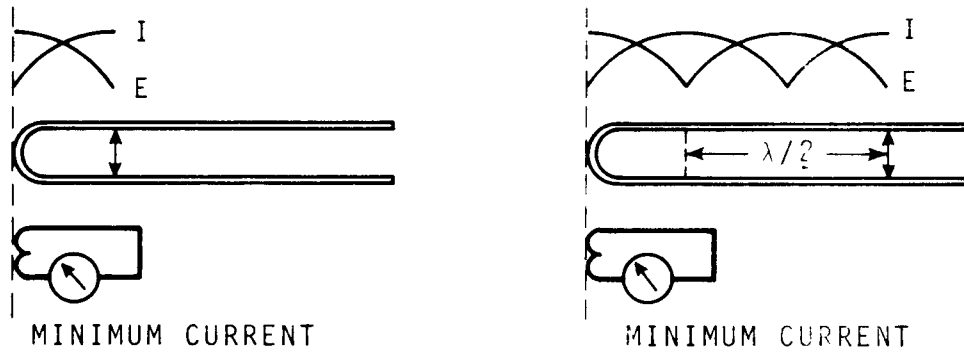


Figure 29 Detection of Voltage Nulls with the Lecher Frame

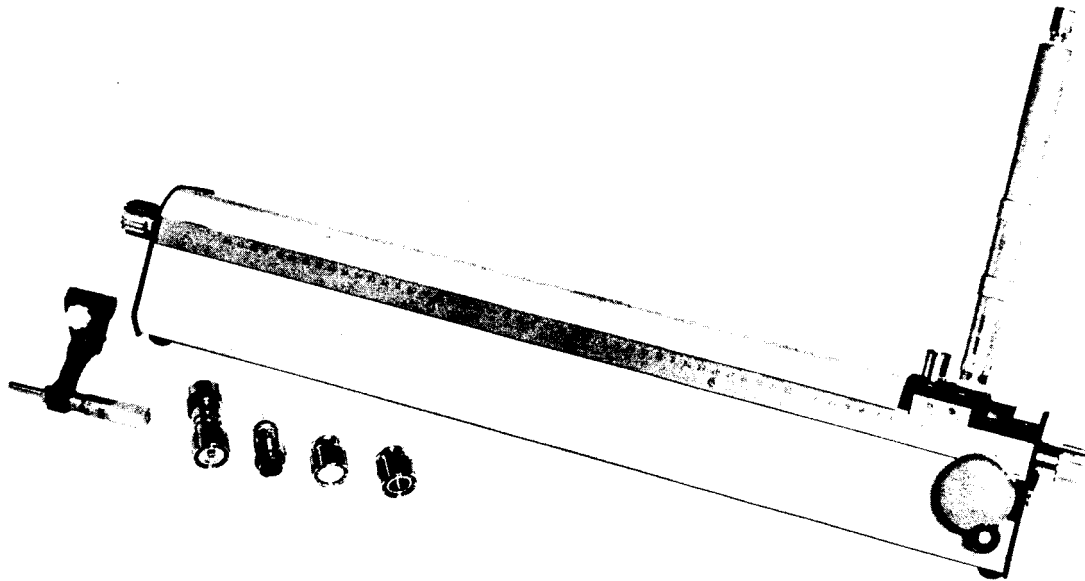
Starting at one end of the frame, the slide is moved along slowly until a point is reached at which the source is sharply disturbed. Here the position of the slider is noted, and then the slider is moved farther until a second disturbance is observed as indicated in figure 29.

The distance  $x$  between the two points is measured and set equal to one-half wavelength. The frequency is computed by dividing the wavelength,  $\lambda = 2x$ , into the speed of propagation. In a well built, air-insulated Lecher frame the wave propagation speed is about 0.975 times the speed of light in free space. Thus the frequency in MHz is equal to  $146.2/x$ , where  $x$  is expressed in meters.

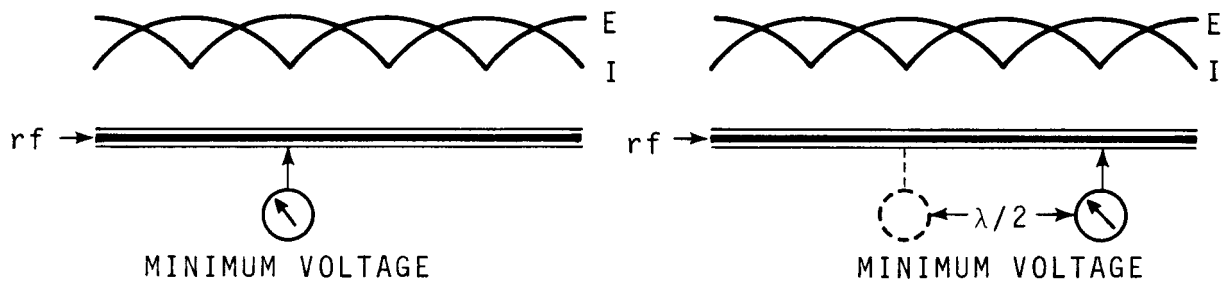
To ensure that two nodes can be located, the Lecher frame should be at least one wavelength long. It may be longer, but a Lecher frame greater than about two meters in length is cumbersome to handle and rather difficult to construct with the necessary rigidity and constant spacing throughout. Generally, the parallel lines are separated by not more than two percent of the shortest wavelength to be measured, but the amount of the spacing is less critical than its uniformity.

As in the case of the lumped-constant device a meter can be incorporated into the Lecher frame to serve as a resonance indicator. In some versions an indicator is inserted directly into the line, but usually a meter is coupled to the line via a separate excitation loop near the pickup end.

As the slider is moved along the line, the meter senses the nodes and nulls of current at its fixed location. Because the nulls are somewhat sharper than the nodes, it is preferable to measure the distance between two successive voltage minima whenever this is practicable.



a. A Precision Slotted Line  
(Photo courtesy of General Radio Co.)



b. Detection of Voltage Nulls with an Open-End  
Slotted Line and Traveling Probe

Figure 30 The Slotted Line



### c. Slotted Line and Slotted Section

A variation of the Lecher frame is the slotted coaxial line, which finds use in the measurement of uhf and microwave frequencies. The slotted line is a short length of transmission line consisting of a metal rod mounted coaxially within a hollow metal sleeve.

Energy from the unknown frequency source is applied to one end of the coaxial line while the opposite end is either short-circuited or left open. In either case a standing wave results with the successive nodes (or nulls) spaced one-half wavelength apart. An rf probe, mounted on a carriage, is moved along a slot in the outer sleeve to detect the voltage maxima and minima. The probe travel between adjacent nodes (or nulls) is measured to determine the wavelength and hence the frequency.

Another version of the slotted line uses a fixed probe and a movable disc. The disc is threaded onto the central rod in such fashion as to short-circuit the rod to the inside wall of the surrounding sleeve. As the rod is turned, the shorting disc moves along the coaxial line in a manner exactly analagous to the slider of the Lecher frame.

Slotted lines are useful at frequencies up to about 18 GHz, but higher frequencies can be measured if the coaxial section is replaced with a slotted waveguide section. One wall of the waveguide features a narrow longitudinal slot through which a traveling probe samples the electric field within the section. The slotted waveguide allows measurement of frequencies up to about 40 GHz.

Both slotted lines and slotted sections are available commercially with a vernier head which permits probe travel to be measured with a resolution of 0.1 millimeter. The relation between distance and frequency is dependent upon the velocity constant of the line or section and is most conveniently determined from a calibration chart for the particular instrument being used.

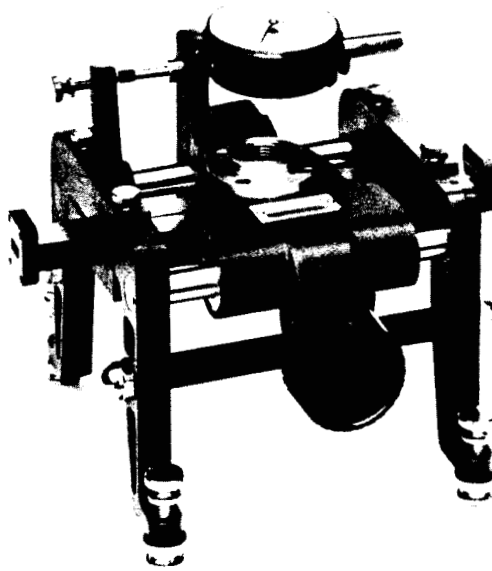


Figure 31 A Precision Slotted Waveguide Section  
(Photo courtesy of Hewlett Packard Co.)

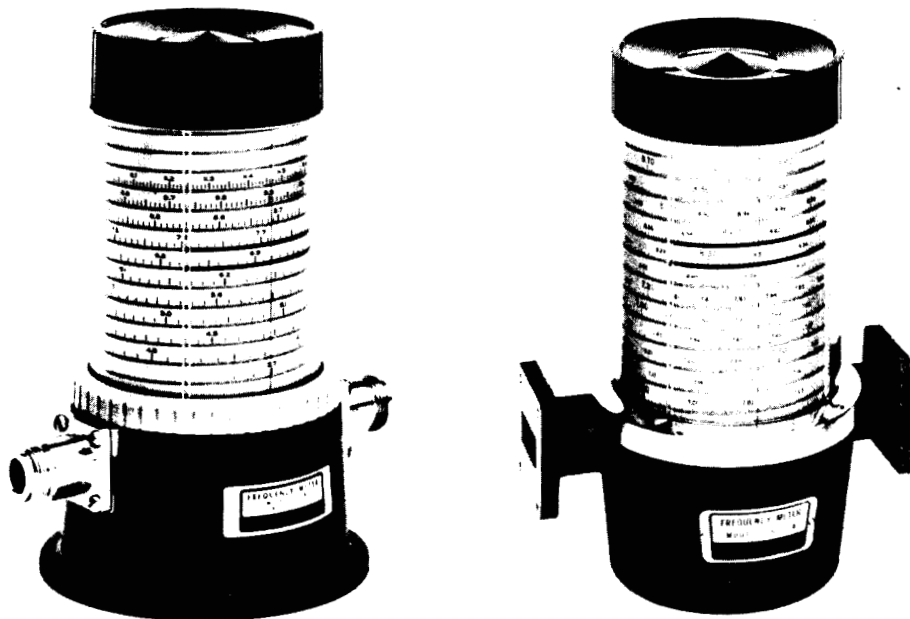


Figure 32 Cavity Wavemeters for Coaxial Cable or Waveguide  
(Photo courtesy of Hewlett Packard Co.)

#### d. Tunable Cavity

Wavemeters find their widest application today in the measurement of microwave frequencies. One of the most popular microwave meters utilizes a tunable cavity, the physical dimensions of which determine its range of resonance. Cavity-type wavemeters can be obtained for frequency measurements from about 1100 MHz to as high as 140 GHz.

The cavity is usually tuned by means of an internal choke plunger at the end of a precision lead screw. A calibrated dial indicates the position of the plunger and hence the resonant frequency of the cavity. Resolution is enhanced by arranging the scale in the form of a long spiral.

The operation of a cavity wavemeter is similar to that of an LC absorption wavemeter. The cavity is coupled to the frequency source via coaxial connections or suitable waveguide flanges. A frequency measurement is carried out by tuning the cavity until its impedance change at resonance causes a sharp observable reaction. At this point the unknown frequency (or wavelength) is read directly from the tuning dial. When out of resonance the high-Q cavity has little effect on the frequency source; consequently, it may be detuned and left permanently connected to the system under test.

### 3.2 Wavemeter Accuracy

To attain the best accuracy with a wavemeter it is essential that coupling to the frequency source and any neighboring objects be kept very loose, for otherwise the external impedance reflected into the wavemeter might alter its calibration. With direct coupling it may be necessary to insert an attenuator between the frequency source and the wavemeter to reduce frequency-pulling effects as the wavemeter is tuned through resonance.

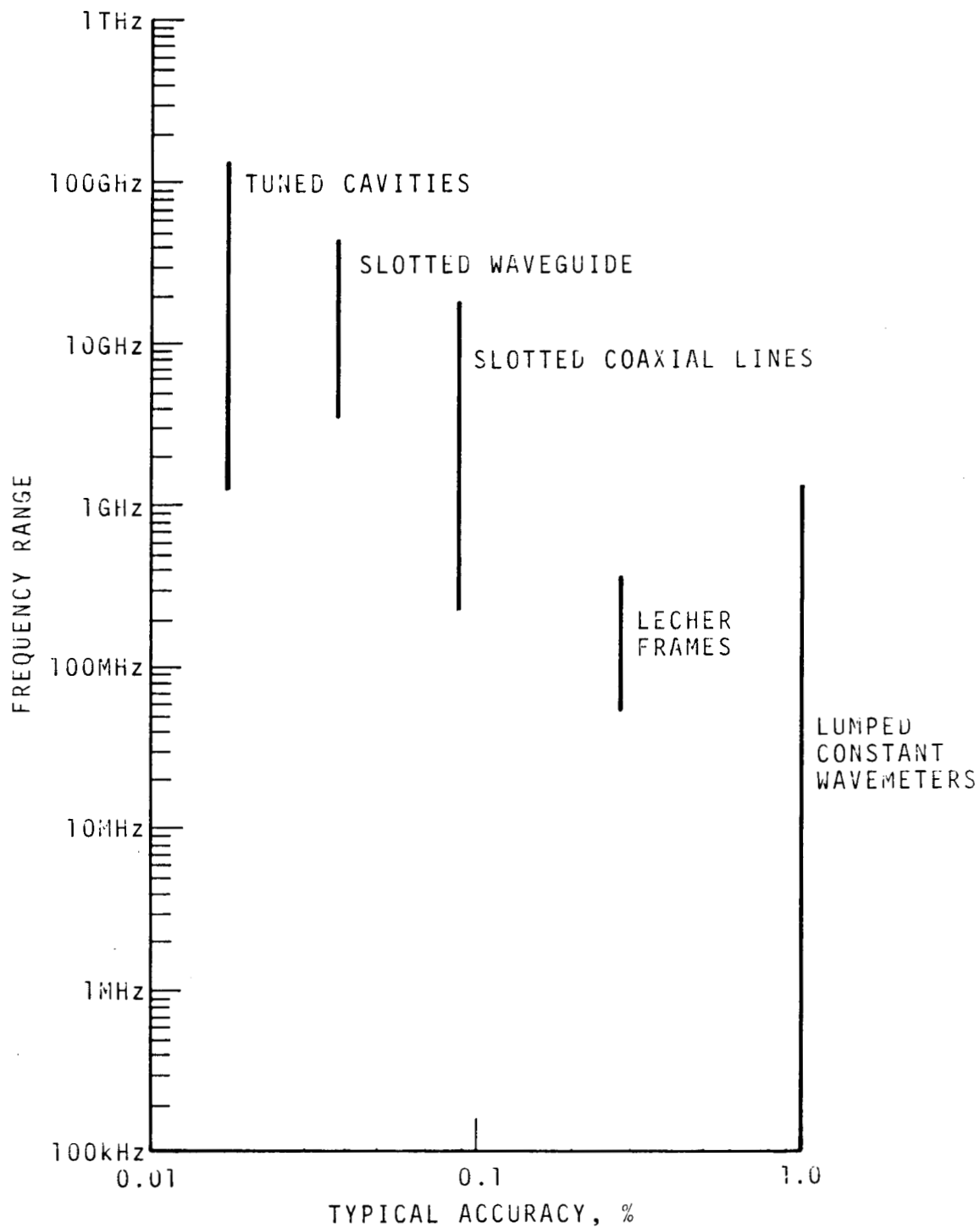


Figure 33 Frequency Range and Typical Accuracy for Various Types of Wavemeters

The accuracy of a wavemeter is limited by the  $Q$  and stability of its resonant elements. As mentioned earlier the instrument's resolution can be improved by increasing the effective length of the tuning scale; but in most wavemeters the resolution of the dial scale is better than the accuracy with which exact resonance can be determined due to the relatively flat peak of the resonance curve.

The typical accuracy of an absorption wavemeter having lumped-constant elements is about 1% when no correction is made for temperature. When a suitable correction is applied, the accuracy can be improved to near 0.1% in some instances.

A wavemeter with distributed-impedance elements is especially dependent upon the stability of its physical dimensions. To a lesser extent its accuracy is also affected by instability of the dielectric constant under fluctuations in humidity. In high-quality instruments both effects are minimized by such construction features as the use of temperature-compensating materials and dessicants.

Slotted sections and cavities are no better than the precision with which their elements are machined and fitted together. Interior surface blemishes and discontinuities in the slot can be very detrimental. The accuracy attainable with slotted sections and cavities may range from 0.1% to 0.01%, the exact figure in any particular case depending upon the care exercised in the design, construction, and use of device.

Figure 33 summarizes the applicable frequency range and typical accuracy of the various types of wavemeters discussed in this section. On a percentage basis the accuracy tends to improve as the frequency increases.

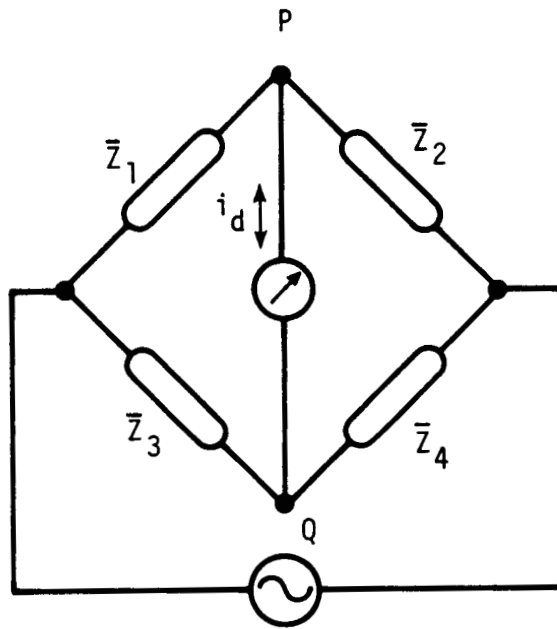


Figure 34 General Four-Arm Uncoupled AC Bridge

### 3.3 Frequency Bridges

An ac bridge may be utilized for frequency measurement provided the bridge current is supplied by the unknown frequency source and suitable reactive elements are employed in one or more of the arms. In essence the unknown frequency is compared against the resonance frequency of the bridge. Alternatively, two frequencies may be compared one against the other by measuring each one individually with the same bridge. A functional diagram of the general four-arm ac bridge is shown in figure 34.

The bridge is balanced by adjusting the impedance arms until the detector current ( $i_d$ ) becomes zero. This condition implies that points P and Q are at the same potential, or

$$\bar{Z}_1 \bar{Z}_4 = \bar{Z}_2 \bar{Z}_3 .$$

The foregoing equation requires the voltages at points P and Q to be equal in both magnitude and phase, i.e.

$$\begin{aligned} Z_1 Z_4 &= Z_2 Z_3 \\ \text{and} \\ \varphi_1 + \varphi_4 &= \varphi_2 + \varphi_3 . \end{aligned}$$

The first of these conditions is known as resistive (or steady state) balance; the second condition is called reactive (or variable state) balance. Both conditions must be satisfied before the ac bridge is completely balanced. Care must be taken in designing the bridge to keep the two conditions independent of each other, for otherwise the process of balancing the bridge would be quite tedious.

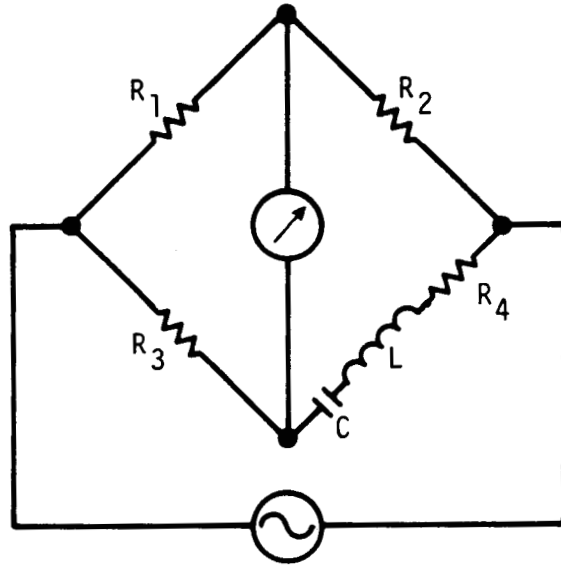


Figure 35 Basic Resonant-Frequency Bridge

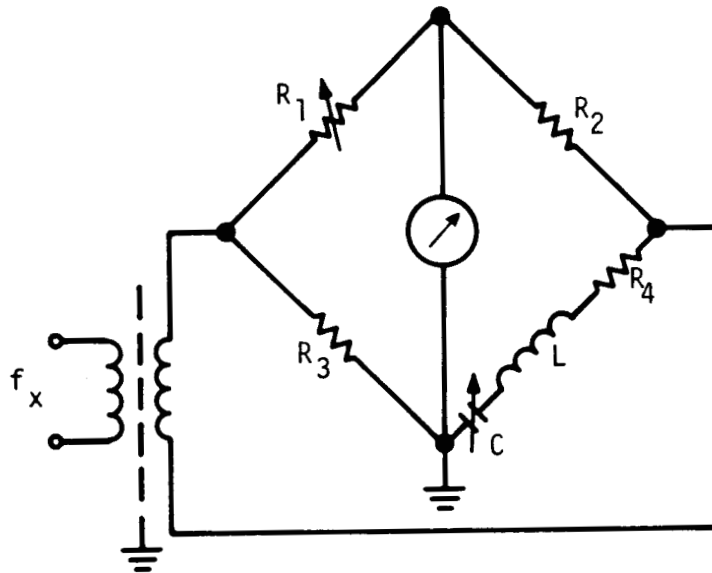


Figure 36 Practical Resonant-Frequency Bridge



### a. LCR Resonant Frequency Bridge

A basic network for frequency measurement is the resonant-frequency bridge in which a tuned LCR filter comprises one of the arms. Because the reactive components (L and C) are confined to one arm, it may be expected that the bridge is frequency sensitive since reactive phase relations cannot be nullified by adjustment of the three solely resistive arms. Perfect balance can occur only when the frequency of the bridge current is equal to the resonant frequency of the LCR filter. In this case the inductive reactance exactly equals and cancels the capacitive reactance, thereby reducing the bridge effectively to one with purely resistive components. Thus the two necessary conditions for complete balance are

$$R_1 R_4 = R_2 R_3 \quad (\text{resistive balance})$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} \quad (\text{reactive balance}).$$

To thoroughly isolate the bridge elements from the frequency source being measured, it is preferable to connect the frequency source to the bridge by means of a high-quality, electrostatically shielded transformer. The use of a coupling transformer between source and the bridge also permits one side of the null detector to be grounded. A practical version of the resonant-frequency bridge is shown schematically in figure 36.

$R_4$  is considered to be the ohmic resistance of the inductor and hence is quite small for high-Q coils. From the resistive balance equation it is seen that an extremely small value for  $R_4$  implies a very large value for  $R_1$ . In fact as  $R_4$  approaches zero ohms,  $R_1$  must become infinite to preserve balance.

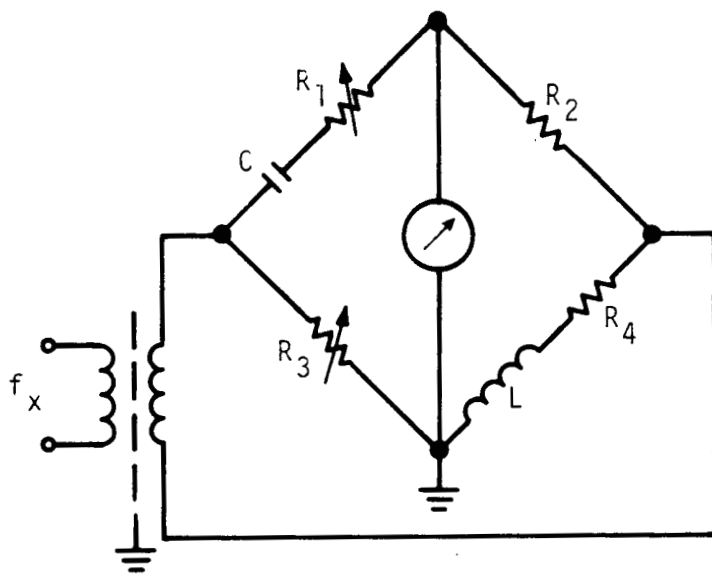


Figure 37 The Hay Bridge

Because of insulator leakage and similar limitations, it may be impossible to obtain a sufficiently large value for  $R_1$  when a high-Q filter is used. For this reason the resonant-frequency bridge is more appropriate with low-Q filters of the type normally encountered at the higher radio frequencies. An effective Q of 10 or less is typical of inductors used in this circuit.

#### b. Hay Bridge

For the measurement of lower radio frequencies, at which inductors of  $Q > 10$  are commonplace, the Hay bridge is more practical than the resonant-frequency bridge. Balance conditions for the Hay bridge are expressed by the equations

$$R_1 R_4 + \frac{L}{C} = R_2 R_3 \quad (\text{resistive balance})$$

and

$$f = \frac{1}{2\pi} \sqrt{\frac{R_4}{R_1 LC}} = \frac{1}{2\pi QR_1 C} \quad (\text{reactive balance})$$

Again,  $R_4$  denotes the resistance of L. Recognizing that  $R_4$  is very small in relation to  $R_2$  and  $R_3$ , one may ignore the  $R_1 R_4$  term in the resistive balance equation with little loss of accuracy. The equation then reduces to  $L/C = R_2 R_3$ . To ensure that the  $R_1 R_4$  term remains negligible, the maximum value of  $R_1$  should be restricted to a few ohms — the exact opposite of the requirement imposed by the resonant-frequency bridge. Thus the Hay bridge can function with more convenient values of resistance and sharper balance in conjunction with high-Q inductors.

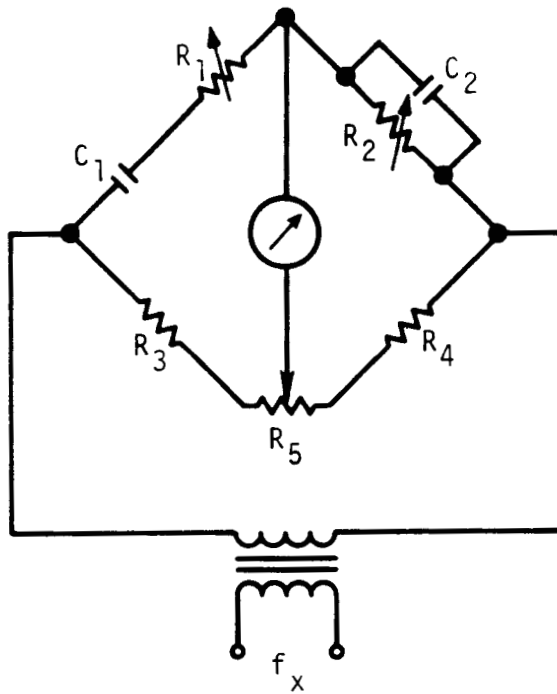


Figure 38 The Wien Bridge

**Example:** What is the frequency as measured with a Hay bridge if complete balance is attained when

$$R_1 = 8.95\Omega, R_2 = 1000\Omega, R_3 = 734\Omega,$$

$$R_4 = 0.103\Omega, L = 41.2\mu\text{h}, \text{ and } C = 56.1\text{ pf ?}$$

**Solution:** After the bridge is balanced,  $R_1$  and  $R_2$  have no further bearing on the determination of  $f$ .

$$f = \frac{1}{2\pi} \sqrt{\frac{R_4}{R_1 LC}} = \frac{1}{2\pi} \sqrt{\frac{0.103}{(8.95)(41.2 \times 10^{-6})(56.1 \times 10^{-12})}}$$

$$= \frac{1}{2\pi} \sqrt{4.97 \times 10^{12}} = 0.357 \times 10^6 \text{ Hz, or } 357 \text{ kHz.}$$

### c. Wien Bridge

LC bridges become somewhat impractical at audio frequencies because of the large value of inductance required and the shielding difficulties encountered. By virtue of their time constants, however, RC networks can be made frequency sensitive and therefore applicable to frequency-measuring bridges. The Wien bridge is one of the most satisfactory for audio and ultrasonic frequencies.

The Wien bridge is balanced by adjusting rheostats  $R_1$  and  $R_2$  for a null. With the components chosen so that  $R_1 = R_2$ ,  $C_1 = C_2$ , and  $R_3 = 2R_4$ , a single adjustment of the ganged resistors satisfies simultaneously the conditions

$$R_1 R_4 = R_2 R_3 \quad (\text{resistive balance})$$

and

$$f = \frac{1}{2\pi R_1 C_1} \quad (\text{reactive balance}).$$

Potentiometer  $R_5$  serves as a trimmer to compensate for any imperfect tracking of  $R_1$  and  $R_2$ . It is adjusted only to enhance the sharpness of the null and in no way enters into the calculation of frequency.

In a typical Wien bridge  $R_1$  and  $R_2$  are selected to cover a 10:1 tuning range with an accuracy of about 1 percent. Decimal multiplying factors can be introduced by changing the capacitance of  $C_1$  and  $C_2$  equally in integral powers of ten. With three sets of capacitors it is possible to cover the entire audio spectrum from 20 Hz to 20 kHz.

### 3.4 Null Detection and Bridge Accuracy

The usefulness of a bridge for frequency measurement accrues from its property that a separate null is produced for each frequency. A serious difficulty arises then if electrical noise, harmonics, or other extraneous waves are superimposed on the frequency being measured.

Because the bridge can be balanced for only one frequency at a time, the null is broadened by the influence of any additional frequencies that are present. Moreover, since the bridge is unbalanced for extraneous frequencies, they can become very prominent at the detector terminals even though they represent only a small percent of the bridge input current. The problem may be augmented or diminished largely by the type of null detector employed.

Although a meter is indicated as the null detector in the foregoing bridge diagrams, several other devices lend themselves satisfactorily to this application. In selecting a null detector it is of course necessary to choose one that is responsive to the frequency being measured; but it is equally important, insofar as possible, to select one that is insensitive to frequencies outside the expected range of measurement.

An ac galvanometer may suffice for frequencies below 200 Hz or thereabouts. High-impedance headphones, electronic voltmeters, and magic-eye tubes are commonly used as null detectors within the audio frequency range. Tuned radio receivers are applicable at higher frequencies. The use of a selective bandpass amplifier in conjunction with the null detector permits greater rejection of hum, noise, and harmonics, thereby providing a sharper null and better resolution.

For frequencies in the mid-audio range (300-5000 Hz), headphones offer a distinct advantage in allowing the operator to discriminate aurally against undesired noise or harmonics. By listening only to the fundamental tone, it is often possible to balance the bridge in spite of other frequencies which may be present.

Though sometimes less sensitive than other devices a cathode ray oscilloscope is especially suitable as a null detector for sinusoidal waves because it will indicate separately the extent of the resistive and reactive balances. The manner in which the oscilloscope may be connected for this purpose is shown in figure 39.

With the internal sweep of the oscilloscope turned off and the bridge initially unbalanced, the oscilloscope will display a tilted elliptical pattern, as in figure 40 a or 40 b . As the bridge arms are adjusted the ellipse will rotate either clockwise or counter-clockwise and will become either flatter or more circular. Reactive balance is indicated by a pattern wherein the major axis of the ellipse is horizontal. When the ellipse is reduced to a straight line, resistive balance is also achieved. By adjusting the variable elements of the bridge arms, it should be possible to obtain perfect balance, as indicated by the pattern in figure 40 f . Inability to reduce the pattern to a horizontal straight line usually indicates the presence of harmonics or other extraneous voltages across the detector terminals.

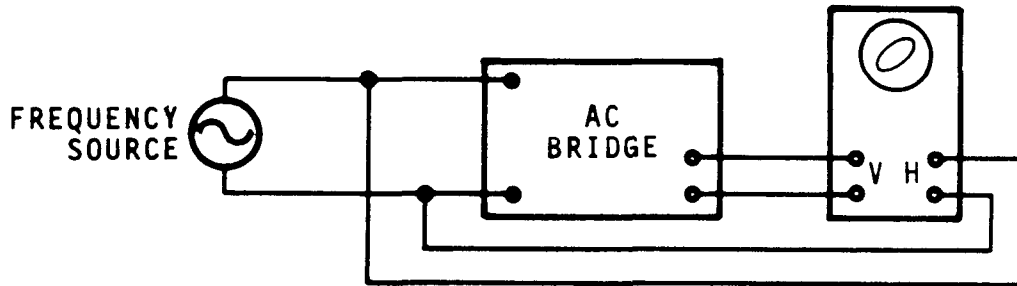


Figure 39 Oscilloscope Connected for Use as a Null Detector

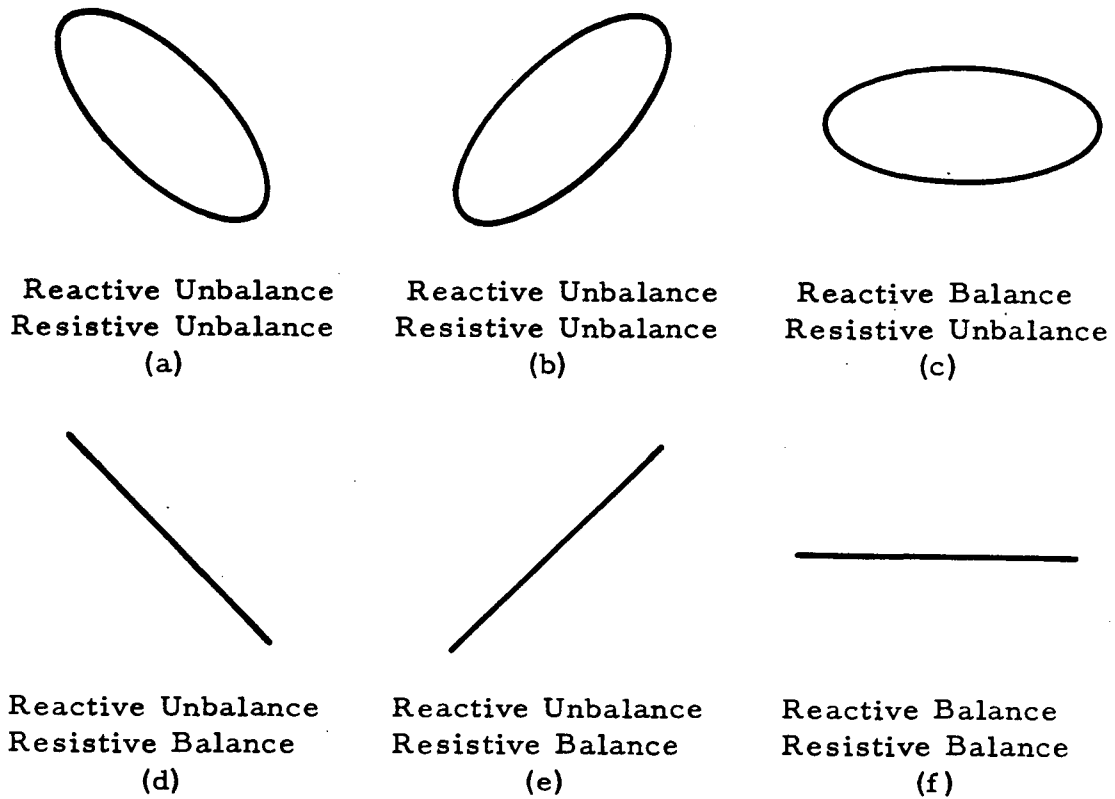


Figure 40 Null Detector Patterns Obtained with an Oscilloscope



Bridge accuracy is dependent upon the sharpness of the null and therefore is a combined function of detector sensitivity and circuit  $Q$ . A highly significant error results from poor resolution in locating the exact null point. Another limiting factor is the inaccuracy of the known value of critical components that constitute the bridge arms.

Various amounts of stray resistance, inductance, and capacitance are always present in the bridge arms. These distributed parameters have a definite effect on null location yet are not usually accountable in the equations for resistive and reactive balance.

Other sources of error may arise from changes in the value of components due to heating and aging, to thermal potentials resulting from unequal temperatures in different portions of the bridge circuit, and to human mistakes in operating the bridge or in carrying out the computations.

Except for highly specialized bridges operated under the most stringent laboratory conditions the overall accuracy is limited to about 0.1%, with 1% being more typical. The relatively poor accuracy serves to compromise the advantages of bridge circuits for frequency measurement.

If unshielded the bridge arms tend to exhibit capacitive effects between themselves and neighboring objects. Although a slight amount of constant stray capacitance can be tolerated, the effects change in an unshielded bridge as the operator or the surroundings move about. Consequently, thorough shielding of the bridge elements is essential for accuracy and operating convenience.

#### 4. FREQUENCY METERS, COMPARATORS AND RELATED EQUIPMENT

Whereas wavemeters and bridges provide a quick means of measuring frequency by comparison with the resonance characteristics of passive networks, other analog frequency meters allow direct comparison with the reference frequency from a standard oscillator. Instruments of this type include heterodyne frequency meters, direct-reading frequency meters, and frequency comparators.

##### 4.1 Heterodyne Frequency Meters

The heterodyne frequency meter is basically a nonlinear mixer partially driven by a reference signal from a self-contained, calibrated, tunable oscillator. It also includes an audio amplifier with suitable provisions for operating a zero-beat indicator, such as headphones, an ac voltmeter, a magic-eye tube, or an oscilloscope. Utilizing the familiar heterodyne principle, it compares the internal oscillator frequency with the unknown frequency of an external rf source.

The unknown signal is applied through the rf input terminals to the mixer stage, where it combines with the reference signal to produce a beat note. The internal oscillator is then carefully tuned for a zero beat, whereupon its frequency is read directly from the calibrated tuning dial. If the unknown frequency falls within the fundamental tuning range of the oscillator, its value will be the same as the oscillator frequency under the zero-beat conditions.

Because the nonlinear mixer gives rise to harmonics, it is also possible to detect beat notes between the fundamental of one signal and harmonics or subharmonics of the other. If the frequency of the external signal matches a known harmonic of the oscillator fundamental, the dial setting must be multiplied by the appropriate harmonic number  $N$ .

On the other hand if the unknown frequency equals a subharmonic, the dial reading must be divided by  $N$ . Thus from a knowledge of harmonic relations, heterodyne measurements can be made of frequencies far above or below the fundamental tuning range of the frequency meter.

Figure 41 shows the block diagram of a basic heterodyne frequency meter. The internal variable-frequency oscillator must exhibit good stability, which calls for close temperature regulation, complete shielding, and rugged construction. The oscillator should also be thoroughly buffered to minimize frequency-pulling tendencies when strong signals are applied from the external source or when other factors would cause the oscillator load to change.

Some heterodyne meters are equipped with a whip antenna so the internal oscillator signal can be radiated over short distances. For checking the frequencies of transmitters or other high-power sources, a safe method is to tune in the transmitter with a radio receiver and then adjust the frequency meter until a zero beat is heard through the receiver's loudspeaker. The unknown frequency can be measured just as though it were coupled directly to the frequency meter.

The overall accuracy of measurement depends upon the calibration accuracy of the tunable oscillator and the resolution attainable through the zero-beat process. At high frequencies the error introduced by imperfect zero-beat detection is generally negligible.

Provided the temperature is kept nearly constant the variable-frequency oscillator may retain its accuracy within 0.1% over long periods of time without recourse to a calibration standard. Because aging is the other major factor that affects calibration accuracy, the short-term behavior is usually much better.

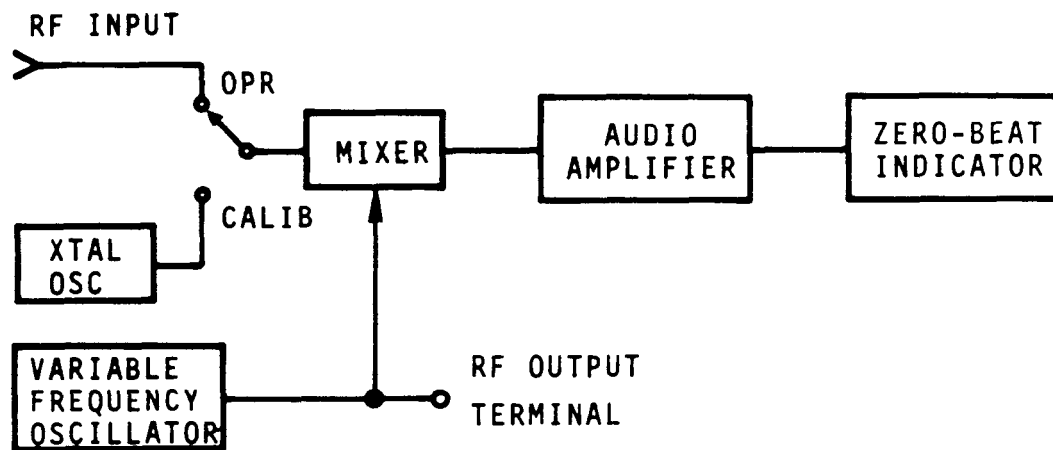


Figure 41 Block Diagram of Heterodyne Frequency Meter

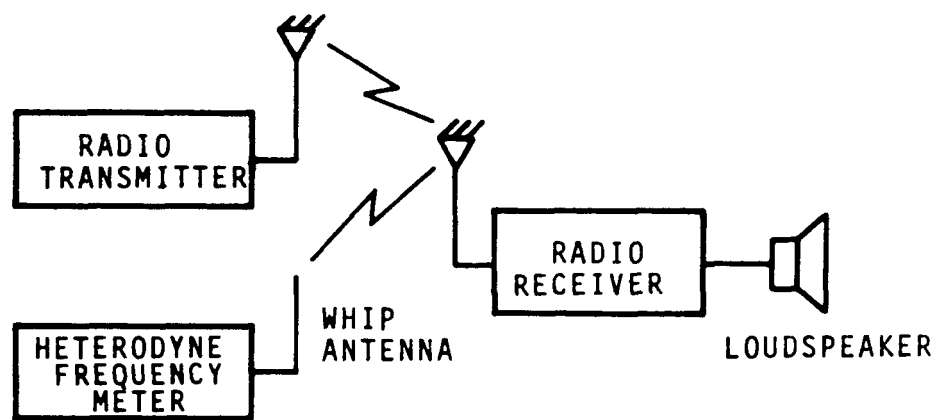


Figure 42 Arrangement for Checking Transmitter Frequency with Heterodyne Frequency Meter and Receiver

Most heterodyne meters contain a highly stable quartz crystal oscillator for use in generating markers at fixed check points on the tuning dial. By correcting the dial calibration at the check point nearest the frequency to be measured, it is possible to achieve an accuracy approaching one part per million after the instrument has reached stable operating temperature.

Heterodyne frequency meters combine wide frequency coverage with good accuracy in a portable instrument. They can be obtained for operation over fundamental frequency ranges from approximately 100 kHz to 200 MHz in disjointed bands. The typical accuracy varies from about 0.01% at low frequencies to 0.00025% or better at very high frequencies. Harmonics up to 3 GHz may be generated with a precision of  $1 \times 10^{-6}$ .

Because they are tunable devices, the heterodyne frequency meters offer better noise immunity and signal sensitivity than wideband instruments such as oscilloscopes and counters. The heterodyne meter is one of the most useful of all frequency-measuring instruments for checking transmitter frequencies, for calibrating rf signal generators, and for other applications which do not require a high degree of signal purity.

#### 4.2 Direct-Reading Analog Frequency Meters

Analog frequency meters operate by transforming the unknown frequency into a different physical quantity, such as voltage amplitude or current amplitude, which can be measured directly by conventional metering instruments. Thus the output indicator might be an ordinary d'Arsonval milliammeter movement whose deflection is somehow made proportional to the input frequency.

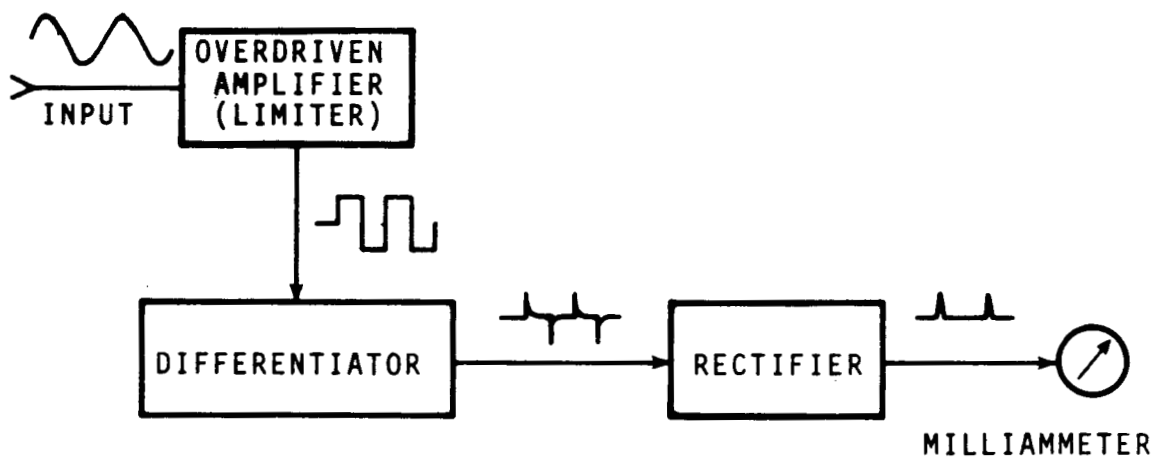


Figure 43 Electronic Audio Frequency Meter

#### a. Electronic Audio Frequency Meter

A meter-type instrument that reads directly in audio frequency units and requires no manipulations of any kind is diagrammed in figure 43. The input signal is shaped into a rectangular waveform, differentiated, rectified, and then applied to a milliammeter. The current spikes have the same frequency as the input signal; but since their peak amplitude is constant, the meter deflection is proportional only to the number of spikes that occur per unit of time. Deflection therefore is determined by the frequency of the input signal, so the meter scale can be calibrated directly in hertz.

Operation of the instrument is no more complicated than the operation of an electronic voltmeter. Besides its simplicity of operation, the instrument has an additional advantage in that frequency indications are independent of both waveform and amplitude of the input signal over very wide limits. Its overall accuracy is comparable to that of the meter movement, typically 1 to 5 percent.

#### b. Radio Frequency Meter

The operating range of the audio frequency meter may be extended to the vhf region or higher by adding a series of decade frequency scalars ahead of the instrument. Figure 44 depicts the block diagram of such a device capable of measuring radio frequencies up to 50 MHz directly.

The audio frequency meter is preceded by four mixer stages, each excited by a signal derived from a standard 1 MHz crystal oscillator. The injection frequency for the first mixer consists of the 25th to the 49th harmonic of 1 MHz as determined by the tuning of harmonic selector A.

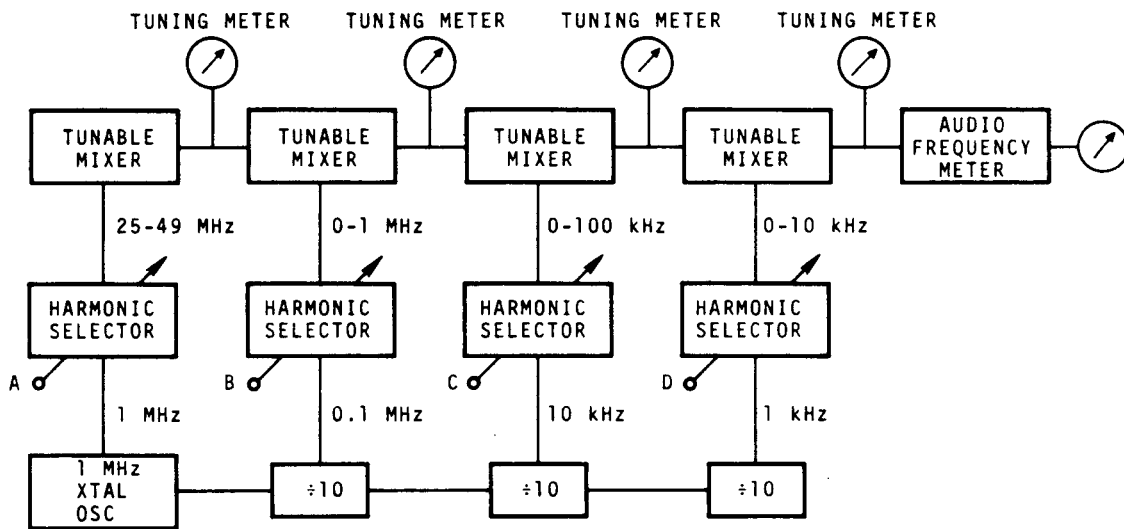


Figure 44 Direct-Reading Radio Frequency Meter



The difference between the input frequency and the selected harmonic is then applied to a second mixer, where it is heterodyned against a subharmonic of 1 MHz as determined by the tuning of harmonic selector B. The process continues until the input frequency is eventually reduced to a value that falls within range of the direct-reading audio meter.

The instrument is operated by manually tuning each harmonic selector and mixer in succession for peak deflection on the associated tuning meter. Each harmonic selector dial is calibrated in such a manner that the input frequency can be read directly from the dial settings in conjunction with the reading of the audio meter. For example, if optimum tuning occurs when dial A is set at 32, dial B is at 7, dial C is at 5, dial D is at 9, and the audio meter is reading 170, then the input frequency is 32.759170 MHz. Typical accuracy of the instrument with an analog audio meter is  $5 \times 10^{-7}$  at vhf. If the audio meter is replaced by an electronic digital counter, the accuracy may be improved to  $2 \times 10^{-8}$  or better.

### 4.3 Frequency Comparators

Available commercially are so-called frequency comparators, which are versatile devices for making frequency measurements and comparisons. These instruments combine phase error multipliers, phase detectors, integrating circuits and frequency multipliers into useful and accurate laboratory aids. The block diagram of this instrument is shown in figure 45. The heart of the frequency comparator is a variable-multiplication phase error multiplier of special design.<sup>3</sup>

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<sup>3</sup> See section 4.4 b for a discussion of phase error multipliers.

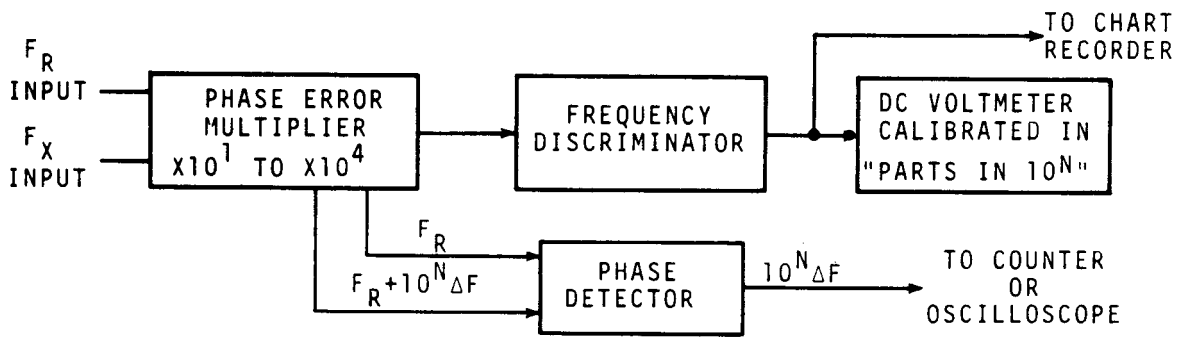


Figure 45 Basic Frequency Comparator

The phase error multiplier, like all electronic devices, is subject to noise. Through each stage of error multiplication the noise at the input is amplified. Eventually the noise out of the phase error multiplier would be stronger than the signal, so to maintain a useful signal-to-noise ratio the multiplication is usually limited to 10,000.

Probably the most important part of this instrument is the frequency discriminator, which works against a tuned-circuit. The output is a dc voltage proportional to the difference between the frequency of the discriminator input signal and the resonant frequency of the tuned circuit. The Q of the tuned circuit determines the sensitivity of the discriminator.

As an example of the Q required, consider a discriminator operating at 1 MHz with an error multiplication of 10,000. If the comparator is to respond to an initial frequency difference of 1 part in  $10^{12}$ , the discriminator will have to detect a frequency offset of  $\Delta f = 10^{-12} \times 10^4 \times 10^6 \text{ Hz} = 0.01 \text{ Hz}$ . If 0.01 Hz corresponds to 1% of full-scale meter deflection, the bandwidth of the discriminator must be  $100 \times 0.01 \text{ Hz} = 1 \text{ Hz}$ . This requires a Q of  $10^6$ . One commercial comparator solves this problem by using a quartz crystal as the tuned circuit for the discriminator.

The comparator usually contains phase detectors<sup>4</sup> that also are connected to the output of the phase error multiplier chain. The phase detectors have various integrating times, or bandwidths, that allow one to optimize the accuracy of the particular measurement being made. As shown in block diagram, the comparator can be used with an external chart recorder, an electronic digital counter, or an oscilloscope.

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<sup>4</sup>

Phase detectors are discussed in section 4.4c.

#### 4.4 Auxiliary Equipment

Some specialized types of equipment, though not in themselves complete systems for the measurement of frequency or time, are nevertheless very useful as auxiliary devices to increase the versatility, accuracy, and range of other measuring instruments. These auxiliary devices include frequency synthesizers, phase error multipliers, phase detectors, frequency multipliers, frequency dividers, signal averagers, and phase-tracking receivers.

##### a. Frequency Synthesizers

Not all frequency comparisons and measurements are performed at even frequencies like 100 kHz, 1 MHz, 5 MHz, etc. Many electronic applications involve odd frequencies. An example is the television color subcarrier frequency of  $(63/88) \times (5 \text{ MHz})$ . Vhf and uhf communications channel frequencies are another example.

Within the operating limits of the instrument, a frequency synthesizer can generate any frequency with very high precision and with the same accuracy as the standard input signal. There are two general types of synthesizers, each with points in its favor.

The direct synthesizer uses internal frequency multipliers, adders, subtracters, and dividers in combination to produce the output frequency, which may also contain energy at some of the component frequencies. The indirect method of synthesis uses a phase-lock oscillator to provide the actual output signal. The spectral purity of the latter method is more a function of the phase-lock loop than of the spectral components of the locking signal. Both types of synthesizer are widely used in modern electronic equipment for the generation of accurately-known frequencies.

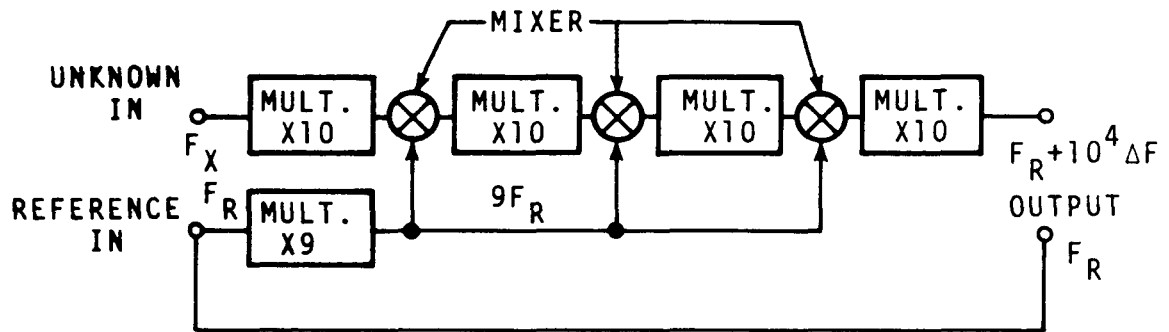
The synthesizer has two parameters which determine its usefulness in any application. One is range and the other is resolution. If the instrument has a range of 10 MHz and a resolution of 0.1 Hz, any frequency up to 10 MHz may be generated in steps of 0.1 Hz. The dials or buttons on the front of the instrument display the output frequency. These controls actually interconnect the various multipliers, mixers, etc. that are necessary to produce the output frequency.

As mentioned previously, the measurement and comparison of odd frequencies is simplified by the use of a frequency synthesizer. As an example, assume that one wishes to determine the frequency of a crystal oscillator. The output frequency may be measured with an electronic counter if one is available. Barring the availability of a counter, a Lissajous pattern may be displayed on an oscilloscope with some reference frequency; but the pattern on the oscilloscope might be too complex for an accurate interpretation.

On the other hand if a frequency synthesizer is available, a low-ratio Lissajous pattern may be obtained. Once the pattern is stabilized, the oscillator frequency may be read off the synthesizer dials. If the pattern is other than 1:1, the oscillator frequency may be computed by using the dial value and the pattern ratio. Similarly, phase or frequency stability of odd-frequency oscillators may be determined using a synthesizer in conjunction with phase detectors, oscilloscopes and chart recorders.

#### b. Phase Error Multipliers

Oscilloscopes, phase detectors, chart recorders, and similar devices used to make frequency comparisons are limited by their frequency response. This can be a serious handicap when frequency multiplication is practiced. A phase error multiplier, however, multiplies the phase or frequency difference between two signals without increasing the frequencies themselves as far as the user is concerned.



$$\Delta F = |F_R - F_X|, F_X = F_R \pm \Delta F$$

Figure 46 Phase Error Multiplier

The block diagram of a phase error multiplier is shown in figure 46. It consists of one or more decade frequency multipliers in conjunction with one or more mixers tuned to the difference frequency between the mixer input signals.

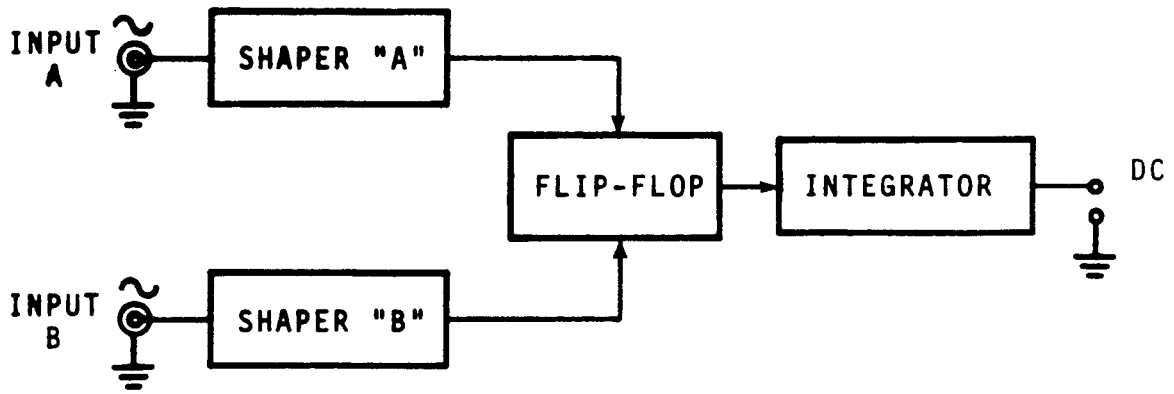
The reference frequency  $f_r$  is multiplied by nine to produce  $9f_r$ . The unknown frequency  $f_x$  is multiplied by 10 to produce  $10f_x$ . But  $f_x$  may be redefined as  $f_x = f_r \pm \Delta f$ . Then  $10f_x = 10f_r \pm 10\Delta f$  and the mixer output will be  $10f_r \pm 10\Delta f - 9f_r = f_r \pm 10\Delta f$ .

The difference between the reference frequency and the mixer output is now 10 times what it was initially. This process can be repeated, as shown in the block diagram, until the amplified noise renders the whole process useless. Four such decades seem to be about the limit. This is equivalent to multiplying two 1-MHz signals to 10 GHz. The output of the phase error multiplier is now treated the same way as the input would have been, but the greatly amplified frequency difference simplifies and quickens the measurement process. An application of the phase error multiplier is discussed in section 4.3, which deals with frequency comparators.

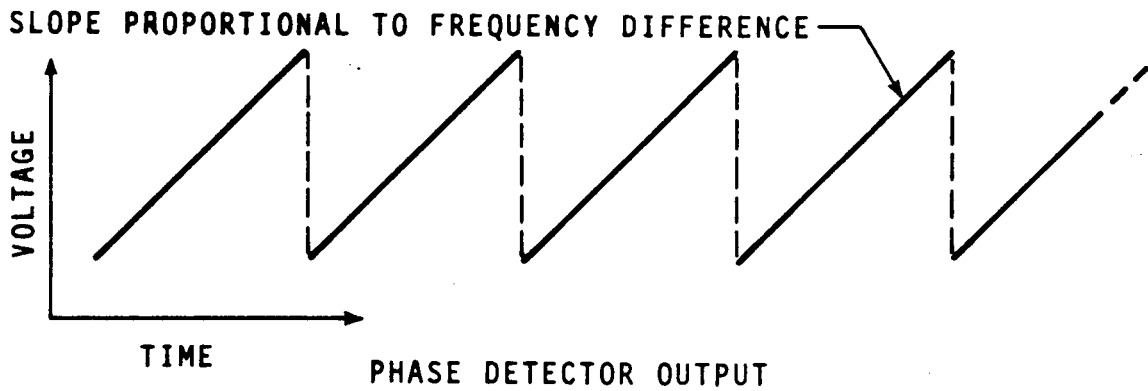
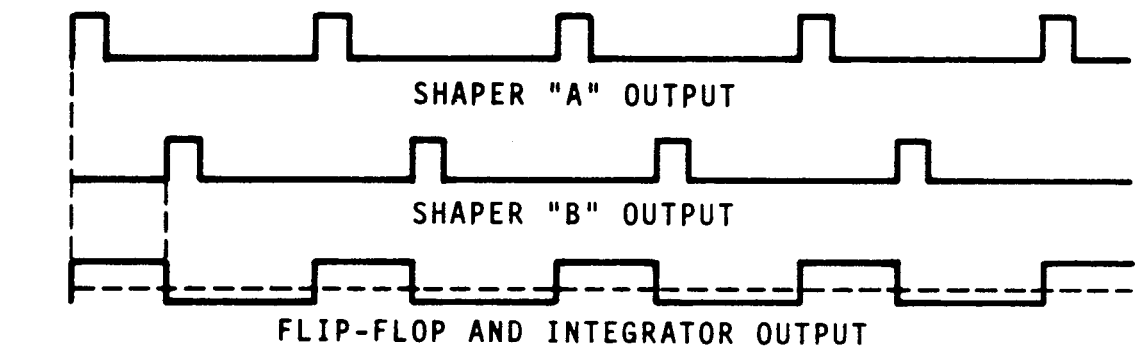
### c. Phase Detectors

There are two main types of phase detectors — the linear type and the nonlinear type. They are discussed separately in the following paragraphs.

(1) Linear Type. The linear phase detector is designed to be used with phase error multipliers and strip chart recorders. In the previous section it was noted that phase detectors are limited in their frequency response. This is one very good reason for combining a phase detector with a phase error multiplier.



a. Block Diagram



b. Waveforms

Figure 47 Linear Phase Detector

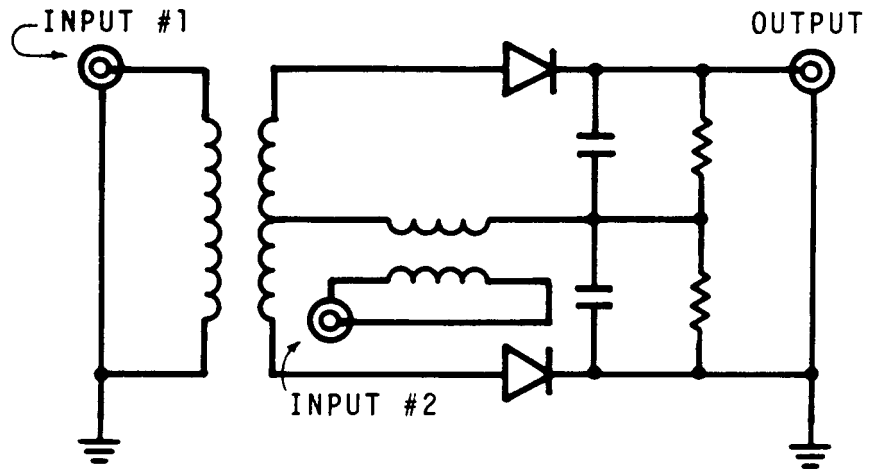


A block diagram of the linear phase detector is shown in figure 47a. Examination shows that the heart of this type of phase detector is a flip-flop. The action, or duty cycle, of the flip-flop determines the output voltage of the detector.

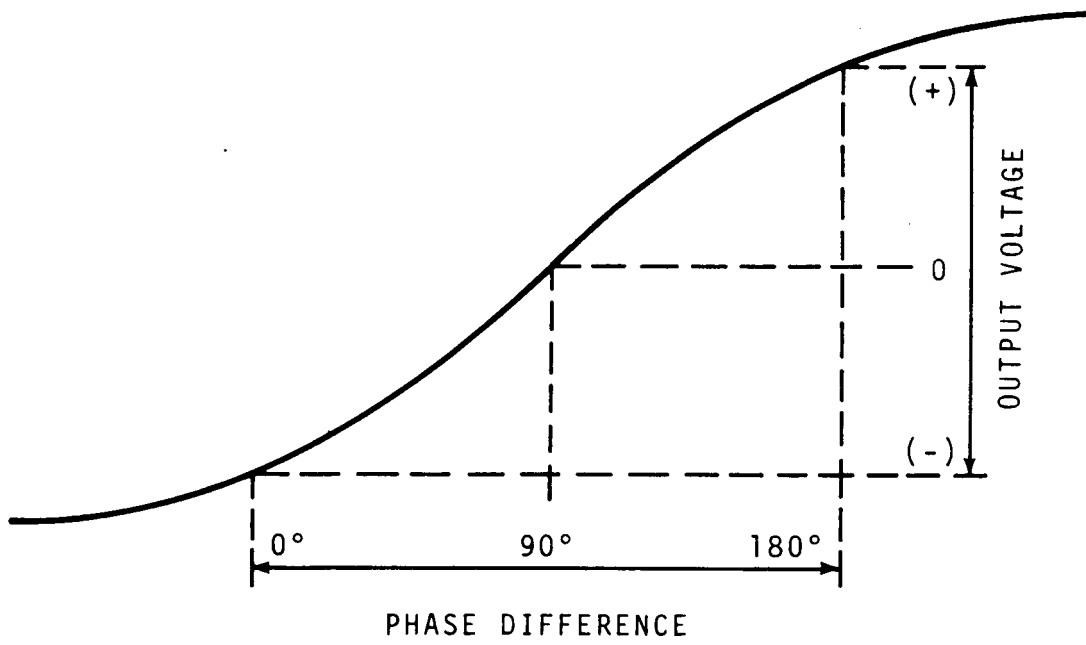
Frequency limiting occurs because of the restricted speed of transition of the flip-flop. If the shaped signals from the two inputs are less than  $0.1\mu\text{s}$  apart, for instance, the flip-flop will not react at all. Consequently the dc voltage to the chart recorder will not be a true time representation of the phase difference. In fact, what happens is that the saw-tooth output shown in figure 47b begins to shrink. The waveform shifts upward from zero and downward from full-scale. This shrunken response could be calibrated, but it is much more convenient to use the phase detector at a frequency with which it is perfectly compatible. This is where the phase error multiplier proves helpful. With the phase error multiplier one is able to have very small full-scale values of phase shift, say  $0.01\mu\text{s}$ , with inputs in the 100-kHz range.

(2) Nonlinear Type. A nonlinear phase detector is essentially a phase discriminator similar in behavior to the frequency response of the discriminator used in FM receivers. The circuit diagram of a typical nonlinear detector is presented in figure 48a; its phase response characteristic is shown in figure 48b.

Because the response characteristic is nonlinear, this phase detector is not used extensively with chart recorders. It has, however, one important characteristic that the linear phase detector lacks — its output voltage can reverse polarity. In other words the output is a dc voltage proportional to the phase difference between the input signals, and the polarity of the output voltage depends on the lead or lag condition of the reference signal with respect to the unknown signal phase.



a. Schematic Diagram



b. Response Curve

Figure 48 Nonlinear Phase Detector

If the input signals are different in frequency, the output voltage is periodic like a sine wave and there is no dc component. This ac output can be measured with a frequency counter, or it can be applied to an oscilloscope to form a Lissajous pattern.

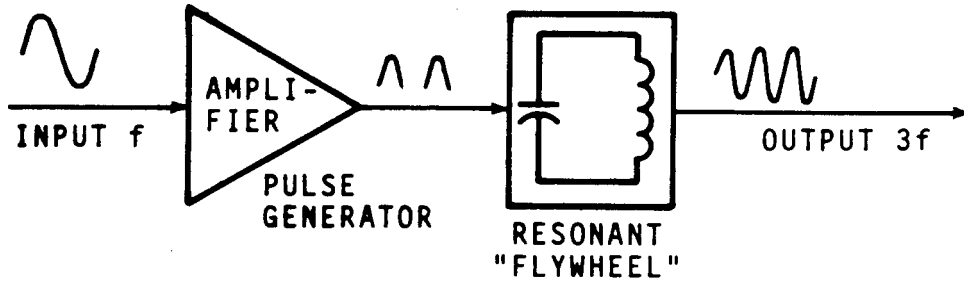
#### d. Frequency Multipliers

Reference has been made throughout this report to multipliers and multiplier chains. These types of equipment should be a part of every well equipped laboratory engaged in measurement and comparison of frequency or time. The block diagram of a frequency multiplier is shown in figure 49a.

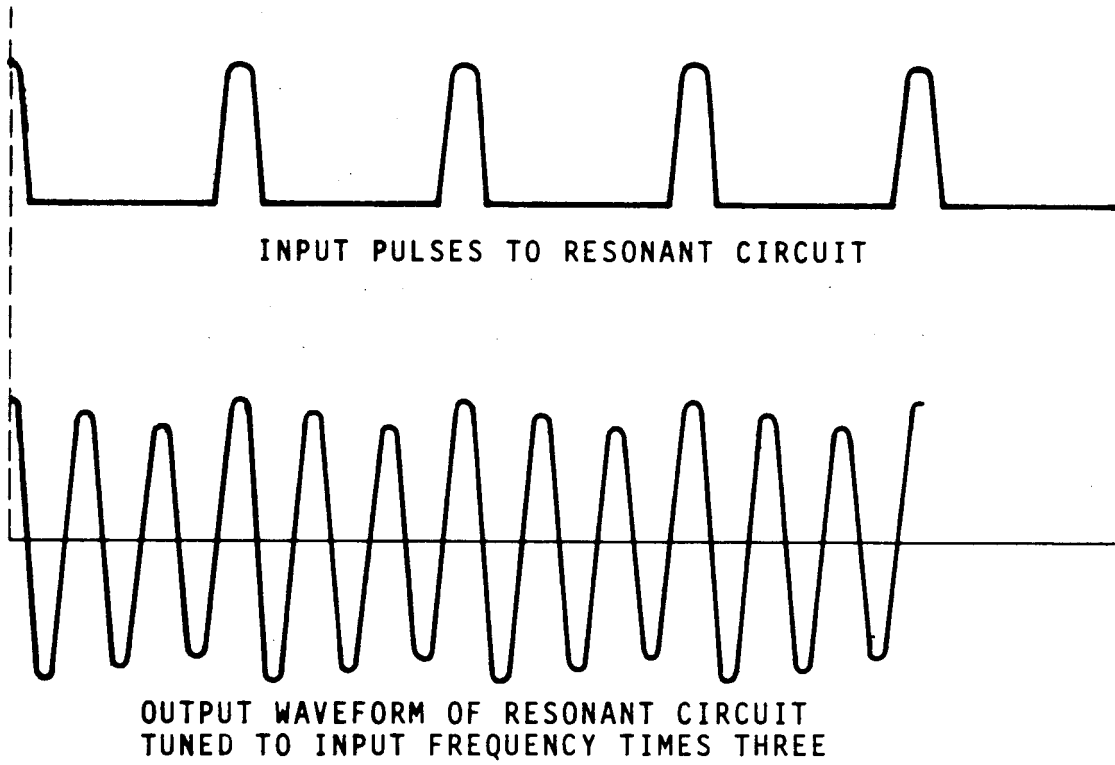
If one were to consider a black box concept of this device, the term "multiplier" would describe what is observed. A frequency  $f$  is applied to the input and a frequency  $Mf$  is observed at the output, where  $M$  denotes the multiplication factor. The terminology is misleading, however, when one considers how the device actually functions.

One way to look at frequency multiplication is to consider the concept of a reinforced flywheel. The ringing of a parallel resonant circuit can be thought of as flywheel action. The idea is to excite the resonant circuit with an electrical impulse to make it ring, or oscillate. If this impulse kicks the resonant circuit once every three cycles of the ringing frequency, we have effectively multiplied the excitation frequency by three.

The output waveform of the resonant circuit is shown in figure 49b. The decrease in amplitude of the output waveform between excitation pulses is caused by the resistance of the resonant circuit. The damping effect is not appreciable, however, if the resonant circuit has a reasonably high  $Q$ .



a. Simplified Diagram



b. Waveforms

Figure 49 A Frequency Tripler

There is a second way of looking at the multiplier action that lends itself more readily to mathematical analysis. The frequency multiplier can be thought of as having two main parts: (1) a harmonic generator and (2) a narrow band filter. The harmonic generator receives the input signal, which is usually a sine wave of frequency  $f$ . The sine wave is converted by the generator into a series of pulses of width  $w$  and frequency  $f$ . Analysis of this pulse train shows that it contains energy at frequency  $f, 2f, 3f, 4f \dots nf$ . In other words there is a spectrum associated with the pulse train. All that remains is to filter off the harmonic desired; but because each harmonic suffers some loss in amplitude, one normally amplifies the pulse train before filtering it.

#### e. Frequency Dividers

Frequency dividers are special forms of frequency synthesizers.<sup>5</sup> They may be grouped into two general types: (1) analog dividers that use regenerative feedback and (2) digital dividers that use binary arithmetic together with logic functions provided by bistable circuits. Analog dividers lend themselves to rf applications in conjunction with receivers and transmitters. Digital types are found in electronic counters, computers and slewable dividers.

(1) Analog Type. Figure 50 shows the block diagram of a regenerative divider. Frequency  $f$  is supplied to the input of the divider. After amplification this signal becomes one of two inputs to a mixer stage. The mixer output at  $f/10$  is multiplied to  $9f/10$  and then fed back to the other input of the mixer.

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<sup>5</sup>See section 4.4 a for a discussion of frequency synthesizers.

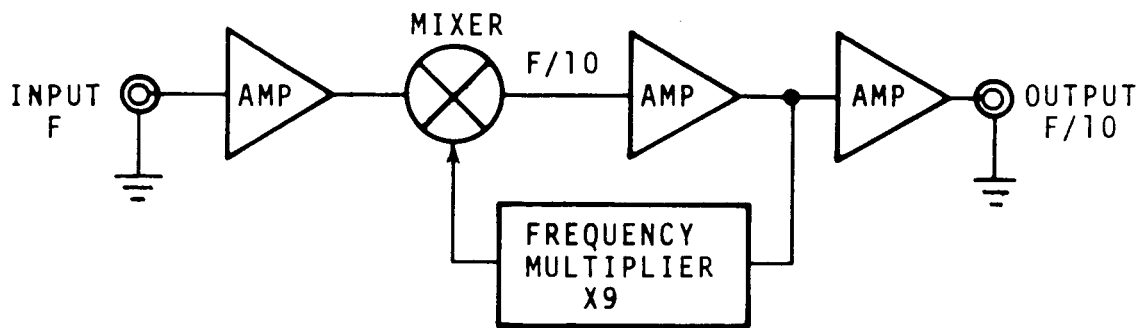
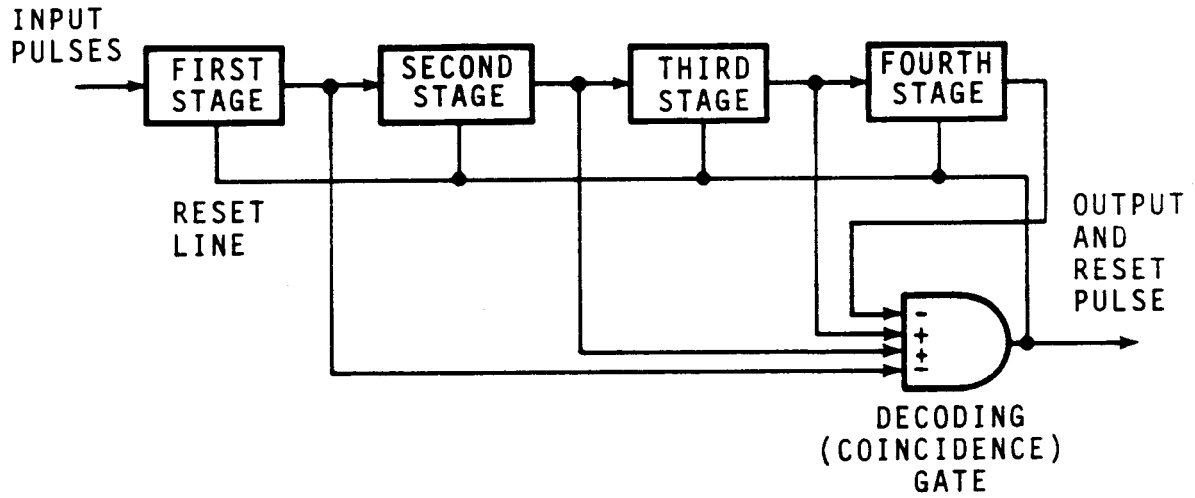


Figure 50 Regenerative Type of Decade Frequency Divider

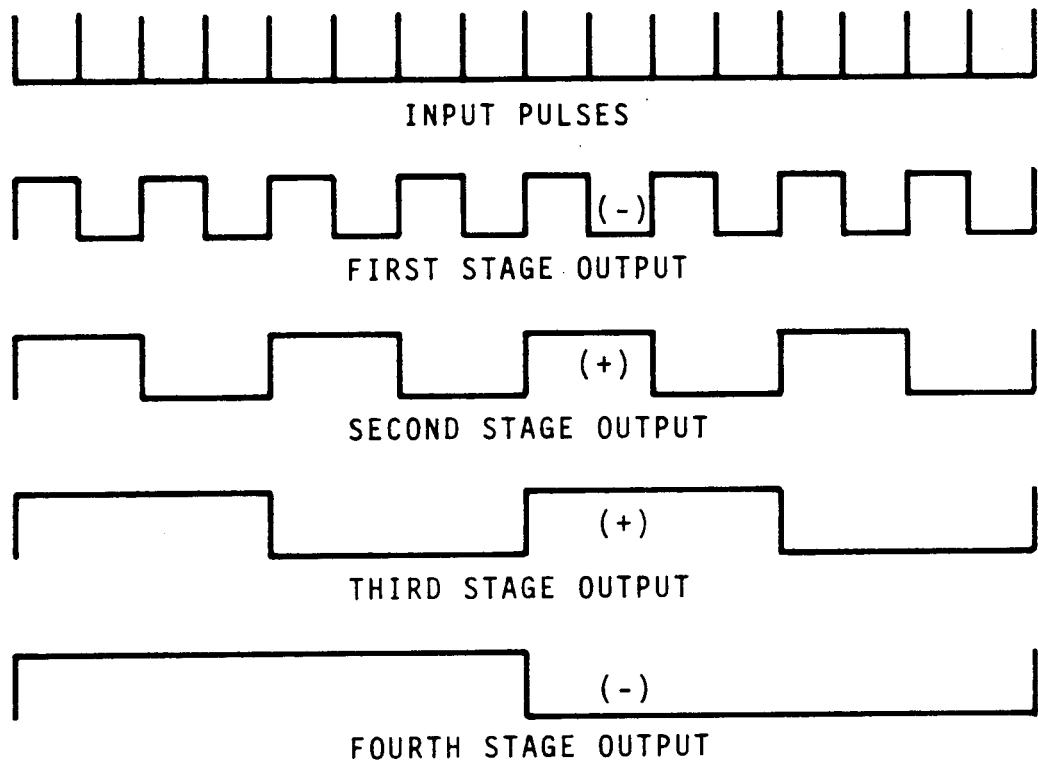
The gain of the various stages is very critical. The regenerative divider is actually an oscillatory circuit though not self-oscillating. If the loop gain around the feedback circuit is too high, the unit will continue to divide even with no input. The output frequency in this condition is determined by the tuning of the various circuits, and any resemblance to the desired output frequency is purely coincidental! Conversely, if the circuit gain is too low the unit will not function at all. The proper condition, as one would suppose, is where the unit divides under conditions of input signal but ceases operation when that input signal is removed.

Some commercial units have their gain set so that the dividers are not self-starting. This feature is designed to eliminate the problems of division with no input signal. Also since the unit isn't self-starting, the output phase should not shift around due to momentary loss of input or dc power. Once the divider stops, it remains stopped until the start button is pressed. Correct operation of an analog divider is easily checked using either the trigger method or a Lissajous pattern on an oscilloscope.

(2) Digital Type. Simple bistable circuits such as flip-flops may be combined to perform digital division. Any flip-flop, whether assembled from discrete components or an integrated circuit, performs the same basic function. The output assumes two possible states, first one and then the other, when the circuit is triggered by two successive input pulses. The input pulses may be considered as positive (+) signals. On the other hand, the output may be either a positive (+) level or a negative (-) level. For two successive (+) input pulses, the output will first be a (+) and then a (-) or visa-versa. Two (+) inputs equal one (+) output. Thus we have division by two.



a. Block Diagram



b. Waveforms

Figure 51 Four-Stage Flip-Flop Divider



If the output of one flip-flop is connected to the input of another, the output of the second flip-flop will be a (+) signal for every four (+) input signals. A third flip-flop would allow division by eight, and so on. Each succeeding flip-flop provides an additional division by two. The divisors are 2, 4, 8, 16, 32, etc.

A decade ( $\div 10$ ) divider is formed by using extra circuitry. One such divider uses internal feedback to supply six extra pulses to a four stage flip-flop chain, thereby forcing the circuit into supplying an output pulse after only ten input pulses rather than the normal sixteen.

Another version makes use of the fact that there is a unique combination of output (+) and (-) signals within the chain itself that indicates when ten input pulses have been counted. Upon detecting this unique combination a special AND gate delivers an output pulse and simultaneously resets the chain to its initial condition. Figure 51a shows a simple decade digital divider of this type; its waveforms are shown in figure 51b.

When using an oscilloscope to observe noisy periodic waveforms, such as pulses or short bursts of sine waves extracted from radio receivers, it is often advantageous to trigger the oscilloscope from a separate source. The problem arises as to how one might position the pattern for best viewing or measurement. A slewable digital divider can solve this problem.

Let us assume we have a digital divider that accepts a 1-MHz input signal, counts  $10^6$  cycles of this input, and then delivers a narrow output pulse. If this process is repeated continuously, the output pulses will occur once every second with an accuracy limited mainly by the accuracy of the 1-MHz input signal. If we use the 1-pps output to trigger an oscilloscope, we may need to shift this pulse around in the time domain to fully observe the desired waveform.

Because the divider counts individual cycles of the input frequency, the timing of the output pulse is determined by the instant when the divider starts counting. We can alter the timing of the output pulse by turning the divider on and off, by momentarily removing the 1-MHz input signal, or by having internal circuitry that causes the output pulse to shift in time. A slewable divider accomplishes the latter result in a simple, straightforward manner.

If during the counting cycle we were to add a pulse to the input pulse train, the dividers would tabulate this additional pulse along with the normal pulses. The divider chain would count along as usual until the number 1,000,000 is reached. At that instant the unit would send out its normal seconds pulse; but, since we added one extra pulse to the input train, the output pulse would occur  $1\mu\text{s}$  earlier than it would have otherwise. Were we to add a pulse to the input train every second, the output pulse would advance in time by one microsecond per second. Similarly, if we subtract one pulse from the input train each second, the output pulse would be retarded one microsecond per second. By adding or subtracting pulses at different places in the division or counting process, we can make the output pulse move forward or backward in time. This technique is the basis on which the slewable divider operates.

#### f. Signal Averager

The term "signal averager" can be applied to several types of equipment, but the discussion here will be limited to the averager whose block diagram is shown in figure 52. For the purpose of understanding the signal averaging process let us assume a periodic event, such as a seconds pulse, that occurs in the presence of random noise. This situation is actually encountered when time signals, as received via radio or landline, are accompanied by static or other random interference.

If the same point on the seconds pulse is examined each time the pulse occurs, an average voltage for that point will emerge. This is because the signal amplitude at the point is constant, and ultimately the random noise voltage at that point will average out to zero. The length of time required for the average signal value to appear will depend upon the amount and nature of the noise.

The signal averager examines many points on the seconds pulse. The instantaneous value of voltage at each point is stored in a memory circuit comprised of a bank of high-quality capacitors. Each time the pulse occurs, the same points are examined and stored in the same memory elements. After a sufficient length of time, each capacitor will have stored the average value of voltage from its respective point on the pulse.

Each of the memory capacitors is in series with an electronic switch. The switches are closed and opened by a control circuit, which operates each switch in succession so that only one is closed at any given time. Resistors between the input and output amplifiers allow the time constant of the memory elements to be optimized with respect to the length of the seconds pulse.

As each switch is closed, its associated capacitor charges to the instantaneous voltage present at the point on the waveform being examined. That voltage is simultaneously displayed on an oscilloscope. The waveform of the displayed pulse is composed then of many discrete voltage levels read from many storage capacitors.

As one can imagine, the entire process must be carefully synchronized. The switch controller is triggered simultaneously with the oscilloscope trigger, and the switching rate is adjusted nominally to allow a complete display of all capacitor voltages during one sweep of the oscilloscope beam.

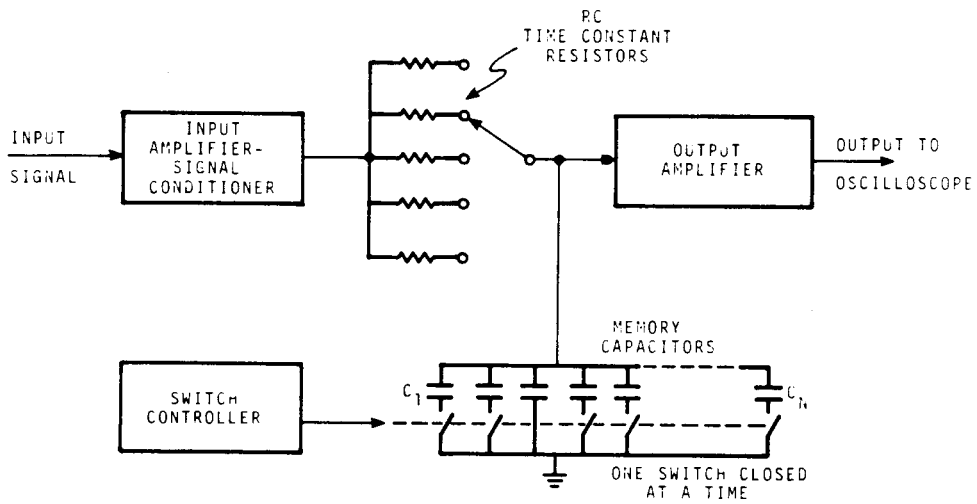


Figure 52 Simplified Diagram of Signal Averager

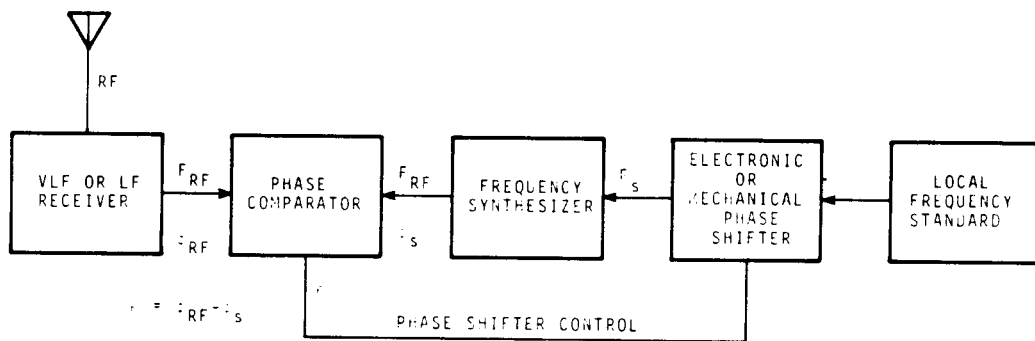


Figure 53 Simple Phase-Tracking Receiver

### g. Phase-Tracking Receivers

The block diagram of a simple phase-tracking receiver is shown in figure 53. Basically this receiver adjusts the phase of a local frequency standard to agree with that of a received radio signal. The phase adjustment is performed by a phase shifter, which also accepts the output signal of the local standard. This shifted signal becomes the input signal for an internal frequency synthesizer, whose output frequency is set equal to the received frequency. Comparison between these two signals is then made by a phase detector which in turn controls the action of the phase shifter.

A time constant in the servo loop limits and controls the rate of phase change that can be accomplished by the shifter. This time constant permits the receiver to have very narrow bandwidth. Commercial receivers feature adjustable time constants which allow the effective bandwidth to be reduced to as low as 0.001 Hz. With bandwidths this narrow, the receiver can follow a signal that is much weaker than the neighboring noise levels.

Another way to view the action of a phase-tracking receiver is to consider it a signal-averaging process. The long time constants (100 seconds or more) characteristic of these receivers may be thought of as averaging times. The phase-locked output frequency of the receiver is a faithful representation of the received frequency and phase. Only the noise has been removed.

The stability of a long distance data or telephone line, for instance, may be determined by the use of a vlf phase-tracking receiver. The only requirements are that the received signal and the local standard be sufficiently stable to allow tracking with the long time constants involved.

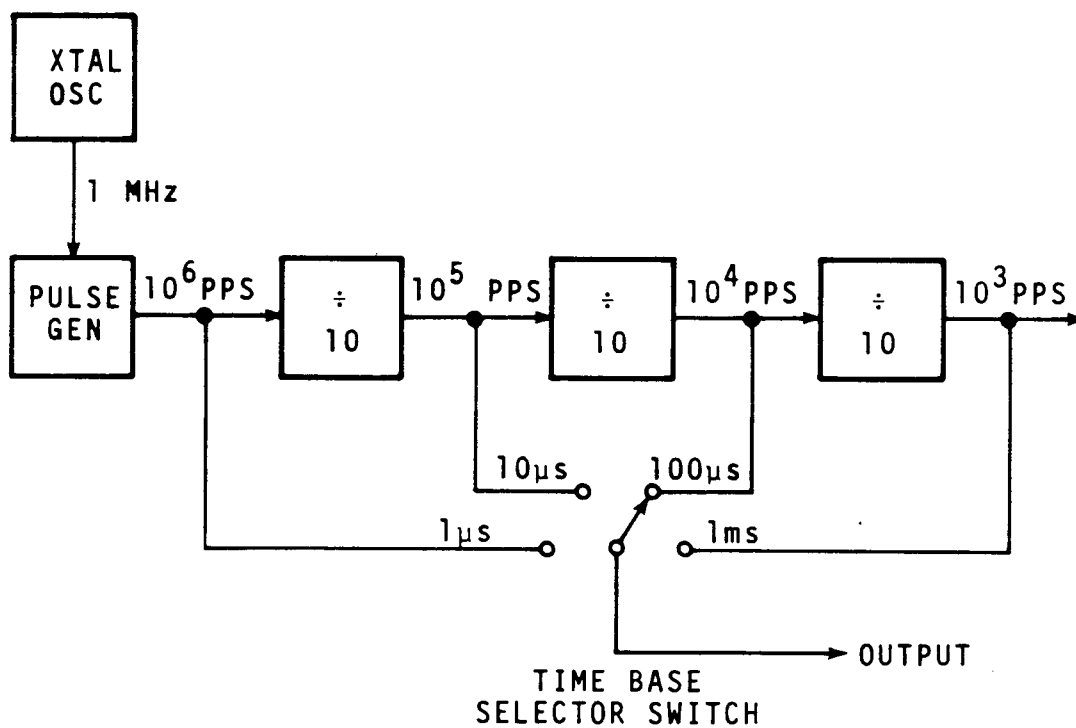


Figure 54 Simplified Diagram of Counter's Time Base

## 5. ELECTRONIC DIGITAL COUNTERS

The electronic digital counter is perhaps the most convenient, versatile, and accurate instrument available for the measurement of frequency and time. Functioning semi-automatically at electronic speeds it permits easy, rapid counting of electrical impulses at either periodic or random rates regardless of whether the waveforms are sinusoidal or complex. The results of the count are displayed clearly and precisely on a numerical readout assembly.

Modern general-purpose counters have a large variety of measurement features built into the basic instrument. Many also have provisions for the insertion of auxiliary plug-in modules which adapt the already versatile counter to an even greater range of applications. Although counters differ widely in scope of performance and in details of design, most contain several major functional sections in common. The basic constituents of a typical counter are (1) the time base, (2) the main gate, and (3) the decade counting assembly.

The time base provides uniform, precisely spaced pulses for synchronization and control of the logic circuitry within the counter. The time interval between pulses must be accurately known because the time base also serves as a reference for frequency-time measurements. Whereas timing pulses are sometimes obtained from the power line frequency or from an external frequency standard, most counters incorporate a highly stable, internal, quartz crystal oscillator for this purpose.

The oscillator may be followed by a chain of decade dividers to afford the user a choice of several different time increments. By adding more dividers to the chain it is possible to obtain longer time increments; by increasing the oscillator frequency one may obtain shorter time increments.

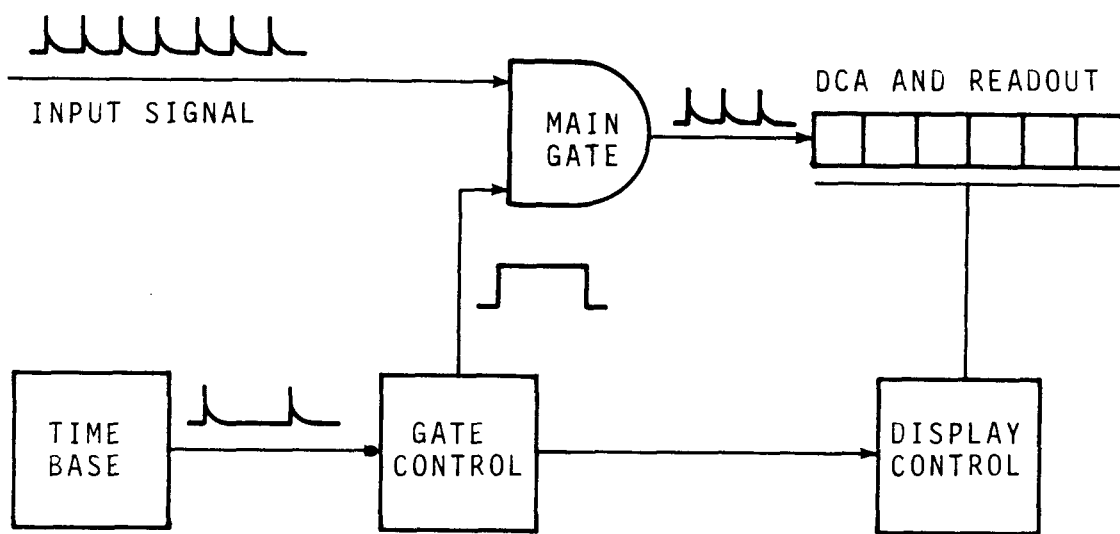


Figure 55 Simplified Block Diagram of Electronic Counter



The main gate controls the time at which the count begins and ends. The gate may be actuated automatically by pulses derived from the time base or else manually by means of start-stop buttons on the control panel. The impulses which pass through the gate are directed to the decade counting assembly (DCA) where they are totalized and registered as a sum on the visual numerical readout. After a pre-determined display period, the instrument resets itself and commences another count.

Other important functional sections include amplifiers, signal-shapers, power supplies, and control circuits for the various logic assemblies. The input channels are usually provided with trigger level controls in conjunction with step-attenuators to establish the exact amplitude limits at which the counter becomes responsive or unresponsive to input signals. Proper setting of the trigger level control(s) permits accurate measurement of complex waveforms and also provides discrimination against noise or other extraneous signals.

## 5.1 Frequency Measurement

Modern counters are capable of measuring frequencies from dc to several hundred megahertz directly. With suitable auxiliary devices and indirect methods of measurement the frequency range of the counter may be extended into the microwave region. The maximum counting rate of the basic instrument is limited by the inability of its switching circuits to follow extremely fast polarity reversals.

### a. Direct Measurement

For direct frequency measurements the input signal is first amplified, clipped, and converted into a series of uniform pulses. The pulses are then applied to the main gate, which is enabled for a known time interval by its control circuit under influence of the time base.

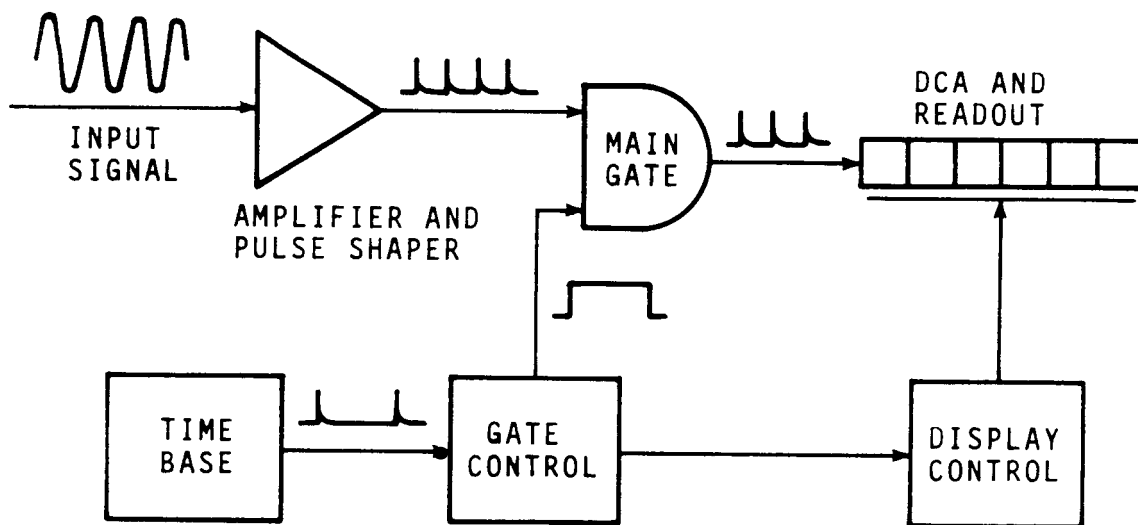


Figure 56 Diagram of Counter in the Frequency-Measurement Mode

The gating interval may be  $1\mu\text{s}$ ,  $1\text{ms}$ ,  $1\text{s}$ ,  $10\text{s}$ , etc., as determined by the setting of the time base selector switch. During the gating interval the signal pulses are conducted through the main gate and on to the DCA, where the pulses are actually counted. The count obtained in a given gating interval is a measure of the average input frequency for that interval. As such the frequency is displayed on the numerical readout and retained there until supplanted by a new count.

#### b. Prescaling

The usable frequency-measuring range of the counter can be extended by a technique known as prescaling. The method of prescaling is to divide the frequency of the input signal by some known factor before the signal is applied to the counter. Instruments which accomplish the division are called prescalers and are available as accessories for many commercially made counters.

If the signal frequency is prescaled by a factor of  $D$ , the reading displayed by the counter is  $\frac{1}{D}$  times the actual frequency of the signal. Hence, by simple multiplication, the counter reading may be converted to the true value of the measured frequency.

As an example let us suppose we wish to measure a frequency in the range between 200 and 300 MHz with a counter capable of measuring to only 50 MHz directly. By means of a scale-of-8 divider we reduce the unknown frequency to a value well within the capability of the counter. If the counter indicates a frequency of 35.180 MHz at the output of the prescaler, the actual value of the frequency being measured is  $8 \times 35.180 \text{ MHz} = 281.44 \text{ MHz}$ . Whereas in principle any number may be chosen for the divisor  $D$ , prescalers are usually designed to divide the input frequency by an integral power of two.

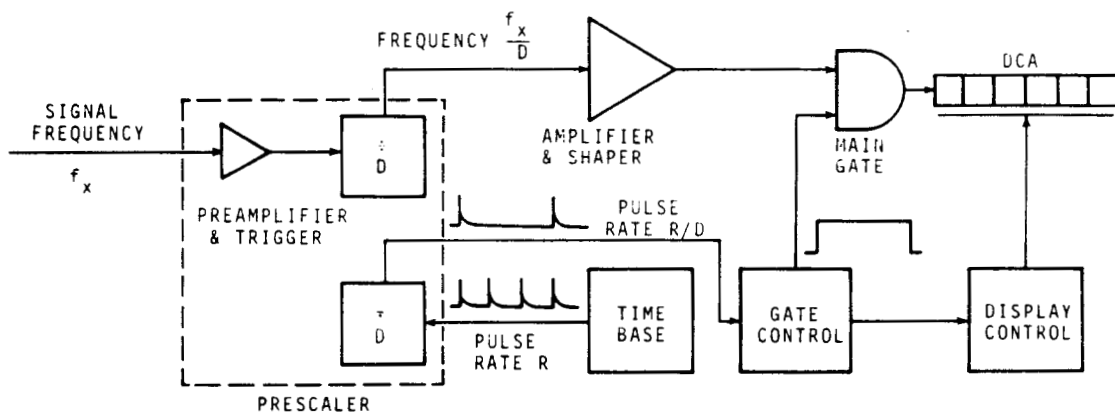


Figure 57 Prescaler-Counter Combination

The popularity of base two for this application results from the relative economy with which high-speed binary dividers can be assembled from discrete components as compared to the cost of decimal dividers having the same switching capabilities and same mode of construction. With the rapid growth of integrated circuit technology, however, decimal dividers have become quite inexpensive and are finding ever wider application in prescalers.

To retain the counter's direct-reading feature and thus avoid the necessity of multiplying by the factor  $D$ , some prescalers incorporate a provision for lengthening the counter's gate time by the same factor. If the gating interval is increased  $D$  times while the input frequency is divided by  $D$ , the scale factor becomes self-cancelling and the counter displays the correct input frequency directly. Figure 57 depicts a basic counter equipped with a prescaler which divides the signal frequency and the time-base frequency by the same amount.

Prescaling offers a convenient way of covering wide frequency ranges without elaborate accessory equipment and/or tedious operating procedures. Accuracy of the prescaler-counter combination is the same as that of the basic counter alone; but since the technique requires an increase in measurement time by virtue of the scale factor  $D$ , it is somewhat slower than the direct method.

### c. Heterodyne Converters

Another way of extending the frequency range of a counter is to use a heterodyne converter. The converter translates the unknown signal frequency downward by beating it against a precisely known signal of different frequency. The principle is the same as that employed in the familiar superheterodyne radio receiver to produce an intermediate frequency.

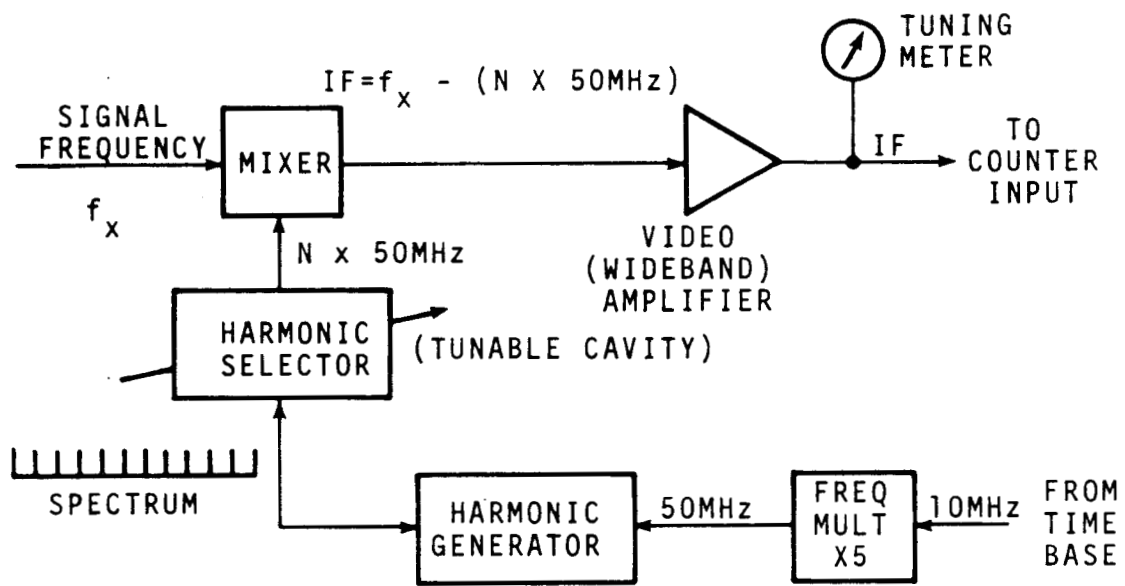


Figure 58 Typical Manually-Tuned Heterodyne Converter

The beat, or difference frequency, is applied to the counter and measured directly. The reading displayed by the counter is added algebraically to the known frequency in order to determine the unknown signal frequency.

Heterodyne converters are available either as plug-in modules or as external accessories for many commercial counters. The block diagram of a representative, manually-tuned converter appears as figure 58.

In this particular example a 10 MHz frequency from the counter's internal time base is multiplied to 50 MHz and applied to a harmonic generator. The harmonic generator produces a wide spectrum of frequencies in 50-MHz increments. By means of a tunable cavity the desired Nth harmonic can be selected from among the many harmonics available at the output of the generator. In practice it is preferable to select the harmonic that is nearest to, but lower than, the signal frequency. When this condition is met, the intermediate frequency from the mixer falls within the passband of the video amplifier as indicated by a strong upscale movement of the tuning meter. The intermediate frequency,  $f_x - (N \times 50 \text{ MHz})$ , is fed to the counter and measured. A calibrated dial on the cavity tuning control indicates the number to be added to the counter reading.

For example, consider a signal frequency of 873 MHz. As the harmonic selector is tuned upward from its lowermost setting, successively higher harmonics of 50 MHz are injected at the mixer. When the tuning dial reaches a setting of 850 MHz, or the 17th harmonic of 50 MHz, the tuning meter will rise sharply and the counter will display a reading of 23 MHz. Thus the signal frequency is the sum of the dial setting and the counter reading.

For the dial setting and the counter reading to be additive, it is important that the tuning point always be approached from the lower side and the adjustment stopped at the first peak. If the tuning control is turned too far, so that the  $(N + 1)$ th harmonic is selected, the injection frequency will exceed the signal frequency and the counter reading must then be treated as a negative number. Again referring to the previous example we note that the 18th harmonic of 50 MHz is 900 MHz. With a signal frequency of 873 MHz, the resultant counter reading of 27 MHz must be subtracted from the dial setting of 900 MHz to obtain  $f_x$ .

Heterodyne converters are widely used to extend the basic frequency range of counters. Those containing harmonic generators and tuned cavities are usable at microwave frequencies. For lower frequencies the harmonic generator may be replaced by a series of frequency multipliers whose harmonic output is controlled via switchable tuned circuits. In either case the underlying heterodyne principle is the same.

If the harmonic injection frequency is derived from the counter's internal time base, the heterodyne technique offers the same accuracy as direct counting. Of all known frequency-extension methods, the heterodyne technique provides the greatest resolution per gating interval; i.e., it allows the operator to achieve accurate microwave measurements in a minimum of time. Moreover, this method can tolerate large amounts of frequency modulation on the measured signal without jeopardizing the accuracy of the measurement.

In some of the most modern converters the manually-tuned cavities are replaced with electronically-tuned filters of the yttrium-iron garnet (YIG) type. These are known as automatic heterodyne converters. Whether manual or automatic, every converter has a limited range of frequencies over which it will function. In order to cover an extremely wide frequency range, therefore, two or more converters may be needed.



#### d. Transfer Oscillators

Transfer oscillators are used to extend the frequency measuring range of counters into regions that are normally beyond the workable limit of prescalers and heterodyne converters. At the present state of the art, transfer oscillators are usable at frequencies up to 40 GHz and higher. Their wide bandwidth allows coverage by a single plug-in unit over a great range of frequencies. Furthermore, most versions permit accurate frequency measurement of complex waveforms, such as frequency-modulated signals and pulsed rf.

A typical transfer oscillator consists of a highly-stable variable frequency oscillator (VFO), a harmonic generator, a mixer, and a null detector. Like the heterodyne converter, the transfer oscillator mixes the unknown signal frequency with a selected harmonic of some internal reference frequency. In the latter case, however, the reference frequency is derived from the VFO; and the counter is used, sometimes in conjunction with a prescaler or converter, to measure the VFO fundamental.

In operation the VFO is carefully tuned until one of its harmonics matches the unknown frequency of the signal being measured. The frequency match is indicated by a zero-beat condition on the null detector. At this point the counter reading  $f_1$  is observed and recorded.

Then the VFO is slowly tuned upward or downward until the next adjacent harmonic produces a null, whereupon a new counter reading  $f_2$  is noted. The harmonic number applicable to  $f_1$  can be calculated from the formula

$$N_1 = \frac{f_2}{|f_1 - f_2|} .$$

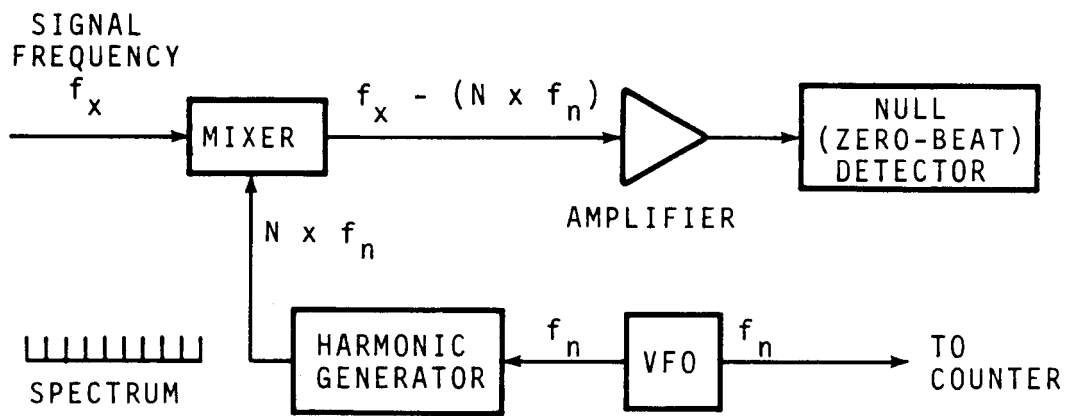


Figure 59 Typical Manually-Tuned Transfer Oscillator

Having determined both  $N_1$  and  $f_1$  it is an easy task to compute the signal frequency,  $f_x = N_1 f_1$ . To illustrate the computation process, let us assume that the first observed null occurs when  $f_1 = 88.0$  MHz. Let us further assume that as the VFO is tuned slowly upward, the next null is found at  $f_2 = 90.0$  MHz. The applicable harmonic number then has the value

$$N_1 = \frac{90.0}{|88.0 - 90.0|} = \frac{90.0}{2.0} = 45$$

and the unknown frequency is therefore

$$f_x = 45 \times 88.0 \text{ MHz} = 3960 \text{ MHz}.$$

The same final result would be obtained if the roles of  $f_1$  and  $f_2$  were interchanged, i.e., if the VFO were tuned downward from 90.0 MHz to 88.0 MHz in arriving at the second null.

For any given  $f_x$  many null points may be observed as the VFO is tuned through its entire range. Again using the input frequency of 3960 MHz as an example, an adjacent null pair may be found at VFO settings of  $f_1 = 84.25531914\dots$  MHz and  $f_2 = 82.5$  MHz. Because the fractional part of  $f_1$  does not terminate, however, the counter cannot display that number in its entirety. Although  $N_1 = 47$  can be deduced from the formula (rounded off to the nearest whole number since harmonic numbers must be integers),  $f_x$  cannot be calculated with extreme accuracy from the relation  $f_x = N_1 f_1$  because there is no way of knowing the missing portion of  $f_1$ . In this case it is better to calculate  $f_x$  from  $f_2$ , since  $f_2$  is evidently an exact number. As an alternative to  $f_x = N_1 f_1$  the formula  $f_x = (N_1 \pm 1)f_2$  can be used. The plus sign is applicable if  $f_2$  is less than  $f_1$ , whereas the minus sign is chosen if  $f_2$  is greater than  $f_1$ .

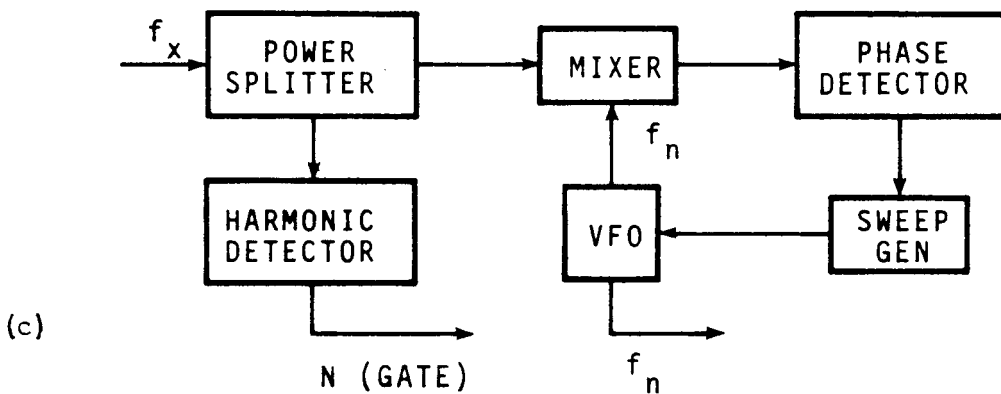
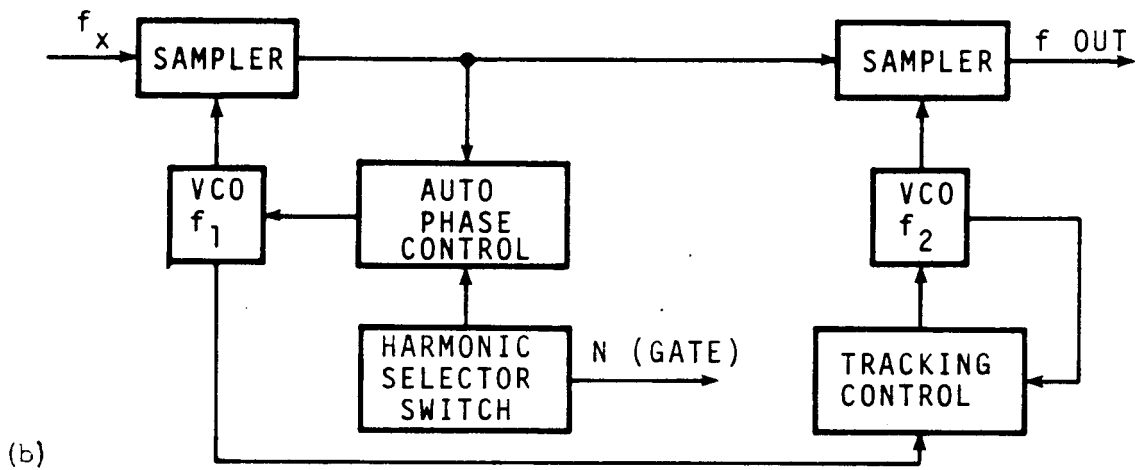
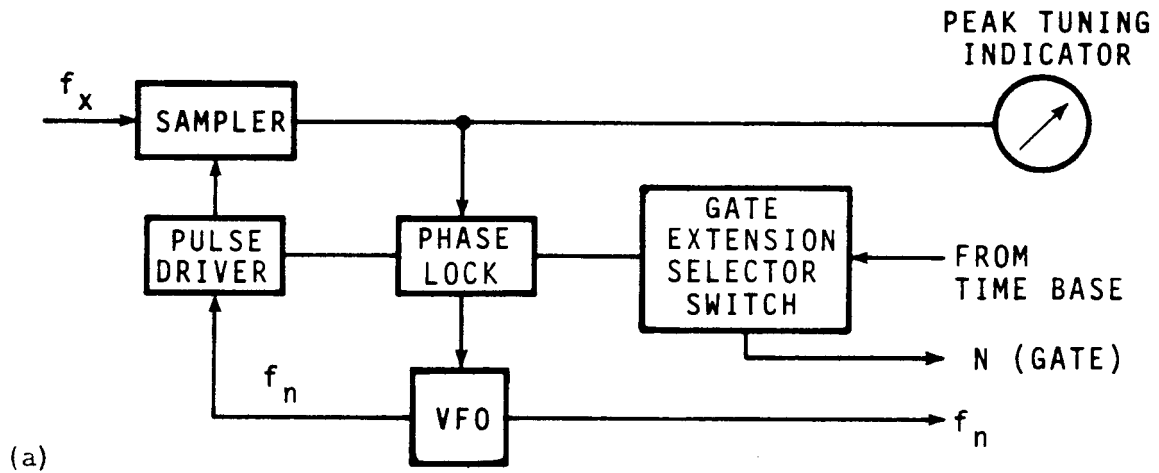


Figure 60 Automatic Transfer Oscillators

For  $f_1 = 84.25531914\dots$  MHz and  $f_2 = 82.5$  MHz, we have  $f_x = (47 + 1)(82.5 \text{ MHz}) = (48)(82.5 \text{ MHz}) = 3960 \text{ MHz}$  exactly. If it is impossible to find an adjacent null pair without both readings being truncated by the counter's display, an unavoidable source of error is introduced.

Another source of error may arise from the operator's inability to discern exactly where the zero beat occurs. In this regard the resolution of the null detector is all important. The total error is the sum of the fractional error in the counter reading plus the fractional error caused by imperfect null detection. Typically, an uncertainty of several parts in  $10^7$  accompanies microwave measurements made with manually-tuned transfer oscillators.

Some of the more elaborate transfer oscillators employ feedback methods which automatically lock the VFO to an exact submultiple of  $f_x$ . The phase lock feature essentially eliminates human error caused by improper zero-beat detection. Most automatic transfer oscillators also have provisions for extending the counter's gating interval by the factor  $N$  so that the counter can display  $f_x$  directly. Figure 60 shows block diagrams of three automatic transfer oscillators which are presently available on the commercial market.

Phase-locked transfer oscillators are normally used to measure only stable cw signals and often are not applicable to complex waveforms containing frequency or pulse modulation. Even in those versions which are able to measure pulsed carriers, the accuracy is not as great as when measuring continuous waves. The automatic feature is designed for rapid phase lock and fast sampling, but a major disadvantage arises from the fact that resolution varies with the input frequency.

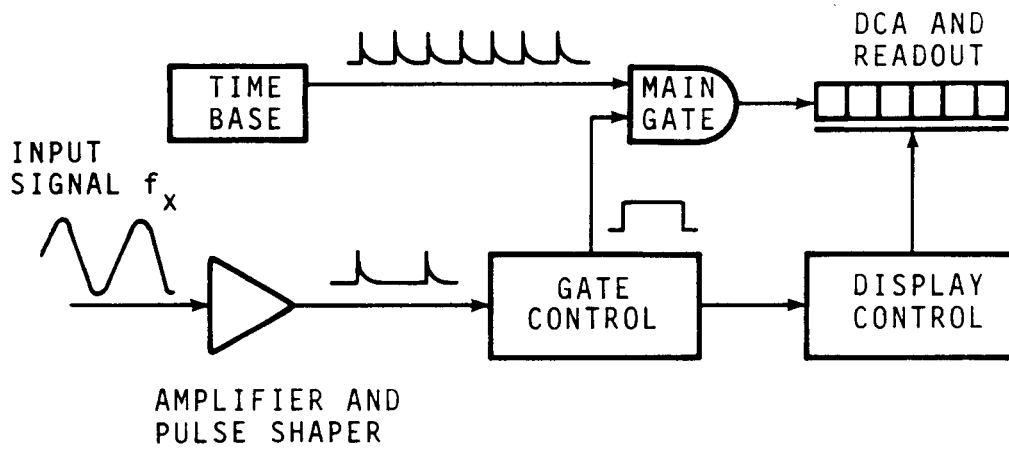


Figure 61 Diagram of Counter in the Period-Measurement Mode

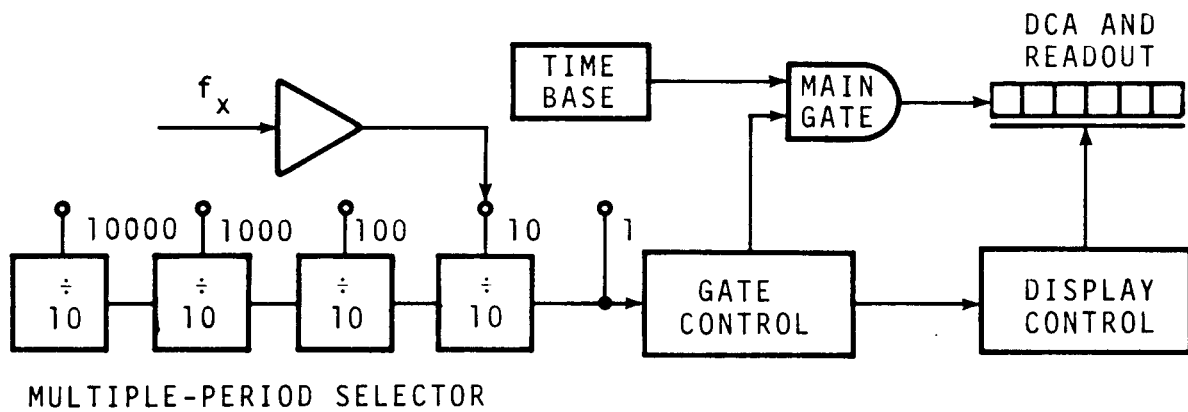


Figure 62 Diagram of Counter in the Multiple-Period Mode

## 5.2 Period Measurement

Sometimes it is preferable to measure the period of a signal rather than its frequency. The basic components of a frequency counter can be rearranged to provide period measurements simply by exchanging the roles of the time base and the input signal. The exchange is accomplished in modern counters by means of a function switch on the control panel.

In the period-measurement mode the output of the time base generator is counted during a time interval controlled by the period of the input signal  $f_x$ . If, for example,  $f_x$  has a period of 1 ms and the time base output is  $10^5$  pps (or  $10\mu\text{s}$ ), the counter then totalizes  $(1 \times 10^{-3} \text{ s})(1 \times 10^5 \text{ pulses/s}) = 100$  pulses during each gating interval. If the period of  $f_x$  increases from 1 ms to 10 ms, the counter then totalizes 1000 pulses. Thus the counter readout is proportional to the period of  $f_x$  and can be designed to display the period directly in units of time ( $\mu\text{s}$ , ms, s, etc.).

As a general rule low frequencies can be determined more accurately by measuring period rather than frequency. This is true because longer periods allow more counts to accumulate in the DCA; hence the resolution is better than if low frequencies are measured directly. Because frequency is the reciprocal of period, an unknown frequency  $f_x$  can be calculated from its measured period  $T_x$  by means of the formula  $f_x = 1/T_x$ .

The accuracy of a period measurement can be improved by using the multiple-period mode of operation. In this mode the output of the signal amplifier is routed through one or more decade frequency dividers before reaching the gate control circuit. Thus the main gate is held open for 10, 100, 1000, or more periods depending upon the number of divider stages used. The more periods over which a signal is averaged, the better the resolution which can be achieved.

PERIOD MULTIPLE	TIME BASE				
	1ms	0.1ms	10 $\mu$ s	1 $\mu$ s	0.1 $\mu$ s
X1	2 ms	2.4 ms	2.40 ms	2408 $\mu$ s	2409.0 $\mu$ s
X10	2.4 ms	2.41 ms	2409 $\mu$ s	2409.0 $\mu$ s	2408.91 $\mu$ s
X100	2.41 ms	2.408 ms	2408.8 $\mu$ s	2408.91 $\mu$ s	2408.912 $\mu$ s
X1000	2.409 ms	2.4089 ms	2408.91 $\mu$ s	2408.911 $\mu$ s	2408.9113 $\mu$ s
X10000	2.4089ms	2.40890ms	2408.912 $\mu$ s	2408.9112 $\mu$ s	*408.91124 $\mu$ s

\*LEADING DIGIT OFF SCALE

Figure 63 Readings Obtained with Various Settings of Time Base and Period Multiplier Controls

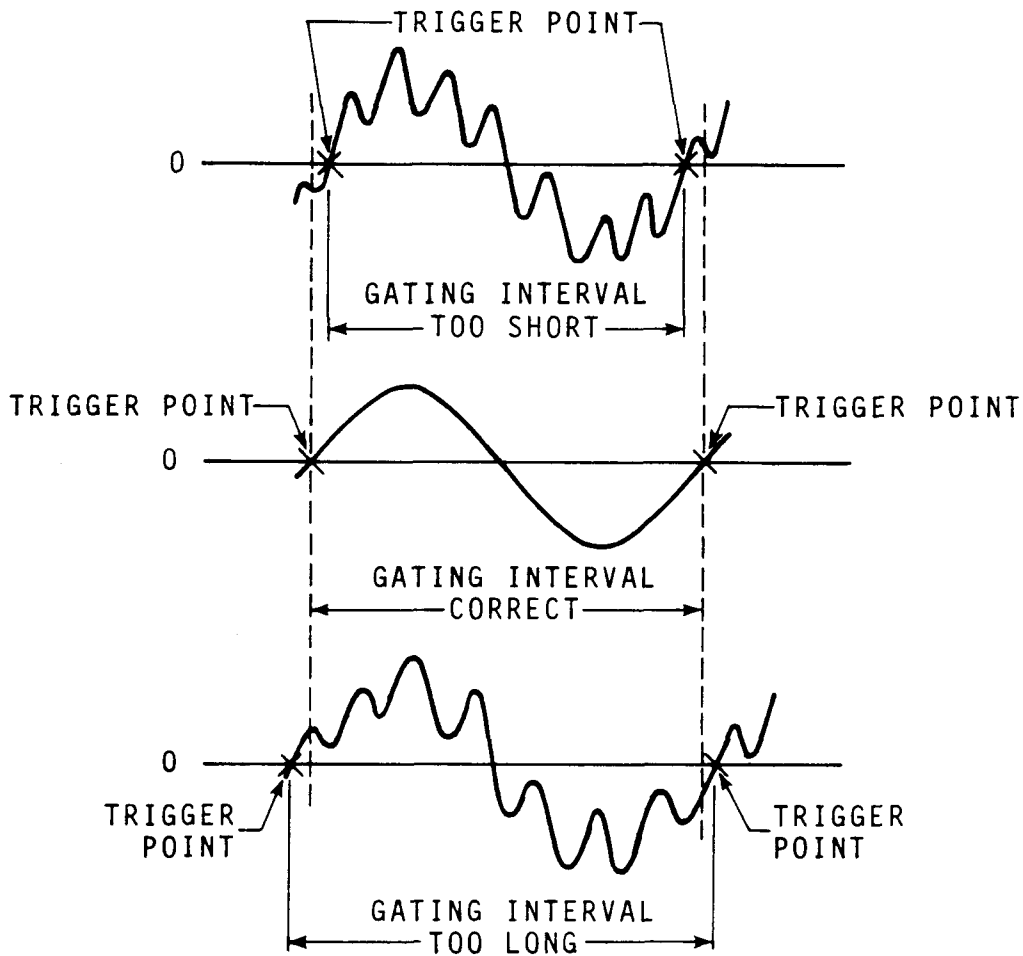


Figure 64 Effect of Noise on Trigger Point



To illustrate the improved accuracy attainable by multiple-period averaging at lower frequencies, consider a counter reading of 0.415 kHz, which we assume was taken in the frequency-measurement mode. The same frequency, if divided by 10 and then subjected to a period measurement using a  $10^7$  pps (or  $0.1\mu\text{s}$ ) time base, may produce a six-digit reading of  $2408.91\mu\text{s}$ .<sup>6</sup> By taking the reciprocal of  $2408.91\mu\text{s}$ , it is seen that  $f_x$  averages 0.415125 kHz (or 415.125 Hz) over ten complete periods.

The accuracy of the measurement may be enhanced still further by increasing the time base frequency or the gating interval or both. The table in figure 63 shows the results of repeated period measurements of the same sinusoidal wave. The measurements were made with a counter having an 8-digit display and with various settings of both the time base selector switch and the period multiplier switch. The improved resolution at higher time base rates and at longer gating intervals is quite apparent.

Noise is one of the major sources of error in period measurements. A noisy input signal may cause the main gate to open or close prematurely at one instant and late at another instant. In the period measurement mode most counters are designed to trigger the main gate at a positive-going zero-axis crossing of the input signal. Any jitter at this critical point produces erratic gating, as evidenced by figure 64.

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<sup>6</sup>Generally, the display control section is designed to position the decimal point and to scale the units of measurement in accordance with the setting of the period selector switch so that the reading is correct for a single period.

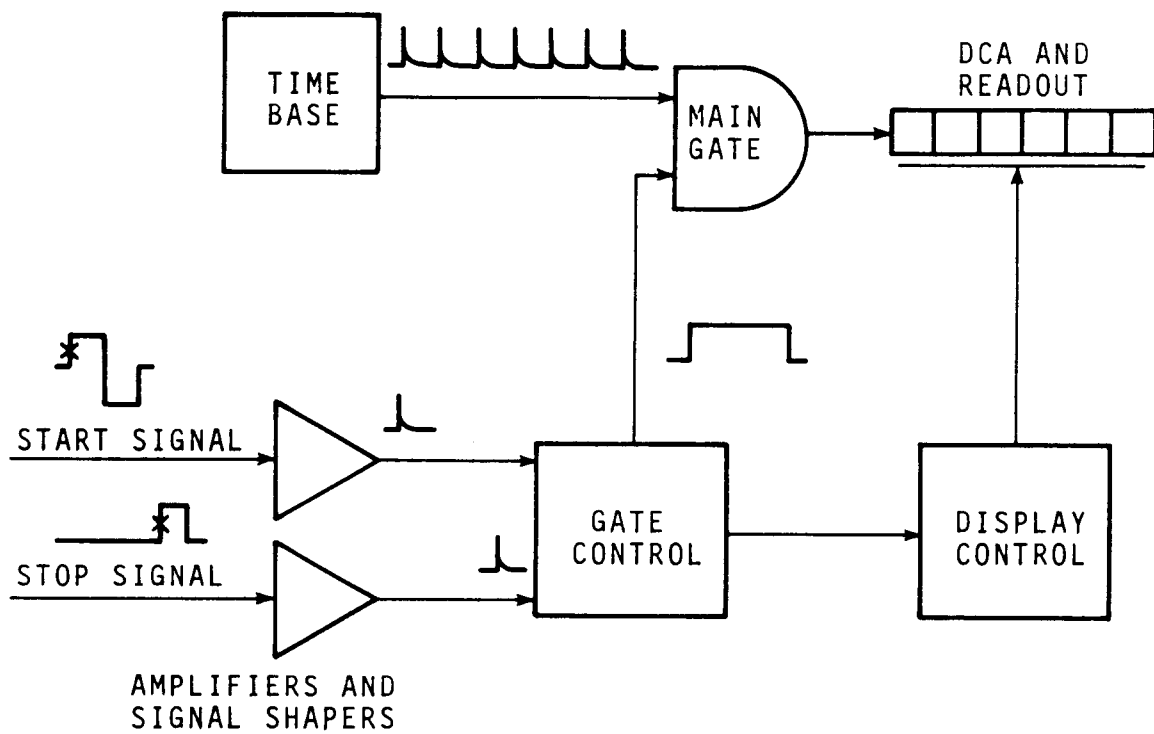


Figure 65 Diagram of Counter in the Time-Interval Mode

The fractional error resulting from noise modulation is independent of frequency and may be reduced by extending the measurement to more than one period. A ten-period measurement, for instance, is ten times more precise than a single-period measurement, because the same uncertainty in the time of opening and closing the main gate is distributed over an interval ten times as long.

### 5.3 Time Interval Measurement

Electronic counters are widely used for the measurement of time intervals, both periodic and aperiodic, ranging in length from less than a microsecond up to hours and even days. At the end of the measured interval the elapsed time is displayed in digital form by the readout assembly.

Counters vary considerably in their time-measuring ability. Some are designed to measure the duration of a single electric pulse, whereas others measure the interval between the occurrence of two different pulses. The more versatile models have individually adjustable start and stop controls which allow either mode of time interval measurement to be chosen by the operator.

As with period measurements the basic components of the frequency counter are reconfigured in such a manner that the output pulses from the time base are counted. To define the interval being measured, however, separate start-stop pulses are utilized to control the main gate.

During the interval between the start and stop signals the main gate is held open, and counts are accumulated by the DCA in direct proportion to the length of the gating interval. Thus at the end of the gating interval the readout assembly can be made to display the elapsed time directly in microseconds, seconds, minutes, or other appropriate units.

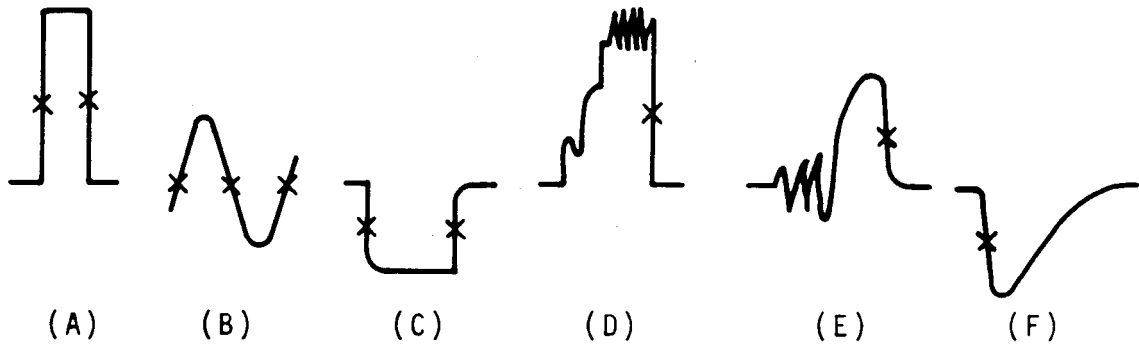


Figure 66 Optimum Trigger Points for Start-Stop Pulses

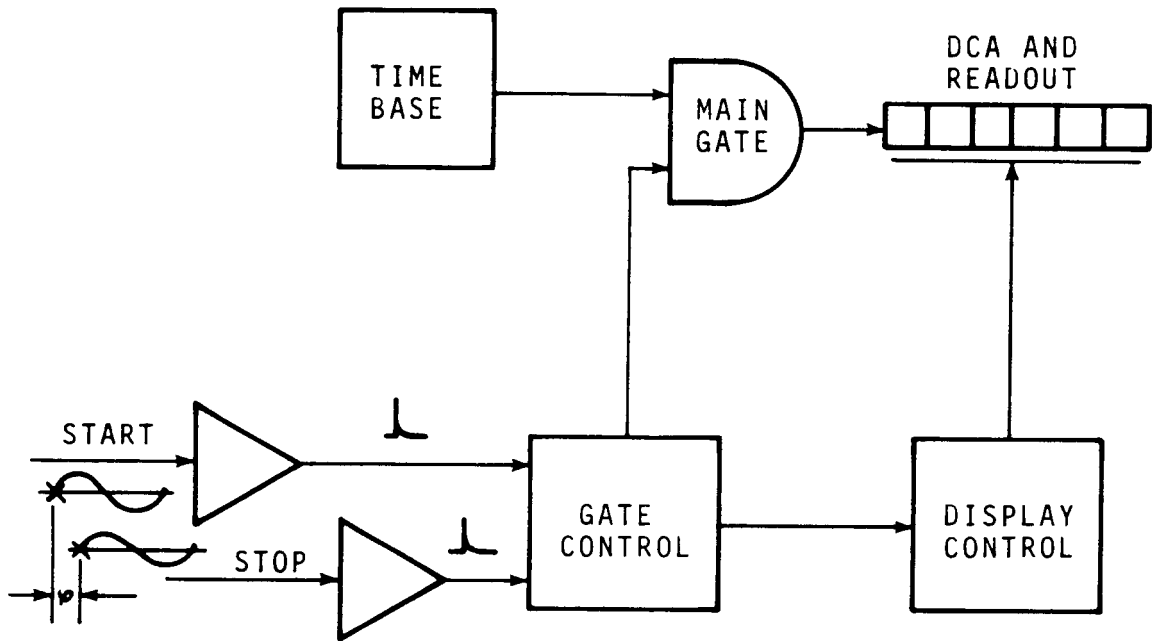


Figure 67 Use of Time-Interval Unit for Phase Measurement

The input channels for the start-stop signals are designed to produce sharp, uniform trigger pulses from a variety of pulse shapes; but the best results are obtained if steep, clean pulses are applied to the input terminals first. If either channel is outfitted with a trigger level control, it is possible to select the input amplitude at which a trigger pulse from that channel is generated. If also a slope control is incorporated, it is possible to make the trigger point coincide with a given amplitude on either the positive-going or the negative-going side of the input signal at will. If both channels are equipped with trigger level and slope controls, the trigger points for the start pulse and the stop pulse can be adjusted independently.

Intelligent use of the trigger level and slope controls enhances the chance of making measurements that are free of error caused by the superposition of noise, jitter, harmonics, and other forms of distortion on the input signal. Figure 66 depicts the best triggering points for six different input waveshapes. In each case the desired trigger point (as indicated by an x) is at the center of the steepest, cleanest portion of the signal. It is worth noting that for noise-free sinusoidal signals as in waveform (B), the optimum trigger points occur at the zero crossings.

#### 5.4 Phase Measurement

The time interval feature lends itself readily to the determination of phase relationships between any two input signals of the same known frequency. For this application the start channel is fed one of the signals while the second signal is directed to the stop channel. The level and slope controls are adjusted to trigger at exactly the same point on both input waveforms. For sinusoidal waves the ideal triggering point is the zero-crossover .

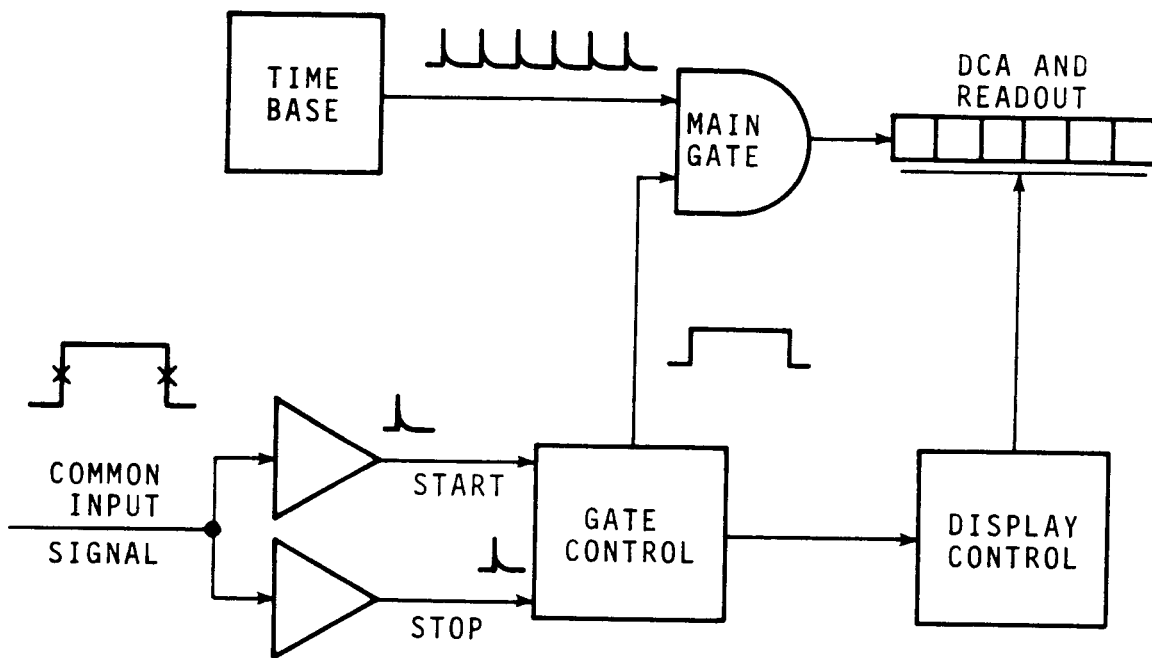


Figure 68 Time-Interval Counter with Start-Stop Channels Connected to Common Source for Pulse-Width or Period Measurements

The time interval ( $t$ ) between corresponding points on the two input waveforms is then read directly from the counter. The time interval can be converted into a phase angle by means of the formula

$$\varphi = 360 t/T \text{ degrees} \quad \text{or} \quad \varphi = 2\pi t/T \text{ radians} ,$$

where  $T$  is the known period of the two signals expressed in the same units as  $t$ .

### 5.5 Pulse Width Determination

Trigger controls are seldom calibrated with sufficient accuracy to measure the rise time or decay time of a single pulse. However by selecting one polarity of slope for the start trigger and the opposite polarity for the stop trigger, it is often possible to measure the total duration of a pulse. The duration of pulse (A) in figure 66, for example, could be determined by selecting point  $x$  on the positive slope for the start trigger and point  $x$  on the negative slope for the stop trigger.

Carrying the idea one step further, the period of waveform (B) could be measured readily by choosing the positive zero-crossing as the trigger point for both the start and stop pulses. When attempting to measure pulse width or waveform period with a time interval counter, it is necessary of course to feed both input channels from the common source.

The smallest measureable time interval is limited by the minimum resolution time of the counter. In general, the precision of time interval measurements is influenced by the same factors that affect period measurements. The presence of noise, however slight its amount, can seriously impair the precision of pulse width determinations.

## 5.6 Counter Accuracy

Depending both upon the characteristics of the counter and its method of use, the precision of measurement may range from 1% to a few parts in  $10^{10}$  or better. Although the possible causes for inaccurate measurement are many, the main sources of error are (1) time-base instability, (2) uncertain gating, and (3) faulty triggering. The total measurement error is the algebraic sum of all individual errors.

### a. Time Base Error

The time base oscillator is apt to change frequency very slightly during the course of a single measurement or series of measurements. The principal causes of oscillator instability are temperature variation, line voltage fluctuation, internal noise, and crystal aging.

The temperature effects can be minimized by keeping the counter in a constant-temperature environment and by allowing ample space for air circulation all around the instrument. In localities where line voltage fluctuations are severe, it may be necessary to operate the counter from a regulated power source, not only to improve the time base stability but also to safeguard the instrument from damage caused by voltage transients.

Any noise generated internally by the time base oscillator produces a random jitter in its output frequency. Unless restricted to very low magnitudes the jitter may cause erroneous counts, especially when a measurement is performed during a brief time span. As the measurement time is extended to longer intervals, the perturbations tend to cancel out on the average. For this reason, the jitter is often called short-term instability. A typical time base stability graph is shown in figure 69.



In modern counters the amount of oscillator noise is generally so small that short-term stability is specified only for the most precise instruments. When interpreting specifications it should be realized that the averaging time has an important bearing on the actual meaning of stability figures. For example, an oscillator having a stability specification of  $5 \times 10^{-10}$  (per minute) might be noisier than one with a specification of  $5 \times 10^{-9}$  (per second). To be absolutely meaningful, short-term stability figures should be compared for the same averaging period, preferably of only a few seconds or less.

Long-term instability, on the other hand, refers to a slow predictable drift in the average frequency of an oscillator due to gradual changes in the resonant elements. Precision quartz crystals exhibit drift rates typically of  $10^{-10}$  to  $10^{-8}$  per day as a result of crystal aging. After a brief warm-up period when the oscillator is first energized, a high-quality crystal assumes an essentially linear drift characteristic, the slope of which reveals the aging rate.

Because aging produces a consistent and cumulative error as time goes on, it is necessary to recalibrate the oscillator periodically if the time base is to remain accurate. To ensure utmost accuracy it may be wise to check the oscillator calibration immediately before and after each critical measurement.

For measurements which require a time base accuracy exceeding that of the counter's internal oscillator, it is advisable to use an external frequency standard to drive the time pulse generator. Most counters have a built-in connector and switch for convenience in hooking to an external standard. With such arrangements the internal oscillator is automatically disconnected when the time base excitation is switched to an external source.

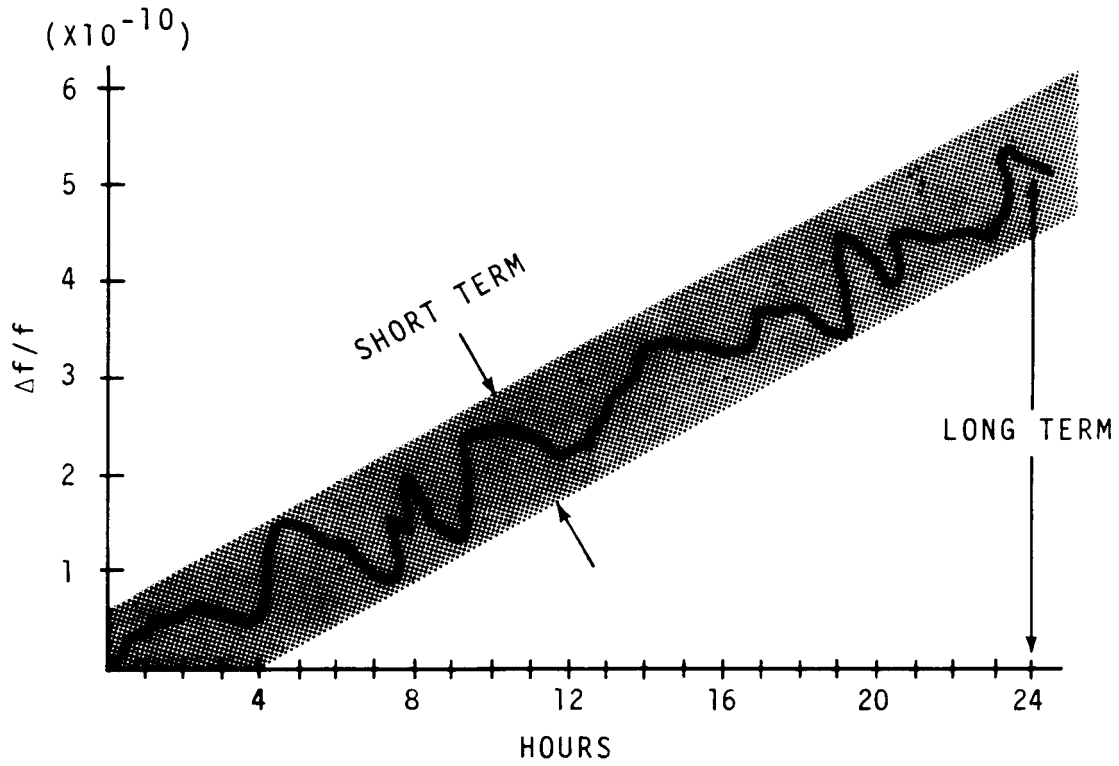


Figure 69 Typical Time Base Stability Curve

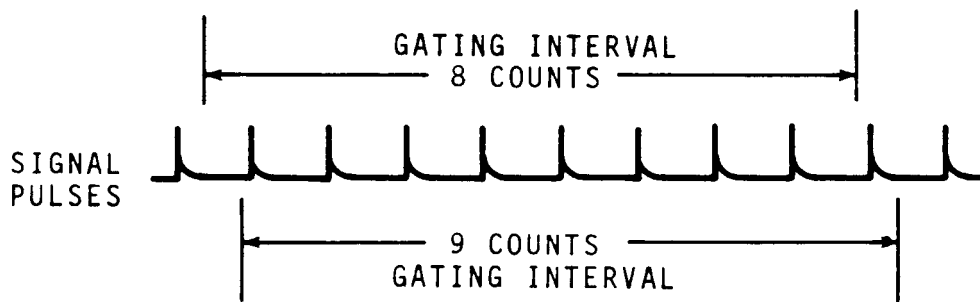


Figure 70 Constant Gating Interval with Ambiguity of  $\pm 1$  Count

## b. Gate Error

An uncertainty of plus or minus one count is inherent with all conventional electronic counters because the time base pulses are not usually synchronized with the signal being measured. Due to the indefinite phase relation between the pulses being counted and the exact beginning or end of the gating interval, it is possible for the displayed reading to be in error by one count even if all other sources of error were nonexistent. As an example, figure 70 shows how the same signal might produce a count of either 8 or 9, depending upon the particular instant at which the gating interval commences.

The fractional uncertainty ( $\epsilon_g$ ) arising from the count ambiguity is

$$\epsilon_g = \pm \frac{1}{f \cdot t} = \pm \frac{1}{n}$$

where  $f$  denotes the average frequency of the signal pulses,  $t$  denotes the duration of the gating interval, and  $n$  denotes the total number of pulses counted.

It is apparent that the uncertainty becomes smaller as more pulses are counted. The inverse proportionality explains why long gating intervals permit greater accuracy of measurement. Clearly, a deviation of  $\pm 1$  count is less profound if  $n$  is large than if  $n$  is small.

In the counting of a 1 MHz frequency for 1 second,  $n$  has the value of  $1 \times 10^6$  and therefore  $\epsilon_g = \pm 1 \times 10^{-6}$ . If either the measured frequency is increased to 10 MHz or the gating interval is lengthened to 10 seconds,  $n$  also increases tenfold and  $\epsilon_g$  reduces to  $\pm 1 \times 10^{-7}$ . In measurements of high frequencies, whether performed directly or with the aid of heterodyne converters, the uncertainty of  $\pm 1$  count may be less significant than the counter's time base error. The same is true of prescalers which incorporate gate extension features.

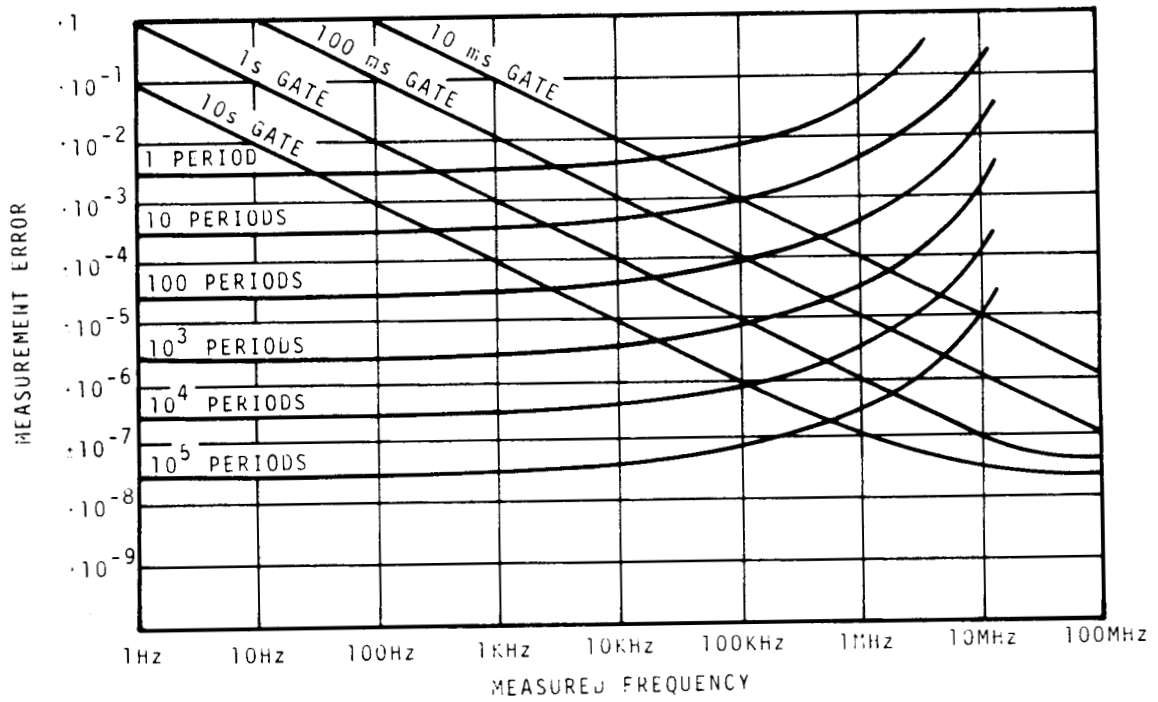


Figure 71 Accuracy Chart for Period and Frequency Measurements

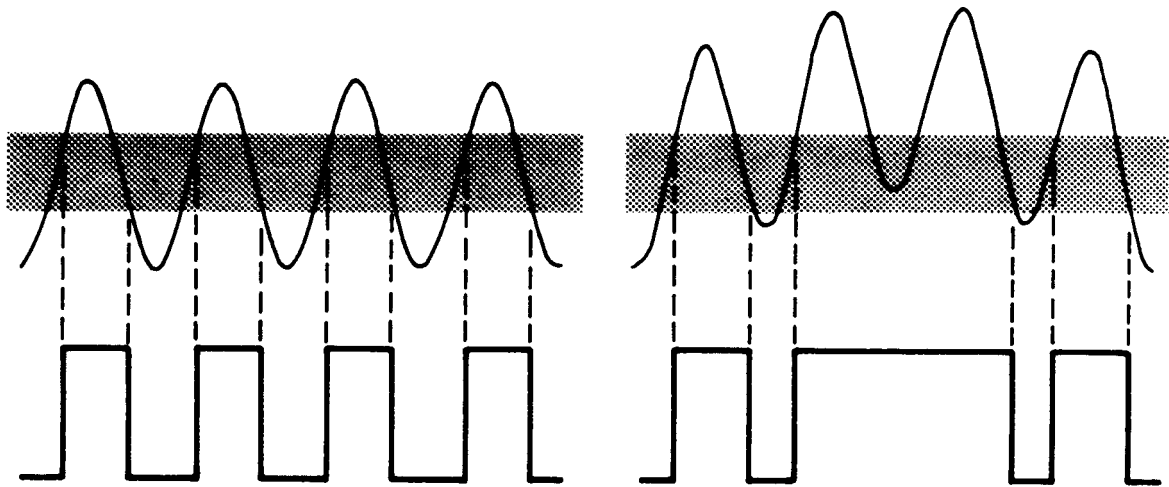
Other techniques may be used to reduce the effect of gate error. For instance, both the time base frequency and the unknown frequency may be multiplied to higher frequencies, mixed, and then subjected to measurement via their beat frequency. By counting the beat frequency and applying the known multiplying factor, it is possible to determine the unknown frequency with very high precision.

In a previous section it was indicated that low frequencies can be determined more accurately by period measurements than by direct frequency measurements. For high frequencies, however, the reverse is true. At some point in the frequency spectrum, therefore, the effect of gate error must be identical for either mode of operation. The chart in figure 71 depicts the crossover point as well as the accuracy which can be expected both from multiple-period measurements and direct frequency measurements up to 100 MHz.

### c. Trigger Error

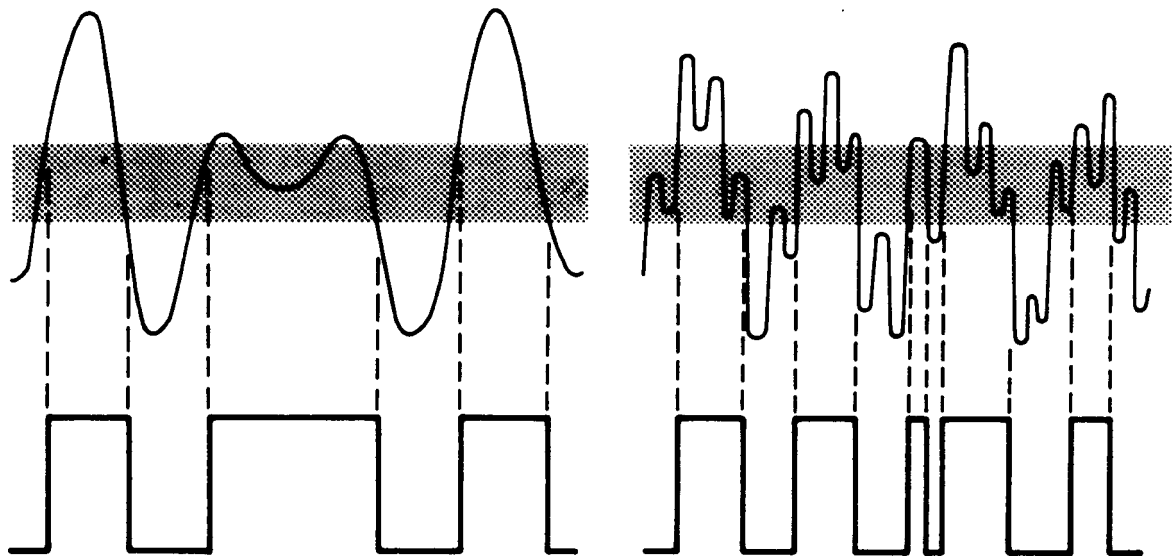
Trigger error usually arises from the presence of noise or other modulation on the gate-control signal. This source of error has been mentioned previously in conjunction with period measurements and time interval measurements. For frequency measurements the trigger error is almost always negligible because the gate-control signal, obtained directly from the time base, is virtually noise-free.

For period and time interval measurements the trigger error ( $\epsilon_{tr}$ ) is expressed mathematically as the ratio of the gate time deviation ( $\Delta t$ ) to the total duration ( $t$ ) of the gating interval. Trigger error is also proportional to the fraction of peak noise voltage ( $E_n$ ) over peak signal voltage ( $E_s$ ).



(a)  
Continuous Sine Wave Input  
(four pulses produced)

(b)  
Low-Frequency Ripple Superimposed  
(three pulses produced)



(c)  
Heavy Amplitude Modulation  
(three pulses produced)

(d)  
High-Frequency Interference  
(five pulses produced)

Figure 72 Undifferentiated Schmitt-Trigger Waveforms

Hence,

$$\epsilon_{tr} = \frac{\Delta t}{t} = k \frac{E_n}{E_s}$$

where  $k$  is a proportionality constant determined by the nature of the modulation.

Although noise can sometimes be removed from the input signal by external devices, e.g., limiters, filters, etc., its effects can be reduced more conveniently by multiple period averaging. As in the case of gate error ( $\epsilon_g$ ) the formula for trigger error ( $\epsilon_{tr}$ ) contains the gating interval ( $t$ ) in the denominator. Consequently, as more periods are averaged, the effects of both the  $\pm 1$  count ambiguity and the trigger error diminish proportionately.

Most counters incorporate a Schmitt trigger or similar circuit to generate the trigger pulses. Because of a slight hysteresis or lag in switching action which characterizes such circuits, the rise and decay of the trigger pulses are not initiated at exactly the same voltage. Instead the width of the trigger pulse is determined by two voltage levels — one that establishes the point at which the pulse begins to rise and the other that dictates the point where the pulse starts to fall. A complete pulse is produced, therefore, only when the input signal crosses both critical levels.

Figure 72 shows how counts may be lost or gained as a result of signal modulation. The shaded area in each example demarks the hysteresis zone of the trigger circuit. Incorrect counts are likely to result whenever the input waveform reverses direction within the shaded zone.

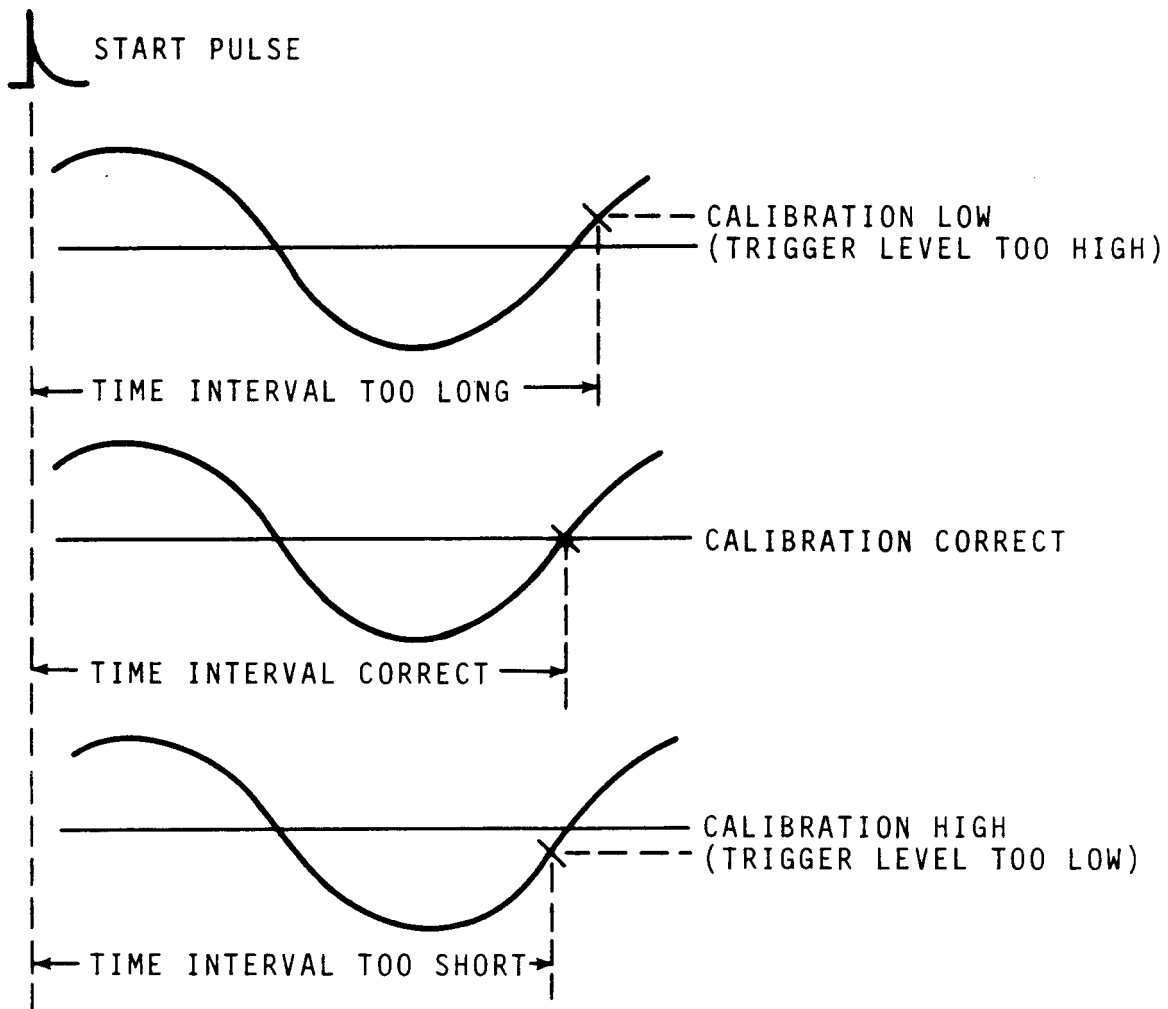


Figure 73 Time Error Produced by Improper Calibration of Trigger Level Control



Having no definite period, noise causes the trigger pulses to be gained or lost in a random manner. A systematic error might arise, however, from incorrect calibration of the trigger level controls or any change in behavior of the input or gating circuitry.

To measure the time interval between the occurrence of a sharp pulse and the next positive-going zero crossing of a continuous sine wave, it would be logical to start the counter with the pulse and adjust the stop-channel control(s) to trigger at zero volts on the positive slope. If the trigger level control for the stop channel is improperly calibrated, however, the stop pulse may be produced at a point that is actually positive or negative when the control is set at zero. This means that the measured time interval will be too long or too short as indicated by figure 73.

Of course, an opposite error will result if the trigger level control for the start channel reads too low or too high. Should the controls for both channels be misadjusted in opposite directions, the combined error will be cumulative. Furthermore, the effect of trigger calibration error is made worse if instead of a steep wavefront the input signal has a gradual slope at the selected triggering point.

Unequal amplitudes or rise times of the input signals may produce slightly different delays in the generation of trigger pulses. Also, the gate control circuit might respond more promptly in one direction than the other. To ensure accurate performance, therefore, it is advisable to check the triggering levels occasionally by measuring the period of a known test signal. Because the check can be no better than the quality of the testing source, only high-grade instruments should be used to generate the test signal and to determine its true period.

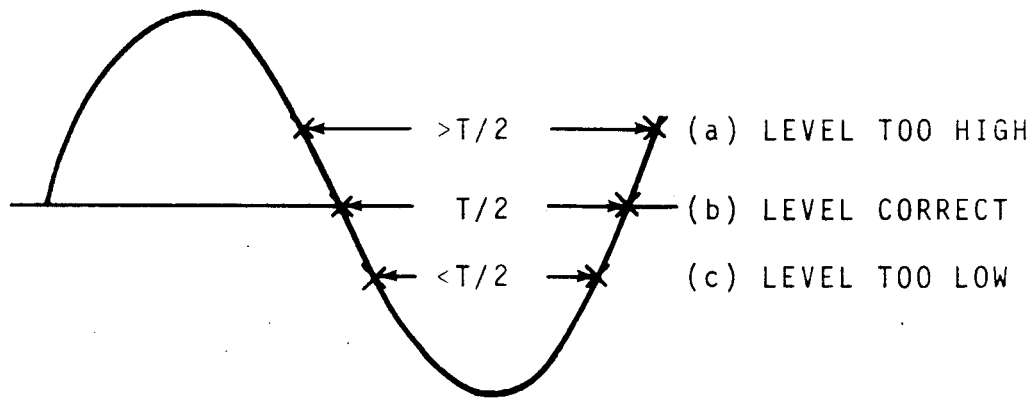


Figure 74 Sine Wave Method of Checking Trigger Level Calibration

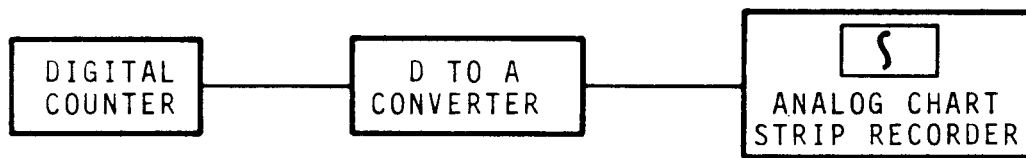


Figure 75 Digital-to-Analog Arrangement for Chart Recordings

A low-frequency sine wave is convenient for checking the calibration of trigger controls. With the counter in the time interval mode, the gating interval switch in the X1 position, and both input channels adjusted to trigger at zero volts on the same slope, the readout assembly should display exactly one period of the common test signal. Next the slope selector for the stop channel is turned to its opposite position without adjustment of any other controls. If the counter reading changes by exactly one-half period when the opposite slope is selected, the trigger level setting is true zero. If the reading changes by more than one-half period, as in (a) of figure 74, the stop-channel's trigger level is too high; if the change is less than one-half period, as in (c), the trigger level is too low.

The start trigger can be checked in a similar fashion by holding all other controls fixed and switching the slope selector for the start channel only. By testing both channels under all slope combinations, it is also possible to detect differential response of the gate control circuit.

### 5.7 Printout and Recording

Most counters have provisions for connecting a mechanical printer to the output of the display assembly so that a permanent record of the readings can be secured. In conjunction with a digital-to-analog converter, the display section can also be used to drive a graphic recorder. It is necessary of course that the translation circuitry for the printer (or recorder) be compatible with the binary code and the control voltages available from the counter if a valid printout (or recording) is to be obtained.

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