SPECTRAL ANALYSIS OF A PHASE LOCKED LASER AT 891 GHz, AN APPLICATION OF JOSEPHSON JUNCTIONS IN THE FAR INFRARED

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Résumé. — Nous avons utilisé une jonction Josephson pour étudier la pureté spectrale d'un laser HCN utilisé dans une chaîne de synthèse de fréquence infrarouge. Pour obtenir une largeur de raie du laser plus étroite, on le synchronise avec une onde hyperfréquence issue d'une chaîne de multiplication de fréquence. Une valeur calculée de cette largeur de raie est obtenue à l'aide d'une mesure du spectre de bruit de la source hyperfréquence, et d'une théorie due à Middleton. Les propriétés caractéristiques des jonctions Josephson en tant que mélangeur et multiplicateur de fréquences permettent de les utiliser pour mesurer cette largeur de raie. La jonction Josephson est excitée par un signal en bande X, issu d'un générateur à klystron stabilisé par une cavité spéciale, de haute pureté spectrale. La jonction génère le 92^e harmonique et effectue le mélange avec le signal à 891 GHz issu du laser HCN. Le battement est amplifié puis envoyé dans un analyseur de spectre. On présente les détails de l'expérience, les résultats et on indique la relation qui permet de calculer la largeur de raie.

Abstract. — We have used a Josephson junction to investigate the spectral purity of an HCN laser which is used in an infrared frequency synthesis chain. To obtain a narrower linewidth from the laser it has been phase locked to a multiplied microwave reference chain. A calculated value for this linewidth was based upon the measured noise spectrum of the microwave source and a theory due to Middleton. One can take advantage of the unique properties of the Josephson junction as a frequency multiplier and mixer for use in measuring this linewidth. The Josephson junction is driven by an X-band signal which is derived from a specially designed cavity stabilized klystron system of high spectral purity. The 92nd harmonic of the X-band signal is generated in the Josephson junction. In addition, the Josephson junction acts as a mixer of the harmonic signals and the 891 GHz output of the HCN laser. The 92nd harmonic beat signal is taken from the Josephson junction, amplified, and sent to a spectrum analyzer for frequency domain analysis. Details of the experiment, results, and relation to predicted linewidths are presented.

1. **Introduction.** — The motivation for developing and spectrum analyzing the phase locked HCN laser is for use in infrared frequency synthesis experiments ; hence we give a brief explanation of that topic to set the work in perspective.

The objective of infrared frequency synthesis [1] is to measure or control the frequencies of lasers in the infrared. To measure accurately the frequency of a laser, ν_M, one must synthesize a frequency ν_s which is close to the frequency to be measured. The difference between these two frequencies is an intermediate frequency, ν_{IF}, typically in the 10 to 100 MHz range. The synthesized frequency is :

$$\begin{aligned} \nu_s &= \nu_M \pm \nu_{IF} \\ &= l\nu_1 + m\nu_2 + n\nu_3 \end{aligned}$$

where ν_1 and ν_2 are basis laser frequencies which have been determined by prior synthesis measurements, ν_3 is a microwave frequency. The quantities l, m and n are harmonic numbers with m and n allowed both positive and negative values. The harmonic generation, as well as the mixing which produces the intermediate frequency to be measured, occurs in a suitable diode, typically a tungsten catwhisker on a nickel base [2] or a Josephson junction [3]. One then determines the frequency of the beat note center with a spectrum analyzer-tracking generator or, when the beat note signal quality permits one can use a frequency counter.

A typical infrared frequency synthesis chain is shown in figure 1 ; it was used in measuring the frequency of the 3.39 μm methane transition [4]. One can measure the frequency of the HCN laser at 337 μm, use that to measure the water vapor laser at 28 μm, then extend the measurements to CO_2 frequencies at 9.3 and 10.2 μm and on to methane at 3.39 μm. Chains similar to these have also been developed at NPL in England [5] and MIT in the US [6]. All these chains have the HCN laser as the lowest frequency basis laser and it is desirable to narrow this

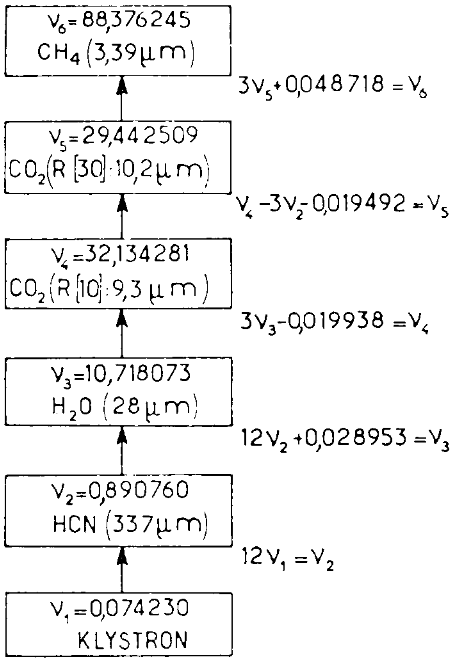

FIG. 1. — Simplified block diagram of an infrared frequency synthesis chain for measuring the frequency of methane.

laser's RF power spectral linewidth to reduce the uncertainty in locating its center [7]. We can reduce this linewidth if we can servo it to a narrower line.

2. **HCN laser stabilization.** — Figure 2 shows a block diagram of an improved scheme for stabilizing the laser. This basic reference chain is shown in the upper left. An X-band klystron is phase locked to some harmonic (say the 90th) of a quartz crystal oscillator with good long term stability. The output of the phase locked oscillator is fed through a variable attenuator and used to injection lock a cavity stabilized klystron which has good short term stability. An E-band klystron is then phase locked to the 7th harmonic of the second X-band source. The HCN laser is then compared with the 12th harmonic of the E-band source. The difference frequency at 30 MHz is used in the control apparatus. The control circuitry consists of two servo loops, one for frequency locking, the other for phase locking. The frequency lock loop consists of a frequency discriminator, preamplifier and filter and a translator driver with a slow (up to 100 Hz frequency response) correction applied to the piezoelectric translator (PZT). This loop serves as the acquisition aiding loop for the phase lock system.

The phase lock portion of the scheme is shown on the right. It consists of a filter, a 29.75 MHz quartz oscillator, a balanced mixer, a 250 kHz reference oscillator, a phase detector and filter, a current controller, and the power supply. The 30 MHz signal is mixed with 29.75 MHz, and the 250 kHz difference

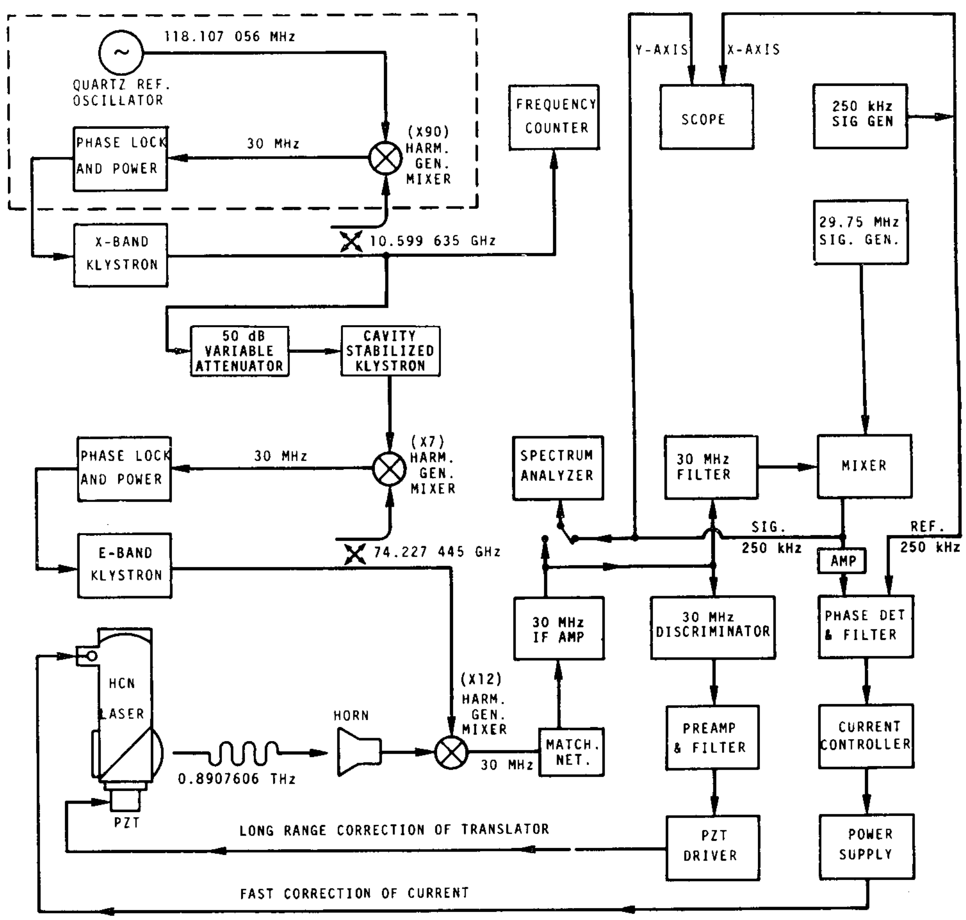

FIG. 2. — Block diagram of laser stabilization scheme.

signal goes to the phase detector and to the Y-axis of the scope. Both the 29.75 and 250 kHz oscillators are of sufficient quality to avoid contributing to the linewidth. The X-axis of the scope is driven by the phase detector reference.

The phase detector output is filtered and is fed as a correction signal to a pass tube in the series controller which controls the current through the laser. (The index of refraction of the laser discharge depends upon the current.) The upper response frequency of the open loop-controller laser combination [8] has been extended to 5 kHz and gives better performance than previously reported [7].

Figure 3 shows the beat note between the HCN laser and the reference when the laser is frequency locked, *A*, and when the laser is phase locked *B*. Figure 4 shows the beat note on an expanded scale and a Lissajous figure where the Y-axis is derived from the beat between the laser and microwave reference chain (down converted to 250 kHz) while the X-axis is driven by the phase detector reference oscillator. From this, we infer that the laser can be made to phase-track the reference quite closely.

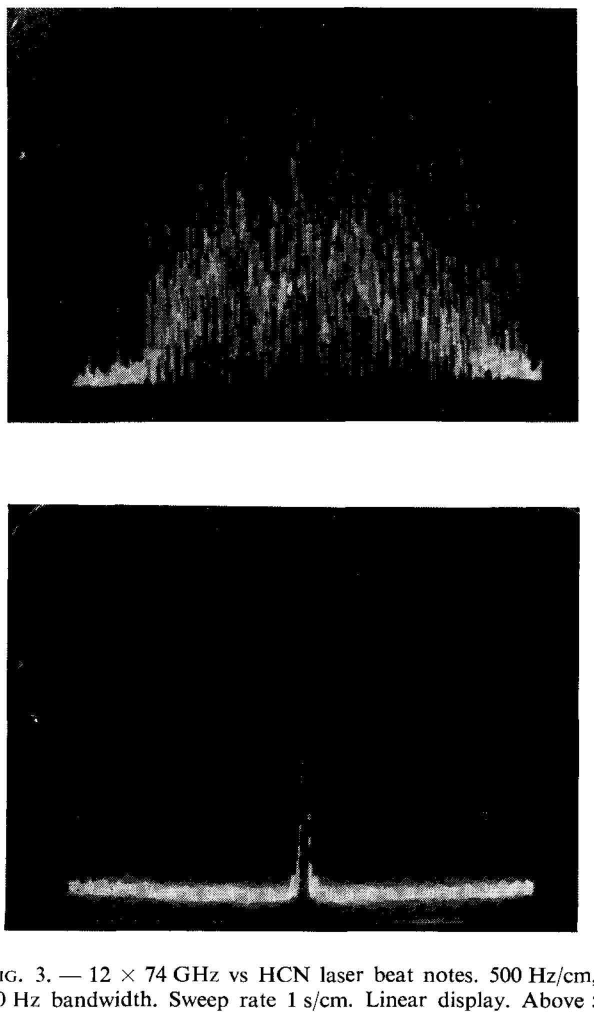

FIG. 3. — 12 × 74 GHz vs HCN laser beat notes. 500 Hz/cm, 30 Hz bandwidth. Sweep rate 1 s/cm. Linear display. Above : frequency lock, 10 mV/cm. Below : phase lock, 20 mV/cm. Oscilloscope trace corresponds to 10 cm.

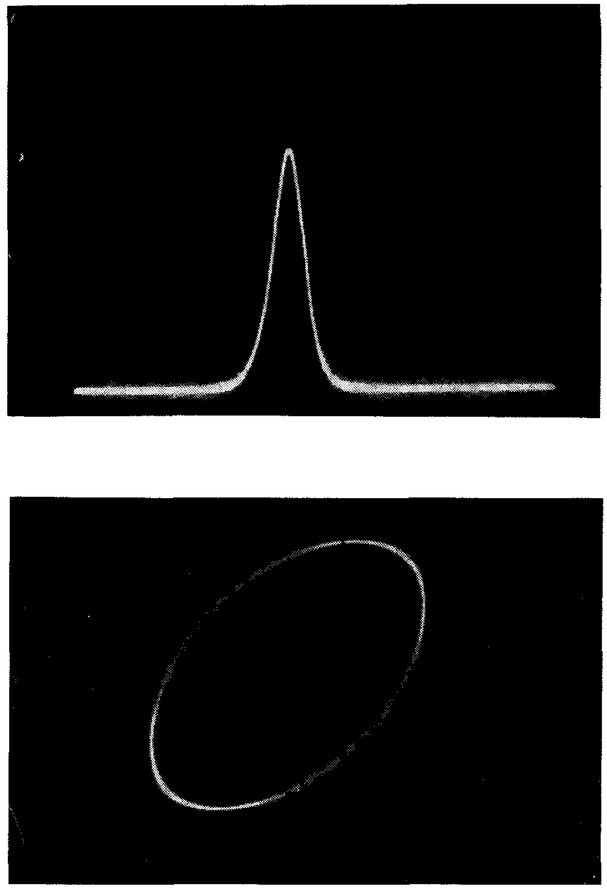

FIG. 4. — Top : RF power spectrum of beat between HCN laser and 12th harmonic of 74 GHz klystron with laser phase locked to 12th harmonic. Vertical scale is linear, dispersion is 20 Hz/cm, bandwidth 10 Hz and sweep rate 1 s/cm (30 MHz is center frequency). Below : Lissajous figure at 250 kHz where Y-axis is driven by down converted beat note and X-axis is driven by phase detector reference (10 s exposure).

3. **Josephson junction spectrum analyzer.** — A suitable reference frequency is needed to analyze the results discussed above. The straightforward approach would be the comparison of two phase locked HCN lasers, and to examine the beat note. Instead, we took advantage of the extremely high efficiency [3], [9] of the Josephson junction as a frequency multiplier (details of apparatus in Fig. 5). A block diagram for this experiment is shown in figure 6. Shown below the laser is a simplified version of the block diagram shown previously for phase locking the laser. Since the laser is tightly locked to the output of multiplier chain *A*, the display on spectrum analyzer *B* really compares the output of the two multiplier reference chains at 0.9 THz. Both multiplier chains are similar in that the same model cavity stabilized klystron is used as the X-band source at 10 GHz and each of these is injection locked to another X-band klystron which is phase locked to a quartz crystal oscillator for good long term stability (min. to min.). Multiplier chain *A* was shown in

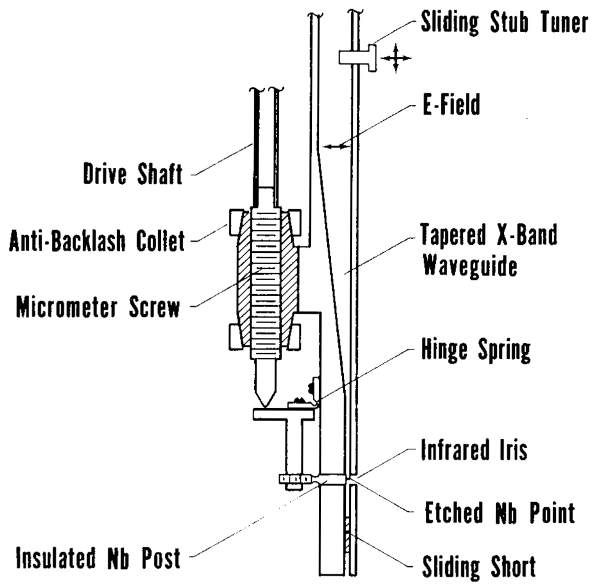

FIG. 5. — Line drawing of Josephson junction multiplier mixer indicating microwave tuning devices for impedance matching to junction. The device is immersed in superfluid helium. An Irtran II window (with an indium seal) transmits the laser radiation to the junction.

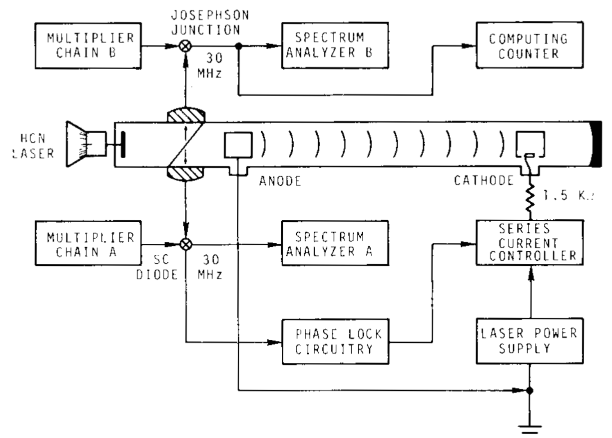

FIG. 6. — Block diagram of scheme for measuring linewidth and stability of a phase locked HCN laser with a Josephson junction.

figure 2. Multiplier chain B has a 5 MHz quartz oscillator and multiplier chain of the same design used in the NBS cesium beam frequency standard. The output of this chain (which is at X-band) injection locks the cavity stabilized klystron which is used to drive the Josephson junction. By increasing the level of the injecting signal, the reference tends to assume the noise spectra of the injecting signal. For example, figure 7 shows the beat note between the laser and the 92nd harmonic of klystron B with minimal attenuation between the injecting multiplier chain output cavity stabilized klystron B. This beat note is about 150 kHz wide. With a quartz filter (which eliminates some phase noise) between the 5 MHz driving oscillator and the multiplier chain, and

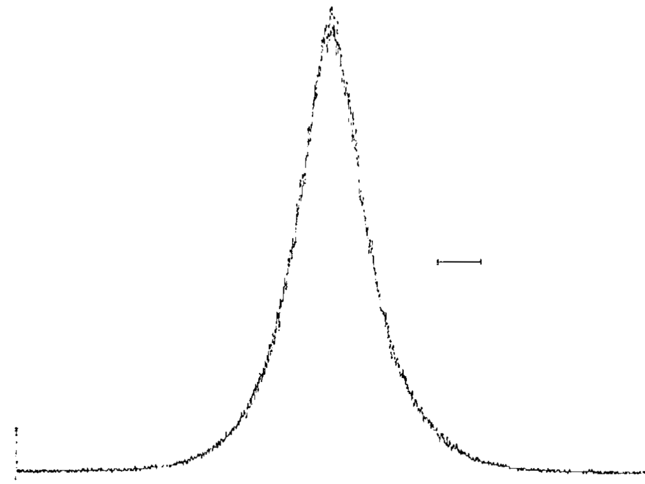

FIG. 7. — Beat at spectrum analyzer B between phase locked HCN laser and 93rd harmonic of the cavity stabilized source when injection locked to unfiltered multiplier. The bar indicates 100 kHz on frequency axis. Bandwidth was 3 kHz. This represents 200 sweeps at 0.2 ms per channel over 1 024 channels of a multichannel analyzer.

a much lower injection level (about 20 dB) this beat note drops below 10 kHz.

We conclude that the Josephson junction spectrum analysis provides an experimental means for checking the theoretical prediction [10] of the linewidth based upon examination of noise spectra at radio and microwave frequencies and helps to indicate the quality of the reference required to achieve the desired linewidth for the HCN laser.

4. **Cavity stabilized references.** — In order to predict the linewidth at the multiplied frequency (see Appendix) the reference sources must be analyzed at 10 GHz. The system with the best behavior is then selected. The cavity stabilized klystron scheme was chosen as the result of such a procedure [11]. In operation, the injecting level is decreased to the point where it barely holds lock and thus avoids significant deterioration of the (unlocked) cavity stabilized klystron spectrum. In order to predict the linewidth at high multiplication factors one must know the Fourier spectrum at the fundamental signal from these sources. The quantity of interest is the spectral density, S_{δ_ν}, of the frequency fluctuations, δ_ν, which is related to the measurement bandwidth, B, by

$$S_{\delta_\nu} = \frac{(\delta_\nu)^2}{B}.$$

The case of white frequency noise (multiplied linewidth goes as n^2) and flicker of frequency noise (multiplied linewidth varies as n) do not apply here since the Fourier spectrum is not described by these two simple models. Three different experimental methods were used to evaluate the actual Fourier spectral density of multiplier chain B. Since multiplier chain A is similar up to 10.6 GHz (a nearly identical cavity stabilized klystron which is injection locked by a klystron which is in turn phase locked to a quartz oscillator harmonic) we make the assumption that its spectrum is similar. Figure 8 shows the composite spectrum based on these three measurement techniques. The circles represent frequency domain data obtained by using a cavity discriminator method. For frequencies of 100 Hz and smaller, time domain measurements were made. The solid line is an estimate of S_{δ_ν} based on the time domain data and a second frequency domain method (spectral analysis of the correction voltage when one oscillator is phase-locked to the other). Since the amplitude of the spectral density varies by orders of magnitude and some of the points are widely separated and have substantial uncertainty, we have used an approximation for purposes of further calculation. This approximation

$$S_A = P_1 + e^{-\alpha\omega} \sum_{n=2}^{Np} P_n \omega^{n-2}$$

is evaluated (curve fitted) in a computer and the results indicated by the squares shown here. Details are in the Appendix.

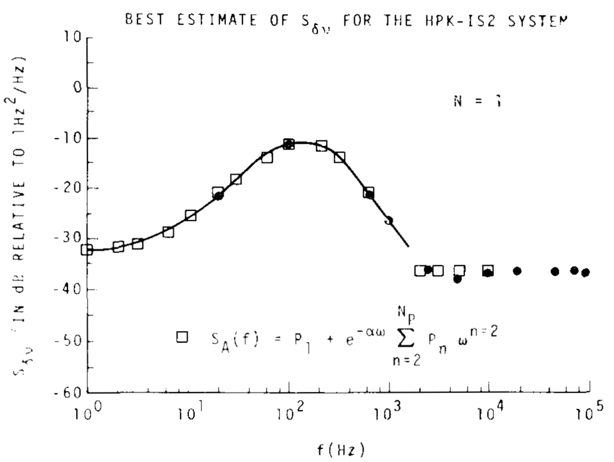

FIG. 8. — Best estimate of the Fourier spectrum of cavity stabilized klystron B with minimal injection signal. The solid line and points are the results of three different measurement techniques. The square blocks indicate the computer generated values for $S_A(f)$ which is fitted to the data.

The reference signal of interest from multiplier chain B is the 92nd harmonic of 9.6 GHz. The 92nd harmonic is generated in a Josephson junction which also serves as the mixer to combine the reference signal with the laser radiation.

5. **Theoretical and experimental linewidths.** — The theory and requisite computer programs for calculating linewidths at harmonic frequencies are described in the Appendix. The predicted linewidths are shown in figure 9 for $N = 100$ and $N = 1\,000$. The $N = 100$ result is the one relevant to our experiment. The

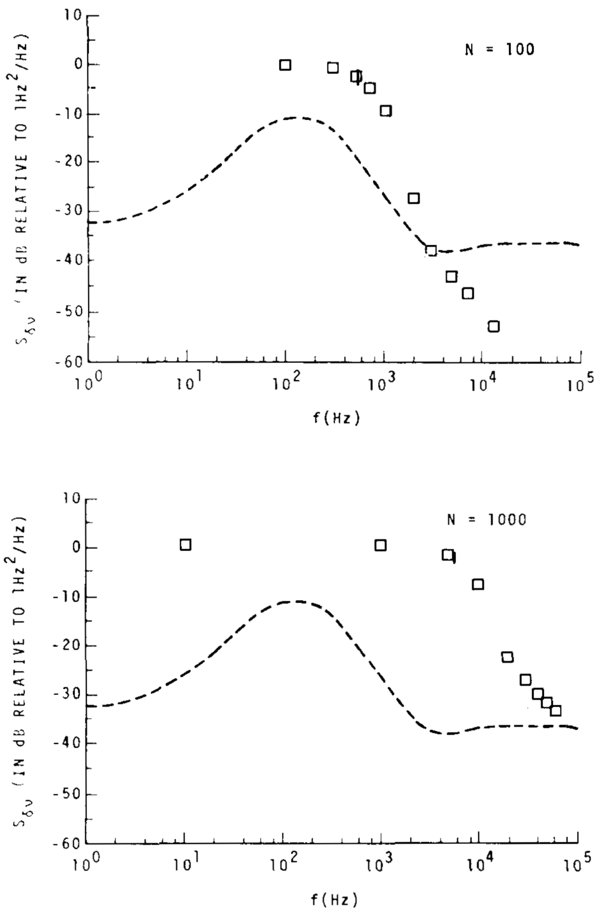

FIG. 9. — Computer evaluated predictions of the Fourier spectrum for multiplication factors of $N = 100$ and $N = 1\,000$ are indicated by squares. The -3 dB linewidth of the RF power spectrum is multiplied by N is equal to the — 3 dB width of the spectral density of the frequency fluctuations S_{δ_ν}. The short 3 dB bar gives linewidth values of 680 Hz and 6 800 Hz, respectively. The fundamental of figure 8 is repeated in the broken line.

calculated linewidth is about 700 Hz and is in agreement with an independent calculation by Halford [10]. (We note that at the fundamental the linewidth is 1 mHz.) If we naively assume that both cavity stabilized klystrons give rise to identical widths at 1 THz, the resulting theoretical beat note linewidth between the two sourves at 1 THz should be about $700 \times \sqrt{2}$ or 1 kHz.

In the experimental results that follow, the phase locked laser is always examined with the Josephson junction spectrum analyzer. Figures 10 and 11 show the experimental results. Figure 10 shows a comparison between the frequency and phase locked case where the results were recorded on a multichannel analyzer. The respective linewidths are 8 and 4 kHz. The averaging time is about a minute and part of the linewidth is possibly due to relative drift between the two cavity stabilized klystrons. Since a very weak injection lock was used, a residual drift of the klystrons may be expected.) Figure 11 shows frequency and phase

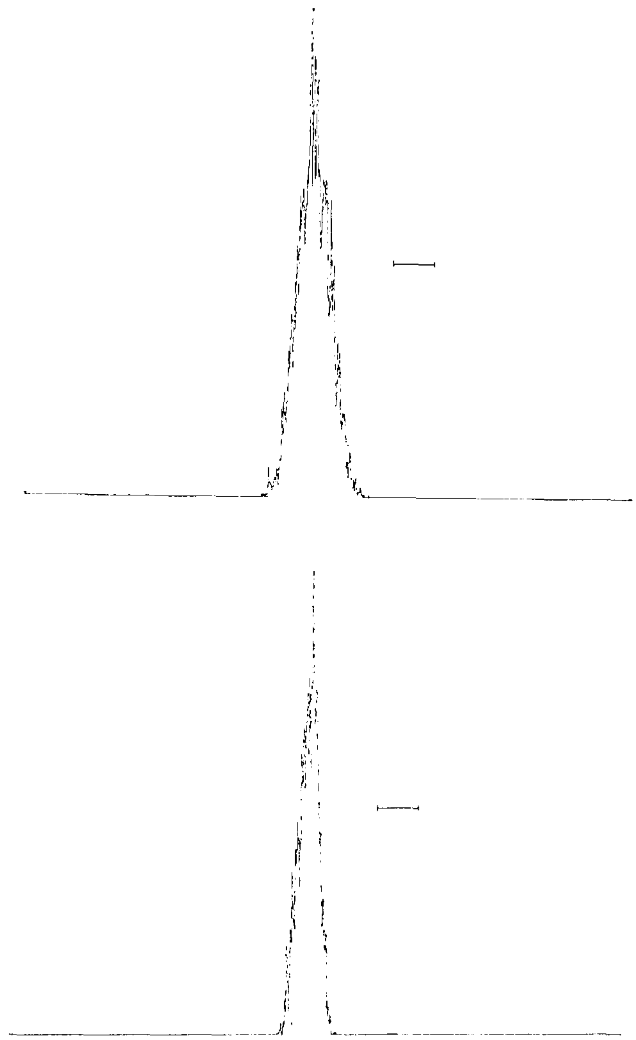

FIG. 10. — Multichannel analyzer display of RF power spectrum on analyzer B when laser is a) frequency locked and b) phase locked to chain A. Bar indicates 10 kHz and bandwidth is 300 Hz. 50 sweeps with 1 ms per channel over 1 024 channels comprise input.

locked beat notes over one second averaging with respective linewidths of 3.5 and 1.5 kHz. The latter is in reasonable agreement with the predicted 1 kHz value. An exact comparison is not possible at this time since the response function of the Josephson junction is not know with sufficient accuracy and the chains are not exactly equal.

In summary we make the following points :

1. The Josephson junction provides a relatively simple means for analyzing spectra of signals in the infrared due to its extremely high efficiency as a harmonic generator.

2. The agreement between experiment and theory is reasonably good for averaging times of 1 s. This encourages one to try the superconducting cavity [12] for a reference. Its multiplied output has been calculated [13] to have a linewidth of a few hertz at 1 THz, based on measurements in the gigahertz region.

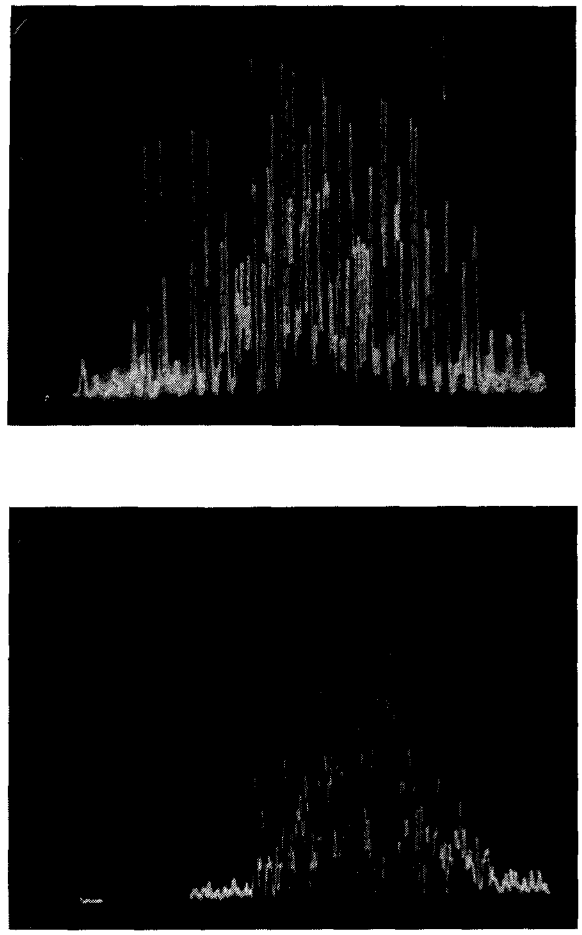

FIG. 11. — Oscilloscope display of single sweep of RF power spectrum on analyzer B of beat between laser and multiplier chain B when laser is (a) frequency locked and (b) phase locked to chain A. Dispersion is 500 Hz/cm, bandwidth is 100 Hz, and sweep rate is 0.1 s/cm.

3. In the interim, the techniques described for locking a laser provide long term stabilization of the HCN laser frequency while reducing the linewidth.

Acknowledgments. — We would like to acknowledge valuable discussions with Donald Halford on the reference problem and are grateful to Lyman Elwell for electronics support.

APPENDIX

We assume two signals, $V_L(t)$ and $V_x(t)$, with

$$V_L(t) = V_{L_0} \cos \omega_L t$$

and

$$V_x(t) = V_{x_0} \cos \left(\omega_x t + \varphi_x(t)\right),$$

with $\varphi_x(t)$ a zero-mean normally-distributed stationary phase noise modulation (no amplitude modulation is admitted). The signal $V_x(t)$ is subject to frequency multiplication by N, and mixed with the signal $V_L(t)$; the beat is $V_M(t)$. Assuming

$$\omega_0 \equiv N\omega_x - \omega_L \ll \omega_L ,$$

low-pass filtering gives the resulting signal $V_M(t)$

$$V_M(t) = A \cos(\omega_0 t + N\varphi_x(t)) .$$

Middleton has shown [14] that near the frequency ω_0, he spectral density of V_M, $S_M(f)$, is given by

$$S_M(f) = A_0^2 \int_0^\infty d\tau \cos(\omega - \omega_0) \tau \, e^{-[K_\varphi(0) - K_\varphi(\tau)]N^2} \tag{1}$$

in terms of the convariance of the phase noise:

$$K_\varphi(\tau) \equiv \overline{\varphi_x(t) \varphi_x(t + \tau)} .$$

The spectral density $S_{\delta_v}(f)$ (in Hz²/Hz) of the frequency fluctuations

$$\delta_v = \frac{\dot{\varphi}(t)}{2\pi} ,$$

$$\overline{\delta_v(t) \delta_v(t + \tau)} = \int_0^\infty df S_{\delta_v}(f) \cos \omega\tau$$

so that

$$K_\varphi(\tau) = (2\pi)^2 \int_0^\infty df \frac{S_{\delta_v}(f) \cos \omega\tau}{\omega^2} .$$

Thus

$$K_\varphi(0) - K_\varphi(\tau) = 2\pi \int_0^\infty d\omega \frac{S_{\delta_v}(f)(1 - \cos \omega\tau)}{\omega^2} . \tag{2}$$

To avoid the double integration involved in (1) and (2), and since the curve $S_{\delta_v}(f)$ is imprecisely known (widely separated points with substantial error), we have used an approximating form

$$S_A(f) = P_1 + e^{-\alpha\omega} \sum_{n=2}^{Np} P_n \omega^{n-2} . \tag{3}$$

Since $S_{\delta_v}(f)$ exhibits changes of orders of magnitude, we use an error estimate E over a set of frequencies $(f_1, ..., f_M)$ spanning the measured curve which measures the difference from unity of the ratios $S_A(f_m)/S_{\delta_v}(f_m)$:

$$E \equiv \frac{1}{M} \sum_{m=1}^{M} \left(\frac{S_A(f_m)}{S_{\delta_v}(f_m)} - 1 \right)^2 .$$

Thus

$$E = \sum_{m=1}^{M} w_m (S_{\delta_v}(f_m) - S_A(f_m))^2 , \tag{4}$$

with weights,

$$w_m = \frac{1}{M(S_{\delta_v}(f_m))^2} . \tag{5}$$

A program calculates optimum values $(P_1, ..., P_{Np})$ for a given α to minimize $E(P_1, ..., P_{Np}; \alpha)$. The program is repeated to minimize E over α.

Figure 8 shows the measured points (circles) and the interpolated curve $S_{\delta_v}(f)$, and the optimum fit $S_A(f)$ squares at the chosen frequency values for the optimum values. (The number of points $Np = 6$.)

$$\begin{aligned} \alpha &= 1.5 \quad \times 10^{-3} \\ p_m &= 2.087\,2 \times 10^{-4} \\ & 4.035\,7 \times 10^{-4} \\ & 2.470\,5 \times 10^{-7} \\ & 5.985\,5 \times 10^{-7} \\ & -2.985\,1 \times 10^{-10} \\ & 4.758\,8 \times 10^{-14} \end{aligned}$$

(The negative value for P_5 merely indicates that $\Omega(t)$ is not in fact composed of independent processes with spectra of type used in (3).)

Using the approximation (3), the function

$$R_M(\tau) = K_\varphi(0) - K_\varphi(\tau) \geqslant 0 ,$$

$$R_M(\tau) = 2\pi \left\{ \frac{\pi\tau}{2} P_1 + P_2 \left(\tau \tan^{-1} s - \frac{\alpha}{2} \ln(1 + S^2) \right) + \right.$$

$$+ P_3 \frac{1}{2} \ln(1 + s^2)$$

$$\left. + \sum_{n=4}^{Np} P_n \frac{\Gamma(n-3)}{\alpha^{n-3}} \left[1 - \frac{\cos(n-3) \tan^{-1} s}{(1 + s^2) \frac{n-3}{2}} \right] \right\} ,$$

where

$$s \equiv \frac{\tau}{\alpha} .$$

A second program calculates (1) by numerical integration for an input set of frequency differences

$$\Delta \equiv \omega - \omega_0 .$$

Fifty-point Gaussian integration is used on τ-intervals $\Delta\tau$ selected to prevent $N^2 R_M(\tau)$ from changing by more than a specified amount (e. g., 0.1) over the interval. For large values of Δ, where several oscillations of cos $\Delta\tau$ can occur over a interval $\Delta\tau$, integration is broken into periods to eliminate the error introduced in calculating trigonometric functions of large argument. The crossover point Δ_c may be varied to test the accuracy of the results.

Figure 9 shows the spectrum $S_M(f)$ (squares) for values of $N = 100$ and $N = 1\,000$, respectively. The short vertical bar marks the -3 dB point.

For $N = 1$, the half-width turns out to be about a millihertz.

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