SPECTRAL ANALYSIS OF A PHASE LOCKED LASER AT 891 GHz, AN APPLICATION OF JOSEPHSON JUNCTIONS IN THE FAR INFRARED

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Abstract. — We have used a Josephson junction to investigate the spectral purity of an HCN laser which is used in an infrared frequency synthesis chain. To obtain a narrower linewidth from the laser it has been phase locked to a multiplied microwave reference chain. A calculated value for this linewidth was based upon the measured noise spectrum of the microwave source and a theory due to Middleton. One can take advantage of the unique properties of the Josephson junction as a frequency multiplier and mixer for use in measuring this linewidth. The Josephson junction is driven by an X-band signal which is derived from a specially designed cavity stabilized klystron system of high spectral purity. The 92nd harmonic of the X-band signal is generated in the Josephson junction. In addition, the Josephson junction acts as a mixer of the harmonic signals and the 891 GHz output of the HCN laser. The 92nd harmonic beat signal is taken from the Josephson junction, amplified, and sent to a spectrum analyzer for frequency domain analysis. Details of the experiment, results, and relation to predicted linewidths are presented.

1. Introduction. — The motivation for developing and spectrum analyzing the phase locked HCN laser is for use in infrared frequency synthesis experiments; hence we give a brief explanation of that topic to set the work in perspective.

The objective of infrared frequency synthesis [1] is to measure or control the frequencies of lasers in the infrared. To measure accurately the frequency of a laser, \( v_M \), one must synthesize a frequency \( v_s \) which is close to the frequency to be measured. The difference between these two frequencies is an intermediate frequency, \( v_{IF} \), typically in the 10 to 100 MHz range. The synthesized frequency is:

\[
v_s = v_M \pm v_{IF} = /v_1 + m v_2 + n v_3
\]

where \( v_1 \) and \( v_2 \) are basis laser frequencies which have been determined by prior synthesis measurements, \( v_3 \) is a microwave frequency. The quantities \( l, m \) and \( n \) are harmonic numbers with \( m \) and \( n \) allowed both positive and negative values. The harmonic generation, as well as the mixing which produces the intermediate frequency to be measured, occurs in a suitable diode, typically a tungsten catwhisker on a nickel base [2] or a Josephson junction [3]. One then determines the frequency of the beat note center with a spectrum analyzer-tracking generator or, when the beat note signal quality permits one can use a frequency counter.

A typical infrared frequency synthesis chain is shown in figure 1; it was used in measuring the frequency of the 3.39 \( \mu \)m methane transition [4]. One can measure the frequency of the HCN laser at 337 GHz, use that to measure the water vapor laser at 28 \( \mu \)m, then extend the measurements to \( \mathrm{CO}_2 \) frequencies at 9.3 and 10.2 \( \mu \)m and on to methane at 3.39 \( \mu \)m. Chains similar to these have also been developed at NPL in England [5] and MIT in the US [6]. All these chains have the HCN laser as the lowest frequency basis laser and it is desirable to narrow this
2. HCN laser stabilization. — Figure 2 shows a block diagram of an improved scheme for stabilizing the laser. This basic reference chain is shown in the upper left. An X-band klystron is phase locked to some harmonic (say the 90th) of a quartz crystal oscillator with good long term stability. The output of the phase locked oscillator is fed through a variable attenuator and used to injection lock a cavity stabilized klystron which has good short term stability. An E-band klystron is then phase locked to the 7th harmonic of the second X-band source. The HCN laser is then compared with the 12th harmonic of the E-band source. The difference frequency at 30 MHz is used in the control apparatus. The control circuitry consists of two servo loops, one for frequency locking, the other for phase locking. The frequency lock loop consists of a frequency discriminator, preamplifier and filter and a translator driver with a slow (up to 100 Hz frequency response) correction applied to the piezoelectric translator (PZT). This loop serves as the acquisition aiding loop for the phase lock system.

The phase lock portion of the scheme is shown on the right. It consists of a filter, a 29.75 MHz quartz oscillator, a balanced mixer, a 250 kHz reference oscillator, a phase detector and filter, a current controller, and the power supply. The 30 MHz signal is mixed with 29.75 MHz, and the 250 kHz difference
signal goes to the phase detector and to the Y-axis of the scope. Both the 29.75 and 250 kHz oscillators are of sufficient quality to avoid contributing to the linewidth. The X-axis of the scope is driven by the phase detector reference.

The phase detector output is filtered and is fed as a correction signal to a pass tube in the series controller which controls the current through the laser. (The index of refraction of the laser discharge depends upon the current.) The upper response frequency of the open loop-controller laser combination [8] has been extended to 5 kHz and gives better performance than previously reported [7].

Figure 3 shows the beat note between the HCN laser and the reference when the laser is frequency locked, A, and when the laser is phase locked B. Figure 4 shows the beat note on an expanded scale and a Lissajous figure where the Y-axis is derived from the beat between the laser and microwave reference chain (down converted to 250 kHz) while the X-axis is driven by the phase detector reference oscillator. From this, we infer that the laser can be made to phase-track the reference quite closely.

**Fig. 3.** — 12 x 74 GHz vs HCN laser beat notes. 500 Hz/cm, 30 Hz bandwidth. Sweep rate 1 s/cm. Linear display. Above: frequency lock, 10 mV/cm. Below: phase lock, 20 mV/cm. Oscilloscope trace corresponds to 10 cm.

**Fig. 4.** — Top: RF power spectrum of beat between HCN laser and 12th harmonic of 74 GHz klystron with laser phase locked to 12th harmonic. Vertical scale is linear, dispersion is 20 Hz/cm, bandwidth 10 Hz and sweep rate 1 s/cm (30 MHz is center frequency). Below: Lissajous figure at 250 kHz where Y-axis is driven by down converted beat note and X-axis is driven by phase detector reference (10 s exposure).

3. **Josephson junction spectrum analyzer.** — A suitable reference frequency is needed to analyze the results discussed above. The straightforward approach would be the comparison of two phase locked HCN lasers, and to examine the beat note. Instead, we took advantage of the extremely high efficiency [3], [9] of the Josephson junction as a frequency multiplier (details of apparatus in Fig. 5). A block diagram for this experiment is shown in figure 6. Shown below the laser is a simplified version of the block diagram shown previously for phase locking the laser. Since the laser is tightly locked to the output of multiplier chain A, the display on spectrum analyzer B really compares the output of the two multiplier reference chains at 0.9 THz. Both multiplier chains are similar in that the same model cavity stabilized klystron is used as the X-band source at 10 GHz and each of these is injection locked to another X-band klystron which is phase locked to a quartz crystal oscillator for good long term stability (min. to min.). Multiplier chain A was shown in
figure 2. Multiplier chain $B$ has a 5 MHz quartz oscillator and multiplier chain of the same design used in the NBS cesium beam frequency standard. The output of this chain (which is at X-band) injection locks the cavity stabilized klystron which is used to drive the Josephson junction. By increasing the level of the injecting signal, the reference tends to assume the noise spectra of the injecting signal. For example, figure 7 shows the beat note between the laser and the 92nd harmonic of klystron $B$ with minimal attenuation between the injecting multiplier chain output cavity stabilized klystron $B$. This beat note is about 150 kHz wide. With a quartz filter (which eliminates some phase noise) between the 5 MHz driving oscillator and the multiplier chain, and a much lower injection level (about 20 dB) this beat note drops below 10 kHz.

We conclude that the Josephson junction spectrum analysis provides an experimental means for checking the theoretical prediction [10] of the linewidth based upon examination of noise spectra at radio and microwave frequencies and helps to indicate the quality of the reference required to achieve the desired linewidth for the HCN laser.

4. Cavity stabilized references. — In order to predict the linewidth at the multiplied frequency (see Appendix) the reference sources must be analyzed at 10 GHz. The system with the best behavior is then selected. The cavity stabilized klystron scheme was chosen as the result of such a procedure [11]. In operation, the injecting level is decreased to the point where it barely holds lock and thus avoids significant deterioration of the (unlocked) cavity stabilized klystron spectrum. In order to predict the linewidth at high multiplication factors one must know the Fourier spectrum at the fundamental signal from these sources. The quantity of interest is the spectral density, $S_{f_{n}}$, of the frequency fluctuations, $\delta_{f_{n}}$ which is related to the measurement bandwidth, $B$, by

$$S_{f_{n}} = \frac{\langle \delta_{f_{n}} \rangle^2}{B}.$$

The case of white frequency noise (multiplied linewidth goes as $n^2$) and flicker of frequency noise (multiplied linewidth varies as $n$) do not apply here since the Fourier spectrum is not described by these two simple models. Three different experimental methods were used to evaluate the actual Fourier spectral density of multiplier chain $B$. Since multiplier
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The reference signal of interest from multiplier chain $A$ is similar up to 10.6 GHz (a nearly identical cavity stabilized klystron which is injection locked by a klystron which is in turn phase locked to a quartz oscillator harmonic) we make the assumption that its spectrum is similar. Figure 8 shows the composite spectrum based on these three measurement techniques. The circles represent frequency domain data obtained by using a cavity discriminator method. For frequencies of 100 Hz and smaller, time domain measurements were made. The solid line is an estimate of $S_n$ based on the time domain data and a second frequency domain method (spectral analysis of the correction voltage when one oscillator is phase-locked to the other). Since the amplitude of the spectral density varies by orders of magnitude and some of the points are widely separated and have substantial uncertainty, we have used an approximation for purposes of further calculation. This approximation

$$S_n = P_1 + e^{\omega_{0p}} \sum_{n=2}^{N} P_n e^{\omega_{n-2}}$$

is evaluated (curve fitted) in a computer and the results indicated by the squares shown here. Details are in the Appendix.

The reference signal of interest from multiplier chain $B$ is the 92nd harmonic of 9.6 GHz. The 92nd harmonic is generated in a Josephson junction which also serves as the mixer to combine the reference signal with the laser radiation.

5. Theoretical and experimental linewidths. — The theory and requisite computer programs for calculating linewidths at harmonic frequencies are described in the Appendix. The predicted linewidths are shown in figure 9 for $N = 100$ and $N = 1000$. The $N = 100$ result is the one relevant to our experiment. The calculated linewidth is about 700 Hz and is in agreement with an independent calculation by Halford [10]. (We note that at the fundamental the linewidth is 1 mHz.) If we naively assume that both cavity stabilized klystrons give rise to identical widths at 1 THz, the resulting theoretical beat note linewidth between the two sources at 1 THz should be about $700 \times \sqrt{2}$ or 1 kHz.

In the experimental results that follow, the phase locked laser is always examined with the Josephson junction spectrum analyzer. Figures 10 and 11 show the experimental results. Figure 10 shows a comparison between the frequency and phase locked case where the results were recorded on a multichannel analyzer. The respective linewidths are 8 and 4 kHz. The averaging time is about a minute and part of the linewidth is possibly due to relative drift between the two cavity stabilized klystrons. Since a very weak injection lock was used, a residual drift of the klystrons may be expected.) Figure 11 shows frequency and phase

![Figure 8](image8.png)

**Fig. 8.** — Best estimate of the Fourier spectrum of cavity stabilized klystron $B$ with minimal injection signal. The solid line and points are the results of three different measurement techniques. The square blocks indicate the computer generated values for $S_n(f)$ which is fitted to the data.

![Figure 9](image9.png)

**Fig. 9.** — Computer evaluated predictions of the Fourier spectrum for multiplication factors of $N = 100$ and $N = 1000$ are indicated by squares. The —3 dB linewidth of the RF power spectrum is multiplied by $N$ is equal to the —3 dB width of the spectral density of the frequency fluctuations $S_n$. The short 3 dB bar gives linewidth values of 680 Hz and 6800 Hz, respectively. The fundamental of figure 8 is repeated in the broken line.
locked beat notes over one second averaging with respective linewidths of 3.5 and 1.5 kHz. The latter is in reasonable agreement with the predicted 1 kHz value. An exact comparison is not possible at this time since the response function of the Josephson junction is not know with sufficient accuracy and the chains are not exactly equal.

In summary we make the following points:

1. The Josephson junction provides a relatively simple means for analyzing spectra of signals in the infrared due to its extremely high efficiency as a harmonic generator.

2. The agreement between experiment and theory is reasonably good for averaging times of 1 s. This encourages one to try the superconducting cavity [12] for a reference. Its multiplied output has been calculated [13] to have a linewidth of a few hertz at 1 THz, based on measurements in the gigahertz region.

3. In the interim, the techniques described for locking a laser provide long term stabilization of the HCN laser frequency while reducing the linewidth.

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APPENDIX

We assume two signals, \( V_1(t) \) and \( V_x(t) \), with

\[
V_1(t) = V_{1o} \cos \omega_1 t
\]

and

\[
V_x(t) = V_{xo} \cos (\omega_x t + \varphi_x(t)),
\]

with \( \varphi_x(t) \) a zero-mean normally-distributed stationary phase noise modulation (no amplitude modulation is admitted). The signal \( V_x(t) \) is subject to frequency
multiplication by \( N \), and mixed with the signal \( V_L(t) \); the beat is \( V_M(t) \). Assuming
\[
\omega_0 \equiv N \omega_s - \omega_L \ll \omega_L,
\]
low-pass filtering gives the resulting signal \( V_M(t) \)
\[
V_M(t) = A \cos (\omega_0 t + N \phi_s(t)).
\]
Middleton has shown [14] that near the frequency \( \omega_0 \),
the spectral density of \( S_M(f) \), is given by
\[
S_M(f) = A_0^2 \int_0^\infty d\tau \cos (\omega - \omega_0) e^{-j(K_\phi(0) - K_\phi(\tau))N^2}
\]
in terms of the covariance of the phase noise :
\[
K_\phi(\tau) = \langle \phi_s(t) \phi_s(t + \tau) \rangle.
\]
The spectral density \( S_\delta(f) \) (in Hz²/Hz) of the frequency fluctuations
\[
\delta_\nu = \frac{\phi_s(t)}{2 \pi},
\]
\[
\delta_\nu(t) \delta_\nu(t + \tau) = \int_0^\infty df S_\delta(f) \cos \omega \tau
\]
so that
\[
K_\phi(\tau) = (2 \pi)^2 \int_0^\infty df S_\delta(f) \cos \omega \tau \frac{\omega^2}{\omega^2}
\]
Thus
\[
K_\phi(0) - K_\phi(\tau) = 2 \pi \int_0^\infty d\omega S_\delta(f) \left( 1 - \cos \omega \tau \right) \omega^2.
\]
To avoid the double integration involved in (1)
and (2), and since the curve \( S_\delta(f) \) is imprecisely
known (widely separated points with substantial
error), we have used an approximating form
\[
S_\delta(f) = P_1 + e^{-\alpha v} \sum_{n=2}^{N_p} P_n \omega^{-n/2}.
\]
A program calculates optimum values \((P_1, ..., P_Np)\)
for a given \( \alpha \) to minimize \( E(P_1, ..., P_Np; \alpha) \). The
program is repeated to minimize \( E \) over \( \alpha \).

Figure 8 shows the measured points (circles) and
the interpolated curve \( S_\delta(f) \), and the optimum fit
\( S_A(f) \) squares at the chosen frequency values for the
optimum values. (The number of points \( Np = 6 \).)

The spectral density
\[
S_\delta(f) \quad \text{(in Hz²/Hz)}
\]
\[
\text{of the frequency fluctuations}
\]
\[
(1)
\]
\[
(2)
\]
\[
(3)
\]
\[
(4)
\]
\[
(5)
\]
\[
A \equiv \omega - \omega_0.
\]

Fifty-point Gaussian integration is used on \( \tau \)-
intervals \( \Delta \tau \) selected to prevent \( N^2 \) \( R_M(\tau) \) from changing
by more than a specified amount (e. g., 0.1) over the
interval. For large values of \( A \), where several oscillations of \( \cos \Delta \tau \) can occur over a interval \( \Delta \tau \), integration
is broken into periods to eliminate the error introduced in calculating trigonometric functions of
large argument. The crossover point \( \Delta \alpha \) may be varied
to test the accuracy of the results.

Figure 9 shows the spectrum \( S_M(f) \) (squares) for
values of \( N = 100 \) and \( N = 1000 \), respectively. The short vertical bar marks the \(-3 \) dB point.
For \( N = 1 \), the half-width turns out to be about a millihertz.
References