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NOTE

COMMENTS ON ION MICROFIELD DISTRIBUTIONS AS USED IN PLASMA LINE BROADENING THEORIES*

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Abstract—It is shown that the use of an extended ion microfield function gives rise to improved line shapes in the transition region between the line center and the asymptotic wings.

IN MOST of the modern theories for spectral line broadening in plasmas,(1–3) the calculation of the line shape involves an average over all possible static ion fields. It can be shown(3) that the line shape, $I(\omega)$, is given by

$$I(\omega) = \int Q(\mathscr{E})J(\omega, \mathscr{E})\, \mathrm{d}^3\mathscr{E} \tag{1}$$

where $Q(\mathscr{E})$ is the probability of finding an ion field $\mathscr{E}$ at the radiating atom and $J(\omega, \mathscr{E})$ represents the line shape due to the electron–atom interactions in the presence of this field. Since the ion field is static and since, in the absence of external fields, there is no preferred direction in space, both Q and J are spherically symmetric functions of the ion field. We therefore write

$$I(\omega) = \int_0^\infty P(\mathscr{E})J(\omega, \mathscr{E})\, \mathrm{d}\mathscr{E} \tag{2}$$

where

$$P(\mathscr{E}) = 4\pi\mathscr{E}^2 Q(\mathscr{E}). \tag{3}$$

The microfield function, $P(\mathscr{E})$, has been calculated by statistical methods,(4–6) and it has been tabulated for relative field strengths, ε (i.e. $\mathscr{E}$ divided by the Holtsmark normal field strength(4–6)), ranging from zero to ten. It is generally felt that this range is sufficient for calculations near the center of a line. For calculations in the line wings, the asymptotic

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Holtsmark function

$$P_H(\varepsilon) = 1.496/\varepsilon^{5/2} \tag{4}$$

is used(7) for $\varepsilon > 10$. The purpose of this note is to show that such a procedure is not valid for calculations in the transition region between the line center and the line wings. In this region it is necessary to use more accurate values of $P(\varepsilon)$ for $\varepsilon \geq 10$. Extended tabulations of the microfield functions and their connection with the asymptotic Holtsmark function are given in Ref. 8; asymptotic expressions which may be used for $\varepsilon \geq 10$ are given in Ref. 6.

In order to illustrate the importance of integrating beyond $\varepsilon = 10$, the profile of the Lyman alpha line from hydrogen has been calculated, using the relaxation theory(3) for $J(\omega, \mathscr{E})$ (corrected for electron correlations(9)), with a microfield function which is tabulated to $\varepsilon = 30$. In Fig. 1, this profile is compared with the theoretical results of the impact theory(7) and with experimental observations.(10) The line shape functions are divided by the asymptotic Holtsmark profile(7) and plotted against $\Delta\lambda$, the wavelength separation from the center of the natural line. The relaxation theory profile was obtained by averaging the intensities of the red and blue wings since this theory produces asymmetric line shapes. It should be noted that the profiles obtained by the impact theory were calculated with improved Φ-matrix elements (see Ref. 7), resulting in a closer agreement with the relaxation theory than that reported in previous comparisons.(3,9)

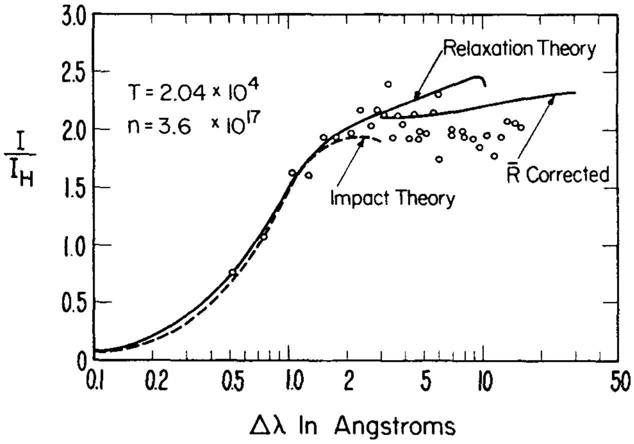

FIG. 1. A comparison of Ly-α line profiles at a temperature of 2.04×10^4 with a density of 3.6×10^{17}.

The relaxation theory profile shows a sharp drop at 10 Å which results from stopping the integration at $\varepsilon = 30$. If we had integrated out to $\varepsilon = 60$, for example, this drop would be even sharper and would occur at 20 Å. If the integration were stopped at $\varepsilon = 10$, a smoother drop would occur near 3.5 Å and the curve would closely follow the impact theory profile (which used the Mozer–Baranger microfield(4) to $\varepsilon = 10$). The drop is smoother near the line center because this region of the profile is dominated by electron collision broadening and the effect of the static ion field is less significant.

From Fig. 1 it is obvious that the present version of the relaxation theory is not valid in the far wings; nevertheless, the extended microfield integration permits a more reasonable connection with the asymptotic wing profile ($\bar{R}$-corrected) obtained by GRIEM.(7)

It may be argued that one has only to use the Holtsmark function (equation (4)) for $\varepsilon \geq 10$ in order to extend the microfield integration. This procedure has been tested on the Ly–α line, and it is found that a 5 per cent increase in intensity occurs at 3.5 Å, as shown in Fig. 2. For high series members, this effect will be less pronounced because the large number of Stark components smears out the discontinuities.(11)

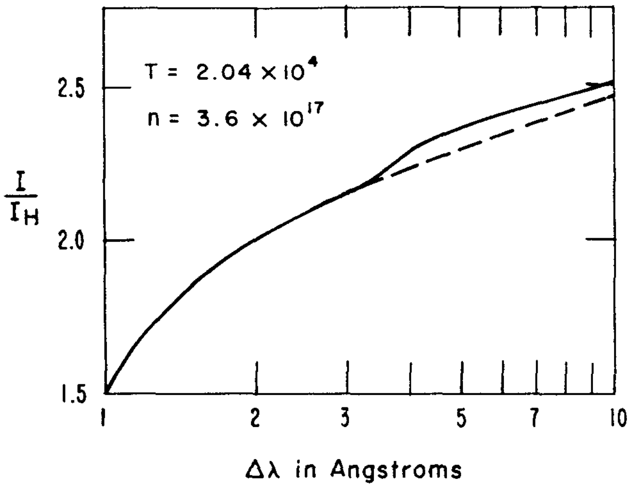

FIG. 2. A comparison of the line profile obtained by using $P_H(\varepsilon)$ for $\varepsilon > 10$ (solid line) with the results obtained using an extended microfield function (dotted line).

While we have given only a single example of the use of an extended microfield, similar tests have been made for other cases with the same general results. We thus conclude that the microfield average in equation (2) should be extended beyond $\varepsilon = 10$ when calculating line profiles in the transition region (between the line center and the asymptotic wing). The use of $P_H(\varepsilon)$ for $\varepsilon > 10$ will produce a jump in intensity at some point in each Stark component; hence, for low series members, more accurate values of $P(\varepsilon)$ for $\varepsilon > 10$ are required.

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