tee. This reduces the effect of reflections in the test circuit on the power delivered to the stabilization circuit.

The cavity of one of these circuits might be used as a device for measurement of very small thicknesses. A small distortion in the shape of the cavity produces a measurable change in the beat frequency of two stabilized oscillators, and cavities that are very sensitive in this respect could easily be designed. The wavemeters used in the experimental systems changed frequency by 100 c.p.s. for a change in the position of the end plate of 10 angstrom units.

Oscillators having such narrow-band output frequencies as these could be used as carriers for voice communication in narrow frequency channels. If the carrier frequency contains deviations from a discrete frequency of less than 100 c.p.s., as appears to be possible, a frequency modulation producing a deviation of 100 kc. would give a transmitted signal-to-noise ratio of 60 db. A tremendous number of channels wide enough for such signals could be created in a microwave band only a few per cent in width. Another paper by the author will describe a special duplex communication system built up around these frequency-stabilization systems.

These oscillators would also be useful in fundamental research on the interactions between gases and high-frequency fields. There are several gases having quantum mechanical transitions giving rise to resonance absorption in the microwave region. The stabilized oscillators make possible investigation of the details of the structure of these absorption spectra, and ultimately one might use one such absorption line to obtain, with a stabilized oscillator, an absolute standard of frequency.

Synchronization of Oscillators*

ROBERT D. HUNTOON†, SENIOR MEMBER, I.R.E., AND A. WEISS†

Summary—A theory is presented which predicts the behavior of any self-limiting oscillator in the presence of an injected sinusoidal voltage or current of small but constant magnitude. The internal mechanism responsible for synchronization is not needed, and the theory is thus applicable to any source of alternating current. Experimental verification of the theory is presented for the case of a low-power Hartley oscillator operating at 11.5 Mc.

The theory is extended to include the mutual synchronization of two oscillators of arbitrary properties, and applied to a number of examples to indicate briefly the properties of a synchronized oscillator when used as (a) a linear voltmeter for small voltages, (b) a field-intensity meter, (c) a linear a.m. demodulator for small signals, (d) an f.m. demodulator, and (e) a synchronous amplifier-limiter. The use of a synchronized oscillator is of particular interest because microwave generators can be used in addition to the more conventional triode oscillators.

I. Introduction

THE EARLY EXPERIMENTS of Vincent,[1] followed by Appleton's[2] theoretical treatment, have led to a considerable interest in possible practical applications of the synchronization of oscillators.[3] Since the publication of these early papers, there has been a continually growing literature on the subject, with attention now primarily centered on (a) the use of an oscillator as a synchronous-amplifier limiter for f.m. reception, and (b) the use of a chain of synchronous oscillators to drive a linear accelerator for the production of high-energy atomic particles. There are, of course, numerous other applications, some of which are discussed in the light of the theory which is the subject of this paper.

Following Appleton, theoretical treatments of oscillator synchronization have been concerned with the internal mechanism within a triode oscillator which accounts for synchronization. The phenomenon of synchronization with a disturbance impressed from an external source is not limited to triode oscillators. Rather, any source of alternating e.m.f. whose frequency and amplitude are continuous functions of the load impedance attached to it (the magnetron, for example) will exhibit similar behavior. It should thus be possible to discuss certain general features of synchronization without reference to the internal mechanism which accounts for it. The theory so derived will be generally applicable to all types of oscillators.

In a recent paper Adler[4] has developed a differential equation whose solution accounts for many of the observed phenomena of synchronization. Again, the triode oscillator mechanism has been the basis of the discussion. However, the scheme used by Adler can be extended in a manner which does not involve the particular generator. The result is a differential equation similar to his but more general. In addition, amplitude behavior as well as frequency behavior can be included.

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† Ordnance Development Division, National Bureau of Standards, Washington, D. C.

[1] J. H. Vincent, "On some experiments in which two neighboring maintained oscillatory circuits affect a resonating circuit," *Proc. Roy. Soc.*, vol. 32, part 2, pp. 84–91; 1919–1920.

[2] E. V. Appleton, "The automatic synchronization of triode oscillators," *Proc. Camb. Phil. Soc.*, vol. 21, pp. 231–248; 1922–1923.

[3] The term "oscillator" as used here means a source of harmonic vibration whose steady-state amplitude is limited to a finite value by some internal nonlinear characteristic.

[4] Robert Adler, "A study of locking phenomena in oscillators," Proc. I.R.E., vol. 34, pp. 351–357; June, 1946.

The performance of the oscillator is specified in terms of a set of compliance coefficients which show how amplitude and frequency depend upon the load impedance. The values of the coefficients are not derived here but are assumed to be given as constants of the problem. They may be derived theoretically or measured for the particular oscillator.

The injected voltage is considered as equivalent to the IZ drop on a fictitious increment in the load impedance. The oscillator's frequency and amplitude shift in accordance with its compliance coefficients and the magnitude and phase of the incremental load impedance. If the disturbance due to the injected voltage is small and its frequency is close to that of the oscillator, replacing the actual voltage by a fictitious impedance of varying phase and magnitude is valid and the synchronization behavior can be calculated.

II. Synchronization by an Impressed Voltage

In the discussion to follow, complex quantities will be represented by boldface italic characters; quantities not so designated will denote absolute magnitudes. The factor $e^{j\omega t}$ will usually be omitted.

A. Compliance Coefficients

Let Fig. 1 represent an energy source of the type which converts d.c. energy to a.c. energy, such as a typical triode oscillator or magnetron. We will be interested in two pairs of terminals. Those marked E–E are the output terminals of the device for delivering a.c.

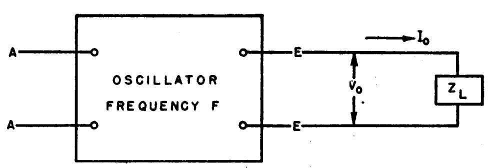

Fig. 1—Oscillator for synchronization studies.

power to a load impedance Z_L. The terminals A–A represent any pair of terminals which give a d.c. or a.c. indication of the amplitude of oscillation, such as grid bias or d.c. plate current.

Assume that there are also available, when necessary, instruments which indicate either the voltage V_0 across the load or current I_0 through it. Let $\boldsymbol{V_0}$ and $\boldsymbol{I_0}$ be the initial values of these quantities when the oscillator is feeding its load circuit. Similarly, let F represent the frequency of the oscillator, and F_0 its undisturbed value. When a small impedance $\boldsymbol{z}$ is added to the load, the frequency and amplitude change. The compliance coefficients are defined in terms of these changes; thus,

$$A_r = \left.\frac{\partial A}{\partial r}\right|_{z=0} \qquad A_x = -\left.\frac{\partial A}{\partial x}\right|_{z=0} \tag{1}$$

$$F_r = \left.\frac{\partial F}{\partial r}\right|_{z=0} \qquad F_x = -\left.\frac{\partial F}{\partial x}\right|_{z=0} \tag{2}$$

where

$$\boldsymbol{z} = r + jx. \tag{3}$$

The negative sign in A_x and F_x is incorporated here for reasons of symmetry in later expressions.

A and F are expanded in a Taylor series about A_0 and F_0, keeping only first-order terms. This gives

$$A - A_0 = rA_r - xA_x \tag{4}$$

$$F - F_0 = rF_r - xF_x. \tag{5}$$

Complex compliance coefficients for amplitude, $\boldsymbol{C_A}$, and frequency, $\boldsymbol{C_F}$, will be needed. These are

$$\boldsymbol{C_A} = C_A e^{j\alpha} = A_r + jA_x = \sqrt{A_r^2 + A_x^2}\, e^{j\alpha} \tag{6}$$

and

$$\boldsymbol{C_F} = C_F e^{j\beta} = F_r + jF_x = \sqrt{F_r^2 + F_x^2}\, e^{j\beta}. \tag{7}$$

B. Synchronization Equation

Let a small voltage be induced in the load circuit from an outside source. Assume the voltage is small enough so that the change in I can be neglected and we can, with sufficient accuracy, represent I by its initial value I_0. We replace the induced voltage v by a small impedance z where

$$\boldsymbol{z} = \frac{v}{I_0} e^{j\phi}. \tag{8}$$

We may thus write, with the aid of (4) and (5) (keeping only real parts),

$$A - A_0 = \boldsymbol{C_A z} = \frac{C_A v}{I_0} \cos(\phi + \alpha) \tag{9}$$

and

$$F - F_0 = \boldsymbol{C_F z} = \frac{C_F v}{I_0} \cos(\phi + \beta) \tag{10}$$

where

$$\tan\alpha = \frac{A_x}{A_r}, \qquad \tan\beta = \frac{F_x}{F_r}.$$

If the injected voltage v has the frequency F' and the instantaneous frequency of the oscillator is F, we can write

$$\frac{1}{2\pi}\frac{d\phi}{dt} = F' - F = (F' - F_0) - (F - F_0) \tag{11}$$

and

$$\boldsymbol{z} = \frac{v}{I_0} e^{j2\pi(F'-F)t} = \frac{v}{I_0} e^{j\phi(t)}. \tag{12}$$

If $F' - F$ is not too large, the oscillator will follow the impedance changes as shown by (9) and (10). In particular, (10) gives

$$\frac{1}{2\pi}\frac{d\phi}{dt} = (F' - F_0) \mp \frac{C_F v}{I_0}\cos(\phi + \beta), \qquad (13)$$

a differential equation similar to that derived by Adler which shows how the beat frequency, if any, varies with time.

Putting

$$F' - F_0 = f$$

and

$$\frac{C_F v}{I_0} = Kv$$

into (13) yields

$$\frac{1}{2\pi}\frac{d\phi}{dt} = f - Kv\cos(\phi + \beta). \qquad (14)$$

It is immediately evident from (14) that the solution $\phi(t)$ is of a complicated periodic form when

$$f^2 > K^2 v^2 \qquad (15)$$

and reduces exponentially to a steady value of ϕ when

$$f^2 < K^2 v^2. \qquad (16)$$

Condition (16) corresponds to synchronization between the injected voltage and the oscillator current at a fixed phase angle ϕ. Since we are interested primarily in synchronization, the solution of (14) subject to (16) is needed. It is

$$\frac{\cos\psi - \cos(\phi + \beta)}{1 - \cos(\phi + \beta - \psi)} = \text{const. } e^{-2\pi t\sqrt{K^2 v^2 - f^2}} \qquad (17)$$

where

$$\cos\psi = \frac{f}{Kv}\cdot$$

The steady-state value of ϕ for large t is given by

$$\cos(\phi + \beta) = \frac{f}{Kv}\cdot \qquad (18)$$

The equilibrium value is approached in such a manner that the time constant is approximately

$$\tau \cong \frac{1}{2\pi\sqrt{K^2 v^2 - f^2}} = \frac{1}{2\pi Kv\sin\psi}\cdot \qquad (19)$$

There are two values of $(\phi+\beta)$ which satisfy (18). One corresponds to stable equilibrium; the other to unstable equilibrium. From (14),

$$\frac{1}{2\pi}\frac{d}{d\phi}\left(\frac{d\phi}{dt}\right) = Kv\sin(\phi + \beta). \qquad (20)$$

For stability,

$$\frac{d}{d\phi}\left(\frac{d\phi}{dt}\right)$$

must be negative. Thus only values of $(\phi+\beta)$ such that $\sin(\phi+\beta)$ is negative lead to stable synchronization.

Equation (18) shows that synchronization can be obtained over a range of f such that

$$-Kv < f < Kv,$$

or, over a band of frequencies,

$$\Delta f = 2Kv. \qquad (21)$$

C. Amplitude Changes

The quantity $a = A - A_0$ expresses the change of some convenient amplitude parameter, such as plate current, in the presence of an injected voltage. It is evident from (9) and (10) that a and f are functionally related through the parameter ϕ. By defining new quantities

$$\begin{aligned}\delta &= (\phi + \beta)\\ \rho &= (\alpha - \beta)\end{aligned} \qquad (22)$$

we can write (9) and (10) in terms of dimensionless variables U, W,

$$U = \frac{fI_0}{C_F v} = \cos\delta \qquad (23)$$

$$W = \frac{aI_0}{C_A v} = \cos(\delta + \rho), \qquad (24)$$

from which it is evident that the form of the functional relation between a and f is independent of I_0, v, C_A, and C_F for small disturbances. Elimination of δ in (23) and (24) leads to the equation for an ellipse in the U, W plane, which degenerates to a line when $\rho=0$ or π and into a circle when $\rho = \pm\pi/2$.

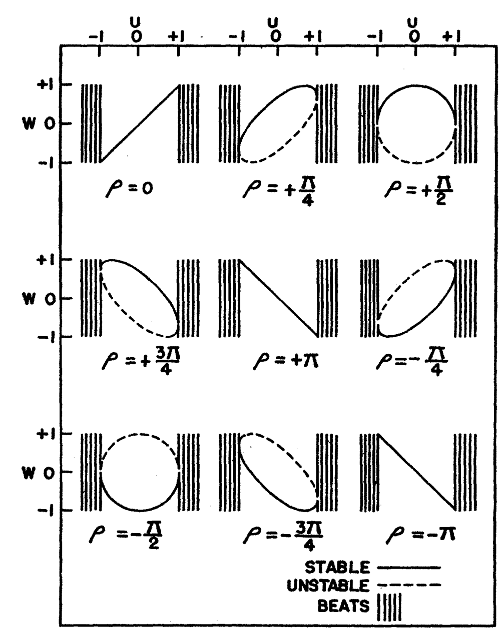

Fig. 2—Forms of the U–W curve in the region of synchronization.

Fig. 2 shows the U–W curves for several typical values of ρ. In most cases the frequency of an oscillator depends more upon the reactance of the load than upon its resistance. Thus ρ will generally be nearly $-\pi/2$ and the U–W curve almost a semicircle. In the figure the broken line shows the condition of unstable equilibrium, the solid line shows stable equilibrium, and the vertical lines indicate the region of beats outside the synchronization band.

It is important to note that, no matter what the value of ρ, the maximum absolute value of a is the same and is given by

$$a_{\max} = \frac{C_A v}{I_0}. \tag{25}$$

Thus, if the frequency of the injected voltage is swept across the synchronization band of the oscillator, there will be a pulse of voltage or current (depending upon the quantity represented by a) whose peak value is independent of ρ.

From (21), (23), (24), and (25), we see that

$$\frac{\Delta f}{a_{\max}} = \frac{2C_F}{C_A}, \tag{26}$$

which shows that the synchronization bandwidth is proportional to $a_{\max}$, or, from (25), to the injected voltage v. It is often convenient to use $a_{\max}$ as a measure of v without measuring v. The bandwidth of synchronization can then be predicted directly from (26).

III. Experimental Measurements

In order to check the foregoing theory, experimental measurements were made on a small Hartley oscillator operating at 11.5 Mc. R.f. voltage for injection was supplied by a push-pull power oscillator operating at ten times the plate voltage of the small oscillator and very loosely coupled to it inductively.

Fig. 3 is a circuit diagram of the test oscillator showing the method of voltage injection and a diode for measuring r.f. plate swing. It will be noted that the plate coil has been used for the load Z_L and that the synchronizing voltage is induced in it.

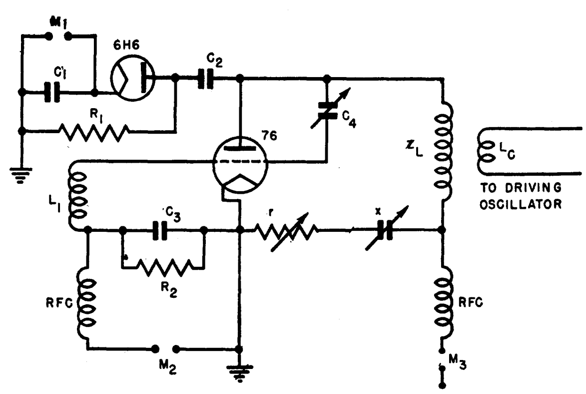

Fig. 3—Circuit diagram of the test oscillator.

Z_L	3.9 μh.	C_1	0.001 μfd.
L_1	5.5 μh.	C_2	0.0001 μfd.
L_C	coupling coil	C_3	0.00007 μfd.
R_1	1.0 megohm	C_4	0.0002 μfd.
R_2	15,000 ohms		r = series resistance
M_1	radio-frequency plate-voltage meter		x = series reactance
M_2	d.c. grid-voltage meter		RFC = 2.5 mh.
M_3	plate-current meter		

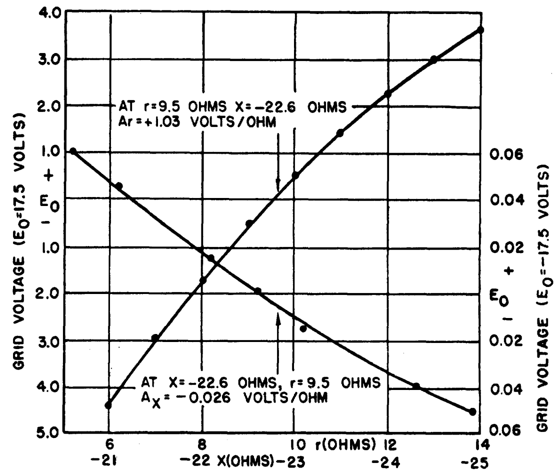

Fig. 4—Experimental curves for evaluation of A_r and A_x. (Use left ordinates for A_r, right ordinates for A_x.)

The compliance coefficients were measured by inserting capacitors x and resistors r in series with the plate tank coil. To allow measurement on both sides of the operating point, this point was specified to be $r = 9.5$ ohms, $x = -22.6$ ohms.

Fig. 4 shows the experimental curves from which A_r and A_x can be obtained. From them we observe that the compliance coefficient $\boldsymbol{C_A}$ has the value

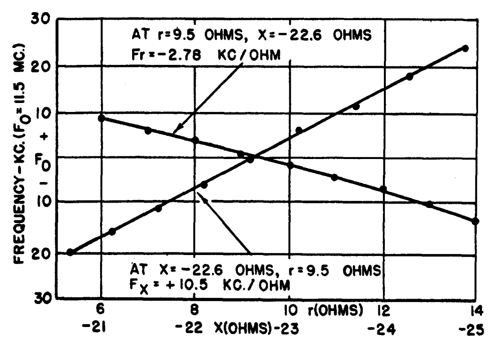

Fig. 5—Experimental curves for evaluation of F_r and F_x.

$$C_A = 1.03 \text{ volts/ohm}$$
$$\alpha = -1.5 \text{ degrees.}$$

Fig. 5 shows similar curves for the evaluation of $\boldsymbol{C_F}$. The appropriate values are

$$C_F = 10.8 \text{ kc./ohm}$$
$$\beta = +105 \text{ degrees.}$$

Calculation of the expected bandwidth of synchronization from these values gives

$$\frac{\Delta f}{a_{\max}} = \frac{2C_F}{C_A} = 21 \text{ kc./volt.}$$

Fig. 6 shows the experimental curve of bandwidth of synchronization as a function of $a_{\max}$. The slope of the curve at the origin is 20.6 kc./volt, in good agreement with the expected value. Note also that the curve is linear over the range of voltages used.

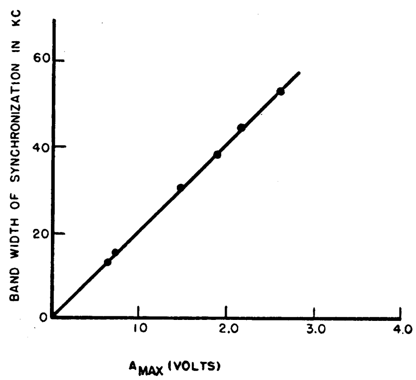

Fig. 6—Experimental determination of bandwidth of synchronization in terms of injected voltage as measured by $a_{\max}$. The slope of the curve is 20.6 kc./volt.

Fig. 7 shows the result of an experimental measurement of the relation between $a_{\max}$ and v. We note again that the relation is linear over the range investigated.

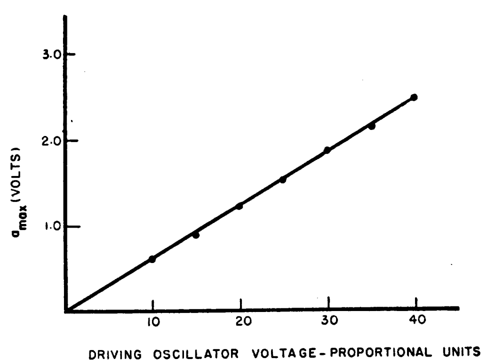

Fig. 7—Relation between $a_{\max}$ and injected voltage.

For this oscillator $\rho = -106.5$ degrees, and the U–W curve should be nearly a semicircle. Fig. 8 shows the exact form of the U–W curve for $\rho = -106.5$ degrees (solid line) and the measured curve when sweeping the power oscillator from high to low frequency across the band (solid dots). To check for possible hysteresis the curve was measured again, sweeping from low to high frequency, with results shown by crosses. The injected signal was increased from $a_{\max} = 0.92$ volts to $a_{\max} = 4.9$ volts and the curves were repeated to observe the effect of a large signal. Results are given by triangles (low to high frequency) and circles (high to low frequency). There is no evidence of hysteresis, although its presence has been mentioned in Appleton's studies.

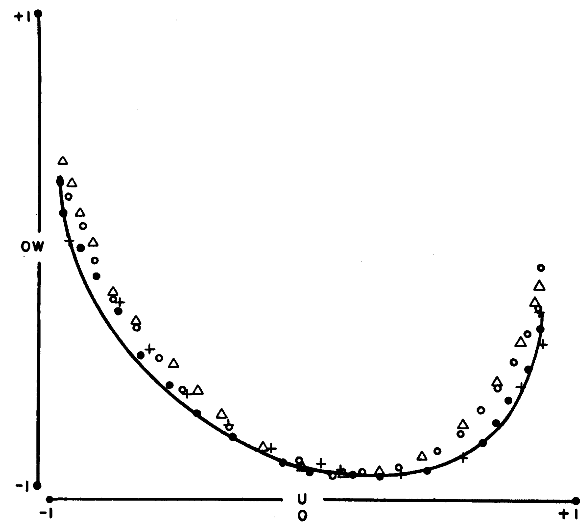

Fig. 8—Experimental and theoretical U–W curves for the test oscillator. Solid line: theoretical curve for $\rho = -106.5°$).

$a_{\max}$ 0.92 volts: Solid dots: experimental values sweeping from high to low frequency. Crosses: experimental values sweeping from low to high frequency.

$a_{\max}$ 4.9 volts: Circles: high to low frequency. Triangles: low to high frequency.

IV. Mutual Synchronization of Two Oscillators

Consider two oscillators of the form shown in Fig. 1 and let them be coupled by a mutual impedance

$$\boldsymbol{Z}_{12} = Z_{12}e^{j\phi_{12}}. \tag{27}$$

Let the two systems to be identified by subscripts 1 and 2. The coupling is assumed to be arranged so that the coupled voltages are induced in the load impedances $\boldsymbol{Z}_L$ of each system.

Both Z_{12} and ϕ_{12} will, in general, be functions of frequency. To simplify the present discussion, we assume that this dependence can be neglected over the narrow range of frequencies covered by the synchronization band.

Since we are interested only in synchronization we assume that both oscillators are synchronized at frequency F, and that their undisturbed frequencies are F_{01} and F_{02}, respectively.

In order to specify phases we refer all phases to the current $\boldsymbol{I}_{01}$ in the load of oscillator 1. We will seek the value of the phase angle θ_{12} between the currents $\boldsymbol{I}_{01}$ and $\boldsymbol{I}_{02}$. We write (omitting the term $e^{j2\pi Ft}$)

$$\begin{aligned} \boldsymbol{I}_{01} &= I_{01}e^{j0} \\ \boldsymbol{I}_{02} &= I_{02}e^{j\theta_{12}} \\ \boldsymbol{v}_1 &= v_1e^{j\phi_1} \\ \boldsymbol{v}_2 &= v_2e^{j(\theta_{12}+\phi_2)} \\ \boldsymbol{Z}_{12} &= Z_{12}e^{j\phi_{12}}. \end{aligned} \tag{28}$$

Now,

$$v_1 = I_{02}\boldsymbol{Z}_{12} = I_{02}Z_{12}e^{j(\theta_{12}+\phi_{12})}, \quad (29)$$

whence

$$\begin{aligned} v_1 &= I_{02}Z_{12} \\ \theta_{12} + \phi_{12} &= \phi_1 + 2n\pi. \end{aligned} \quad (30)$$

Also,

$$v_2 = I_{01}\boldsymbol{Z}_{12} = I_{01}Z_{12}e^{j\phi_{12}}, \quad (31)$$

whence

$$\begin{aligned} v_2 &= I_{01}Z_{12} \\ \phi_{12} &= \theta_{12} + \phi_2 + 2n\pi. \end{aligned} \quad (32)$$

We will drop the $2n\pi$, since it has no further interest.

Each of the oscillators will react to the injected voltage it sees, independently of the other oscillator. Thus, we write two equations like (10) and get

$$F - F_{10} = \frac{C_{F1}v_1}{I_{01}} \cos(\phi_1 + \beta_1) \quad (33)$$

$$F - F_{20} = \frac{C_{F2}v_2}{I_{02}} \cos(\phi_2 + \beta_2). \quad (34)$$

These we can combine, with the aid of (29), (30), (31), and (32), to get

$$F_{20} - F_{10} = \frac{C_{F1}v_1}{I_{01}} \left[\cos(\theta_{12} + \phi_{12} + \beta_1) - \frac{C_{F2}}{C_{F1}} \left(\frac{I_{01}}{I_{02}} \right)^2 \cos(-\theta_{12} + \phi_{12} + \beta_2) \right] \quad (35)$$

which is an equation involving θ_{12} as the only unknown.

We observe immediately from (35) that both oscillators contribute to the bandwidth of synchronization. To see the effect more clearly, we write

$$\begin{aligned} \Phi &= (\theta_{12} + \phi_{12} + \beta_1) \\ \mathcal{E}_1 &= -(B_1 + B_2 + 2\phi_{12}) \\ k &= \frac{C_{F2}}{C_{F1}} \left(\frac{I_{01}}{I_{02}} \right)^2 \end{aligned} \quad (36)$$

and get

$$F_{20} - F_{10} = \frac{C_{F1}v_1}{I_{01}} \left[\sqrt{1 + k^2 - 2k \cos \mathcal{E}_1} \cos(\Phi + \mathcal{E}_2) \right] \quad (37)$$

where

$$\tan \mathcal{E}_2 = \frac{-k \sin \mathcal{E}_1}{1 - k \cos \mathcal{E}_1}.$$

From (37) we see that the two oscillators synchronize over a band of frequencies Δf_{12}, given by

$$\Delta f_{12} = \Delta f_1 \sqrt{1 + k^2 - 2k \cos \mathcal{E}_1}. \quad (38)$$

If oscillator 2 is much more powerful than oscillator 1 but otherwise identical, k will be very small and Δf_{12} becomes equal to Δf_1.

From this it can be seen that it is important to have the driving oscillator more powerful than the test oscillator when making synchronization measurements. If the two are identical, k will be 1 and the band of synchronization can vary from 0 to $2\Delta f_1$, depending on $\mathcal{E}_1$.

The allowed values of Φ and hence of θ_{12} can be obtained from (35), (36), and (37) when the necessary parameters are given.

In a similar manner the equations can be extended to include the case of N oscillators acting upon one another.

V. Applications

Several interesting applications of the synchronized oscillator, some of which have been described elsewhere, may be studied with the aid of this theory. In the following no attempt has been made to make an exhaustive study of any particular application, but rather to indicate as a basis for further investigation some interesting applications of the synchronized oscillator.

A. Linear R.F. Voltmeter

It has been shown that a_{max} is proportional to v and independent of $\boldsymbol{C_F}$, α, or β, and therefore a synchronized oscillator can be used as a linear votmeter giving a d.c. indication of the amplitude of the injected a.c. voltage. The use of a synchronized oscillator provides a linear voltmeter for small voltages at any frequency for which an oscillator can be constructed, including the microwave region, since the treatment is not confined to triodes and lumped circuit elements.

If V is the r.f. voltage (peak) on the load impedance Z_L, then

$$V = I_0 Z_L$$

and

$$a_{max} = C_A Z_L \frac{v}{V}. \quad (39)$$

Typical values measured on the experimental oscillator are $C_A = 1.03$ volts/ohm, $Z_L = 236$ ohms, and $V = 47$ volts. Whence

$$a_{max} = 5.2v,$$

indicating that this oscillator-voltmeter gives about a five-fold amplification of the voltage to be measured.

If the oscillator is properly designed it will be found that, to a good approximation,

$$C_A = SV$$

where S is a constant of proportionality. We can then write

$$a_{max} = SZ_L v$$

and note that a_{max} is independent of V, or that the calibration of the voltmeter is independent of the power-supply voltage driving it. To demonstrate this, the test oscillator was used to measure a fixed injected voltage while its own power-supply voltage was varied from 105 to 225 volts. The result is shown in Fig. 9. By careful design the dependence on power-supply voltage can be further reduced. One of our test oscillators showed no measurable change in reading.

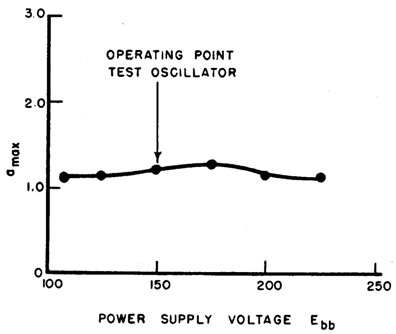

Fig. 9—a_{max} as a function of test oscillator E_{bb} for fixed injected voltage.

While we have been concerned with the behavior of the oscillator under small disturbances, we have seen that the device is linear for a_{max} up to 2.5 volts and therefore for injected voltage of 0.5 volt. Higher voltages can be handled by more powerful oscillators, but it must be remembered (as seen in Section IV) that the source of the voltage to be measured must have a higher power than the test oscillator to avoid complications.

If the frequency of the injected voltage cannot be varied across the synchronization band of the voltmeter, the frequency of the voltmeter can be varied across the synchronization band by a small variable capacitor. The d.c. grid bias or other amplitude indicator can be coupled through a blocking capacitor to a peak voltmeter. As the voltmeter-oscillator is wobbled back and forth across the frequency of the injected voltage to be measured, a pulse will be observed whose peak value is a_{max}. From this pulse the size of the injected voltage can be calculated.

It should also be noted that the synchronized oscillator can be used, as done by Appleton,[2] to measure small voltages by determining the bandwidth of synchronization, which is also linearly related to v by the relation

$$\Delta f = 2C_F \frac{vZ_L}{V}.$$

B. Field-Intensity Meter

The voltmeter properties of the synchronized oscillator lend themselves nicely to the measurement of field intensity at any frequency for which an oscillator is available. Appleton used the synchronization bandwidth of an oscillator to measure field intensities. It is proposed here to use the voltage changes directly, instead of the synchronization band, largely because the power-supply variation no longer enters the calculation and frequency measurements are not needed.

Assume that a small oscillator, like that of Fig. 1, is available and that the grid bias is to be used as indicating voltage. An antenna is coupled to the load Z_L so that its radiation resistance appears as R_S in that load circuit.

If the antenna is in an r.f. field of E peak volts per meter, whose strength is to be measured, the field will induce a voltage v (as already defined) in the load impedance Z_L of which the antenna is now a part. The magnitude of v can be shown to be

$$v = \frac{\lambda}{\pi} E \sqrt{\frac{R_s G}{120}} f(\theta) \tag{40}$$

where G is the gain referred to an isotropic radiator and $f(\theta)$ is the normalized electric-field radiation pattern of the antenna.

C_A and S should be measured about an operating load including the R_s of the antenna used. If a tuning capacitor in the oscillator is wobbled back and forth through the synchronization region, a pulse of peak value a_{max} will be observed, as in the case of the voltmeter. From its magnitude the strength of the field E can be calculated. It will be

$$E = \frac{\pi}{\lambda} \frac{a_{max}}{SZ_L} \sqrt{\frac{120}{R_s G}}. \tag{41}$$

The sensitivity of the field-strength meter will decrease with decreasing λ, but at the higher frequencies an increased gain G can be used to compensate for the loss.

C. Linear A.M. Detector

The synchronized oscillator can be used as a linear demodulator for amplitude modulation by taking the intelligence-frequency component from the A terminals. In this application it will be best to use an oscillator which has $\rho \cong \mp\pi/2$ so that the U–W curve is nearly a semicircle. This will be true if the signal is injected into the plate or grid circuit of a class-C oscillator, and the output is read from the d.c. grid bias. It will be necessary to have sufficient signal strength so that the synchronization band will include all the sideband frequencies.

To achieve this it appears reasonable to require that the time constant of the device be short compared to the shortest period of the modulation to be received. We have seen in (19) that the time constant is approximately

$$\tau = \frac{1}{2\pi\sqrt{\left(\frac{\Delta f}{2}\right)^2 - f^2}} = \frac{1}{\pi \Delta f \sin \psi}. \tag{42}$$

Sin ψ is unity near the center of lock-in where f is nearly zero. Thus the requirement that τ be short compared to $1/f_{\max}$, where $f_{\max}$ is the highest modulation frequency to be reproduced, means that

$$\frac{1}{\pi\Delta f} \ll \frac{1}{f_{\max}}$$

or that

$$\pi\Delta f \gg f_{\max}. \tag{43}$$

From this we conclude that the signal used at the demodulator must be large enough to give a synchronization bandwidth of at least 30 kc. in order to give faithful reproduction of 10 kc. modulation.

If the synchronized demodulator is used it will have the advantage not only of linearity, but it can also give a demodulation voltage amplification, as shown in (39) *et seq.*

When nearly 100 per cent modulation is used the device will lead to distortion of a peculiar form because synchronization may be lost when the signal is small near the peak of modulation. However, the synchronized oscillator-demodulator appears to present interesting possibilities worthy of further investigation.

D. F.M. Discriminator

If the oscillator circuit is arranged so that $\rho=0$ or π, the synchronized oscillator can be used as an f.m. discriminator-demodulator. Reference to Fig. 2 shows that, under these conditions, the $U-W$ curve is a straight line with $U=0$ at center frequency.

One way of achieving this is to couple an auxiliary resonant circuit to the test oscillator and inject the synchronizing signal into this auxiliary circuit. The output can be taken from the d.c. grid bias of the oscillator or from a diode connected across the resonant circuit. Fig. 10 shows the auxiliary resonant circuit and the coupling to the driving oscillator used in the experimental tests.

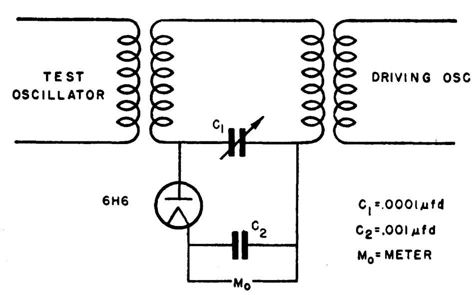

Fig. 10—Auxiliary resonant circuit to obtain behavior characteristic of $\rho=0$.

If the resonant circuit is properly detuned (about 70 per cent of resonance voltage), resistance and/or reactance added in the auxiliary circuit appear as reactance and/or resistance in the oscillator load circuit. If in the original oscillator $\rho=\pm\pi/2$, it will appear to be $\rho=0$ or π when the auxiliary circuit is added and the desired result is attained.

Fig. 11 shows three $U-W$ curves taken from the diode across the resonant circuit; for exact resonance of the diode circuit, 100 per cent; detuned to 70 per cent of resonant voltage, and detuned to an intermediate value, 90 per cent. The linear $U-W$ curve desired was achieved at the 70 per cent detuning adjustment.

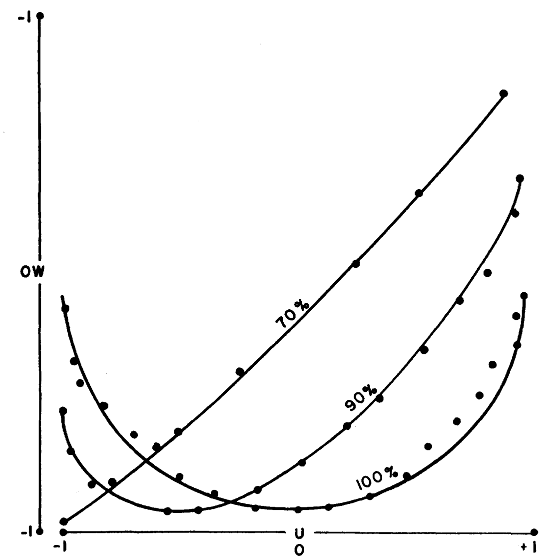

Fig. 11—Experimental $U-W$ curves taken from auxiliary resonant circuit. (Percentages refer to resonant voltage on auxiliary circuit.)

When the arrangement described above is used as a discriminator, a will be zero at the center frequency, and the device is thus insensitive to amplitude modulation in a manner similar to a balanced discriminator.

E. F.M. Synchronous Amplifier-Limiter

In this application the oscillator is locked to an f.m. signal. It follows the frequency variations without serious amplitude change, and hence becomes a combined amplifier and limiter. It has been discussed previously in the literature.[5]

If the synchronized oscillator is capable of following the frequency deviations, the response to an f.m. signal of the form

$$f(t) = f_0 \sin 2\pi f_m t \tag{44}$$

will be a solution of

$$\frac{1}{2\pi}\frac{d\phi}{dt} + \frac{C_F v(t)}{I_0}\cos(\phi+\beta) = f(t) \tag{45}$$

where $v(t)$ represents any amplitude modulation of v that may be present. Direct integration of (45) is compli-

[5] C. W. Carnahan and H. P. Kalmus, "Synchronized oscillators as frequency-modulation receiver limiters," *Electronics*, vol. 17, pp. 108–112; August, 1944.

cated and need not be performed to the approximation needed here. It will be recalled that ϕ responds to changes in f and v with a time constant τ given by (19). If the changes in f or v occur in a time long compared with τ, the oscillator is essentially in equilibrium at each instant, and a succession of steady-state solutions for various fixed f is a good-enough approximation to the actual solution for varying f. If the injected voltage v is always so large that

$$Kv = \eta f_0; \qquad \eta > 1, \tag{46}$$

then

$$\tau \leqq \frac{1}{2\pi f_0\sqrt{\eta^2 - 1}} \cdot \tag{47}$$

Since it is standard practice to have $f_0 > 5f_m$, it is evident that the time constant is short compared with the frequency-modulation period $1/f_m$ and the equilibrium solution (18) is a reasonable approximation. Similar arguments hold for changes in v, but we are not not interested in amplitude modulation here, and will henceforth assume v to be constant.

We see from (18) that changes in f will produce changes in ϕ, so that an additional phase modulation will be added to the impressed signal. This implies that $d\phi/dt \neq 0$, in contradiction to the original assumptions made in solving (14) to get (18). The correction will be small if the frequency variations are slow, and it can be shown that the distortion will be negligible if $Kv \geqq 2f_0$.

Amplitude changes in v will also lead to phase modulation. However, if Kv is kept somewhat larger than f_0, the phase is relatively insensitive to voltage changes and the distortion arising from amplitude modulation is thereby minimized.

If the criterion $Kv = 2f_0$ is set as a design center, the voltage amplification achieved by the use of the synchronized oscillator will be

$$\frac{V}{v} = \frac{C_F Z_L}{2f_0} \cdot \tag{48}$$

To estimate the order of magnitude of the gain that may safely be used, assume (a) that a single LC circuit is controlling the oscillator, (b) that the voltage V is the one across the entire inductance of the oscillating circuit, and (c) that v is injected into this inductance. Then

$$C_F = \frac{1}{4\pi L} \cdot \tag{49}$$

This gives

$$\frac{V}{v} \quad \frac{F}{4f_0} \cdot$$

If the oscillator frequency is 10 Mc. and f_0 is 100 kc. the maximum voltage amplification of the device can be about 25. Of course, if the voltage across a part of the tank inductance is used, as in the experiments already described, the gain is correspondingly reduced. However, gains of 10 or more should be readily obtainable. Also, the voltage v may be injected into the grid circuit of the oscillator and the gain of the tube used to increase the voltage seen in the tank circuit. Equation (49) refers only to the voltage v injected into the oscillating circuit which controls the frequency.

Unless there is some amplitude-regulating device on the synchronized oscillator, there will also be an amplitude modulation in its output. The magnitude of the effect can be calculated from (9) and (24). It is usually small enough to be neglected.

VI. Conclusion

Although the theory and experiments just described have been discussed in terms of a conventional self-limiting source of alternating e.m.f., the concepts involved are quite general, and with appropriate redefinition of symbols the equations can apply equally well to any source of harmonic disturbance, electrical, electromagnetic, mechanical, or acoustical, singly or in combination. The self-limitation implies that some nonlinear element is present to limit the amplitude of oscillation. A truly linear oscillator will not exhibit synchronization effects. It will, however, not appear in practice.

We may then conclude that any source of harmonic disturbance whose steady-state frequency is a continuous function of the load applied to it, and whose frequency can shift with sufficient rapidity, will exhibit synchronization behavior when a harmonic disturbance is impressed upon it from an external source. If the amplitude of the device is also a function of its load, then it will exhibit a characteristic amplitude variation in the synchronization region. We may also conclude that the source will synchronize with an impressed disturbance, however small, if the frequency of the outside disturbance is close enough to that of the undisturbed source.

It is important to remember that the properties of the external source, supplying the synchronizing signal and the coupling impedance, are important in determining the bandwidth and phase of synchronization. If there is a considerable disparity in the power output of the two sources, the weaker determines the synchronization properties of the system. If the power outputs are nearly equal, both sources contribute almost equally to the synchronization properties.

VII. Acknowledgment

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