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ABSTRACT

We demonstrate a high finesse, microfabricated mirror-based, air-gap cavity with volume less than 1 ml, constructed in an array, that can support low-noise microwave generation through optical frequency division. We use the air-gap cavity in conjunction with a 10 nm bandwidth mode-locked laser to generate low phase noise 10 GHz microwaves, exhibiting a phase noise of −95 and −142 dBc/Hz at 100 Hz and 10 kHz offset frequencies, respectively. This is accomplished using the 2-point lock optical frequency division method, where we exploit 40 dB common-mode rejection of two lasers separated by 1.29 THz and locked to the same air-gap cavity. If used with an octave spanning comb, the air-gap cavity is capable of supporting 10 GHz phase noise below −160 dBc/Hz at 10 kHz offset, a level significantly lower than electronic synthesizers. These results show how extremely small optical reference cavities, operated without the benefit of vacuum enclosures or thermal insulation, can, nonetheless, support state-of-the-art microwave phase noise in compact and portable systems.

I. INTRODUCTION

Microwave signals with low timing noise and high frequency stability play a crucial role in various applications, including high-performance radar, position and navigation, communications, and sensing.[1–4] In many cases, a compact and portable microwave source is required while maintaining ultralow-noise performance, and it is becoming increasingly difficult to meet ever more stringent system requirements using traditional electronic oscillators. To address this, the extremely low loss and high quality factors of optical systems have been exploited to generate microwave signals with extremely high frequency stability and low timing noise. A number of architectures have been proposed and demonstrated with trade-offs in complexity, size, and performance.[5–14] Of these optical techniques, optical frequency division (OFD) produces the lowest noise microwaves by frequency-dividing an ultrastable optical oscillator with an optical frequency comb.[5] Using OFD, ultralow 10 GHz phase noise better than −170 dBc/Hz at 10 kHz offset[15] and fractional frequency instability reaching 10^{-18} have been demonstrated.[16]

The phase noise performance of an OFD system is ultimately limited by the noise of the optical frequency reference. The lowest frequency-noise references are constructed by locking a laser to a vacuum-gap Fabry–Pérot resonator, thereby transferring the cavity length stability to the laser frequency stability. With long cavity lengths, high-vacuum enclosures, temperature stabilization, and, in some cases, cryogenic operation, lasers locked to cavities have reached a fractional frequency instability of 10^{-16} and below.[17–19] While these methods lead to extraordinarily low-noise lasers, the volume of the cavity and its environmental isolation enclosure (sometimes exceeding 1 m^3), as well as the power requirements to

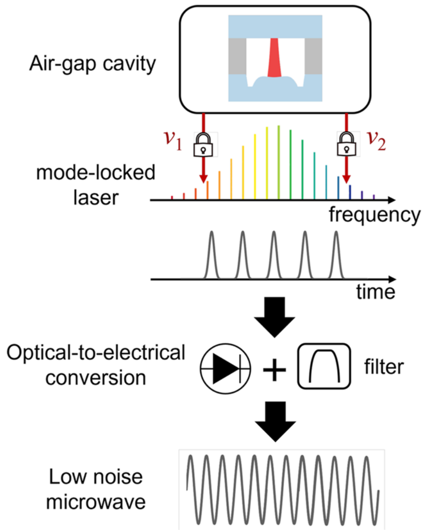

FIG. 1. Conceptual diagram of 2-point optical frequency division with an air-gap optical reference cavity. Two lasers locked to the same air-gap cavity are used to stabilize the repetition rate of a mode-locked laser. The fractional frequency stability of the generated microwave inherits the stability of the cavity free spectral range.

keep the cavity temperature stabilized and under vacuum, severely limit the employment of these systems in applications that require portability and out-of-lab operation. Thus, there is a strong need for a reduction in the footprint of optical references used in OFD.

In this paper, we demonstrate a large reduction in the volume and complexity of a low phase noise optical frequency reference while still achieving low phase noise microwaves with OFD. We introduce a compact high-finesse air-gap Fabry–Pérot cavity, where multiple cavities with finesse over 750 000 can be produced in parallel using micromirror fabrication techniques.[20] We construct a 3×3 cavity array on 50.8 mm (2 in.) diameter wafers, where the volume of each individual cavity is <0.2 ml. We then establish the performance of our air-gap cavities for OFD in two ways. First, we show that the optical phase noise of a laser locked to an air-gap cavity supports ultralow phase noise microwaves when combined with an octave spanning frequency comb, with a projected noise level of < -160 dBc/Hz at 10 kHz offset from a 10 GHz carrier. Second, we show how common-mode rejection of the noise of an air-gap Fabry–Pérot cavity can be exploited in a "2-point" OFD system (shown conceptually in Fig. 1), where the OFD footprint is reduced by using a mode-locked laser spanning just 10 nm. In this case, we generate 10 GHz microwaves with phase noise < -140 dBc/Hz at 10 kHz offset from a 10 GHz carrier. These results demonstrate how state-of-the-art microwave phase noise can be generated with much simpler systems than commonly employed while meeting the size and power draw requirements of many field applications.

II. AIR-GAP REFERENCE CAVITY

The reference cavity we use is one of nine high-finesse Fabry–Pérot cavities within an array of microfabricated cavities, of which photographs are shown in Figs. 2(a) and 2(b).[21] The cavity array has a mechanically rigid structure and is optical contact bonded in air. As shown in Fig. 2(c), the cavity assembly consists of three 50.8 mm (2 in.) diameter substrates made of ultra-low-expansion (ULE) glass, with one substrate containing nine curved, high reflectivity (HR) microfabricated mirrors, one 3 mm-thick spacer with nine holes, and one HR-coated flat substrate serving as the second cavity mirror. The free spectral range (FSR) of each cavity is ~50 GHz. Since the spacer does not have vent holes, air molecules at atmospheric pressure are sealed inside the individual cavities after the bonding process is completed. The curved mirrors have a radius of curvature (ROC) of 35 cm with a surface roughness of about 1 Å, leading to ultralow loss and high finesse.

The curved mirrors are lithographically fabricated through a reflow-based technique,[20] which we briefly describe here. First, patterns of photoresist disks are created on the substrate. Then, the photoresist disks undergo a reflow process in a solvent-vapor chamber at a high temperature to form a near-parabolic shape. A reactive ion etch process is then used to transfer the reflowed pattern to the substrate. The patterned substrate is subsequently coated by a high-reflectivity (>99.999%) dielectric coating centered at 1560 nm. Whereas high-reflectivity coatings are typically masked such that the optical contact area is uncoated, here optical contact bonding is performed directly on the coated surfaces. In this way, we avoid complicated masking of our mirror substrates at the expense of a slightly higher coefficient of thermal expansion of our cavity. Ring-down measurements on the cavity array yield an average finesse of over 650 000. Six of the nine cavities have finesse at or above 750 000, with the highest finesse measured to be 854 000 with a corresponding quality factor of 3.3×10^9. The differences in finesse across the array could be due to roughness variations of the wafer substrate, occasional point defects in the coatings, or imperfections in the mirror

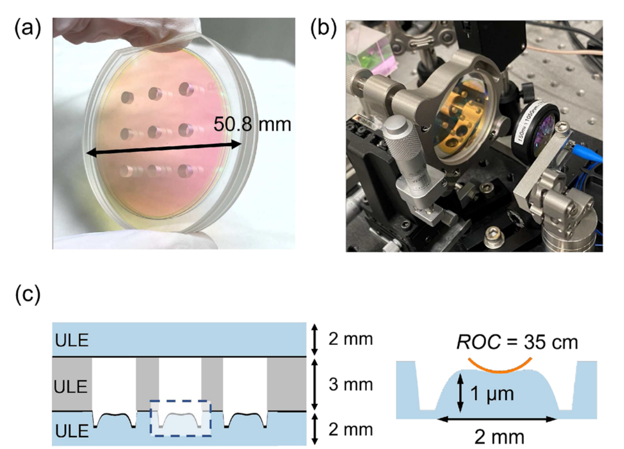

FIG. 2. (a) Photograph of the 50.8 mm (2 in.) diameter cavity array, with nine individual Fabry–Pérot cavities with a 3×3 layout. (b) The cavity is mounted in a standard 2 in. mirror mount in open air without any enclosure for all laser locking and microwave generation experiments. (c) The cavity array, consisting of three super-polished ultra-low-expansion (ULE) glass substrates, is optical contact bonded in air and seals air molecules inside. The curved mirrors, which have a radius of curvature (ROC) of 35 cm, are lithographically fabricated using a reflow-based technique.

shape that cause misalignment or beam clipping. The cavity array can be diced into individual cavities of size ~0.5 ml,[14] demonstrating a path toward the mass production of high-finesse and small-size Fabry–Pérot cavities.

The cavity array is held in air by a standard 2 in. mirror mount without any enclosure or temperature control, as shown in Fig. 2(b). The cavity with the highest finesse of 854 000 was chosen for the following experimental studies, though we note that the high finesse of the majority of the cavities within the array could support laser locking of similar fidelity. The high finesse enables sufficient suppression on the residual electronic noise of the laser locks, which is crucial to the common-mode-rejection measurement discussed later in this section.

We lock two lasers to the cavity, one at 1545 nm and the other at 1555 nm (1.29 THz separation). We first measure the optical phase noise of the individual lasers and, then, the relative phase noise between them. An Er:fiber-based optical frequency comb, stabilized to an ultrastable optical cavity,[22] serves as the phase noise reference, as depicted in Fig. 3(a). The optical phase noise of the 1545 nm fiber laser stabilized to the cavity by the Pound–Drever–Hall (PDH) technique is shown in Fig. 3(b). While the phase noise is reduced from that of the narrow-linewidth free-running laser from 8 to 2 kHz (calculated with the β-separation line method[23]), the cavity-stabilized phase noise level is 30 dB above the cavity thermal noise limit given by the Brownian noise of the dielectric coatings.[24] This is to be expected, since the air molecules sealed inside the cavity cause fluctuations in the optical path length of the mode. However, even with this level of phase noise, a full OFD with an octave-spanning frequency comb would give $-20\log(194\ \text{THz}/10\ \text{GHz}) = -86$ dB reduction in phase noise for a 10 GHz microwave output, resulting in lower than -160 dBc/Hz phase noise at 10 kHz offset frequency, a noise value well below most commercial microwave synthesizers.

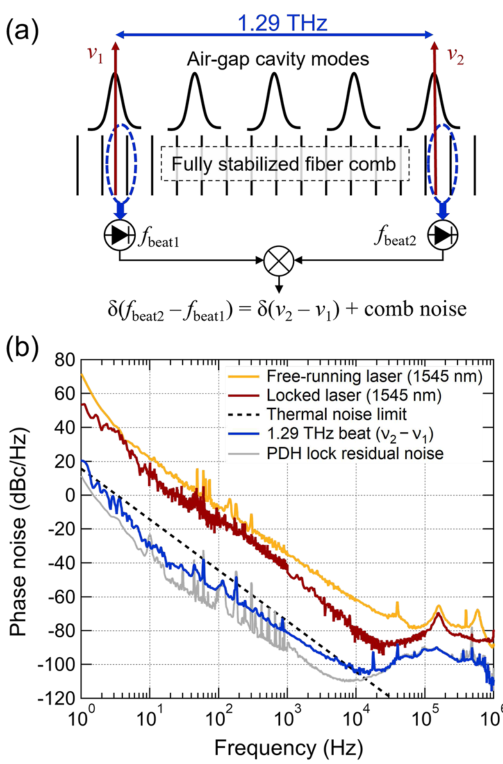

FIG. 3. (a) Phase noise measurement architecture for lasers locked to the air-gap cavity. The noise of the individual CW lasers is determined by beating with a separately stabilized optical frequency comb, and the relative phase noise of two CW lasers locked to the same cavity is extracted by mixing the individual beat tones. (b) Measured phase noise of one of the CW lasers and the relative noise between two CW lasers. Blue: relative phase noise of the two lasers separated by 1.29 THz. Red: phase noise of the 1545 nm laser stabilized to the air-gap cavity, measured against a fully stabilized comb. Phase noise of the 1555 nm laser stabilized to the same cavity is at a comparable level, as expected (not shown). At offset frequencies higher than 20 kHz, the phase noise is dominated by the reference comb. Gray: combined noise of the PDH locks, including the residual noise of the two lasers and contributions from the PDH photodetector and locking electronics. Yellow: phase noise of the 1545 nm free-running laser. The phase noise of 1555 nm free-running laser is at a comparable level.

To better utilize the compactness of the air-gap cavity, we pair it with a mode-locked laser, which serves as a more compact and portable optical frequency divider than an octave-spanning comb. Since a mode-locked laser without super-continuum generation has a much narrower bandwidth, we use two lasers locked to the common optical reference cavity to stabilize the repetition rate, referred to as the 2-point lock method.[8] In this case, the frequency division ratio of the laser noise is given by the ratio of the frequency separation between the two CW lasers to the microwave frequency, which is lower than that in full OFD. However, the noise on the CW lasers locked to the same cavity is largely common, leading to a large common-mode rejection (CMR) that benefits microwave generation. This is an important distinction from the free-running lasers themselves: although the free-running laser phase noise is only slightly higher, there is no CMR of their noise without locking to a common reference cavity.

The amount of CMR can be quantified by comparing the noise of a single laser to the relative noise between the two lasers. As a result of CMR, the relative noise of the two lasers is lower than that of a single laser. Ideally, if two lasers are locked to two cavity modes at frequencies ν_1 and ν_2, their fractional frequency noise would be equal to the fractional frequency fluctuation of the cavity FSR, which is equivalent to the fractional length fluctuation of the cavity,

$$\frac{\delta\nu_1}{\nu_1} \approx \frac{\delta(m \times FSR)}{m \times FSR} \approx \frac{\delta(FSR)}{FSR} \approx \frac{\delta L}{L}, \tag{1}$$

$$\frac{\delta\nu_2}{\nu_2} \approx \frac{\delta(n \times FSR)}{n \times FSR} \approx \frac{\delta(FSR)}{FSR} \approx \frac{\delta L}{L}, \tag{2}$$

where L is the cavity length and m and n are the integers corresponding to the cavity modes ν_1 and ν_2. When the difference frequency between the two lasers at $\nu_2 - \nu_1 = (n - m) \times \text{FSR}$ is taken, its fractional frequency fluctuation is

$$\frac{\delta(\nu_2 - \nu_1)}{\nu_2 - \nu_1} \approx \frac{\delta[(m - n) \times (FSR)]}{(m - n) \times FSR} \approx \frac{\delta(FSR)}{FSR}. \tag{3}$$

The frequency fluctuation of the difference frequency is then related to that of a single laser by

$$\left(\frac{\delta\nu_2}{\delta(\nu_2 - \nu_1)}\right)^2 = \left(\frac{\nu_2}{\nu_2 - \nu_1}\right)^2. \tag{4}$$

A similar relationship holds for ν_1. While this relation describes frequency fluctuations, the phase fluctuation of the difference frequency is related to that of a single laser by the same ratio. This ratio represents the maximum amount of CMR achievable under ideal conditions. When combined with the frequency division of 2-point locking to generate a microwave frequency at f_μ, an additional noise reduction of $(\nu_2 - \nu_1)^2/f_\mu^2$ is obtained. This would result in a total phase noise reduction factor equal to that of full OFD, namely $(\nu_2/f_\mu)^2$.

Importantly, there are two complicating factors that limit the predicted noise reduction to less than that given by full OFD. First, there is always residual noise in the individual PDH locks.[25] This noise is not common between lasers and does not enjoy any common-mode rejection. However, this noise is reduced by the 2-point OFD ratio and is mitigated by increasing the separation between the laser frequencies. Therefore, it is beneficial to set the laser frequency separation as wide as the comb bandwidth. Second, the two lasers necessarily sample the cavity noise slightly differently, limiting the level of CMR of the cavity noise. For example, coating Brownian noise is proportional to the square of the optical spot size, and the spot size, in turn, scales inversely with the square-root of the frequency of the optical mode. Thus, lasers locked to a common cavity see slightly different levels of coating noise, depending on their frequency separation, limiting the extent of the CMR. Similarly, cavity modes at different frequencies will sample the noise of the optical path length in the air gap differently. Interestingly, thermo-optic noise of broadband coatings may exhibit an anticorrelated behavior, where the frequencies of modes separated by ~100 THz have been observed to drift in opposite directions.[26] While thermo-optic noise for our cavities is much lower than the noise from the air gap or coating Brownian, it may become manifest in 2-point OFD if it exhibits a lower level of CMR.

To test the CMR we can achieve with the air-gap cavity, we stabilize the two narrow-linewidth fiber lasers at 1545 nm and at 1555 nm to our cavity by PDH locking. Then, we combine the two stabilized lasers with a fully stabilized fiber frequency comb to bridge the 1.29 THz gap between the lasers, down-shifting their relative phase fluctuations onto an RF carrier. This is performed by mixing the two beat tones between the cavity stabilized lasers and the nearest comb modes of the fiber comb, as illustrated in Fig. 3(a). To eliminate the comb noise, a second fully stabilized comb was used to generate a second $f_{\mathrm{beat2}} - f_{\mathrm{beat1}}$ and a cross-spectrum measurement was performed [not shown in Fig. 3(a)]. The result of the phase noise measurement is shown in Fig. 3(b). The limit set by the perfect CMR assumption of Eq. (4) gives $20\log(194/1.29\ \mathrm{THz}) \approx 44$ dB maximum CMR. The measured phase noise on the 1.29 THz beat is about 40 dB lower than the individual CW laser phase noise up to 1 kHz offset frequency, which is not far from the estimated 44 dB reduction for a perfect CMR. From 1 to 10 kHz offset frequencies, the measured CMR gradually decreases as the offset frequency increases, resulting in ~20 dB reduction from the single CW laser noise at 10 kHz offset, which may be caused by uncorrelated fiber length fluctuations from non-common fiber paths. After 10 kHz, the phase noise at higher offset frequencies is limited by the residual noise of the laser PDH locks.

In Sec. III, we show how this large CMR can be combined with 2-point OFD to achieve another $20\log(1.29\ \mathrm{THz}/10\ \mathrm{GHz}) \approx 42$ dB reduction in phase noise, which implies 10 GHz phase noise of about −146 dBc/Hz at 10 kHz offset frequency and −98 dBc/Hz at 100 Hz offset frequency.

III. MICROWAVE GENERATION

A schematic illustration of the 2-point lock setup is shown in Fig. 4(a). As mentioned above, the 1545 and 1555 nm fiber lasers are stabilized to the common air-gap cavity on the cavity array by PDH locking. About 5 mW from each CW laser is combined with the ~20 mW output of a commercial 500 MHz-repetition-rate semiconductor saturable absorber mirror (SESAM) Er/Yb:glass mode-locked laser using fiber couplers. The optical spectrum overlap between the mode-locked laser and the CW lasers is shown in Fig. 4(b). The combined output of the lasers is split into two paths and bandpass-filtered for optical beat detection. The frequencies of the two heterodyne beats between the CW lasers and the

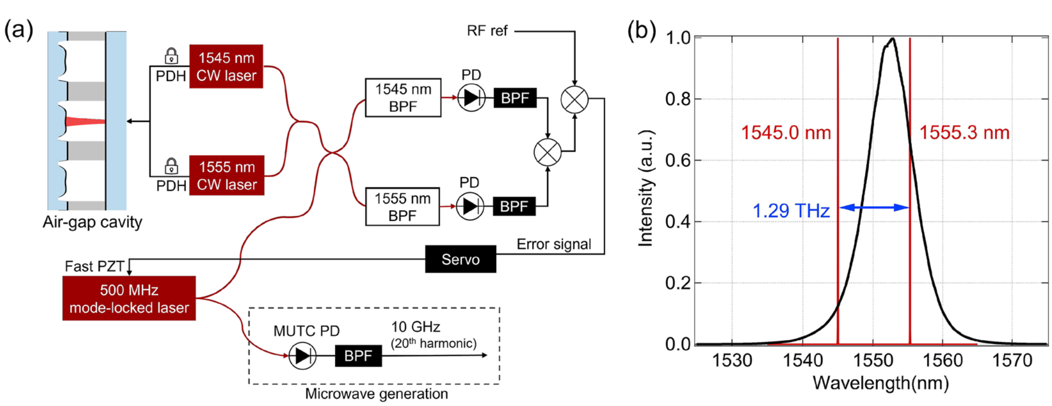

FIG. 4. (a) Schematic diagram of 2-point lock microwave generation with the air-gap cavity. The mode-locked laser repetition rate is stabilized via feedback to a fast piezoelectric transducer (PZT). PDH: Pound–Drever–Hall lock; BPF: bandpass filter; and MUTC PD: modified uni-traveling carrier photodiode. (b) Optical spectrum overlap between the mode-locked laser (black) and CW lasers (red).

mode-locked laser are 420 and 327 MHz, each with a signal-to-noise ratio (SNR) of ~55 dB in 100 kHz resolution bandwidth. The heterodyne beat signals are individually selected with bandpass filters and mixed, producing a difference frequency,[14]

$$f_{\text{beat2}} - f_{\text{beat1}} = N f_{\text{rep}} - (\nu_2 - \nu_1), \tag{5}$$

where f_{beat1} and f_{beat2} are the beatnotes between the mode-locked laser comb lines and the CW lasers and N is an integer corresponding to the number of comb lines spanned by the CW lasers. When $f_{\text{beat2}} - f_{\text{beat1}}$ is locked to an RF reference by feeding back to the mode-locked laser, the fluctuations on f_{rep} within the mode-locked laser locking bandwidth may be expressed as

$$\langle \delta f_{\text{rep}}^2 \rangle = \frac{\langle \delta (f_{\text{beat2}} - f_{\text{beat1}})^2 \rangle + \langle \delta (\nu_2 - \nu_1)^2 \rangle}{N^2}, \tag{6}$$

where the brackets denote averaging. Thus, the noise in the mixed heterodyne beats, due to either the RF reference or the residual noise in the lock, as well as the relative noise between the CW lasers, is divided by N^2 when mapped to the frequency (or phase) noise power of the mode-locked laser repetition rate. When the Mth harmonic of f_{rep} is selected for microwave generation, the noise reduction factor is $20\log(Mf_{\text{rep}}/Nf_{\text{rep}})$, in our case, $20\log(10\text{ GHz}/1.29\text{ THz}) \approx -42$ dB. To detect f_{rep} and its harmonics, we use a high-speed modified uni-traveling carrier photodetector (MUTC PD) with 40 μm diameter.[27] The average photocurrent generated by the MUTC PD is about 0.9 mA, and phase noise measurements are performed on the 20th harmonic of f_{rep} at 10 GHz. The phase noise is measured by heterodyning against another 10 GHz signal that has much lower noise, generated through OFD of a fully stabilized octave-spanning fiber frequency comb that is locked to a 30 cm long optical reference cavity with 10^{-16} fractional frequency instability.[22] Phase noise measurements are performed using a commercial phase noise analyzer.

The phase noise of the generated 10 GHz is shown in Fig. 5(a), along with projected noise contributions from the $f_{\text{beat2}} - f_{\text{beat1}}$ residual noise (due to finite gain in the mode-locked laser feedback loop) and the relative noise between the CW lasers. The 10 GHz phase noise is −142 dBc/Hz at 10 kHz offset frequency and −95 dBc/Hz at 100 Hz offset frequency, close to our previous estimate, which represents an improvement of up to 60 dB from the 10 GHz signal generated by the unstabilized mode-locked laser. For offset frequencies below ~300 Hz, the measured phase noise of the generated 10 GHz signal is limited by the 1.29 THz beat noise, and from ~300 Hz to ~10 kHz, the microwave phase noise follows the $f_{\text{beat2}} - f_{\text{beat1}}$ residual noise. Both the 1.29 THz beat noise and the $f_{\text{beat2}} - f_{\text{beat1}}$ residual noise have been scaled by the 2-point OFD factor of −42 dB. While the noise at offset frequencies above the feedback servo bandwidth of ~5 kHz (as indicated by the noise "servo bump") should follow that of the free-running mode-locked laser noise, the measured white noise floor is limited by the measurement setup.

It is also interesting to compare our 10 GHz noise to the projected noise of a 10 GHz signal generated by full OFD, where the optical noise of a single laser will be reduced by 86 dB. As shown in Fig. 5(a), full OFD would provide nearly the same 10 GHz phase noise for offset frequencies below ~300 Hz, again demonstrating the high level of CMR when locking two lasers to the same optical cavity.

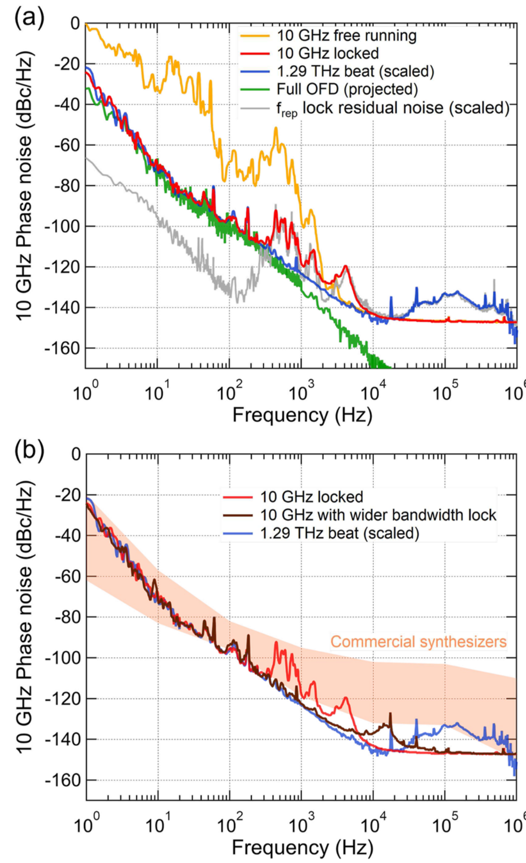

FIG. 5. (a) Single-sideband phase noise on a 10 GHz carrier. Red: 2-point optical frequency division with air-gap cavity. Yellow: free-running mode-locked laser. Gray: residual noise of f_{rep} stabilization, scaled to 10 GHz. Blue: relative phase noise between the two CW lasers, scaled to 10 GHz. Green: projected 10 GHz phase noise with full optical frequency division. (b) With a wider phase lock bandwidth, the 10 GHz phase noise follows the relative phase noise of the CW lasers better. The phase noise of commercial synthesizers is shown for comparison.

It is only for offset frequencies above ~1 kHz that full OFD displays its advantage over the 2-point lock.

If we further increase the bandwidth of the mode-locked laser phase lock, taking advantage of the fast piezoelectric transducer (PZT) used for feedback,[28] we can improve the noise level in some offset frequency ranges while worsening it in others. As shown in Fig. 5(b), the noise between 300 Hz and 6 kHz is improved significantly, though at the expense of higher 10 kHz offset noise. In this case, the 10 GHz phase noise faithfully tracks the projected 1.29 THz beat noise until reaching the servo bump, peaked at about 15 kHz. With a faster PZT,[29] we can expect further improvements in the feedback bandwidth and possibly reach the −146 dBc/Hz limit set by the projected 1.29 THz beat at 10 kHz offset. In either case, the 10 GHz phase noise generated with the air-gap cavity compares favorably with commercially available low-noise synthesizers at offset frequencies above 1 kHz.

There are important and noteworthy trade-offs associated with the CW laser separation for 2-point OFD. As mentioned above, a larger frequency separation results in a larger frequency division factor, leading to a larger reduction in phase noise. Meanwhile, a larger separation also means a smaller amount of CMR. These two effects balance each other such that the total noise reduction factor is equal to that of full OFD. In our case, a 1.29 THz separation results in nearly 44 dB CMR and 42 dB OFD factor, adding up to the full OFD factor of 86 dB. The main advantage of having a larger separation is, therefore, in the noise division of the residual lock noise (PDH locks for the two lasers, as well as f_{rep} lock), which does not experience any CMR. This is especially important when the residual noise of the locks dominates the phase noise of $\nu_2 - \nu_1$. In our case, the residual noise of the locks is well below the limit set by the CMR, thanks to the high finesse of the cavity. However, if the finesse were three times lower than the current value of 854 000, the residual lock noise would limit the CMR we could achieve. Then, it would be necessary to increase the frequency separation of the two CW lasers to make full use of the predicted CMR. Similar situations happen if 2-point OFD is performed on a vacuum-gap cavity.[14] Having a larger frequency separation in such situations would ease the requirement on the residual noise of the locks.

Another consideration when choosing the laser frequency separation is the SNR of the beats between the mode-locked laser and the CW lasers. The SNR of the beats limits the white noise floor if the feedback bandwidth of f_{rep} lock is wide enough. For example, a 55 dB SNR in 100 kHz resolution bandwidth, as in our case, would result in a white noise floor of −147 dBc/Hz for the 42 dB optical frequency division factor. The CW laser separation should thus be as large as possible for a maximum optical frequency division factor while maintaining a good overlap with the mode-locked laser spectrum for sufficient SNR of the beat tones.

IV. CONCLUSION

We have presented a high finesse, compact, air-gap Fabry–Pérot optical reference cavity capable of supporting low-noise microwave generation. Our small cavity volume is enabled by parallel-process manufacturing of high finesse micromirrors. With its sub-ml elemental volume and ability to operate without an enclosure, our cavity represents a volume decrease of at least 100 times compared to the large cavity and high-vacuum environments typical of optical reference cavities, greatly simplifying the optical reference in low-noise OFD systems. We envision that individual cavities will be diced from large cavity arrays, as proposed in Ref. 20 and pictured in Ref. 14. Compact and rugged optical coupling may be achieved by gradient index (GRIN) lens-coupled fibers bonded directly to the cavity, or bonding to a photonic waveguide.[30] Further size reduction may be realized by replacing the narrow-linewidth fiber laser used here with a chip-based laser, previously shown to tightly lock to optical reference cavities,[31] and replacing the mode-locked laser with a THz-spanning microresonator-based frequency comb.[14]

By combining our cavity with a mode-locked laser, we demonstrate the capability of our cavity to generate low-noise microwaves via 2-point OFD that does not require an octave-spanning frequency comb. Here, we take advantage of the large common-mode rejection of the cavity noise that provides a similar optical-to-microwave noise reduction as full OFD. Despite being limited by the noise of the air-gap, the resulting 10 GHz phase noise is comparable to a low-noise commercial synthesizer but with the potential for a smaller system volume.

Finally, we note that further noise improvements may be achieved while keeping the compact volume of our cavity by combining cavities consisting of micromirrors with recent developments in in-vacuum cavity bonding.[32] In this case, an octave-spanning comb could take advantage of the 30 dB optical noise reduction at low offset frequencies to reduce the microwave noise by an equal amount. In a 2-point OFD system, the microwave noise is expected to be limited by the residual lock noise, with further improvements possible with a broader comb span.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yifan Liu: Conceptualization (equal); Investigation (lead); Visualization (lead); Writing – original draft (lead); Writing – review & editing (lead). **Dahyeon Lee**: Conceptualization (equal); Investigation (equal); Software (equal); Visualization (equal); Writing – review & editing (equal). **Takuma Nakamura**: Conceptualization (equal); Investigation (equal); Visualization (equal); Writing – review & editing (equal). **Naijun Jin**: Conceptualization (equal); Investigation (equal); Writing – review & editing (supporting). **Haotian Cheng**: Investigation (equal); Writing – review & editing (supporting). **Megan L. Kelleher**: Conceptualization (supporting); Investigation (supporting); Writing – review & editing (supporting). **Charles A. McLemore**: Conceptualization (supporting); Investigation (supporting); Writing – review & editing (supporting). **Igor Kudelin**: Investigation (supporting); Writing – review & editing (supporting). **William Groman**: Investigation (supporting); Writing – review & editing (supporting). **Scott A. Diddams**: Conceptualization (equal); Funding acquisition (equal); Investigation (equal); Supervision (equal); Writing – review & editing (equal). **Peter T. Rakich**: Conceptualization (equal); Funding acquisition (equal); Investigation (equal); Supervision (equal); Writing – review & editing (equal). **Franklyn Quinlan**: Conceptualization (lead); Funding acquisition (equal); Investigation (equal); Supervision (lead); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

REFERENCES

[1] J. Scheer and J. Kurtz, *Coherent Radar Performance Estimation*, *Artech House Radar Library* (Artech House, 1993).

[2] J. Vig, "Military applications of high accuracy frequency standards and clocks," IEEE Trans. Ultrason. Ferroelectrics Freq. Control **40**, 522–527 (1993).

[3] S. Doeleman, "High frequency very long baseline interferometry: Frequency standards and imaging an event horizon," in *Proceedings of the 7th Symposium Frequency Standards and Metrology*, edited by L. Maleki (World Scientific, 2009), pp. 175–183.

[4] M. R. Khanzadi, D. Kuylenstierna, A. Panahi, T. Eriksson, and H. Zirath, "Calculation of the performance of communication systems from measured oscillator phase noise," IEEE Trans. Circuits Syst. **61**, 1553–1565 (2014).

[5] T. M. Fortier, M. S. Kirchner, F. Quinlan, J. Taylor, J. Bergquist, T. Rosenband, N. Lemke, A. Ludlow, Y. Jiang, C. Oates, and S. A. Diddams, "Generation of ultrastable microwaves via optical frequency division," Nat. Photonics **5**, 425–429 (2011).

[6] D. Kwon, D. Jeong, I. Jeon, H. Lee, and J. Kim, "Ultrastable microwave and soliton-pulse generation from fibre-photonic-stabilized microcombs," Nat. Commun. **13**, 381 (2022).

[7] J. Li, X. Yi, H. Lee, S. A. Diddams, and K. J. Vahala, "Electro-optical frequency division and stable microwave synthesis," Science **345**, 309–313 (2014).

[8] W. C. Swann, E. Baumann, F. R. Giorgetta, and N. R. Newbury, "Microwave generation with low residual phase noise from a femtosecond fiber laser with an intracavity electro-optic modulator," Opt. Express **19**, 24387–24395 (2011).

[9] W. Liang, D. Eliyahu, V. S. Ilchenko, A. A. Savchenkov, A. B. Matsko, D. Seidel, and L. Maleki, "High spectral purity Kerr frequency comb radio frequency photonic oscillator," Nat. Commun. **6**, 7957 (2015).

[10] S. B. Papp, K. Beha, P. Del'Haye, F. Quinlan, H. Lee, K. J. Vahala, and S. A. Diddams, "Microresonator frequency comb optical clock," Optica **1**, 10–14 (2014).

[11] D. Eliyahu, D. Seidel, and L. Maleki, "Phase noise of a high performance OEO and an ultra low noise floor cross-correlation microwave photonic homodyne system," in *2008 IEEE International Frequency Control Symposium* (IEEE, Honolulu, HI, 2008), Vol. 1–2, pp. 811–814.

[12] M. Kalubovilage, M. Endo, and T. R. Schibli, "Ultra-low phase noise microwave generation with a free-running monolithic femtosecond laser," Opt. Express **28**, 25400–25409 (2020).

[13] W. Loh, D. Gray, R. Irion, O. May, C. Belanger, J. Plant, P. W. Juodawlkis, and S. Yegnanarayanan, "Ultralow noise microwave synthesis via difference frequency division of a Brillouin resonator," arXiv:2312.05384 [physics.optics] (2023).

[14] I. Kudelin, W. Groman, Q.-X. Ji, J. Guo, M. L. Kelleher, D. Lee, T. Nakamura, C. A. McLemore, P. Shirmohammadi, S. Hanifi, H. Cheng, N. Jin, S. Halliday, Z. Dai, L. Wu, W. Jin, Y. Liu, W. Zhang, C. Xiang, V. Iltchenko, O. Miller, A. Matsko, S. Bowers, P. T. Rakich, J. C. Campbell, J. E. Bowers, K. Vahala, F. Quinlan, and S. A. Diddams, "Photonic chip-based low noise microwave oscillator," arXiv:2307.08937 [physics.optics] (2023).

[15] X. Xie, R. Bouchand, D. Nicolodi, M. Giunta, W. Hänsel, M. Lezius, A. Joshi, S. Datta, C. Alexandre, M. Lours *et al.*, "Photonic microwave signals with zeptosecond-level absolute timing noise," Nat. Photonics **11**, 44–47 (2017).

[16] T. Nakamura, J. Davila-Rodriguez, H. Leopardi, J. A. Sherman, T. M. Fortier, X. Xie, J. C. Campbell, W. F. McGrew, X. Zhang, Y. S. Hassan, D. Nicolodi, K. Beloy, A. D. Ludlow, S. A. Diddams, and F. Quinlan, "Coherent optical clock down-conversion for microwave frequencies with 10^{-18} instability," Science **368**, 889–892 (2020).

[17] D. G. Matei, T. Legero, S. Häfner, C. Grebing, R. Weyrich, W. Zhang, L. Sonderhouse, J. M. Robinson, J. Ye, F. Riehle, and U. Sterr, "1.5 μm lasers with sub 10 mHz linewidth," Phys. Rev. Lett. **118**, 263202 (2017).

[18] S. Häfner, S. Falke, C. Grebing, S. Vogt, T. Legero, M. Merimaa, C. Lisdat, and U. Sterr, "8×10^{-17} fractional laser frequency instability with a long room-temperature cavity," Opt. Lett. **40**, 2112–2115 (2015).

[19] R. Xiao, Y. Xu, Y. Wang, H. Sun, and Q. Chen, "Transportable 30 cm optical cavity based ultrastable lasers with beating instability of 2×10^{-16}," Appl. Phys. B **128**, 220 (2022).

[20] N. Jin, C. A. McLemore, D. Mason, J. P. Hendrie, Y. Luo, M. L. Kelleher, P. Kharel, F. Quinlan, S. A. Diddams, and P. T. Rakich, "Micro-fabricated mirrors with finesse exceeding one million," Optica **9**, 965–970 (2022).

[21] N. Jin, Y. Liu, C. A. McLemore, Y. Luo, D. Mason, P. Kharel, S. A. Diddams, F. Quinlan, and P. Rakich, "Wafer-level fabrication of Fabry-Pérot resonators with finesse exceeding one million," in *CLEO 2023* (Optica Publishing Group, 2023), p. SW4L.6.

[22] M. Schioppo, R. C. Brown, W. F. McGrew, N. Hinkley, R. J. Fasano, K. Beloy, T. Yoon, G. Milani, D. Nicolodi, J. Sherman, N. B. Phillips, C. W. Oates, and A. D. Ludlow, "Ultrastable optical clock with two cold-atom ensembles," Nat. Photonics **11**, 48–52 (2017).

[23] G. Di Domenico, S. Schilt, and P. Thomann, "Simple approach to the relation between laser frequency noise and laser line shape," Appl. Opt. **49**, 4801–4807 (2010).

[24] K. Numata, A. Kemery, and J. Camp, "Thermal-noise limit in the frequency stabilization of lasers with rigid cavities," Phys. Rev. Lett. **93**, 250602 (2004).

[25] T. Day, E. K. Gustafson, and R. L. Byer, "Sub-hertz relative frequency stabilization of two-diode laser-pumped Nd:YAG lasers locked to a Fabry-Perot interferometer," IEEE J. Quantum Electron. **28**, 1106–1117 (1992).

[26] R. C. Terrien, J. P. Ninan, S. A. Diddams, S. Mahadevan, S. Halverson, C. Bender, C. Fredrick, F. Hearty, J. Jennings, A. J. Metcalf, A. Monson, A. Roy, C. Schwab, and G. Stefánsson, "Broadband stability of the habitable zone planet finder Fabry–Pérot etalon calibration system: Evidence for chromatic variation," Astron. J. **161**, 252 (2021).

[27] X. Xie, Q. Zhou, K. Li, Y. Shen, Q. Li, Z. Yang, A. Beling, and J. C. Campbell, "Improved power conversion efficiency in high-performance photodiodes by flip-chip bonding on diamond," Optica **1**, 429–435 (2014).

[28] T. C. Briles, D. C. Yost, A. Cingöz, J. Ye, and T. R. Schibli, "Simple piezoelectric-actuated mirror with 180 kHz servo bandwidth," Opt. Express **18**, 9739–9746 (2010).

[29] T. Nakamura, S. Tani, I. Ito, M. Endo, and Y. Kobayashi, "Piezo-electric transducer actuated mirror with a servo bandwidth beyond 500 kHz," Opt. Express **28**, 16118–16125 (2020).

[30] H. Cheng, N. Jin, Z. Dai, C. Xiang, J. Guo, Y. Zhou, S. A. Diddams, F. Quinlan, J. Bowers, O. Miller, and P. Rakich, "A novel approach to interface high-Q Fabry–Pérot resonators with photonic circuits," APL Photonics **8**, 116105 (2023).

[31] J. Guo, C. A. McLemore, C. Xiang, D. Lee, L. Wu, W. Jin, M. Kelleher, N. Jin, D. Mason, L. Chang *et al.*, "Chip-based laser with 1-hertz integrated linewidth," Sci. Adv. **8**, eabp9006 (2022).

[32] Y. Liu, C. A. McLemore, M. Kelleher, D. Lee, T. Nakamura, N. Jin, S. Schima, P. Rakich, S. A. Diddams, and F. Quinlan, "Thermal-noise-limited, compact optical reference cavity operated without a vacuum enclosure," arXiv:2307.04758 (2023).