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# Electronic synthesis of light

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**We report on bidirectional frequency conversion between the microwave and optical domains using electro-optics. Advances in communications, time keeping, and quantum sensing have all come to depend upon the coherent interoperation of light wave and microwave signals. To connect these domains, which are separated by a factor of 10,000 in frequency, requires specialized technology that has until now only been achieved by ultrafast mode-locked lasers. In contrast, electro-optic modulation (EOM) combs arise deterministically by imposing microwave-rate oscillations on a continuous-wave laser. Here we demonstrate electro-optic generation of a 160 THz bandwidth supercontinuum and realize $f$-$2f$ self-referencing. Coherence of the supercontinuum is achieved through optical filtering of electronic noise on the seed EOM comb. The mode frequencies of the supercontinuum are derived from the electronic oscillator and they achieve $<5\times10^{-14}$ fractional accuracy and stability, which opens a novel regime for tunable combs with wide mode spacing apart from the requirements of mode locking.**

***OCIS codes:*** (230.2090) Electro-optical devices; (320.6629) Supercontinuum generation; (120.3940) Metrology; (140.3425) Laser stabilization.

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## 1. INTRODUCTION

Optical frequency combs, precise rulers for measuring and controlling light waves, are now an important tool in modern science and technology. The key feature frequency combs provide is a phase-coherent link between optical and microwave frequencies through self-referencing [1,2]. Combs enable diverse applications, including measurement of optical clocks [3]; precise calibration for optical spectroscopy [4], molecular identification [5], and coherent imaging [6]; control of quantum systems [7]; carrier-phase control in ultrafast science [8]; and photonic generation of microwave signals [9]. Existing self-referenced frequency comb technology is based either on mode-locked lasers or Kerr soliton generation in microcombs [10,11]. Combs based on mode-locked lasers mostly operate at fixed <1 GHz pulse repetition rate, while microresonator combs offer much larger mode spacings (tens to hundreds of GHz), with values rigidly fixed by platform geometry. However, an emerging class of novel applications for combs, including coherent light wave communications [12], multiheterodyne optical detection [13–15], optical waveform synthesis [16], frequency calibration, and searches for exoplanets [17], calls for a coincidence of wide tunability and pulse rates at tens of GHz or higher. These are already some of the established features of electro-optic modulation (EOM) combs in which the modes are derived electronically, but the central outstanding question is whether they can support coherent optical-microwave conversion.

Before the introduction of mode-locked-laser frequency combs, a leading approach to comb generation was direct EOM of CW light [18,19]. In such an EOM comb [20–23], the frequency of each comb mode (counted $n$ from the CW laser) is $\nu_n = \nu_p + n f_{eo}$ and arises jointly from the CW-laser frequency $\nu_p$ and frequency multiplication of the microwave modulation $f_{eo}$. The relative phases of comb lines are derived from the modulation index apart from any mode-locking mechanism; hence, the modulation deterministically transforms a CW laser into a train of light pulses. Furthermore, EOM-comb generators are already commercialized and can be straightforwardly reconfigured by simply changing the laser or modulation frequencies. Despite the simplicity of EOM frequency combs, they have not yet been developed into self-referenced frequency combs.

Here, we accomplish electronic synthesis of light with an EOM comb by identifying and solving two major scientific challenges: low spectral broadening efficiency associated with narrow EOM-comb bandwidth and microwave repetition rates, and full conversion of electronic noise to the optical comb that progressively degrades as $n^2$ the first-order optical coherence of the comb lines. Our approach multiplies the frequency of a 10 GHz oscillator to create an effective optical frequency reference for the 193 THz CW laser at the center of the EOM comb. We implement frequency multiplication both with linear EOM-comb generation and with nonlinear-fiber spectral broadening. An optical filter cavity sufficiently reduces the fundamental electro-optic noise of the comb to preserve its coherence.

The carrier-envelope offset frequency of an EOM comb is $f_0 = \nu_p - N f_{eo}$, whereby a multiplicative factor $N \sim 19,340$ relates the exact phase of the electronic oscillator to that of the CW laser. The need for optical filtering arises since microwave oscillators, given their thermal phase noise, do not support frequency multiplication to the optical domain [12,24,25]. Optical filtering reduces the impact of high-frequency oscillator noise and modifies the phase noise Fourier-frequency dependence to $1/f^2$, which is consistent with that of a laser.

To understand the filter cavity's effect, we note that the solution of the integral equation [26] $\int_{\delta\omega}^{2\pi f_{eo}/2} S_\phi^{(N)} F(\omega)\mathrm{d}\omega = 1$ estimates the linewidth $\delta\omega^{(N)}$ of the $N$th EOM-comb mode. Here we focus on the limiting case for an EOM-comb mode with phase noise $S_\phi^{(N)} = N^2 S_\phi$, where $S_\phi$ is a constant microwave oscillator spectrum and $F(\omega)$ is the filter cavity lineshape that we approximate as a rectangle. Even for a state-of-the-art 10 GHz oscillator with thermally limited phase noise of −189 dBc/Hz at +12 dBm at 300 K without filtering, the comb lines needed for self-referencing are estimated to have $\delta\omega^{(19\,340)}/2\pi > 1$ GHz and are practically undetectable. By use of an optical cavity following EOM-comb generation, we sufficiently reduce the contribution from oscillator phase noise. The result is a negligible projected linewidth contribution to the EOM-comb modes needed for self-referencing.

## 2. EXPERIMENTAL STUDIES

### A. EOM Frequency Comb Generation

Our EOM-comb system (Fig. 1) begins with a 1550 nm CW laser that is for convenience stabilized to a high-finesse, low-expansion Fabry–Perot cavity. A frequency comb is derived by way of optical phase and intensity modulation with waveguide lithium-niobate devices at modulation frequency $f_{eo}$ that

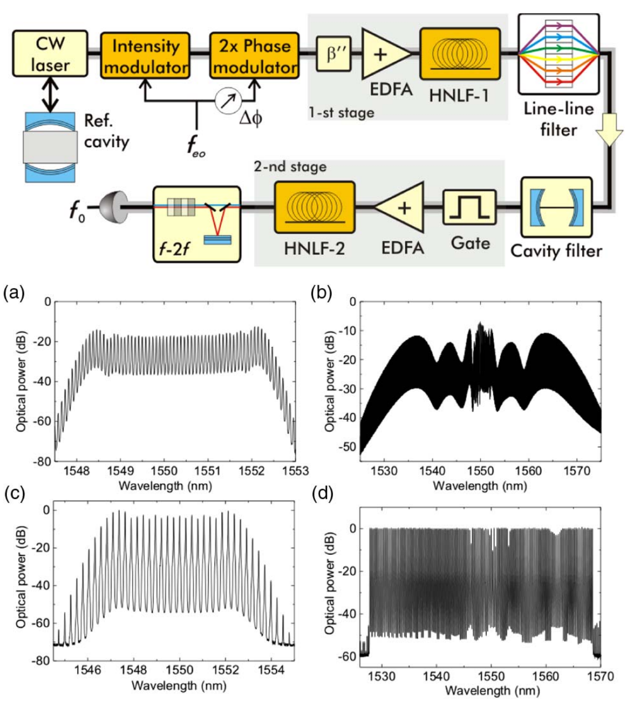


**Fig. 1.** Apparatus for self-referencing a CW laser. The system is comprised of electro-optic phase and intensity modulators, two stages of nonlinear-fiber (HNLF-1 and -2) spectral broadening, a Fabry–Perot optical cavity filter, and $f-2f$ detection. (a) EOM comb with 10 GHz spacing. (b) Idem after HNLF-1. (c) EOM comb with 33 GHz spacing. (d) Idem after HNLF-1 and line-by-line intensity control.

transforms the CW laser into light pulses repeated with each modulation cycle [27–29]. The resulting linear chirp yields pulses whose spectral envelopes are relatively flat, as shown in Figs. 1(a) and 1(c), for combs with 10 and 33 GHz spacing, respectively. Still, the bandwidth of these combs is <1 THz, a factor of 200 too small for self-referencing. Subsequent spectral broadening of the EOM comb exploits the fact that its spectral-phase profile can be compensated using just second-order dispersion [30]. Propagation of the 50% duty cycle pulse train through an appropriate length of 1550 nm single-mode fiber (SMF) allows pulse compression to 1.7 ps, which is near the Fourier-transform limit.

To increase the EOM-comb bandwidth for self-referencing, we utilize two stages of spectral broadening [31] in commercially available highly nonlinear fiber (HNLF). In contrast to earlier results obtained with sub-40-fs titanium–sapphire mode-locked pulses [32], our work opens a new regime in which we realize coherent, octave-spanning frequency combs seeded by relatively long optical pulses. In the first broadening stage, we amplify the EOM comb to 500 mW using a commercial erbium-doped fiber amplifier. Next, the light is guided through a 100 m length of near-zero-dispersion HNLF. The resulting optical spectrum is characteristic of self-phase modulation (SPM); Fig. 1(b) shows the 10 GHz comb following the first broadening stage. In the case of the 33 GHz EOM comb, we show the use of line-by-line power flattening with a spatial light modulator after the SPM broadening [see Fig. 1(d)].

### B. Supercontinuum Generation

Our second broadening stage is designed to coherently increase the EOM-comb span to the >1000 nm needed for $f$-$2f$ self-referencing. Operationally, as shown in Fig. 1, the EOM comb light is reamplified with a cladding-pumped, anomalous dispersion Er/Yb co-doped fiber to yield 140, 400, and 560 pJ pulses for the 33, 10, and 2.5 GHz EOM comb repetition frequencies, respectively. The 2.5 GHz comb is obtained by attenuating three out of every four 10 GHz pulses using a waveguide lithium-niobate intensity modulator [8,21,33,34]. The Er/Yb amplifier offers a maximum average power of 4.5 W, while the temporal intensity autocorrelation of optical pulses exiting the amplifier exhibits <300 fs duration. Prior to the second-stage amplifier, we adjust both the polarization and dispersion of the second stage to maximize supercontinuum generation in HNLF-2. Specifically, second-order dispersion is applied in increments of 0.005 $\mathrm{ps}^2$ using the line-by-line pulse shaper.

The design and implementation of coherent supercontinuum generation at high repetition rate is an advancement of our work. HNLF-2 is composed of two segments of nonlinear fiber with different spectral-dispersion profiles. These are fusion-spliced together with >80% transmission. The first segment has a zero dispersion point near 1300 nm that explains the dispersive wave centered at 1100 nm in Figs. 2(b) and 2(c). The dispersion of the second segment is 1.5 ps/(nm-km), and this fiber generates a dispersive-wave pattern near 1200 nm and broad supercontinuum peaks near 2100 nm. We largely avoid supercontinuum decoherence mechanisms previously associated with >100 fs pulse duration [35]. Figure 2 shows particular examples of the ultrabroad spectra we obtain with our EOM comb. The 10 and 2.5 GHz cases feature more than one octave of bandwidth, and the 10 and 33 GHz comb teeth are prominent across the

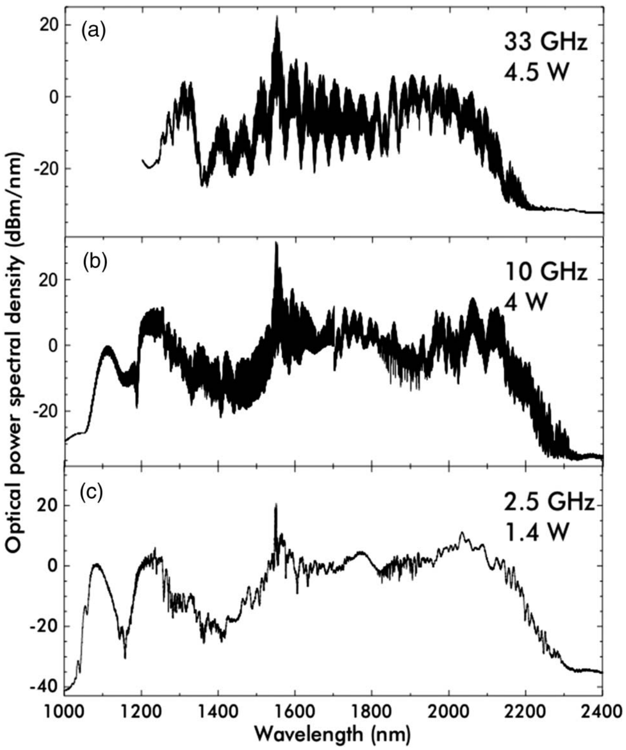


**Fig. 2.** Supercontinuum generated using our second broadening stage with HNLF-2. (a) EOM-comb spacing is 33 GHz. (b) Idem but for 10 GHz spacing. (c) Pulse picking prior to HNLF-2 reduces the initial spacing to 2.5 GHz.

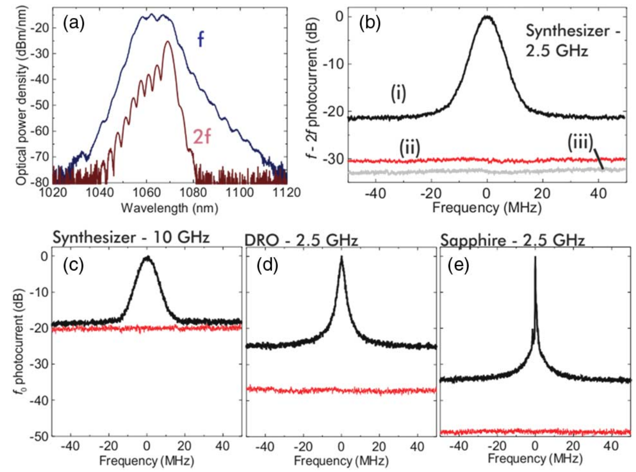


**Fig. 3.** EOM-comb offset-frequency detection. (a) Spectra of fundamental and second-harmonic supercontinuum light at 1070 nm. (b) i, Carrier-offset heterodyne beat ($f_0$) in black; ii, intensity noise in red; iii, detector noise in gray. (c)–(e) Spectrum of $f_0$ with different $f_{eo}$ sources, including a synthesizer, dielectric-resonator oscillator, and sapphire-cavity oscillator. The 2.5 GHz cases utilize pulse picking. All the photocurrent spectra are acquired with 100 kHz bandwidth, and frequency axes are offset 706.6, 1793, 491, and 726 MHz, respectively, in (b)–(d).

entire spectra. (Optical-spectrum-analyzer resolution prevents observation of the 2.5 GHz spaced teeth.)

### C. Carrier-Envelope-Offset-Frequency Detection of an EOM Comb

Following supercontinuum generation, we detect the carrier-envelope-offset frequency ($f_0$) of the EOM comb. A 10 mm sample of periodically poled lithium niobate generates the second harmonic of supercontinuum light at 2140 nm; Fig. 3(a) shows separately obtained optical spectra of the $f$ and $2f$ components at 1070 nm for the 2.5 GHz comb. We photodetect the optical heterodyne beat of these two spectra and optimize alignment of their relative arrival time and polarization. The photocurrent reveals the offset frequency $f_0$ of an EOM comb. This signal represents the generation of an effective optical frequency reference through extreme multiplication of the microwave $f_{eo}$; Fig. 3(b) shows the RF spectrum of the offset beat $f_0$. We have not been able to detect $f_0$ with the filter cavity removed from the system. This Fabry–Perot optical cavity, which includes a mechanical translation stage to match the FSR to $f_{eo}$, features a 7 MHz FWHM linewidth, a ~10 GHz FSR directly locked to $\nu_p$ by feedback to its length, and an insertion loss of 8 dB for the entire SPM-broadened EOM comb. Our unsuccessful search for $f_0$ without the filter cavity involves careful re-optimization of the pulse shaper dispersion setting and all other relevant system parameters. The procedure yields $f$ and $2f$ supercontinuum components comparable to those shown in Fig. 3(a). (Furthermore, we use the highest-performance $f_{eo}$ oscillator available; see our discussion of the sapphire device.) This matches our expectation that the supercontinuum coherence is severely degraded by frequency multiplication of oscillator thermal phase noise, which sits between the modes of the EOM comb. We did not explore placing the filter cavity before HNLF-1 because the dispersion properties of our cavity allow transmission of the complete SPM comb, but future exploration could consider such an arrangement. Future work should consider mode-pulling effects related to the filter cavity, which can be reduced by noise cancelling [36].

We characterize $f_0$ in detail to understand the potential for precision optical measurements with an EOM comb. Electronic spectrum analyzer traces of the 2.5 GHz comb $f_0$ beat and its background contributions are shown in Fig. 3(b); the black trace (i) is the signal, and the red (ii) and gray (iii) traces represent the supercontinuum intensity noise and photodetector noise, respectively. [Fig. 3(c) shows the 10 GHz case, without pulse picking.] Since the EOM comb spacing $f_{eo} = 9.999\ 952$ GHz is locked to a hydrogen-maser frequency reference traceable to the International System second, detecting the center of the $f_0$ spectrum represents an absolute determination of $\nu_p$. Frequency-counting experiments with $f_0$ are enabled by a high signal-to-noise ratio (SNR) and narrow spectral width. The noise floor of $f_0$ exceeds the intensity noise by 8 dB. At present, we cannot determine the relative contributions to this noise floor of supercontinuum generation, which often induces phase noise in $f_0$ [35,37], and electronic noise that is not sufficiently attenuated by the filter cavity. Still, as we show later, the offset beat we obtain is adequate to establish a phase-coherent link between the microwave and optical domains.

The linewidth of $f_0$ is an important consideration for future applications. Previous work has analyzed the relationship between a laser's linewidth and its phase-noise spectrum [38–40]. Building on Ref. [40], we would expect that the EOM comb $f_0$ linewidth is largely determined by the level of $f_{eo}$ phase noise at Fourier frequencies from approximately 10 kHz–10 MHz.

(The filter cavity reduces >3.5 MHz noise.) In this range, a high modulation index leads to the most significant $f_0$ broadening. To directly explore this concept with the EOM comb, we record the power spectrum of $f_0$ using three separate $f_{eo}$ oscillators, which have different 10 kHz–10 MHz phase-noise levels. Figures 3(c)–3(e) are acquired using $f_{eo}$ oscillators with improving phase-noise performance: the same synthesizer as just discussed but excluding pulse picking, a 10 GHz dielectric-resonator oscillator (DRO), and a 10 GHz sapphire-cavity oscillator (sapphire), respectively.

The data traces in Figs. 3(c)–3(e) indicate up to a factor of 40 reduction in linewidth to ~100 kHz with improved $f_{eo}$ performance, which roughly corresponds to the phase-noise specification of these devices. In particular, since pulse picking only divides the pulse-repetition rate, we expect and observe no linewidth change when using the synthesizer for the 10 and 2.5 GHz combs. Our $f_0$ linewidth measurements suggest the DRO (sapphire) devices feature a 10 kHz–10 MHz integrated phase noise at least a factor of ~9(~2200) lower than the synthesizer. Moreover, the filter cavity was included in the setup for these measurements; hence, they highlight the role of frequency-dependent $f_{eo}$ phase noise apart from broadband thermal noise. Future experiments utilizing ultra-low-noise microwave references could realize a <100 kHz linewidth $f_0$ beat that is competitive with some mode-locked lasers [41,42].

### D. Electronic Measurement and Synthesis of an Optical Frequency

Access to the offset frequency of the EOM comb enables precision experiments, including measurement and synthesis of optical frequencies. The schematic in Fig. 4(a) summarizes the optical and electronic connections for our experiments with the synthesizer-driven comb. By measuring $f_0$ with an Agilent 53230A frequency counter, we determine $\nu_p$ with respect to the EOM-comb spacing. The counter records $f_0$ after RF filtering and digital frequency division, which phase-coherently reduce fluctuations. Each 1 s measurement yields $\nu_p$ with a fractional uncertainty of $3 \times 10^{-13}$. As a crosscheck and for comparison, we obtain a separate measurement of $\nu_p$ with respect to the 250.32413 MHz spacing of a self-referenced erbium-fiber frequency comb. Both the EOM-comb and the fiber-comb spacings are referenced to the same hydrogen maser. Figure 4(b) shows a record over 4000 s, during which we monitor the CW laser frequency with the EOM comb (black points) and fiber comb (green points). This data tracks the approximately 65 mHz/s instantaneous linear drift rate of the cavity-stabilized $\nu_p$. Moreover, linear fitting of the two data sets reveals an average offset between them of 17 Hz, which is less than 2 standard deviations of the mean of their combined uncertainty. This represents the accuracy level, in terms of environmental fluctuations that exceed the SNR limits of our digital frequency counting, which our EOM comb system achieves with <22 dB SNR in $f_0$ [43]. Operating the comb with the DRO and sapphire oscillator yield improved results, as expected from their increased SNR.

To understand the precision of such optical frequency-counting measurements, we present their Allan deviation in Fig. 4(c). For less than 100 s averaging, both the EOM and fiber comb instabilities are consistent with their common reference maser's instability of $3 \times 10^{-13}/\sqrt{\tau}$, where $\tau$ is the averaging time.

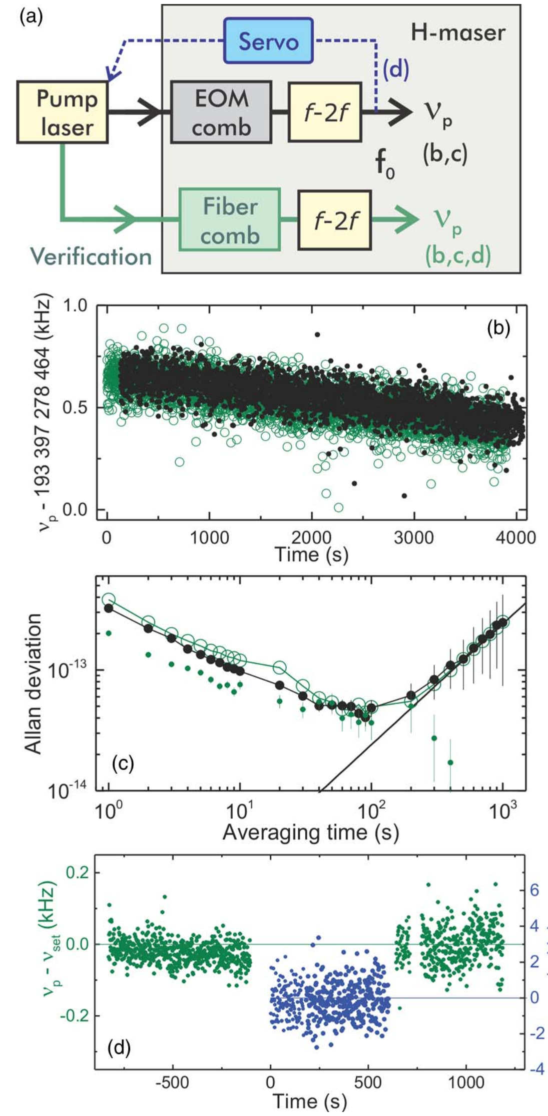


**Fig. 4.** Demonstration of optical frequency measurements and synthesis. (a) System components and connections. Both the EOM comb and an auxiliary fiber comb are referenced to a hydrogen maser. (b) Frequency counting record over 4000 s of the pump laser $\nu_p$ with the EOM (black points) and fiber (green circles) combs. (c) Allan deviation of the frequency count record from (b) for EOM comb (black points) and fiber comb (green circles) and the synthesis data (green points) in (d). The solid line indicates the Allan deviation of a 193 THz laser drifting at 65 mHz/s. (d) Phase-lock synthesis of $\nu_p$ with respect to the maser-referenced $f_{eo}$; $\nu_{set} = 193\,397\,334\,972$ kHz. Green points are measurements of locked $\nu_p$ with the fiber comb, and blue points are the in-loop noise.

Beyond $\tau \sim 100$ s the drift rate of $\nu_p$, indicated by the solid line, becomes apparent. By post-correcting the two datasets with the fitted drift rates, the fractional instability reduces to $3 \times 10^{-14}$ at 1000 s and represents the uncertainty in our determination of the means of the two sets. By comparing $\nu_p$ and $f_{eo}$ of the comb at the input and output of our entire system, we deduce that the near 1 km fiber length of the system contributes to the uncertainty at approximately this level.

Finally, we stabilize the frequency $\nu_p$ of the CW laser using an RF phase-lock of $f_0$ while $f_{eo}$ is held fixed at 9.999 952 GHz.

Operationally, we phase-coherently filter $f_0$ and divide it by 512 prior to discrimination with a digital phase-frequency detector, and feedback with <1 kHz bandwidth is provided to an acousto-optic frequency modulator immediately following the CW laser. The phase-noise spectrum of $f_0$ (see Fig. S5 in Supplement 1) is not substantially changed but the slow drift of $\nu_p$ is reduced. This represents an optimization of our system in which the cavity-stabilized pump laser is much more spectrally pure (but less stable long term) than the effective optical frequency of the multiplied $f_{eo}$. To verify the phase lock, Fig. 4(d) presents two consecutively obtained frequency-counting modalities with $\nu_p$: fiber comb measurements (green points) and in-loop fluctuations in the $\nu_p$ phase lock (blue points). In these experiments, the set point of the $\nu_p$ phase lock is held constant at precisely 193.397 334 972 THz, but this value is adjustable within the CW fiber laser's tuning range of ~30 GHz about the 1.5 GHz FSR modes of the optical reference cavity. The data is semi-continuously acquired at 1 s gate time for 2000 s, as we reconfigure system connections and slightly adjust the phase-lock loop filter. The mean difference of the green points and the phase-lock setpoint is −14(8) Hz, while the frequency offset of the in-loop signal scatters closely about zero. Furthermore, the Allan deviation of the locked $\nu_p$ is presented alongside previously described data in Fig. 4(c). Here the solid green points fall slightly below the other two Allan deviation measurements and do not show the 65 mHz/s cavity drift, since all system components are phase-locked to the same maser reference. This demonstrates the capability to directly synthesize with respect to the maser of the optical frequency of every EOM-comb line.

## 3. CONCLUSION

We have demonstrated optical frequency synthesis and control of a CW laser by way of electro-optic modulation. To accomplish this, we form an EOM comb with 600 GHz initial bandwidth, and then use HNLF-based spectral broadening to create an octave-spanning supercontinuum. A second advancement of our work is the demonstration of optical filtering to preserve the optical coherence of the EOM comb following frequency multiplication from 10 GHz to 193 THz. We demonstrate both precision optical frequency measurements and phase-locked synthesis of all the comb frequencies. These capabilities have already matured in the mode-locked-laser platform. Nevertheless, EOM combs offer important advantages, including wide microwave-rate mode spacing, frequency tuning of the comb, and already-commercialized components that immediately lead to robust systems for a variety of applications in spectroscopy, astronomy, and communications.

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See Supplement 1 for supporting content.

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