J. Quant. Spectrosc. Radiat. Transfer. Vol. 10, pp. 1195–1209. Pergamon Press 1970. Printed in Great Britain

NON-THERMAL EFFECTS IN STARK BROADENING*

W. R. CHAPPELL and J. COOPER

Department of Physics and Astrophysics and Joint Institute for Laboratory Astrophysics†

and

E. W. SMITH

National Bureau of Standards, Boulder, Colorado 80302

(*Received* 7 *April* 1970)

Abstract—When an atom is immersed in a plasma the energy levels are broadened and shifted by the interaction with the fluctuating electric field of the plasma. We discuss the effect of this field on the spectral line shape by the use of results from plasma kinetic theory. In particular we show that in the equilibrium plasma the dominant effect is from single-particle collisions although correlation effects lead to a different result from the usual impact theory. In nonequilibrium plasmas the collective modes can become important and lead to appreciable satellites on the forbidden lines. In the case of a two-temperature electron gas the plasma wave effect is considerably enhanced. The possibility of observing the satellites in unstable plasmas is discussed and the relationship to quasilinear theory is given. In particular, we show that in isotropic systems the line shape can be used to directly observe the turbulent spectrum.

INTRODUCTION

IN AN earlier paper[1] we discussed the fact that the effect of electron correlations on Stark broadening had been incorrectly treated in the usual impact theory.[2] We showed that a well-known result of plasma kinetic theory leads to an expression for $\langle \mathbf{E}(t) \cdot \mathbf{E}(0) \rangle$ which could be interpreted as dynamic double-screening as opposed to the static (or dynamic) single-screened expression proposed by GRIEM *et al.*[2] We indicated very briefly the effect of the additional screening on Griem's Φ_{ab} function and mentioned the possible effects of collective modes.

In the past year there has been increasing interest in the effects of nonequilibrium plasmas[3,4] on line shapes and other applications of plasma kinetic theory[5] to spectral line shapes.

The purpose of this paper is to give a more complete explanation of the "double-screened" result and to discuss the effects of nonequilibrium plasmas on the line shape. We begin by showing that if strong interactions of the atom with the plasma are ignored the line shape can be written in terms of the electric field autocorrelation function $\langle \mathbf{E}(t) \cdot \mathbf{E}(0) \rangle$ in the plasma.

* This work was supported in part by the advanced Research Projects Agency of the Department of Defense and was monitored by Army Research Office-Durham under Contract No. DA-31-124-ARO-D-139.

† Of the National Bureau of Standards and the University of Colorado.

We then introduce ROSTOKER's[6] idea of quasiparticles or dressed test particles and use it to interpret his expression for $\mathbf{E}(t)$. This expression for $\mathbf{E}(t)$ is used to calculate the autocorrelation function by assuming that the plasma consists of noninteracting quasiparticles.

We consider the case of an equilibrium plasma and exhibit the low and high frequency approximations for the autocorrelation function. These approximations are shown to have a wide range of validity when compared to computer results for frequencies in the intermediate region.

We next consider the effect of collective modes (plasma waves, etc.) on the line shape. The main effect is the BARANGER–MOZER[7] plasma satellites on a forbidden line. In the case of a stable plasma the expression derived for the electric field autocorrelation function can be used to obtain the size of these satellites. The more interesting case is probably that of an unstable plasma. We indicate the connection between the line shape theory for unstable plasmas and the theory of unstable or turbulent plasmas. In the case of an isotropic system we show that the turbulent spectrum of the electric field can be directly observed.

THE LINE SHAPE

It is well-known that the effect of a plasma on the line shape can be approximately broken into two regimes. The ions, which move very short distances during the lifetime of the excited state are considered to be static and represented by an electric field probability density $P(\mathbf{E})$. On the other hand, the electrons move great distances during a lifetime and must be treated dynamically. This splitting, although convenient, is very troublesome in that it is well-known that the electron and ion dynamics are intertwined. This coupling is of particular importance with regard to ion acoustic waves which are a low frequency mode. Thus, we might expect to lose some interesting effects by this splitting. It is very unclear how we should proceed if we retain the ion dynamics. However, ion dynamics are known to be important only for frequency separations from the line center less than the ion plasma frequency and this is usually well inside the halfwidth. Consequently, having noted that ion dynamics present an interesting but probably often unimportant problem, we shall henceforth use the static ion approximation (except for further comments in the section on plasma satellites).

The interaction of the atom with the electrons is approximately given by[8]

$$V(\mathbf{r}, t) = -\mathbf{d} \cdot \mathbf{E}(\mathbf{r}, t), \tag{1}$$

where $\mathbf{E}(\mathbf{r}, t)$ is the classical electric field produced by the electrons at the position of the atom at time t. In most theories the atom is taken to be at the origin and the field is written as $\mathbf{E}(t)$. The dipole moment operator is given by $\mathbf{d}$. The only role played by the ions in determining $\mathbf{E}$ is as a neutralizing positive background.

The Hamiltonian for the atom in the plasma can be written as

$$H = H_0 + V, \tag{2}$$

where H_0 is the Hamiltonian for an atom in a static ion field $\mathbf{E}$ possessing eigenstates $\{|a\rangle\}$ and eigenvalues $\{E_a\}$ which are functions of this static ion field.

We consider transitions from a set of upper (initial) states $\{a\}$ to a set of lower (final) states $\{b\}$. Due to interactions with the plasma the line will be broadened and shifted. In many cases the most important contribution to the line shape arises from interactions of atoms in the upper state with the perturbers (the broadening and shift of the lower state being negligible). In such cases the problem is greatly simplified by the assumption of no lower state interactions. In this case, we can write the line shape as[9]

$$I(\omega) = -\frac{1}{\pi}\int \mathrm{d}\mathbf{E}P(\mathbf{E})\,\mathrm{Im}\left\{\sum_{a,a',b}\langle a'|\mathbf{d}|b\rangle\cdot\langle b|\mathbf{d}|a\rangle\langle a|K(\omega)|a'\rangle\rho_{a'}\right\},$$

where $\rho_{a'}$ is the probability the atom was initially in state a' and

$$K(\omega) = [\omega+E_b-H_0-\mathscr{L}(\omega+E^b-H_0)]^{-1} \tag{4}$$

is a resolvant operator and we have chosen $\hbar = 1$.

If we assume that there are only weak interactions between the atom and the plasma, we can take the first term in the expansion of $\mathscr{L}$ in powers of the electric field.[2,8,9]

We then obtain

$$\mathscr{L}(\omega) = -i\int_0^\infty \mathrm{e}^{i\omega t}\langle\tilde{V}(t)\tilde{V}(0)\rangle\,\mathrm{d}t, \tag{5}$$

the operator $\tilde{V}(t)$ is given by

$$\tilde{V}(t) = \mathrm{e}^{itH_0}V(t)\,\mathrm{e}^{-itH_0} \tag{6}$$

and the brackets denote an ensemble average over the coordinates and momenta of the electrons. We can rewrite equation (5) in the form

$$\mathscr{L}(\omega) = -i\int_0^\infty \mathrm{e}^{i\omega t}\mathbf{d}(t)\mathbf{d}(0):\langle\mathbf{E}(t)\mathbf{E}(0)\rangle\,\mathrm{d}t, \tag{7}$$

where

$$\mathbf{d}(t) = \mathrm{e}^{itH_0}\mathbf{d}\,\mathrm{e}^{-itH_0}. \tag{8}$$

We see then that the electron contribution to the line shape is given by the Laplace transform of the electric field autocorrelation function.

Of course, there still remain difficult calculational problems involved with matrix inversion and the ion field average,[8] but these are not of direct concern to us here.

ELECTRIC FIELD AUTOCORRELATION FUNCTION

The electric field autocorrelation function $\langle\mathbf{E}(t)\cdot\mathbf{E}(t')\rangle$ is of fundamental importance to plasma kinetic theory. It has been shown, for example, that its Fourier transform in space and time describes the scattering of radiation by the plasma.[10] The production of bremsstrahlung radiation[11] by electron–ion collisions is closely related to the Fourier transform in time. Because of this important role its properties are reasonably well-known. However, the quantity most intensively studied has been the Fourier transform over space and time of $\langle \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{E}(0, 0) \rangle$ whereas the quantity of interest to us here is the Laplace transform in time of $\langle \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, 0) \rangle$.[12]

ROSTOKER[6] has developed a very useful description of a plasma which involves looking at the plasma as a collection of statistically independent "dressed" test particles (or quasi-particles). This method is particularly appropriate here because the previous work on the effect of electron correlations on line shapes[2,8,13] have relied on a single-particle picture in which the electrons are considered to travel past the atom on a straight line.

In a similar manner the quasiparticle picture depicts the electrons moving along a straight line. However, since the electron polarizes the surrounding plasma, it carries with it a polarization cloud of other particles. The field felt by the atom is the sum of the field due to the electron and the field due to the polarization cloud. The electron plus its polarization cloud is called a quasiparticle and it is possible to write down a phase space density describing the quasiparticle.[14]

It is very easy to write down the field due to the quasiparticle. If the electrons in the plasma did not interact with each other, the field at the origin due to an electron at $\mathbf{r}' = \mathbf{r}_0 + \mathbf{v}_0 t$ would be

$$\mathbf{E}^0(\mathbf{r}') = 4\pi e \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} \frac{i\mathbf{k}}{k^2} \mathrm{e}^{-i\mathbf{k}'\cdot\mathbf{r}'}. \tag{9}$$

This is the field due to a single electron traveling in a straight line initial position and momentum $(\mathbf{r}_0, \mathbf{v}_0)$. If we include the field due to the polarization cloud, we obtain[6,15]

$$\mathbf{E}_{sc}(\mathbf{r}') = 4\pi e \int \frac{\mathrm{d}\mathbf{k} i\mathbf{k} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}'}}{(2\pi)^3 k^2 \varepsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{v})}, \tag{10}$$

where

$$\varepsilon(\mathbf{k}, \omega) = 1 + \omega_p^2/k^2 \int \mathrm{d}\mathbf{v} \frac{\mathbf{k}\cdot\partial f(v)/\partial\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v} + i\eta}. \tag{11}$$

The quantities $\omega_p, f(v)$, and η are the plasma frequency $(4\pi n e^2/m)^{1/2}$, the one-electron distribution function, and the positive infinitesimal, respectively. The quantity $\varepsilon(\mathbf{k}, \omega)$ is the wave number- and frequency-dependent dielectric constant. The effect of the polarization cloud is to give rise to a screening of the ordinary Coulomb interaction. We note that the screening depends on the wavenumber of the particle Fourier component and the velocity of the particle. Because of the velocity dependence, the field is said to be dynamically screened. The total electric field is then given by[6,14,15]

$$\mathbf{E}(t) = \sum_{i=1}^{N} 4\pi e \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} \frac{\mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{r}_i + \mathbf{v}_i t)} i\mathbf{k}}{k^2 \varepsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{v})}, \tag{12}$$

that is, as a sum over all quasiparticles. This expression is valid only for times less than the 90° deflection time and neglects the effects of strong, short-range collisions between the electrons and three body correlations.[6,10,14]

The autocorrelation function implies an average over the initial coordinates and velocities of the particles. In the quasiparticle or dressed test particle picture this is written in the form[6,14,15]

$$\langle \mathbf{E}(t)\cdot\mathbf{E}(t')\rangle = V^{-N}\int\cdots\int d\mathbf{r}_1\cdots d\mathbf{r}_N\, d\mathbf{v}_1\cdots d\mathbf{v}_N$$

$$\times\frac{(4\pi e)^2}{(2\pi)^6}\int d\mathbf{k}\int d\mathbf{k}'\sum_{ij} e^{-i\mathbf{k}\cdot(\mathbf{r}_i+\mathbf{v}_i t)}$$

$$\times e^{-i\mathbf{k}'\cdot(\mathbf{r}_j+\mathbf{v}_j t')}\frac{i\mathbf{k}\cdot i\mathbf{k}' f(\mathbf{v}_1)\cdots f(\mathbf{v}_N)}{k^2k'^2\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_i)\varepsilon(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v}_j)} \tag{13}$$

$$= \frac{(4\pi e)^2 n}{(2\pi)^3}\int d\mathbf{v}\int d\mathbf{k}\frac{k^2 f(v)}{k^4|\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2}\times e^{-i\mathbf{k}\cdot\mathbf{v}(t-t')}, \tag{14}$$

where we used $\varepsilon(-\mathbf{k}, -\omega) = \varepsilon(\mathbf{k},\omega)$.* The statistical independence of the quasiparticles implies that there are no correlations between the initial positions of the electrons. All the effects of interactions are included in the dielectric constant. This result is valid to lowest nontrivial order in the plasma smallness parameter $(n\lambda_D^3)^{-1}$ (the inverse of the number of particles in a Debye sphere). This, as stated before, neglects three body and higher order correlations and strong, close collisions. The result given by equation (14) has been obtained by other methods in the same order of approximation.[10] However, none of the other methods are as capable of a single-particle interpretation as Rostoker's method.

We can rewrite the autocorrelation function in a form that is particularly appropriate for comparison with previous work on line shapes. That is, in the form

$$\langle \mathbf{E}(0)\cdot\mathbf{E}(t)\rangle = n\int d\mathbf{r}_0\int d\mathbf{v}_0 f(\mathbf{v}_0)\mathbf{E}_{sc}(\mathbf{r}_0)\cdot\mathbf{E}_{sc}(\mathbf{r}_0+\mathbf{v}_0 t), \tag{15}$$

where $\mathbf{E}_{sc}(\mathbf{r}_0)$ is the dynamically screened field at the atom due to a quasiparticle at $\mathbf{r}_0$ and is given by equation (10). Griem *et al.*[2] arrived at an expression for the autocorrelation function which involved only one screened field. Since they were working in the impact approximation which corresponds to $\Delta\omega = 0$, they chose that screening to be static. That is, the appropriate dielectric constant was $\varepsilon(\mathbf{k}, 0)$.

We see that the difference between our results and the previous one is that each of the fields in equation (15) is screened dynamically. We therefore will use the term "double-screening" to refer to this result in the remainder of the paper.

It is easy to show that

$$\mathscr{L}(\Delta\omega) = -\frac{i}{3}d^2 g(\Delta\omega), \tag{16}$$

where[6,14,15]

$$g(\Delta\omega) = \int_0^\infty e^{i\Delta\omega t}\langle \mathbf{E}(t)\cdot E(0)\rangle\, dt = \frac{i(4\pi e)^2 n}{(2\pi)^3}\int d\mathbf{k}\int d\mathbf{v}\frac{f(v)}{k^2|\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2(\Delta\omega-\mathbf{k}\cdot\mathbf{v}+i\eta')}. \tag{17}$$

We note that

$$\operatorname{Re} g(\Delta\omega) = g_r(\Delta\omega) = \frac{(4\pi e)^2 n}{2(2\pi)^2}\int \frac{d\mathbf{k}}{k^2|\varepsilon(\mathbf{k},\Delta\omega)|^2}\int d\mathbf{v} f(v)\delta(\Delta\omega - \mathbf{k}\cdot\mathbf{v}) \tag{18}$$

and

$$\operatorname{Im} g(\Delta\omega) = g_I(\Delta\omega) = \frac{(4\pi e)^2 n}{(2\pi)^3}\int \frac{d\mathbf{k}}{k^2} P\int d\mathbf{v}\frac{f(\mathbf{v})}{|\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2(\Delta\omega - \mathbf{k}\cdot\mathbf{v})}, \tag{19}$$

where P denotes the Cauchy principle value integral and $\Delta\omega$ measures either the distance from the allowed line or the distance from the forbidden line depending on the case of interest as we will see in the next section.

The quantity $g_r(\Delta\omega)$ is essentially proportional to the width of the line (there is some mixing of g_r and g_I when the resolvant in equation (4) is evaluated). It also plays an important role in plasma kinetic theory since it is proportional to the resistivity of a plasma. In this context it was studied extensively by DAWSON and OBERMAN.[16]

This quantity can be written in a somewhat simpler form if we assume the plasma is in equilibrium. In that case, we can write

$$g(\Delta\omega) = \left(\frac{8nme^2}{kT}\right)^{1/2} G(\Delta\omega) \tag{20}$$

where

$$G(\Delta\omega) = \int_{z_0}^{\infty} \frac{dz\, e^{-\Omega^2 z^2}}{z|\varepsilon_1(z,\Omega)|^2}, \tag{21}$$

where

$$\Omega = \Delta\omega/\omega_p, \qquad z = (\sqrt{(2)}k\lambda_D)^{-1},$$

and[17]

$$\varepsilon_1(z,\Omega) = 1 + 2z^2\left(1 - 2\Omega z\int_0^{\Omega z} e^{t^2}\, dt + i\sqrt{(\pi)}\Omega z\, e^{-\Omega^2 z^2}\right) \tag{22}$$

and we have introduced a strong collision cutoff z_0 which is equal to $\lambda_m/\sqrt{(2)}\lambda_D$ where λ_m is the minimum impact parameter.

The quantity $G(\Delta\omega)$ is easily evaluated in the limits $\Delta\omega \ll \omega_p$ (which corresponds to the impact theory region) and $\Delta\omega \gg \omega_p$. In these cases $\varepsilon(\mathbf{k},\Delta\omega)$ takes a simple form. For $\Delta\omega \ll \omega_p$, $\varepsilon(\mathbf{k},\Delta\omega)$ is approximately equal to $\varepsilon(\mathbf{k},0) = 1 + (k_D^2/k^2)$.[17] For $\Delta\omega \gg \omega_p$, $\varepsilon(\mathbf{k},\Delta\omega)$ is approximately unity.[17] Consequently, for $\Delta\omega \ll \omega_p$

$$\begin{aligned} G(\Delta\omega) &\simeq \ln\frac{\lambda_D}{\lambda_{\min}} - \frac{1}{2}, \\ &= \ln\frac{\lambda_D}{\sqrt{(e)}\lambda_m} \end{aligned} \tag{23}$$

and for $\Delta\omega \gg \omega_p$,

$$G(\Delta\omega) \simeq \int_{z_0}^{\infty} dz \frac{e^{-\Omega^2 z^2}}{z} \simeq \ln\frac{\sqrt{(2)}\lambda_D}{\lambda_m \Omega} - \frac{\gamma}{2} \tag{24}$$

where γ is Euler's constant 0·577 . . . .[16] The low frequency result differs from the impact theory results by the factor of $\sqrt{e}$ in the logarithm. Using the same minimum impact parameter, this factor leads to about a 15–20 per cent difference between our results and Griem's Φ_{ab}. The high frequency result agrees with the Lewis limit.[18]

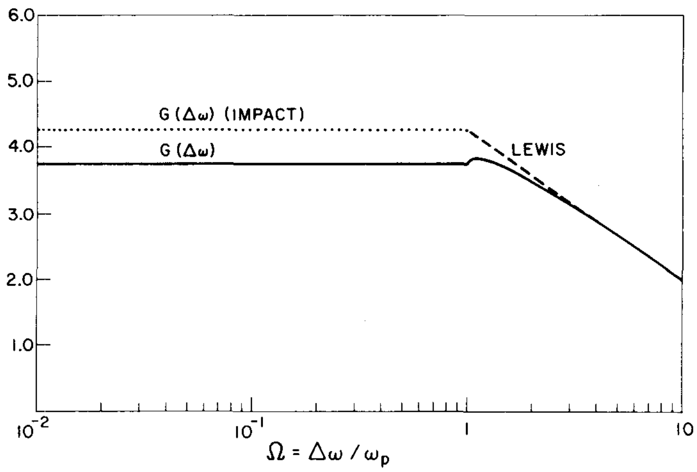

FIG. 1. A comparison of the equilibrium $G(\Delta\omega)$ in equation 21 (solid line) with the "single-screened" impact result[2] (dotted line) and the LEWIS result[18] (dashed line) for $z_0 = 10^{-2}$.

It is quite remarkable that the low frequency approximation is very good over the range $0 \le \Delta\omega \le 1{\cdot}7\omega_p$ and the high frequency approximation is very good over the range $\Delta\omega \ge 1{\cdot}7\omega_p$. This can be seen quite clearly from the results of numerical integration shown in Fig. 1. A similar figure was obtained by DAWSON and OBERMAN.[16] We have also confirmed Dawson and Oberman's observation that the results for a different minimum impact parameter λ_m' are obtained from those for λ_m by adding $\ln \lambda_m/\lambda_m'$ to $G(\Delta\omega)$. We have checked this over the range

$$10^{-7}\lambda_D < \lambda_m < 10^{-1}\lambda_D.$$

It should be noted that our treatment breaks down for times of the order of and greater than the 90° deflection time. However, the electric field autocorrelation function goes to zero after a time $\sim 1/\omega_p$, hence these errors should be negligible when the 90° deflection time is greater than $1/\omega_p$. This limitation also applies to the usual impact theory[2] where the time between strong collisions (which usually is less than or comparable to the 90° deflection time) has to be large compared with the duration of the collision (at most $\sim 1/\omega_p$). This restriction is equivalent to requiring that there be many particles in a Debye sphere.

The integral over k can be decomposed into a part for $k \leq k_D$ and another for $k \geq k_D$. The small k part may be regarded as being due to plasma oscillations and the large k part is interpreted as the "collisional" part. The wave part is responsible for the small bump near ω_p because the dispersion relation for the waves forces the contribution to be concentrated near ω_p. On the other hand, the collisional part contributes over the entire frequency spectrum and in the equilibrium case is the dominant component.

The wave part, as we shall see, can be significantly enhanced by small changes in the velocity distribution function. While the collisional part is largely unaffected by small changes in $f(v)$.

PLASMA SATELLITES

The previous paragraph suggests that we might see structure on the line near the plasma frequency. There is, however, an additional difficulty in that the average over the ion microfield can smear out the effect of plasma peaks appearing in $\mathscr{L}$. The most hopeful cases are those where forbidden lines occur on the wings of the allowed lines.(7) In such cases we can expand the resolvant in inverse powers of $\Delta\omega_{ab}$(= separation from the allowed line) to obtain

$$I(\omega) = I_i(\omega) + I_e(\omega), \tag{25}$$

where

$$I_i(\omega) = \int \mathrm{d}\mathbf{E}P(\mathbf{E}) \sum_{a,b} \delta(\Delta\omega_{ab})|\langle a|d|b\rangle|^2 \tag{26}$$

and

$$I_e(\omega) = -\pi^{-1} \int \mathrm{d}\mathbf{E}P(\mathbf{E})\, \mathrm{Im} \sum_{a,a',b} (\Delta\omega_{ab})^{-1}(\Delta\omega_{a'b})^{-1} \times \langle a'|\mathbf{d}|b\rangle \cdot \langle b|\mathbf{d}|a'\rangle\langle a|\mathscr{L}(\Delta\omega_{ab})|a'\rangle + \cdots, \tag{27}$$

are the ion and electron contributions to the line. We recall that $|a\rangle$ and $|b\rangle$ depend on the ion field strength since the ion field is included in the unperturbed Hamiltonian. This means that radiation will appear at the "forbidden" line. To illustrate the effect of the plasma satellites on the forbidden line which will occur even in the absence of any ion field, we will for simplicity assume that the field dependence of the states is small. We then simplify the problem further by considering a three-state atom consisting of a ground state b and two excited states a and c having the properties that the transition $a \rightarrow b$ by dipole radiation is allowed, the transition $c \rightarrow b$ is forbidden. Moreover ω_{cb} falls on the wings of the line centered at ω_{ab}.

In this case the only significant contribution to $I_e(\omega)$ in the region of the forbidden line is given by

$$I_e(\omega) = \pi^{-1}(\Delta\omega_{ab})^{-2}|\langle a|\mathbf{d}|b\rangle|^2 \int_0^\infty \mathrm{e}^{i\Delta\omega_{ab}t}\langle a|\,\mathrm{e}^{iE_a t}\mathbf{d}(0)\,\mathrm{e}^{-iE_c t}|c\rangle \times \langle c|\mathbf{d}(0)|a\rangle : \langle \mathbf{E}(t)\mathbf{E}(0)\rangle\, \mathrm{d}t, \tag{28}$$

where we have inserted the intermediate state $|c\rangle$ in order to evaluate $\langle a|\mathscr{L}(\Delta\omega_{ab})|a\rangle$. The effect is to change the frequency variable from $\Delta\omega_{ab}$ to $\Delta\omega_{cb}$ (the separation from the forbidden line). As a consequence we find that

$$I_e(\omega) = \frac{|\langle a|\mathbf{d}|b\rangle|^2|\langle a|\mathbf{d}|c\rangle|^2}{3\pi(\Delta\omega_{ab})^2} g_r(\Delta\omega_{cb}). \tag{29}$$

In 1961, BARANGER and MOZER[7] pointed out that the presence of oscillatory behavior in $\mathbf{E}(t)$ at the plasma frequency ω_p could give rise to sidebands on either side of the forbidden line. It is easy to demonstrate this effect. If we simply choose

$$\mathbf{E}(t) = \mathbf{E}_0 \cos \omega_p t, \tag{30}$$

we obtain

$$g_r(\Delta\omega_{cb}) = \frac{\pi E_0^2}{2}[\delta(\Delta\omega_{cb}+\omega_p)+\delta(\Delta\omega_{cb}-\omega_p)] \tag{31}$$

giving rise to lines at $\omega = \omega_{cb} \pm \omega_p$. In fact, the theory of the ac Stark effect shows there will be an infinite number of side bands, coming from higher powers of V, centered about the forbidden line.[19] It is possible to look upon the effect as arising from a second-order process in which, for example, an atom in state c makes a transition to state a by emitting a plasmon and then makes a transition to state b by emitting a photon.

BARANGER and MOZER[7] pointed out that the collective modes (plasma waves) in a plasma could give rise to such an effect. They then discussed the possibility of observing these sidebands on forbidden lines in He. They came to the conclusion that this would be impossible in an equilibrium plasma because of the low level of excitation of the waves, but might well be possible in nonequilibrium plasmas. The problem with their results was that there was no way of predicting what kind of plasma would give rise to enhancements of the plasma bump.

The use of the results of plasma kinetic theory can give us such information. The expression for $g(\Delta\omega)$ in equation (17) is valid as long as the plasma is reasonably stable (i.e. the plasma waves have a finite lifetime). When the plasma is marginally stable or unstable one must appeal to a more complicated theory.

The presence of collective modes is accounted for in equation (17) by the zeros in $\varepsilon(\mathbf{k}, \omega)$ as a function of ω. For a stable plasma these zeros are in the lower half ω-plane and the closest zero to the real axis occurs near $\omega = \pm\omega_p$.[20] These zeros in $\varepsilon(\mathbf{k}, \omega)$ then lead to resonances in the integrand of $g_r(\Delta\omega_{cb})$ near $\Delta\omega_{cb} = \pm\omega_p$ giving rise to the Baranger–Mozer sidebands. In the equilibrium case these sidebands are represented by the small bump at ω_p in Fig. 1 (which is symmetric about $\omega = 0$) thus verifying that the effect is insignificant in an equilibrium plasma.

On the other hand, it is possible to construct models of nonequilibrium plasmas where sufficient plasma waves are excited to produce a noticeable effect.

It is possible to show that the area under the plasma bump in $g_r(\Delta\omega)$ is proportional to the mean energy density of the plasma waves.[21] Thus

$$\int_{\omega_p-\delta}^{\omega_p+\delta} \mathrm{d}\omega g_r(\Delta\omega) \propto \int_0^{k_D} k\,\mathrm{d}k \frac{F(\omega_p/k)}{|F'(\omega_p/k)|} \tag{32}$$

where

$$F(u) = \int \mathrm{d}\mathbf{v} f(v)\delta\left(u - \frac{\mathbf{k}\cdot\mathbf{v}}{k}\right). \tag{33}$$

The denominator in the integrand can be reduced by a nonthermal high velocity tail on the electron distribution.[22] We expect the corresponding increase of area to be due to an increase of the height of the plasma bump at ω_p.

The area under the bump is essentially proportional to the rate of emission of radiation by electron-ion bremsstrahlung,[23,24] hence it is quite natural to consider models that have been proposed for generating enhanced bremsstrahlung by collective effects. These models generally involve placing hot electrons into a background plasma which then generate plasma waves by a Cerenkov-like effect.

We have calculated $g_r(\Delta\omega)$ for TIDMAN and DUPREE's two-temperature electron gas.[23] This system is composed of a background plasma described by a Maxwell–Boltzmann distribution with density n_1 and temperature T_1. Superposed on the background plasma is a group of hot electrons also described by a Maxwell–Boltzmann distribution with temperature $T_2 \gg T_1$ and density $n_2 \ll n_1$. The production of plasma waves is enhanced because of the relative increase in the number of particles in the region $v > \sqrt{(kT_1/m)}$ and a subsequent decrease in the derivative of the distribution function in the same region. In this case the quantity $G(\Delta\omega)$ defined in equation (20) is given by

$$G(\Delta\omega) = \int_{z_0}^{\infty} \mathrm{d}z \frac{\mathrm{e}^{-\Omega^2 z^2} + \alpha^{-1}\gamma\,\mathrm{e}^{-\gamma^2\Omega^2 z^2}}{z|\varepsilon_1(z,\Omega)|^2} \tag{34}$$

where

$$\varepsilon_1(z,\Omega) = 1 + 2z^2\left[1 - 2\Omega z \int_0^{\Omega z} \mathrm{e}^{t^2}\,\mathrm{d}t + i\sqrt{(\pi)}\Omega z\,\mathrm{e}^{-\Omega^2 z^2}\right]$$

$$+ 2\delta^2 z^2\left[1 - 2\gamma\Omega z \int_0^{\gamma\Omega z} \mathrm{e}^{t^2}\,\mathrm{d}t + i\sqrt{(\pi)}\gamma\Omega z\,\mathrm{e}^{-\gamma^2\Omega^2 z^2}\right], \tag{35}$$

and $\gamma = (T_1/T_2)^{1/2}$, $\alpha = n_1/n_2$, and $\delta = (n_2 T_1/n_1 T_2)^{1/2}$. The plasma resonance occurs for $z \gg 1$ at $\omega \simeq \omega_p$. In this region the numerator of the integrand in equation (34) is dominated by the second term which arises from the hot electrons. Similarly the denominator is dominated by the last term in Eq. (35) which also arises from the hot electrons. The integrand is therefore a factor $\alpha\delta^{-4}\gamma^{-1}\,\mathrm{e}^{\gamma^2\Omega^2 z^2}$ greater than it would be if the hot electrons were absent. The effect of this is shown in Fig. 2 which was obtained by numerical integration for the case $n_1/n_2 = 10^8$, $T_1/T_2 = 10^{-2}$, and $z_0 = 10^{-7}$. We note that there is a dramatic enhancement of the plasma bump over its equilibrium value. Since the plasma wave contribution arises from small values of k this contribution is largely independent of the minimum impact parameter. It is important to note that one must build up the resonance over a finite range of $\mathbf{k}$ because of the integration over $\mathbf{k}$.

This is one example of a system where the plasma wave contribution is significant. Another model proposed by TIDMAN *et al.*[24] involves a plasma in a magnetic field. In this case there is a group of hot electrons which has a kinetic temperature perpendicular to the magnetic field which is much larger than the temperature parallel to the field and which has a drift velocity through the thermal plasma along the field. For weak fields ($\omega_c \ll \omega_p$) where ω_c is the electron cyclotron frequency, this gives rise to a splitting of the resonance into two peaks separated by $\omega_c^2/2\omega_p$.

These models indicate that we can find systems in which the plasma waves yield a significant contribution to $g_r(\Delta\omega)$. This large enhancement of the plasma bump at ω_p then gives the Baranger–Mozer satellites. We note that, as Baranger and Mozer pointed out, the $(\Delta\omega_{ab})^{-2}$ factor in equation (29) causes the satellite further from the allowed line to be smaller than the near satellite.

The results given here neglect all ion dynamics. The reason for this simplification as we mentioned earlier, was the fact that the effects of ions and electrons have traditionally

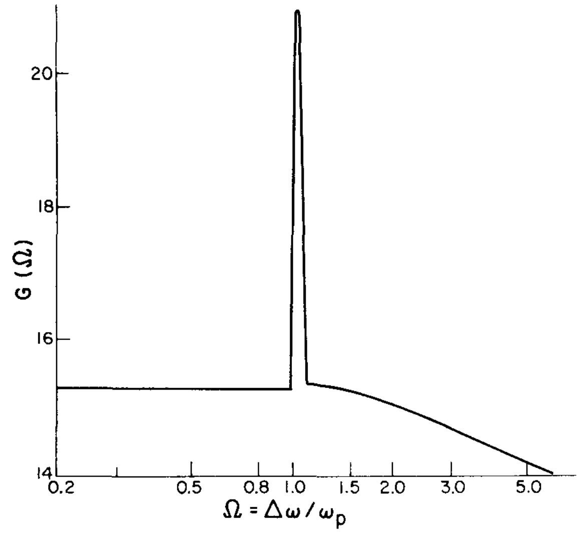

FIG. 2. $G(\Delta\omega)$ for the two-temperature electron gas.

been considered separately. There have been some recent attempts at a theory[5] which might avoid this problem but the general situation is unclear.

Although line broadening theory is at present incapable of a unified treatment of electrons and ions this is not the case with plasma kinetic theory. Rostoker's formalism can easily be extended to include ion motion. In this case there are "dressed" test electrons and "dressed" test ions which travel in straight lines enveloped in polarization clouds consisting of electrons and ions. The electric field due to the electrons, $\mathbf{E}_e$, and that due to the ions, $\mathbf{E}_i$, each have a fast component (due to the electrons) and a slow component (due to the ions). If one wishes to break the broadening into a quasi-static part and an impact part as before presumably the slow components of *both* $\mathbf{E}_e$ and $\mathbf{E}_i$ should be included in the quasi-static theory and the fast components in an impact-like theory. Another point is that there are now four nonvanishing autocorrelation functions $\langle \mathbf{E}_e(0) \cdot \mathbf{E}_e(t) \rangle$, $\langle \mathbf{E}_e(0) \cdot \mathbf{E}_i(t) \rangle$, $\langle \mathbf{E}_i(0) \cdot \mathbf{E}_i(t) \rangle$ and finally $\langle \mathbf{E}(0) \cdot \mathbf{E}(t) \rangle$ where

$$\mathbf{E}(t) = \mathbf{E}_e(t) + \mathbf{E}_i(t). \tag{36}$$

The relative contributions from these various autocorrelation functions vary with frequency. For frequencies, in the range of ω_p, however the dynamics of the ions can be ignored.

There is one case where the electrons and ions can be treated in a unified manner and which could be amenable to experiments. If there is a relative drift of the emitters and the plasma which is fast enough that both the electrons and ions could be treated by the impact theory. This drift velocity should be of the order of the Debye length times the linewidth. In this case the autocorrelation function would become $\langle \mathbf{E}(0, 0) \cdot \mathbf{E}(\mathbf{v}_0 t, t) \rangle$, where $\mathbf{E}(\mathbf{v}_0 t, t)$ is the *total* electric field at the point $\mathbf{v}_0 t$ and time t, and $\mathbf{v}_0$ is the drift velocity. When ion dynamics are included there is another collective mode present known as the ion acoustic mode. This mode can become even more spectacularly enhanced than the electron plasma waves.[22]

UNSTABLE PLASMAS

In the previous section we pointed out that the plasma bump in $g_r(\Delta\omega)$ can be significantly enhanced in certain nonthermal plasmas where additional energy is pumped into the plasma waves. Although we confined ourselves to stable plasmas, it should be clear that the best hunting ground for the plasma satellites would be systems in which some of the collective modes become unstable. In this case the wave amplitudes would grow in time until damped by nonlinear processes. Because this problem essentially involves the solution of the nonlinear Vlasov equation there is no general agreement concerning the theoretical treatment of such systems. However, the problem of weakly unstable systems is often treated by means of the so-called quasilinear theory and we will indicate how this theory applies to the plasma satellite problem.

The collective modes are defined by solutions of the linearized Vlasov equation which lead to oscillations in the Fourier components, $\mathbf{E}_\mathbf{k}(t)$, of the electric field. For these modes we can write

$$\mathbf{E}_k(t') = \mathbf{E}_k(t)\, \mathrm{e}^{-i\omega_k(t-t') - \gamma_k(t-t')}. \tag{37}$$

In the case of an unstable plasma there is some range of wave numbers such that γ_k becomes negative and the waves begin to grow and eventually the linearized Vlasov equation is no longer valid. At this point the growth begins to affect the velocity distribution function which determines γ_k. In the quasilinear theory this leads to a diffusion in velocity space and the velocity distribution function is finally changed to such an extent that γ_k eventually goes to zero and finally becomes negative thereby terminating the growth of the wave amplitudes. This process is assumed to take place over many plasma oscillations and the effect of collisional processes is assumed to be negligible in comparison.

We shall concentrate our attention entirely on the collective modes and write

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{(2\pi)^3} \int \mathrm{d}\mathbf{k}\, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}} \mathbf{E}_\mathbf{k}(t) \tag{38}$$

for the electric field at the point $\mathbf{r}$ and time t due to these modes. If $\mathbf{E}_{\mathbf{k}}(t)$ has the property given in equation (37), we can write

$$\langle \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t') \rangle = \frac{1}{(2\pi)^6} \int \mathrm{d}\mathbf{k} \int \mathrm{d}\mathbf{k}' \, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}} \, \mathrm{e}^{-i\mathbf{k}'\cdot\mathbf{r}} \times \langle \mathbf{E}_{\mathbf{k}}(t) \cdot \mathbf{E}_{\mathbf{k}'}(t) \rangle \, \mathrm{e}^{-i\omega_{k}'(t-t')} \, \mathrm{e}^{-\gamma_k(t-t')}. \tag{39}$$

We assume the system to be homogeneous which allows us to average over the spatial coordinate giving

$$\langle \mathbf{E}(t) \cdot \mathbf{E}(t') \rangle = 8\pi \int \mathrm{d}\mathbf{k} \varepsilon_{\mathbf{k}}(t) \, \mathrm{e}^{-i\omega_k(t-t')} \, \mathrm{e}^{-\gamma_k(t-t')} \tag{40}$$

where

$$\varepsilon_k(t) = \lim_{V\to\infty} \frac{|\mathbf{E}_k(t)|^2}{64\pi^4 V}, \tag{41}$$

where V is the volume of the system. We choose this definition in order to agree with Bernstein and Engelmann.[25]

The quantity $g_r(\Delta\omega)$ is then given by

$$g_r(\Delta\omega) = \mathrm{Re} \int_0^\infty \mathrm{d}\tau \, \mathrm{e}^{i\Delta\omega\tau} \langle \mathbf{E}(t) \cdot \mathbf{E}(t+\tau) \rangle$$

$$= 8\pi \int \mathrm{d}\mathbf{k} \frac{\varepsilon_{\mathbf{k}}(t)\gamma_k}{(\Delta\omega - \omega_k)^2 + \gamma_k^2}, \tag{42}$$

where we assumed $\varepsilon_{\mathbf{k}}(t)$ to be slowly varying in time (quasilinear theory[25] shows it to change over a time period of the inverse of the maximum growth rate which is assumed to be many plasma periods).

The quantity $\varepsilon_{\mathbf{k}}(t)$ is proportional to the energy density in the kth mode at time t. In the quasilinear theory it is determined by the set of coupled equations[25]

$$\frac{\partial \varepsilon_k}{\partial t} = -2\gamma_k \varepsilon_k, \tag{43}$$

and

$$\frac{\partial f(\mathbf{v})}{\partial t} = \nabla_{\mathbf{v}} \cdot (\mathbf{D} \cdot \nabla_{\mathbf{v}}) f(\mathbf{v}), \tag{44}$$

where γ_k is a functional of $f(\mathbf{v})$[25] and

$$\mathbf{D}(v, t) = \frac{8\pi^2 e^2}{m^2} \int \mathrm{d}\mathbf{k} \varepsilon_{\mathbf{k}}(t) \frac{\mathbf{k}\mathbf{k}}{k^2} \frac{\gamma_k}{[(\omega_k - \mathbf{k}\cdot\mathbf{v})^2 + \gamma_k^2]} \tag{45}$$

is the diffusion tensor.

If the system is initially unstable, equations (43) and (44) lead to an initial growth of $\varepsilon_{\mathbf{k}}(t)$ above the thermal level. As $\varepsilon_{\mathbf{k}}(t)$ grows, it begins to affect $f(\mathbf{v})$ through the diffusion tensor in such a way as to slow down and finally reverse the growth. In order to follow this process, we need to know the initial distribution function. We see that equation (42) will then lead to an enhancement of the plasma satellites which are at the frequency ω_k.

It is interesting to note that if the emitters are moving at a uniform velocity $\mathbf{v}_0$ with respect to the plasma the quantity $g_r(\Delta\omega)$ has an appearance very similar to the trace of the diffusion tensor because of the Doppler shift in $\Delta\omega$. That is

$$g_r(\Delta\omega) = 8\pi \int \mathrm{d}\mathbf{k}\frac{\varepsilon_{\mathbf{k}}(t)\gamma_k}{(\Delta\omega+\mathbf{k}\cdot\mathbf{v}_0-\omega_k)^2+\gamma_k^2}. \tag{46}$$

In the case where $|\gamma_k| \ll \omega_k$, i.e. weakly turbulent systems, we can approximate the expression given for $g_r(\Delta\omega)$ in equation (42) by

$$g_r(\Delta\omega) = 8\pi^2 \int \mathrm{d}\mathbf{k}\varepsilon_{\mathbf{k}}(t)\delta(\Delta\omega-\omega_k). \tag{47}$$

If the turbulence is isotropic, $\varepsilon_{\mathbf{k}}$ and $\omega_{\mathbf{k}}$ depend only on $|\mathbf{k}|$, and we obtain

$$g_r(\Delta\omega) = \frac{32\pi^3\kappa^2}{|\omega_\kappa'|}\varepsilon_\kappa(t), \tag{48}$$

where the value of κ is given by $\omega_\kappa = \Delta\omega$ (and we assumed the equation had only one root) and $\omega_\kappa' = (\mathrm{d}/\mathrm{d}\kappa)\omega_\kappa$. This means that the turbulent spectrum $\varepsilon_k(t)$ can be obtained directly from the Baranger–Mozer satellites.

The difficulty with the quasilinear theory is that it breaks down if the energy density in the waves becomes a significant fraction of the total energy density. In such a case there are other more complicated nonlinear processes which compete with the one described here (such as coupling between the different modes). Moreover there are other instabilities than those which arise from the linearized Vlasov equation (nonlinear instabilities).

The importance of looking at the satellites in unstable plasmas lies in the information which can be obtained about the level of excitation from the size of the satellites and the possibility of testing the many competing theories which attempt to describe unstable or turbulent plasmas.

CONCLUSION

We have discussed in this paper the connection between the kinetic theory of plasmas and Stark broadening. We have shown that one of the central results of plasma kinetic theory, namely the electric field autocorrelation function, can be used to calculate the effect of weak electron–atom collisions on the atomic spectra including many body effects. This result leads to a correction to the old impact theory because of the "double screening" of the fields.

The contribution of weak electron–atom collisions can be approximately split into a collisional part and a wave part. In an equilibrium plasma the wave part, which only contributes near frequencies appropriate to the collective mode is insignificant. However, in nonequilibrium plasmas the wave part can be significantly enhanced over its thermal value leading to the Baranger–Mozer satellites on forbidden lines. The observation of these lines can thus be extremely useful in obtaining information about nonequilibrium plasmas. This is particularly true with unstable plasmas where the theory at present is in a state of great flux and the electric field autocorrelation function is a vital piece of information.

There have been two recent reports of observations of plasma satellites on forbidden lines. KUNZE and GRIEM[3] observed the satellites in a Theta pinch by looking in the region of the collisionless shock front. In a separate experiment COOPER and RINGLER[4] observed satellites on forbidden lines arising from an externally imposed ac field. COOPER[26] later looked at a region of instability in an electron beam and observed satellites due to the internal fluctuating fields.

Acknowledgement—One of the authors (W.R.C.) would like to acknowledge helpful comments by Dr. R. E. AAMODT concerning the section on unstable plasmas.

REFERENCES

1. W. CHAPPELL, J. COOPER and E. SMITH, *JQSRT* **9,** 149 (1969).
2. H. R. GRIEM, M. BARANGER, A. C. KOLB and G. OERTEL, *Phys. Rev.* **125,** 177 (1962).
3. H. J. KUNZE and H. R. GRIEM, *Phys. Rev. Lett.* **21,** 1048 (1968).
4. W. S. COOPER III and H. RINGLER, *Phys. Rev.* **179,** 226 (1969).
5. S. RAND, *JQSRT* **9,** 921 (1969); J. W. DUFTY, *Phys. Rev.* (to be published).
6. N. ROSTOKER, *Nucl. Fusion* **1,** 101 (1960); *Phys. Fluids* **7,** 479, 491 (1964).
7. M. BARANGER and B. MOZER, *Phys. Rev.* **123,** 25 (1961).
8. M. BARANGER, *Atomic and Molecular Processes*, Chap. 13. D. BATES, Ed. Academic Press, New York (1962).
9. E. SMITH, C. R. VIDAL and J. COOPER, *Phys. Rev.* **185,** 140 (1969).
10. D. MONTGOMERY and D. TIDMAN, *Plasma Kinetic Theory*, Chap. 14. McGraw-Hill, New York (1964).
11. G. BEKEFI, *Radiation Processes in Plasmas*, Chap. 5. John Wiley, New York (1966).
12. Here $\mathbf{E}(\mathbf{r}, t)$ denotes the field at the point $\mathbf{r}$ at time t. We will consider the atom to be at the origin and suppress the variable $\mathbf{r}$.
13. E. SMITH, *Phys. Rev. Lett.* **18,** 990 (1967).
14. W. R. CHAPPELL, *J. Math. Phys.* **8,** 298 (1967).
15. Ref. 11, p. 126–128.
16. J. DAWSON and C. OBERMAN, *Phys. Fluids* **5,** 517 (1962). The restriction $k < k_D$ which appears in footnote 4, page 518 does not have a direct bearing on the problem at hand. The meaning of the footnote is that $g_r(\Delta\omega)$ is *not* the resistivity unless the wavelength of the applied field is less than the Debye length.
17. Ref. 11, pp. 119–120.
18. M. LEWIS, *Phys. Rev.* **121,** 501 (1960).
19. C. H. TOWNES and A. L. SCHAWLOW, *Microwave Spectroscopy*, pp. 273–279. McGraw-Hill, New York (1955).
20. Ref. 10, pp. 56–61.
21. Ref. 11, p. 116.
22. D. A. TIDMAN and A. EVIATAR, *Phys. Fluids* **8,** 2059 (1965).
23. D. A. TIDMAN and T. H. DUPREE, *Phys. Fluids* **8,** 1860 (1965).
24. D. A. TIDMAN, T. J. BIRMINGHAM and H. M. STAINER, *Astrophys. J.* **146,** 207 (1966).
25. I. B. BERNSTEIN and F. ENGELMANN, *Phys. Fluids* **9,** 937 (1966).
26. W. COOPER III, private communication.