Determining the Uncertainty of Frequency Measurements Referenced to GPS Disciplined Oscillators

Speaker/Author: Michael A. Lombardi
Time and Frequency Division
National Institute of Standards and Technology
325 Broadway, Boulder, CO 80305
lombardi@nist.gov

Abstract: GPS disciplined oscillators (GPSDOs) are commonly used as references for frequency calibrations. Over long intervals, a GPSDO is an inherently accurate source of frequency because it is continuously adjusted to agree with the Coordinated Universal Time (UTC) time scale maintained by the United States Naval Observatory (USNO). However, most frequency calibrations last for intervals of one day or less, and it can be difficult for metrologists to determine the uncertainty of a GPSDO during a short interval, and even more difficult to prove their uncertainty claims to skeptical laboratory assessors. This paper can serve as a guide to metrologists and laboratory assessors who work with GPSDOs as frequency standards. It describes the relationship between GPS time and Coordinated Universal Time (UTC) and explains why GPS time is traceable to the SI. It discusses how a GPSDO utilizes the GPS signals to control the frequency of its local oscillator. It explains how to estimate frequency stability, and how to apply estimates of frequency stability to determine the uncertainty of a GPSDO used as the reference for a frequency calibration.

1. Introduction

In calibration laboratories, frequency measurements generally focus on the electrical signals produced by oscillators. For the purposes of this paper, an oscillator is a device that produces electrical signals at a specific frequency, typically in the form of a sine wave. Several types of mechanical and atomic oscillators exist, but only one, the cesium oscillator, is currently defined as a primary frequency standard. This is because the International System (SI) second (s), the base unit of time interval, has been defined as 9,192,631,770 energy transitions of the cesium atom since 1967 [1]. Frequency, expressed in units of hertz, is the reciprocal of time interval, and is measured by counting the number of repetitive events that occur during the SI second.

National metrology institutes and primary standards laboratories often have sufficient resources to operate one or more cesium oscillators. However, cesium oscillators have a limited life expectancy [2] and a high cost, and not every calibration laboratory wants, or can afford, to own one. The major component of a cesium oscillator, the beam tube, typically lasts for five to ten years, and replacing the beam tube can cost nearly as much as replacing the entire device. In addition, cesium oscillators can require occasional adjustment.

For these reasons, many, if not most, calibration laboratories now operate a Global Positioning System disciplined oscillator (GPSDO) as their frequency standard. A GPSDO has many advantages. For example, it costs much less than a cesium oscillator to initially purchase, sometimes as much as 90 % less. It also costs less to own, because there is no cesium beam tube to replace. If desired, a calibration laboratory can buy multiple GPSDOs for less than the cost of a cesium standard, and use the additional standards for crosschecks and redundancy. Unlike a cesium standard, a GPSDO can recover time by itself, meaning that it can decode time-of-day messages sent by GPS and synchronize an on-time pulse to Coordinated Universal Time. This is important to laboratories that need time synchronization capability. The accuracy and stability of a GPSDO in the long-term can be superior to that of a cesium oscillator. And finally, a GPSDO never requires adjustment, because its frequency is controlled by signals broadcast from satellites.

There are, of course, some disadvantages. For example, a GPSDO requires access to a rooftop location so that an outdoor antenna can be mounted, whereas a cesium oscillator can be operated anywhere where electric power is available. In addition, cesium oscillators are autonomous and independent sources of frequency, which means they can operate without input from another source. A GPSDO can operate properly only where signals from the GPS satellites are available, and are not suitable for applications that need an autonomous frequency source. In most cases, however, calibration laboratories can easily mount an antenna and don't need an autonomous frequency source. Thus, the acquisition of a GPSDO as a practical, lower cost alternative to a cesium oscillator often makes sense, for both economic and technical reasons [3].

The widespread use of GPSDOs as frequency standards in calibration laboratories has created some confusion among laboratory assessors, who sometimes consider the device as a "black box" and doubt a laboratory's traceability and uncertainty claims. This confusion is understandable because the assessor has a hard task, they must determine whether or not a GPSDO establishes traceability to the SI, which, as a prerequisite, requires knowing the measurement uncertainty of the frequency produced by a GPSDO. This paper attempts to eliminate this confusion by discussing how a GPSDO works. It begins by describing the relationship between GPS time; Coordinated Universal Time (UTC); the UTC time scale maintained at the United States Naval Observatory, UTC(USNO); and the UTC time scale maintained at the National Institute of Standards and Technology, UTC(NIST).

2. GPS Frequency and its Relationship to the SI Second

UTC, computed monthly by the Bureau International des Poids et Mesures (BIPM), is the official world time scale. About 70 laboratories, a number that continues to gradually increase, participate in the calculation of UTC by sending data from their local time scales to the BIPM. They collect this data through local clock difference measurements and through international clock comparisons performed with various satellite time transfer techniques. The BIPM publishes a monthly document called the *Circular T* that shows the time differences between each of the local time scales, known as UTC(*k*) where *k* designates the laboratory, and UTC itself.

The BIPM collects and averages data from hundreds of clocks that contribute to the various UTC(*k*) time scales, and also from the primary frequency standards maintained by a few national metrology institutes, before calculating UTC. As a result, UTC represents the best available realization of the SI second. However, UTC is a post processed, virtual time scale that does not generate a physical signal. Fortunately, the local UTC(*k*) time scales operate in real time and do generate physical signals that closely agree with the UTC calculation, often to within 10 ns in time and to within parts in 10^{15} in frequency. The laboratories that contribute to UTC and have their data appear on the *Circular T* are participants in a “key comparison”, currently designated as CTF-K001.UTC. Participation in this key comparison is an internationally accepted method of establishing traceability to the SI. There is no separate key comparison for frequency, thus the *Circular T* results provide traceability to both the second and the hertz [4].

GPS time is referenced to UTC(USNO), and the UTC(USNO) time scale appears on the *Circular T*, so the traceability chain between GPS time and the SI second is always intact. In fact, the USNO contributes more clock data to the UTC calculation than any other laboratory. GPS time differs from UTC(USNO) because GPS is a continuous time scale, whereas all representations of UTC are corrected periodically with the insertion of leap seconds. The zero time-point of the GPS time scale is defined as midnight on the night of January 5, 1980/morning of January 6, 1980, so GPS time has ignored the leap seconds that have occurred since 1980. There is also a small time difference (in nanoseconds) between GPS time and UTC(USNO). However, subframe 4 of the navigation message broadcast by the satellites includes a leap second correction and a UTC(USNO) time difference correction, and both corrections are applied by default by all modern GPSDOs (most models do not even allow the user to turn the corrections off). Therefore, UTC(USNO) is the time scale distributed to users of GPS receivers and GPSDOs worldwide [5, 6].

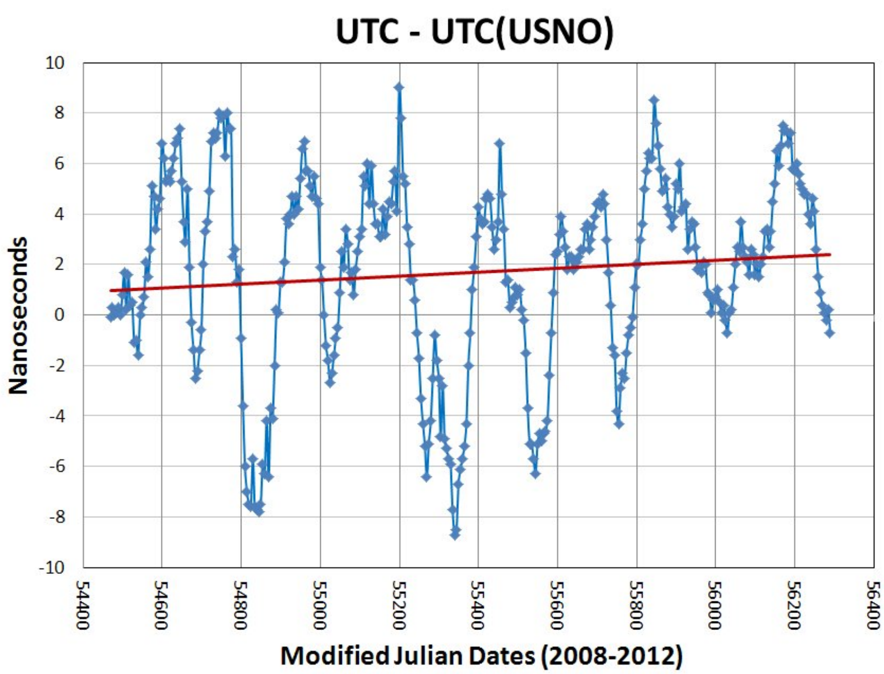

Figure 1. Time differences between UTC and UTC(USNO) from 2008-2012.

Figure 1 illustrates the close relationship between UTC and UTC(USNO) by showing the differences between the two time scales for a five year period (2008-2012). These measurements were obtained from the *Circular T* (www.bipm.org), which provides data at five day intervals. Note that the time difference between UTC and UTC(USNO) was always within ±10 ns, and the average time difference was 1.7 ns. The red trend line represents a linear least squares curve fit. The UTC(NIST) time scale also closely tracked UTC from 2008 to 2012, as indicated by the

Circular T and graphed in Fig. 2. The time difference between UTC and UTC(NIST) was always within ±20 ns and the average time difference was 1.2 ns.

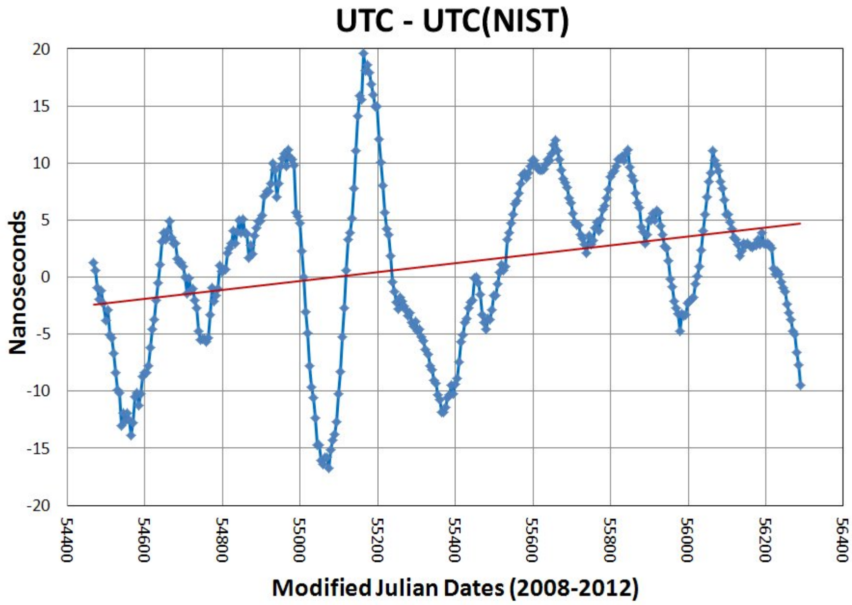

Figure 2. Time differences between UTC and UTC(NIST) from 2008-2012.

The UTC(NIST) and UTC(USNO) time scales are continuously compared to each other using a variety of satellite time transfer methods, and can be considered as equivalent for nearly all metrological and calibration purposes [7]. Figure 3 shows the time difference between NIST and USNO for 2008 to 2012. Again, the time differences always remained within ±20 ns and the average time difference was 0.5 ns.

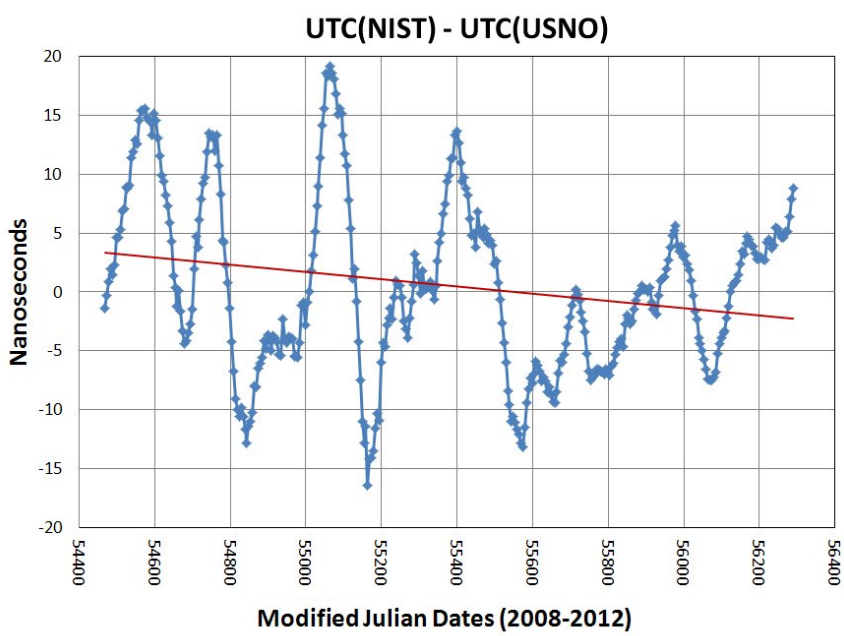

Figure 3. Time differences between UTC(NIST) and UTC(USNO) from 2008-2012.

When the on-time pulse from a calibrated GPS receiver (with the UTC(USNO) corrections turned on) is compared to UTC(NIST), the resulting data are similar to Fig. 3 (slightly different due to receiver noise and because the data were collected at different sampling times). Figure 4 shows such a comparison for a five year interval (2008-2012), with values reported for each day, as opposed to the five day interval of *Circular T*. The receiver is a low cost L1 band device maintained at NIST in Boulder, Colorado. The time differences rarely exceeded ±20 ns and the average time difference was -1.8 ns (there appears to have been a bias of about 2 ns in the receiver delay calibration).

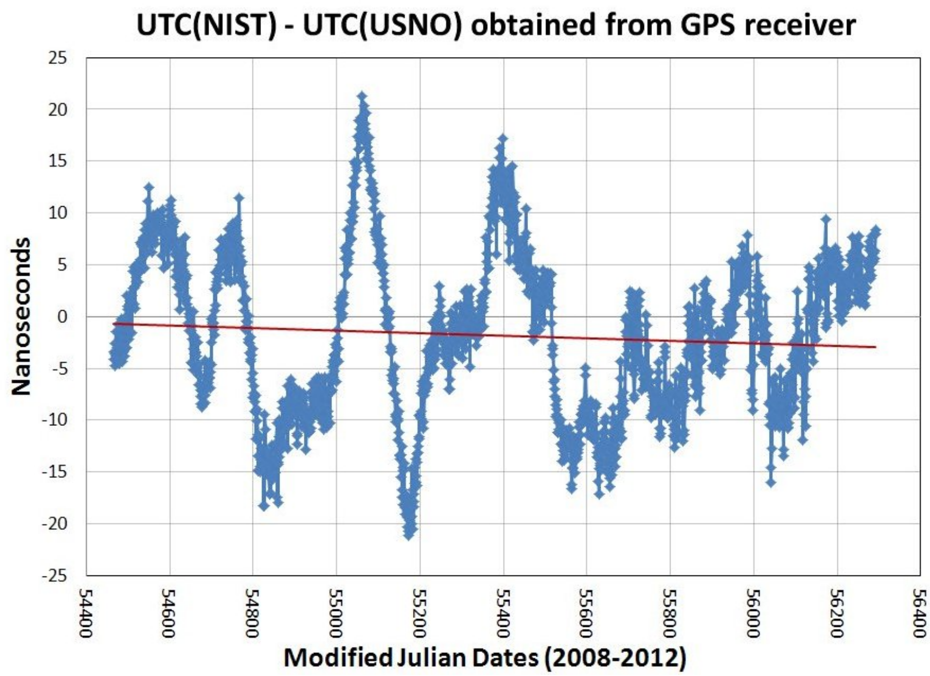

Figure 4. Time differences between UTC(NIST) and UTC(USNO) from GPS receiver, 2008-2012.

3. The Accuracy of GPS Frequency

The graphs shown in Figs. 1 through 4 are known as phase or time difference graphs, and utilize the standard Cartesian *x*/*y* format. The *x*-coordinate indicates elapsed time. The values plotted as the *y*-coordinate represent the change in phase, ΔΦ, between the two electrical signals that are being compared to each other. However, phase changes are usually measured with an instrument, in this case a time interval counter; that displays results in units of time and not in radians or degrees. Thus, the *y*-coordinate in Figs. 1 through 4 is labeled to show the change in time, or Δ*t*.

Graphs of phase or time difference can be used to obtain the difference in frequency, often called the frequency offset, between two signals. The equation for estimating frequency offset in the frequency domain is

$$f_{off} = \frac{f_{meas} - f_{nom}}{f_{nom}} \quad , \qquad (1)$$

where f_{off} is the frequency offset, f_{meas} is the actual frequency in hertz reported by the measurement, and f_{nom} is the nominal frequency in hertz that the oscillator would ideally produce. The nominal frequency is usually listed on the label next to the oscillator's output connector, for example, "10 MHz." Note that f_{meas} always has an associated measurement uncertainty, but that f_{nom} does not; it is an ideal (theoretical) value with no uncertainty. Note also that the nominal frequency is included in both the numerator and the denominator of Eq. (1). Thus, the unit of hertz cancels and the value for f_{off} is dimensionless and will always be relative to the nominal frequency. For example, if f_{off} is 1×10^{-6} then the frequency offset would be 1 Hz for an oscillator with a nominal frequency of 1 MHz, or 10 Hz for an oscillator with a nominal frequency of 10 MHz.

Eq. (1) is often simplified in the literature as

$$f_{off} = \frac{\Delta f}{f} \quad , \tag{2}$$

where f_{off} is the dimensionless frequency offset, Δf is the difference between the measured and nominal frequency in hertz, and f is the nominal frequency in hertz.

Even though the measurements shown in Figs. 1 through 4 (Section 2) were made in the time domain by measuring the time interval difference between two clocks, they can still be used to obtain the frequency offset, f_{off}. This is because frequency is the reciprocal of period, which is expressed as a time interval. A mathematical definition of frequency is

$$f = \frac{1}{T} \quad , \tag{3}$$

where T is the period of the signal in seconds, and f is the frequency in hertz. This can also be expressed as

$$f = T^{-1} \quad . \tag{4}$$

By performing mathematical differentiation on the frequency expression with respect to time and substituting in the result, it can be shown that the average dimensionless difference in frequency is equivalent to the average dimensionless difference in time, or that $\Delta f \ / \ f$ is equivalent to $-\Delta t \ / \ T$ [8]. For example,

$$\Delta f = -T^{-2}\Delta t = -\frac{\Delta t}{T^2} = -\frac{\Delta t}{T} f \quad , \tag{5}$$

therefore

$$f_{off} = \frac{\Delta f}{f} = -\frac{\Delta t}{T} \quad , \tag{6}$$

where Δt is the change in time, and T is the duration of the measurement. If there were only two time interval measurements (TI_1 and TI_2), then T would be the interval between the two measurements and frequency offset in the time domain could be expressed as

$$f_{off} = \frac{TI_2 - TI_1}{T} = -\frac{\Delta t}{T} \quad . \tag{7}$$

To keep the sign correct and in agreement with the slope of the phase graph, note that the first reading must be subtracted from the second reading.

In practice, not just two, but instead a large number of time interval measurements are usually recorded to estimate frequency offset. In that case, T is simply the interval between the first and last measurement. It is common practice to fit a linear least squares line to the phase data, and to use the slope of the least squares line to estimate Δt. Therefore it is also common for metrologists, and only a slight oversimplification, to state that frequency offset is equivalent to the slope of the phase.

Table 1 shows the frequency offset, f_{off}, of UTC(USNO) and UTC(NIST) with respect to UTC, and for UTC(NIST) with respect to UTC(USNO) as obtained from *Circular T*, and for UTC(NIST) with respect to UTC(USNO) as obtained from a GPS receiver. The frequency offset values were obtained from Figs. 1 through 4 by estimating Δt with a linear least squares line. The frequency differences between the various time scales are very small when measured over a long interval, parts in 10^{17}. These numbers indicate that a GPSDO is inherently accurate, at least three orders of magnitude more accurate in the long term than commercial cesium standards. The accuracy of a cesium standard with respect to UTC is usually no better than a few parts in 10^{14}, because they do not have the benefit of being steered to agree with UTC.

Table 1. Frequency comparisons between UTC, UTC(USNO), and UTC(NIST).

Comparison	Frequency Offset	Data Source
UTC – UTC(USNO)	0.9×10^{-17}	Fig. 1
UTC – UTC(NIST)	4.5×10^{-17}	Fig. 2
UTC(NIST) – UTC(USNO)	-3.6×10^{-17}	Fig. 3
UTC(NIST) – UTC(USNO) from GPS receiver output	-1.4×10^{-17}	Fig. 4

4. Basic Principles of a GPS Disciplined Oscillator

Disciplined oscillators allow accurate frequency and time signals, controlled by an external reference, to be simultaneously generated at multiple sites. A disciplined oscillator has at least three parts: a local oscillator (LO), a receiver that collects data transmitted from a reference source, and a frequency or phase comparator. The comparator measures the difference between the LO and the reference, and this difference is converted to a frequency correction that is periodically applied to the LO. By continuously repeating this process, the LO is disciplined so that it replicates the performance of the reference. No manual adjustment is ever necessary.

In the case of a GPSDO, the external reference consists of signals from the GPS satellites, and the information contained in these signals is used to control the frequency of a local quartz or rubidium oscillator. As noted in Section 2, GPS signals are kept in agreement with UTC(USNO). Nearly all GPSDOs use the coarse acquisition (C/A) code on the L1 carrier frequency (1575.42 MHz) as their incoming reference signal. Frequency and time are byproducts of GPS, because the system's main purpose is to serve as a positioning and navigation service. It is for this reason, however, that the frequency and time signals from GPS can be trusted. They must be accurate and stable to within parts in 10^{14} over a 12-hour averaging period in order for GPS to meet its positioning and navigation specifications. The best GPSDOs transfer as much of the inherent accuracy and stability of the satellite signals as possible to the signals generated by the LO.

GPSDO manufacturers seldom disclose how their products work, but a few basic mechanisms are found in most designs. The GPS signals are typically received with a small receiver and antenna. The receiver outputs a 1 pulse per second (pps) signal. A phase detector measures the difference between the 1 pps signal from the GPS receiver and a signal from the LO. The LO typically has a nominal frequency of 10 MHz, so its signal is divided to a lower frequency (often to 1 pps) prior to this phase comparison. A microcontroller reads the output of the phase detector and records the phase difference. It then sends this difference to a control loop, which is often some variation of a proportional-integral-derivative (PID) controller [9] that is typically implemented in software. The PID serves as the control portion for a phase locked loop (PLL) that keeps the device locked to GPS by issuing frequency corrections to the LO [10, 11]. The correction interval can be either fixed or variable, depending upon the design and time constant of the PID. If the GPSDO is properly designed, corrections should be issued before the before the uncorrected LO becomes less stable than the GPS reference. As a result, the correction interval is usually less than one hour, and in cases where the LO is not particularly stable, it could be just a few seconds.

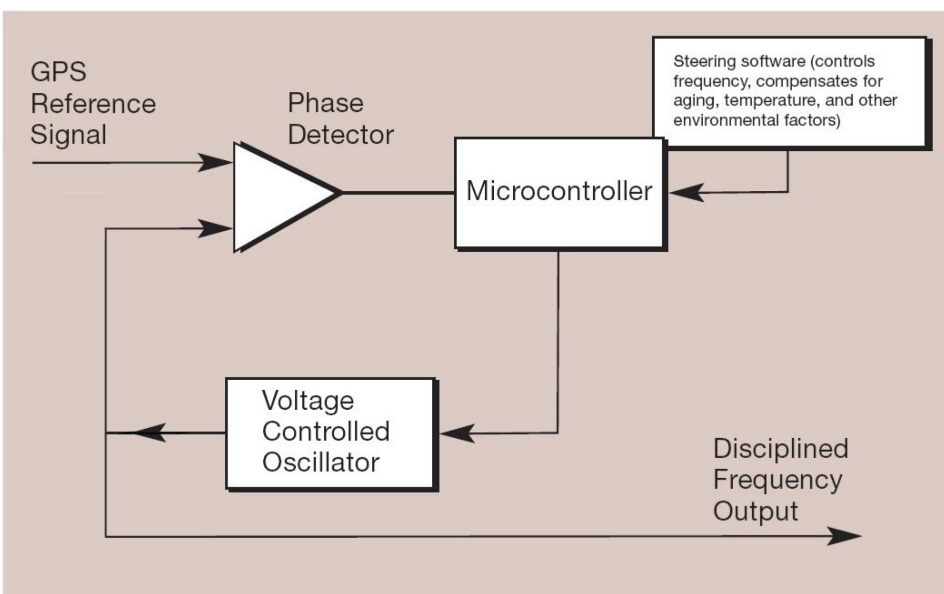

Figure 5. A GPSDO where the local oscillator is adjusted through voltage corrections.

If the LO is a voltage controlled oscillator (VCO), the frequency correction is sent by varying the control voltage sent to the VCO, to keep the phase difference within a given range (Fig. 5). The GPSDO is locked when the phase of the LO has a constant offset relative to the phase of the GPS signals. Ideally, the control loop must be loose enough to ignore the short-term fluctuations of the GPS signals, reducing the amount of phase noise and allowing the LO to provide reasonably good short-term stability. However, the loop must be tight enough to respond rapidly to conditions when the LO is unlocked, and to allow the GPS signals to control the LO frequency so that it is accurate and stable in the long term. The control software often compensates not only for the phase and frequency changes of the LO, but also for the effects of aging, temperature and other environmental parameters.

Another type of GPSDO design does not correct the frequency of the LO. Instead, the output of a free running LO is sent to a frequency synthesizer and corrections are applied to the output of the synthesizer (Fig. 6). A high resolution direct digital synthesizer (DDS) allows for very small frequency corrections. For example, 1 μHz resolution at 10 MHz allows instantaneous frequency corrections of 1×10^{-13}. In addition, allowing the local oscillator to free run often results in better short-term performance than the VCO method, where unexpected shifts in the control voltage can produce unwanted adjustments in the output frequency.

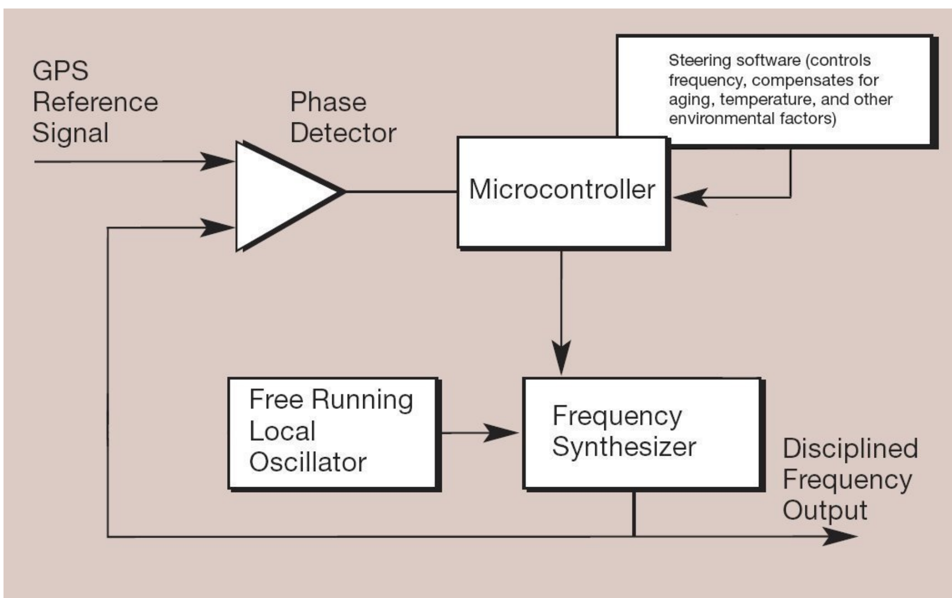

Figure 6. A GPSDO where the output frequency is synthesized.

5. The Stability of Frequency Produced by a GPSDO

Sections 2 and 3 demonstrated that the frequency produced by a GPSDO is inherently accurate when measured over a long interval, because GPS time is constantly adjusted to agree with UTC. Therefore, if a GPSDO is used as the reference for a frequency calibration that lasts for days or weeks, it should be sufficiently accurate to calibrate any device that a calibration laboratory is likely to encounter, including any cesium oscillator.

In practice, however, a frequency calibration doesn't last for days or weeks. The actual duration might be as short as one second and will seldom exceed one day. Therefore, determining the uncertainty of a frequency calibration requires knowledge of the GPSDO's stability over a period that equals the period of the calibration. In other words, if our frequency calibration lasts for one hour, we need to know how stable our reference GPSDO is during a one hour period.

Frequency stability differs from frequency offset or accuracy. Frequency stability indicates how well an oscillator can produce the same frequency offset over a given time interval. Any frequency that stays the same is a stable frequency, regardless of whether the frequency is "right" or "wrong" with respect to its nominal value. It is important to realize that the accuracy of an oscillator over a given interval can never be better than its stability over that same interval. Several techniques for measuring stability are described in [12], and GPSDO manufacturers normally include stability estimates in their specifications.

Frequency metrologists generally rely on non-classical statistics to estimate the frequency stability of oscillators. The most common statistic employed for stability estimates is often called the Allan variance in the literature, but because it actually is the square root of the variance, its proper name is the Allan deviation (ADEV), expressed mathematically as $\sigma_y(\tau)$. Similar to the standard deviation, ADEV is better suited for frequency metrology because it has the advantage of being convergent for most types of oscillator noise [12, 13, 14]. The equation for ADEV using frequency measurements and non-overlapping samples is

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(M-1)}\sum_{i=1}^{M-1}(\bar{y}_{i+1} - \bar{y}_i)^2} \quad , \tag{8}$$

where y_i is the ith in a series of M dimensionless frequency offset measurements averaged over a measurement or sampling interval designated as τ. Note that while standard deviation subtracts the mean from each measurement before squaring their summation, ADEV subtracts the previous data point. Since stability is a measure of frequency fluctuations and not of frequency offset, the differencing of successive data points is done to remove the time-dependent noise contributed by the frequency offset [13]. Also, note that the $\bar{y}$ values in the equation do not refer to the average or mean of the entire data set, but instead imply that the individual measurements in the data set can be obtained by averaging.

The equation for ADEV using phase measurements and non-overlapping samples is

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(N-2)\tau^2}\sum_{i=1}^{N-2}(x_{i+2} - 2x_{i+1} + x_i)^2} \quad , \tag{9}$$

where x_i is the ith in a set of N phase measurements spaced by the measurement interval τ.

To improve the confidence of a stability estimate, ADEV is normally used with overlapping samples that allow estimating stability with all possible combinations of the data set. This is the form of ADEV most widely found in the frequency metrology literature. The equation for ADEV using frequency measurements and overlapping samples is

$$\sigma_y(\tau) = \sqrt{\frac{1}{2m^2(M-2m+1)} \sum_{j=1}^{M-2m+1} \left\{ \sum_{i=j}^{j+m-1} [y_{i+m} - y_i] \right\}^2} \quad . \qquad (10)$$

The equation for ADEV using phase measurements and overlapping samples is

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(N-2m)\tau^2} \sum_{i=1}^{N-2m} (x_{i+2m} - 2x_{i+m} + x_i)^2} \quad . \qquad (11)$$

The overlapping versions of ADEV add an averaging factor, *m*, that is found in both Eq. (10) and Eq. (11). To understand the averaging factor, consider that τ_0 is the shortest interval at which data are taken. For example, if the stability of the device was estimated by measuring its frequency or phase every second, then $\tau_0 = 1$ s. To obtain stability estimates for longer intervals, τ_0 is simply multiplied by *m*, thus $\tau = m\tau_0$. Even though the overlapping samples are not statistically independent, the number of degrees of freedom still increases, thus improving the confidence in the stability estimate [12, 14].

One important advantage of ADEV over standard deviation is its ability to estimate stability over different intervals from the same data set. A typical ADEV graph plots log τ on the *x*-coordinate to indicate the averaging period, and log $\sigma_y(\tau)$ on the *y*-coordinate to indicate dimensionless frequency stability. These graphs are often referred to colloquially as "sigma-tau" graphs.

Most ADEV graphs found in the literature were generated using the octave method, where each successive value of τ is twice as long as the previous value. This method saves computational time, but as computers have become faster it has become more common to estimate ADEV for all possible values of τ. An "all-tau" plot can thus be used to estimate the frequency stability at any interval, making it possible to exactly match the period of the calibration where the GPSDO was used as the reference.

Figure 7 shows an "all tau" graph from a GPSDO that was calibrated at NIST. The device was stable to less than 2×10^{-12} at all averaging periods ($\tau_0 = 1$ minute). This indicates a stable LO, which is this case was a rubidium device. The initial "bump" in the red line indicates the correction interval described in Section 4 when the LO is being adjusted to agree with the GPS signals. The correction interval appears to be near 1000 s. Smaller "bumps", which have been attenuated by averaging for longer intervals, appear at multiples of the correction interval. This type of structure is typically found in frequency stability plots of disciplined oscillators.

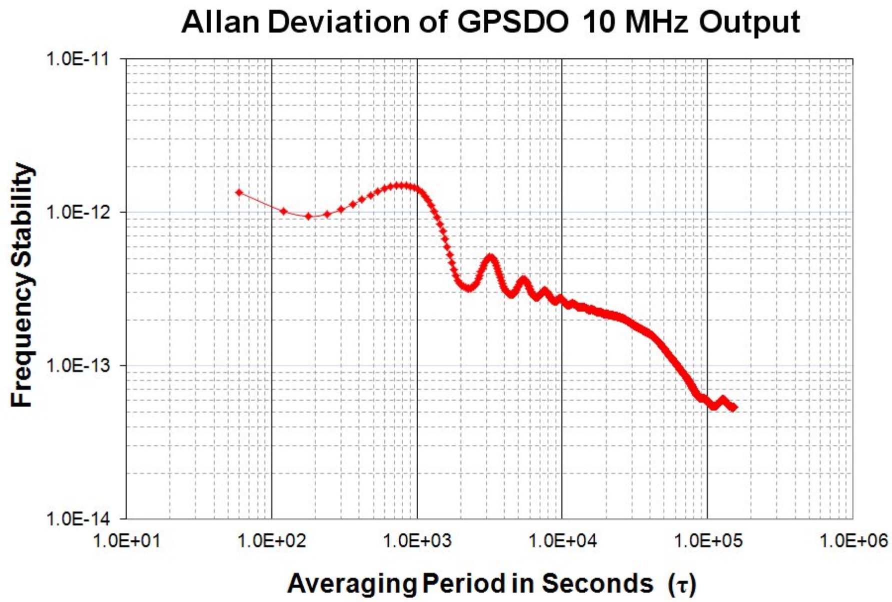

Figure 7. Overlapping ADEV plot ("all-tau") from a GPSDO calibration at NIST.

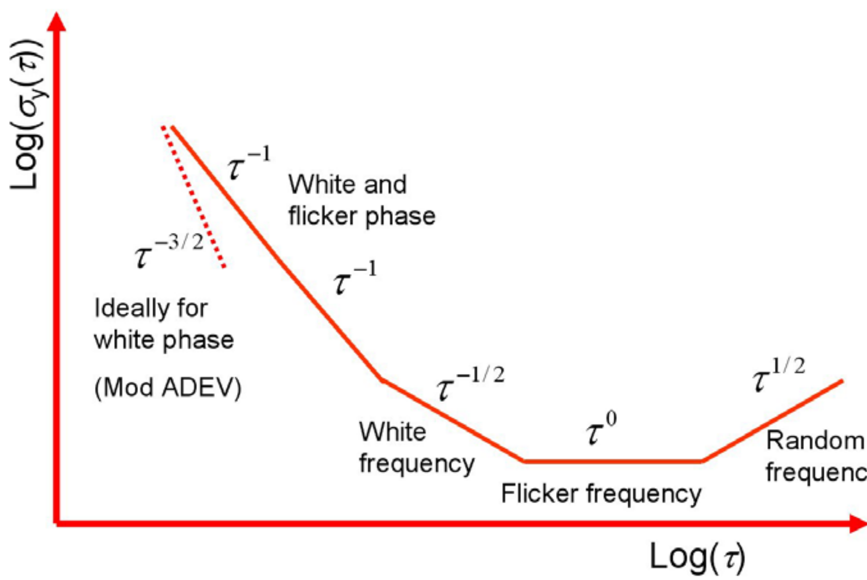

Figure 8. Determining the type of oscillator noise from a frequency stability graph.

In addition to estimating stability, ADEV can help identify the types of oscillator noise by taking the slope of the line on an ADEV graph (Fig. 8). Five noise types are commonly discussed in the frequency metrology literature: white phase and flicker phase (both have a slope of τ^{-1} when used with standard ADEV), white frequency (a slope of $\tau^{-1/2}$), flicker frequency (τ^{0}, no slope), and random walk frequency (a slope of $\tau^{1/2}$). For both types of phase noise, the stability is improving at a rate proportional to the averaging period. For white frequency noise, the stability is still improving, but the rate of improvement has slowed down and is now proportional to the square root of the averaging period. When the flicker frequency region of an ADEV graph is reached, the oscillator has reached a noise floor that shows its best possible stability (often called the “flicker” floor). When this point is reached, there is nothing to be gained by more averaging. In fact, if you continue to average, the stability will start to get worse because the noise type becomes random walk, meaning that the oscillator is now producing successive random steps in frequency [12].

The noise analysis shown in Fig. 8 is commonly found in the literature, but it is important to know that the intended use of ADEV was to estimate the frequency stability of free running, rather than disciplined oscillators. The flicker frequency region of an ADEV graph will never be reached with a GPSDO because its frequency is continuously being corrected to agree with UTC(USNO), which, as indicated in Table 1, is essentially equivalent to UTC and UTC(NIST). Notice, for example, that the stability in Fig. 7 has dropped below 1×10^{-13} at $\tau = 1$ day, and would continue to get smaller indefinitely if more data were collected. There are periods when this rate of improvement will slow down, and other periods where the stability temporarily gets worse due to frequency corrections or very low frequency noise sources such as sunrise and sunset, but in theory a GPSDO will not reach a noise floor. Its stability, and therefore its frequency uncertainty, will continue to get smaller if the calibration period gets longer. This means, for example, that if a cesium oscillator were calibrated over the course of a year, it should make little difference which make or model of GPSDO was used as the reference. All models would produce similar results (with very small uncertainties) if they were operating properly and remained locked to the GPS satellites during the entire calibration.

There can be, however, significant differences in the uncertainty of a GPSDO during a frequency calibration, which usually lasts for one day or less. The short-term stability of a GPSDO, at intervals shorter than the correction interval, should be identical to the short-term stability of its free running LO. Its medium-term stability, at intervals longer than the correction interval but shorter than one day, is design dependent and influenced by many factors. These factors include the quality of the receiver and antenna, the stability of the LO, the resolution of the comparator, the correction method, the correction uncertainty, and the correction interval.

Table 2 shows the frequency stability of ten GPSDOs calibrated at NIST, where τ is equal to one second (when data were available), one minute, one hour, and one day. The ten devices were each manufactured by different companies, but are provided only as an example, and of course do not represent all available models.

As Table 2 indicates, a number of GPSDOs are stable to within parts in 10^{12} at $\tau = 1$ s. Even so, some GPSDOs have low cost local oscillators that are not particularly stable at short averaging periods, and there are advantages in collecting more data, so it is best if a frequency calibration lasts for at least a few seconds, regardless of the measurement requirements. Each GPSDO reached a stability of at least 1×10^{-11} at $\tau = 1$ minute and of at least 6×10^{-12} at $\tau = 1$ hour. A good metric to use when evaluating GPSDO performance is their frequency stability at $\tau = 1$ day. Stability of 1×10^{-13} or less at $\tau = 1$ day normally indicates an instrument of high quality, and six of the ten devices tested reached or exceeded this specification. It is interesting to note that a few GPSDOs even approach the stability of the best commercially available cesium standards at $\tau = 1$ day, which, according to their manufacturer’s specification, is about 0.3×10^{-13}.

Table 2. Frequency stability of various GPSDOs at intervals of one day or less.

GPSDO ID Code	Overlapping Allan Deviation, $\sigma_y(\tau)$				
	GPSDO Local Oscillator Type	1 second	1 minute	1 hour	1 day
SYX	Rubidium	NA	1×10^{-11}	7×10^{-13}	0.5×10^{-13}
ET6	Rubidium	NA	1×10^{-12}	4×10^{-13}	0.6×10^{-13}
ERT	Rubidium	6×10^{-12}	1×10^{-12}	6×10^{-13}	0.7×10^{-13}
AR1	Quartz	6×10^{-12}	1×10^{-11}	1×10^{-12}	0.9×10^{-13}
BR8	Quartz	5×10^{-10}	4×10^{-12}	4×10^{-12}	1×10^{-13}
GD3	Quartz	NA	6×10^{-12}	3×10^{-12}	1×10^{-13}
PT1	Quartz	4×10^{-12}	1×10^{-11}	4×10^{-12}	3×10^{-13}
HPZ	Quartz	1×10^{-12}	2×10^{-12}	1×10^{-12}	4×10^{-13}
TS6	Rubidium	6×10^{-12}	8×10^{-13}	6×10^{-12}	4×10^{-13}
FL9	Rubidium	7×10^{-12}	3×10^{-12}	3×10^{-13}	7×10^{-13}

6. Proposed Method for Uncertainty Estimation

The uncertainty analysis of frequency measurement referenced to GPS is much simpler than the uncertainty analysis of a time measurement. This is mainly because equipment delays (the delays through cables, antennas, GPS receivers, and so on) do not have to be calibrated if they are known to be constant. The uncertainty of the antenna survey is also much less of a problem. The survey of the antenna position, usually done automatically by the GPSDO when first installed, is usually accurate to < 1 m for the determination of latitude and longitude. The antenna survey can produce errors in altitude estimation that sometimes exceed 10 m. This altitude determination error can contribute significant uncertainty to time measurements (> 30 ns in some cases), but does not impact frequency calibrations.

Sections 2 and 3 demonstrated that GPS time is traceable to the SI and is inherently accurate. This accuracy is transferred to the LO with the techniques described in Section 4. These techniques essentially "tune" the GPSDO frequency to agree with UTC, and we can assume that a GPSDO is accurate if locked. However, as noted previously, the accuracy of an oscillator over a given interval can never be better than its stability over that same interval. Therefore, Section 5 described how to estimate the frequency stability of a GPSDO at various intervals by use of the Allan deviation. Assuming that a GPSDO is locked and working properly, this stability information is all that is necessary to report the expanded uncertainty of the frequency produced by a GPSDO. By international recommendation [15], expanded measurement uncertainty is reported in the form

$$Y = y \pm U \quad , \tag{12}$$

where Y is the measurand or the quantity being measured (in this case frequency), y is the best estimate of the measurand (in this case the average frequency with respect to the SI), and U is the expanded measurement uncertainty.

As indicated in Table 1, the frequency offset of a GPSDO with respect to UTC is small, measured in parts in 10^{17} over a long interval, thus for all practical purposes y can be regarded as 0. The range from $y – U$ to y + U is expected to "encompass a large fraction of values than reasonable be attributed to Y [15]." This is what ADEV accomplishes; it shows the deviation in the average frequency at a given interval by encompassing nearly all types of oscillator noise. Thus, a convenient and robust way to estimate U for a GPSDO, is simply to multiply ADEV by 2 to obtain a k = 2 coverage factor. The value chosen for ADEV should be at $\tau = fc_d$, where fc_d is the duration of the frequency calibration that utilized the GPSDO as its reference.

The frequency stability of a GPSDO for periods from out to one day or longer can be obtained by having the device calibrated by a national metrology institute such as NIST, from laboratory measurements [12], or if available, from calibration data or specifications provided by the manufacturer.

It is important for metrologists and for laboratory assessors to be able to distinguish between reasonable and unreasonable uncertainty claims. For example, if a laboratory performs frequency calibrations over a one day interval with a GPSDO, a measurement uncertainty claim of 1×10^{-14} is probably not possible, and should be closely scrutinized. On the other hand, an uncertainty of 1×10^{-13} is possible with the right device, and 1×10^{-12} should be achievable with any model of GPSDO, provided that it has been properly installed and is not malfunctioning. Calibrations performed over intervals of less than one day will, of course, result in larger uncertainties as indicated in Table 2. Finally, all calibration laboratories should have procedures in place to determine whether a GPSDO is locked and working properly, and should be prepared to show those procedures to assessors [3].

7. Summary

Many, if not most, calibration laboratories now operate a GPSDO as their frequency standard, a decision that makes sense, for both economic and technical reasons. GPSDOs produce signals that are inherently accurate and traceable to the SI, because the devices are continuously being adjusted to agree with UTC, the best possible realization of the SI second. Even so, laboratory assessors are sometimes skeptical of the traceability and uncertainty claims made by calibration laboratories that employ GPSDOs as frequency standards, especially when the duration of a frequency calibration is one day or less. To help alleviate this skepticism, this paper has described the relationship between GPS time and UTC, and provided a simple but robust method for determining the uncertainty of frequency measurements referenced to a GPSDO.

8. Acknowledgments

The author thanks Joseph Petersen of Abbott Laboratories in Chicago, Illinois for suggesting the topic of this paper and for many valuable comments.

9. References

1. Resolution 1 of the *13th Conference Generale des Poids et Mesures (CGPM)*, 1967.

2. J. Kusters, L. Cutler, and E. Powers, "Long-Term Experience with Cesium Beam Frequency Standards," *Proceedings of the IEEE Frequency Control Symposium and European Frequency and Time Forum (EFTF)*, pp. 159-163, April 1999.

3. M. Lombardi, "The Use of GPS Disciplined Oscillators as Primary Frequency Standards for Calibration and Metrology Laboratories," *NCSLI Measure J. Meas. Sci.*, vol. 3, no. 3, pp. 56-65, September 2008.

4. P. Whibberly, J. Davis, and S. Shemar, "Local representations of UTC in national laboratories," *Metrologia*, vol. 48, pp. S154-S164, July 2011.

5. Global Positioning Systems Directorate, "Systems Engineering & Integration Interface Specification," *IS-GPS-200F*, September 2011.

6. M. Miranian, "UTC dissemination to the Real-Time User: The Role of USNO," *Proceedings of the 27th Annual Precise Time and Time Interval (PTTI) Systems and Applications Meeting*, San Diego, California, pp. 75-85, November 1995.

7. V. Zhang, T. Parker, R. Bumgarner, J. Hirschauer, A. McKinley, S. Mitchell, E. Powers, J. Skinner, and D. Matsakis, "Recent Calibrations of UTC(NIST) – UTC(USNO)," *Proceedings of the 44th Annual Precise Time and Time Interval (PTTI) Systems and Applications Meeting*, Reston, Virginia, pp. 35-42, November 2012.

8. G. Kamas and S. Howe, eds., "Time and Frequency User's Manual," *National Bureau of Standards Special Publication 559*, 256 p., November 1979.

9. K. Åström and T. Hägglund, *PID Controllers: Theory, Design, and Tuning*, 2nd ed., Instrument Society of America, Research Triangle Park, North Carolina, 1995.

10. B. Shera, "A GPS-based Frequency Standard," *QST Magazine*, vol. 82, pp. 37-44, July 1998.

11. B. Cui, X. Hou, and D. Zhou, "Methodological Approach to GPS Disciplined OCXO Based on PID PLL," *Proceedings of the 9th International Conference on Electronic Measurements and Instruments (ICEMI)*, Beijing, China, pp. 1-528 to 1-533, August 2009.

12. W. Riley, "Handbook of Frequency Stability Analysis," *NIST Special Publication 1065*, July 2008.

13. J. Jespersen, "Introduction to the time domain characterization of frequency standards," *Proceedings of the 23rd Annual Precise Time and Time Interval (PTTI) Systems and Application Meeting*, Pasadena, California, pp. 83-102, December 1991.

14. IEEE, “Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology - Random Instabilities,” *IEEE Standard 1139*, 2008.

15. JCGM, “Evaluation of measurement data – Guide to the expression of uncertainty in measurement,” *JCGM 100*, 2008.