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The Theory of the Optical Wedge Beam Splitter

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The Theory of the Optical Wedge Beam Splitter

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THE THEORY OF THE OPTICAL WEDGE BEAM SPLITTER

Yardley Beers

An optical wedge beam splitter consists of a prism of transparent material with a very small apex angle, usually about one degree. If a pencil beam of radiation is incident upon it, a portion enters the material and undergoes a series of reflections at the surfaces. At each reflection a refracted beam emerges from the material. This paper gives the basic theory for computing the ratio of the intensity of the incident beam to the intensity of any selected emerging beam and also for computing the direction of the emerging beam, assuming that the wedge angle, index of refraction, angle of incidence, and number of reflections are known.

The paper also gives the results of numerical calculations based upon this theory for sample situations which are of interest. It is shown that polarization effects can be minimized by the use of a small wedge angle and by the proper selection of the angle of incidence. In particular, it is shown that it is possible by the use of four reflections and a wedge angle of one degree to obtain attenuation factors of about 400,000 (56db), and that the effect of changes in polarization on the attenuation factor can be held down to about one percent.

Key Words: Optical attenuation, optical beam splitter.

1. INTRODUCTION

A wedge beam splitter is a prism of transparent material such as glass with a very small apex angle. If a narrow pencil beam is approximately at normal incidence to one face, it pentrates the prism and undergoes multiple reflections. At each reflection a portion of the beam is refracted and gives rise to an external beam, as illustrated in figure 1. The various beams are identified by a number, which we shall call the "order" of the beam, which is the number of reflections the radiation has encountered between incidence upon the prism and emergence from it. It is to be noted that there are two first order beams, denoted by -1 and +1, respectively. The + and - signs are arbitrary labels used to distinguish between these beams using a notation which is in the vocabulary of any computer:

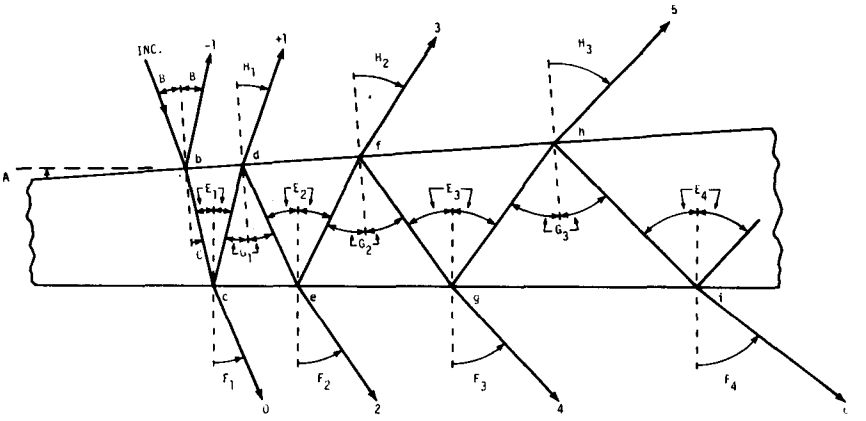


Figure 1. Propagation of a pencil beam in a wedge of small angle.

otherwise they have no significance. The -1 beam is reflected from the first surface. Therefore its properties are independent of the wedge angle, and they are an even function of the angle of incidence B: that is, they are the same for +B as for -B. The properties of all the other beams depend upon the wedge angle, and, in general they do not have the same values at -B as at +B.

There are two principal uses for such a device. One is to permit the monitoring of a source of radiation which varies in the emitted power. In such a case, one beam is incident upon a power meter, while another is used for some other purpose. The other is to provide a fixed amount of attenuation, which can be determined by both experiment and theory, and it is our purpose to indicate how the attenuation can be determined from theory.

In practice, the experimental value of the attenuation factor is more reliable in most cases, since the theory, for simplicity, must make some assumptions such as the neglect of surface scattering and imperfections in the glass. Also we are assuming that the incident beam is large enough to be considered a portion of a plane wave but small enough such that the various beams do not overlap and produce interference effects. Laser beams can not always be accurately described as plane waves. Thus our assumption implies an approximation. In fact it has been [1,2] shown that the mode structure of a laser beam can change on reflection. On the other hand, the theory gives guidance as to which beam to employ for any given purpose, and what conditions to use to make the attenuation as nearly independent of variations in angle of incidence or the state of polarization as possible. Furthermore the theory serves as a check upon experiment: any large discrepancy may be the result of scattered light.

It is desirable to make the attenuation factor as nearly independent of the state of polarization as possible, as often the state of polarization is unknown, or we may wish to compare sources of different polarizations. Also, there has come to our attention the case of a laser whose power output was reasonably constant but whose state of polarization was subject to large fluctuations. We shall show that the attenuation ratio depends very little upon the state of polarization if the wedge angle is very small and if approximately normal incidence is used at the first surface, and under these conditions errors due to polarization effects usually are negligible.

In principle, polarization effects can be eliminated by using a symmetrical arrangement of two identical beam splitters in cascade with their apex lines perpendicular to one another. Such an arrangement in a particular configuration is described in a U.S. Patent [3], and is incorporated in a commercial device. However, such an arrangement involves a number of obvious complications. We, therefore, are focusing our attention on the use of a single beam splitter and in finding under what conditions the errors are negligible.

We shall refer to the points where the various beams emerge, (such as b, c, d, e..... in figure 1) as "ports". If we assume as an approximation, that the energy left in the internal beam after the m'th reflection is negligible, the splitter can be considered as the analog of a 2m port microwave junction. The significance of the factor of 2 will become apparent later when we consider in detail the properties of the plane interface between two transparent media.

In developing the theory it is convenient to keep in mind the limiting situation when the wedge angle goes to zero: i.e., when the sides or the glass are parallel, as illustrated in figure 2. All of the beams emerging from the far side, the even orders, are all parallel

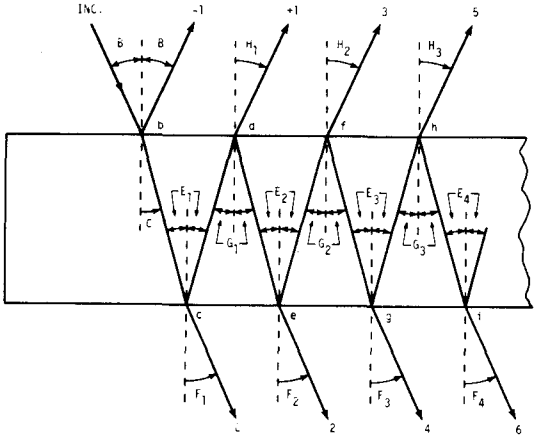


Figure 2. Propagation of a pencil beam in transparent material with parallel plane sides.

to the incident beam INC but displaced from it by an amount which depends upon the angle of incidence, the thickness, and the index of refraction. Under practical conditions this displacement is small, and the effect of changing the angle of incidence is minor. The odd order beams are parallel to each other but deviate from the incident beam by twice the angle of incidence.

The reflectivity R is defined as the ratio of the power per unit area of the interface in the reflected beam to the power per unit area of the interface of the incident beam. For beams of finite cross section this is the same as the ratio of the total power in the reflected beam to the total power in the incident beam. Similiarly, the transmissivity T is the ratio of the total power in the refracted beam to that in the incident beam. In general, these are functions of the angle of incidence B, the index of refraction N, and the state of polarization. Since the angle of refraction C is given by Snell's law in terms of B and N, it may be used as an independent variable instead of N.

Let us define the attenuation factor F as the ratio of the power in the incident beam to the power in the m'th order beam. Assuming that the sides are parallel, that the glass is homogeneous, and that scattering and absorption are negligible, for the -1 beam,

$$F = 1/R, \tag{1}$$

and for any other order m

$$F = 1/(T^2R^m). \tag{2}$$

In eq (2) it is assumed, as will be shown later, that T has the same value for the incident beam as for the emerging beam. For glass of low index of refraction (e.g., 1.5) R is about 1/25 while for high index (e.g., 1.75) it is about 1/13. For rough calculations T may be taken to be approximately unity. In terms of decibels, the attenuation factor is about 14 db per reflection for low index glass and about 11 db per reflection for high index glass. These figures can serve as a rough guide in preliminary design.

Because of the parallelism of the beams, a piece of glass with parallel sides is not generally useful as a beam splitter, since the beams are insufficiently separated to be distinguished by most detectors. Therefore, a practical splitter requires a small but finite angle between the sides allowing the adjacent beams to diverge as shown in figure 1. Later it will be shown that successive angles of incidence for beams inside the glass on the same

side (going from left to right in figure 1) increase by 2A, where A is the prism angle. For angles small enough such that the sine of an angle can be replaced by the angle in radians, it follows that successive emerging beams in air deviate by an angle 2NA, approximately. However, because of the addition of successive increments of 2A, the angle of incidence ultimately gets so large that this approximation does not hold. In fact, even with modest values of A, it may get so large that it exceeds the critical angle, and then the internal beams are totally reflected. R and T are independent of the state of polarization for normal incidence, and their variation with the polarization increases with the angle of incidence. Therefore, in general, polarization effects are more serious with the higher order beams.

As far as the forward (even order) beams are concerned, rotation of the beam splitter has little effect on their directions if the wedge angle is small: the beam splitter, other than giving rise to this divergence of the beams acts approximately like a piece of glass with parallel sides. However, the position of any backward (odd order) beam may be shifted to any arbitrary direction by rotating the prism. In general, as suggested by figure 1, the higher order beams are deviated towards the base of the prism and away from the apex line, under conditions which normally prevail in practice.

2. PROPERTIES OF THE PLANE INTERFACE BETWEEN TWO MEDIA

The calculation of the attenuation factor F for any arbitrary beam requires knowledge of the coefficients R and T at each encounter between the radiation and the interface between glass and air. And to evaluate R and T we must determine, using the laws of geometrical optics, the angle of incidence at each encounter.

The calculation of R and T for the plane interface between two homogeneous isotropic media is discussed in most books in optics and is well known. For convenience we summarize the results here. In spite of all that has been written on the subject, there are also some details which we shall supply that do not seem to be well known.

In figure 3 f't and fs denote the boundary rays of a pencil beam incident on the interface in medium 1 (which for the moment we shall assume to be air) at an angle of incidence B. For brevity we shall denote this beam as f, and we shall use an analogous notation for the

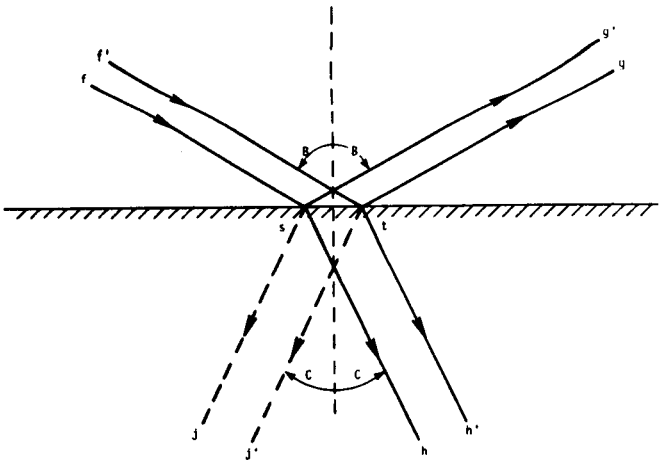


Figure 3. Reflection and refraction at plane interface.

other beams. This gives rise to a reflected beam g at an angle of reflection B and a refracted beam h at an angle of refraction C, where, of course, from Snell's Law,

$$\sin B = N \sin C, \tag{3}$$

where N is the index of refraction of the second medium (glass) relative to the first (air).

If we send the light on to the interface backwards along the direction of g, we produce a beam backwards along f. The beam h is not produced, but we produce a different refracted beam j at an angle C measured in the other direction. Similarly if the incident light is sent backwards along the beam direction h, the beam j is excited by reflection and f by refraction, while incident light along j excites beams h and g. Thus we have a situation analogous to a hybrid circuit in electrical circuit theory, the best known special case of which is the microwave "Magic Tee" junction.

In general for calculating the transmissivity T and the reflectivity R, we must distinguish between the case where the electric vector of the incident radiation is perpindicular to the plane of incidence (denoted by the subscript e) and when it is parallel (denoted by the subscript a). As can be found in many textbooks, [4]

$$R_e = \sin^2 (B-C)/\sin^2 (B+C), \tag{4}$$

$$R_a = \tan^2 (B-C)/\tan^2 (B+C), \tag{5}$$

$$T_e = (\sin 2B)(\sin 2C)/\sin^2 (B+C), \tag{6}$$

and

$$T_a = (\sin 2B)(\sin 2C)/(\sin^2 (B+C) \cos^2 (B-C), \tag{7}$$

In principle, by the use of Snell's Law, eq (3), these coefficients can be given as functions of B and N rather than of B and C, but the form of these equations is not convenient.

When B and C are small, the sines and tangents can be approximated by the appropriate angles in radians, and we note, approximately, $R_e = R_a$, $T_e = T_a$, and polarization effects disappear.

However, at normal incidence itself, B = 0, C = 0, and eqs (4-7) become indeterminant. In such a case it can be shown that [4]

$$R = (N-1)^2/(N+1)^2 \tag{8}$$

and

$$T = 4N/(N+1)^2 . \tag{9}$$

Also, for all cases, in accordance with conservation of energy,

$$R + T = 1. \tag{10}$$

It is to be noted, that in eqs (4-7), R and T are functions only of B + C and the magnitude (not the sign) of B - C. Also in eqs (8) and (9), R and T are invariant with the replacement of N by 1/N. Therefore, for a beam reflected in medium 1 at an angle B, R has the same value as for a beam reflected in medium 2 with an angle C, where B and C are related by eq (3). Similarly, for a beam incident at an angle B in medium 1 and transmitted into medium 2, T has the same value for a wave incident at an angle C in medium 2 and transmitted into medium 1.

Therefore, in principle, all of the R and T coefficients involved in a high order beam attenuation factor F can be determined experimentally. For example, if we want to know R and T at the point f in figure 1, we send in a calibrated beam backwards along the direction that the third order beam had emerged. This generates a reflected beam at f (which is not shown in the diagram). If the power in this beam is measured by a suitable calibrated detector, and if this power is divided by that in the incident beam, the

ratio is R, and this is the same for the internally reflected beams shown in the diagram. At the same time the transmission coefficient at f for the emerging third order beam is given by 1 - R by eq (10).

3. STEPWISE DETERMINATION OF ATTENUATION FACTOR

For a finite wedge angle A, eq (2) must be replaced by

$$F = 1/(T_i R_0 R_1 R_2 \cdots R_{m-1} T_m) \ , \tag{11}$$

where T_i is the transmission coefficient at the point of entrance of the incident beam, T_m is the transmission coefficient at the point of exit of the m'th order beam, and R_k is the reflection coefficient at the point of emergence of the k'th order beam. These coefficients, of course, are assumed to be evaluated at the appropriate respective angles of incidence.

Now let us suppose that the incident beam had been backwards along the direction of the original emergent beam of order m. From the discussion of the properties of a single interface, it can be inferred that a beam would emerge in the direction of the original incident beam. Since the order of the factors in eq (11) is unimportant, the attenuation factor F is the same as for the original configuration. In other words, in the language of circuit theory, the beam splitter is a reciprocal device.

Next let us multiply numerator and denominator of eq (11) by $T_{m'}{}^2$, where m' is intermediate between 0 and m, and group the factors as follows:

$$F = \left[\frac{1}{T_i R_0 \cdots R_{m'-1} T_{m'}}\right] \left[\frac{T_{m'}{}^2}{R_{m'}}\right] \left[\frac{1}{T_{m'} R_{m'+1} \cdots R_{m-1} T_m}\right] \ . \tag{12}$$

In principle, each quantity in brackets can be determined experimentally. The first quantity is just the attenuation factor of the m' order beam. The last factor is the attenuation factor which applies when the incident light is sent backwards along the direction of the m order beam, and when the observation is made on a beam emerging from m' port (but leaving the surface with an opposite angle to the normal). The middle quantitity can be found by sending a beam backwards along the original m' beam direction and observing its -1 reflection. This gives $R_{m'}$ directly, and $T_{m'}$ can be found from it by eq (10).

Therefore if the original factor is too large to be measured directly because of the limited range of the instruments, it can be broken down into the product of quantities which can be brought within range, and yet it is not necessary to measure individually every coefficient, as had been suggested earlier.

4. CALCULATION OF ANGLES

In applying the formulas that have been given for calculating the individual R and T coefficients and thus for calculating the attenuation factor, it remains for us to develop formulas for the angles of incidence. For this purpose we refer to figure 4, which duplicates the first few beams of figure 1 with the same notation but, for clarity, with an expanded scale and with the angles made much larger.

By applying the theorem that the sum of the angles of a triangle is 180° to the triangle abc, we find that

$$E_1 = A + C \quad . \tag{13}$$

By applying this theorem to the triangle bcd, we find that

$$G_1 = 2E_1 - C$$

$$= C + 2A.$$

Thus, G_1 exceeds C by 2A.

We can see by inspection that if we were to repeat this argument to the triangles cde and def we can show that

$$G_2 = G_1 + 2A \ ,$$

and, in general,

$$G_L = C + 2LA, \tag{14}$$

where L is any integer.

By a similar argument we can show that

$$E_L = 2(L - 1)\, A + E_1 \tag{15}$$

$$= C + (2L - 1)A \quad . \tag{16}$$

It is important to point out that there is an implied sign convention with regard to the angles B, C, E_L, and G_L. These angles are all positive as shown in figures 1 and 4. If the incident beam associated with any of these beams should be on the opposite side of the respective normal to the interface, it should be considered as a negative angle.

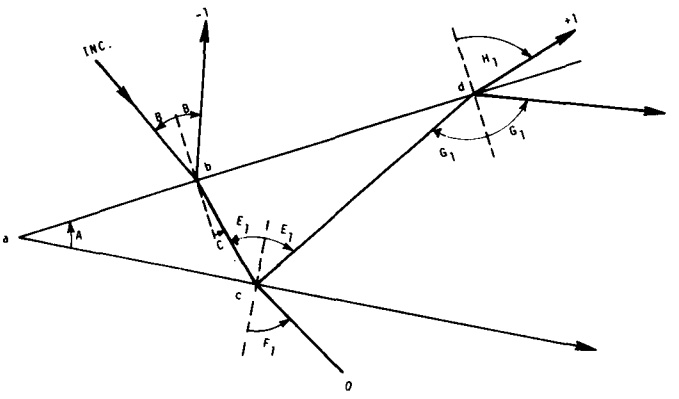


Figure 4. Details of propagation in a wedge.

For negative values of the angle of incidence of the entering beam B, some of these angles may become negative, and in extreme cases the light may be propagated internally towards the apex of the prism. However, ultimately, when L becomes large enough, there are enough increments of 2A to make E_L and G_L positive, and the light is reversed in direction. Then the higher order beams emerge directed towards the base. In the usual situation prevalent under practical conditions, <u>all</u> even order beams emerge directed towards the plane of the base, and, the odd order ones emerge in the sequence shown in figure 1.

Since E_L and G_L are smaller for negative values of B than for positive ones, the minimum polarization effects occur at small negative values of B, as to be shown later by the graphs which have been based upon calculations from the theory. Therefore, especially, in working with higher order beams, one should avoid using a positive value of B.

It is convenient to record the relationship between the beam order m and the index L. For odd orders,

$$m = 2L - 1, \tag{17}$$

and for even orders,

$$m = 2(L - 1). \tag{18}$$

The angles of refraction of the emerging beams, F_L and H_L may be calculated by Snell's law from E_L and G_L respectively. In principle, it is possible to derive expressions for explicit formulas for these in terms of A, B, and N, but if the exact form

of Snell's law is used, these formulas become very cumbersome, and we shall not derive them. Instead, for convenience, we shall assume that the angles are small enough so that their sines may be approximated by the angles in radians. Under practical conditions, this approximation is excellent, at least for beams of low order. On the other hand, with numerical computations carried out by a computer it is entirely practical to use the exact form of the law throughout, and such has been done in the preparation of the graphs to be presented later.

With this approximation, it can be shown that

$$F_L = B + (2L - 1)NA, \tag{19}$$

and

$$H_L = B + 2LNA \quad . \tag{20}$$

therefore

$$F_L - F_{L-1} = 2NA,$$

and

$$H_L - H_{L-1} = 2NA \ .$$

The deviation of the -1 beam from the incident beam is given by

$$D = 2B \ . \tag{21}$$

For other odd orders it is given by

$$\begin{aligned} D &= B + H_L \\ &= 2B + 2LNA. \end{aligned} \tag{22}$$

For even orders it is given by

$$\begin{aligned} D &= F_L - B - A \\ &= (2L - 1)NA - A \ , \end{aligned} \tag{23}$$

which is independent of the angle of incidence B. Experimentally, these even order beams are found to shift in position slightly with changes in B. These shifts are due to the breakdown of the approximation that the sines of angles be replaced by the angles.

The well known case of the minimum deviation of the zero order beam occurs when $C = E_1$, giving

$$B = -NA/2 \ . \tag{24}$$

5. PRACTICAL MEASUREMENTS WITH BEAM SPLITTERS

The measurement of the attenuation factor for the beam of order m, which now we shall denote by F_m, requires putting a detector first in the incident beam and then in the M'th order beam. In general, the power of the beam can not be expected to remain constant, although, if it is reasonably constant, the effects of instabilities can be eliminated to a large extent by alternating the observations and taking the average. However, the effect can be eliminated by observing two beams simultaneously, but neither of these obviously can be the incident beam, as a detector placed in it prevents the radiation in reaching the beam splitter. Therefore, in many applications which are too obvious to enumerate, one interested in the ratio of two other beams m and m'. Therefore, it is convenient to define the beam splitter ratio $S_{mm'}$, as such a ratio, and

$$S_{mm'} = F_m/F_{m'} \quad , \tag{25}$$

which can be calculated theoretically by application of eq (11).

In order to apply the theory, it is necessary of course, to have values of N and A. The measurement of these quantities is contained in a standard course in elementary optics. However, when the wedge angle A becomes very small the standard methods are difficult or impossible to apply.

The standard method of measuring the angle A is to mount the prism on a spectrometer with an angular scale and to use with it a telescope with a Gauss eyepiece. With this, one face is made perpendicular to the axis of the telescope, and then the prism is rotated through an angle until the other face is perpendicular. Then A is just 180 degrees minus the angle through which the prism is rotated. The modern version of this method is to use a laser beam instead of a telescope and to adjust the positions of the prism to produce beams that are reflected along the incident beam. When the angle A is large, there is no difficulty in recognizing when the beam is reflected off the front face (i.e., the -1 beam). However, there are also two positions where the +1 beam is also reflected along the incident beam, and when A is small it may be difficult to identify which beam is which. Here the laser method has a distinct advantage in identifying the beams, since even with a low powered laser it is usually possible to observe the second and third order beams, and, since under practical conditions, these are in the sequence shown in figure 1, they may be used to dis-

tinguish between the two first order beams. At any rate, this method of determining A is likely to have low precision, since it is necessary to take the small difference between two large angles. In practice, it may be better to obtain A indirectly as discussed below and to use the direct value as a check.

The standard method for measuring the index of refraction of a prism of large angle A, which has previously been determined, is to measure the angle of minimum deviation D and with the formula for it to calculate N. However, as we have seen, with small values of A, the angle of deviation is essentially independent of the angle of incidence, and the formula can not be applied.

As a more practical procedure, it is suggested that the index of refraction N be determined by measuring the beam splitter ratio S_{0-1}, which is independent of the angle A. To reduce the effect of inhomogenieties, measurements should be made on both faces and the average computed. By application of eqs(25), (1), and (11) it can be seen that

$$S_{0-1} = T_i T_0 / R_i \quad . \tag{26}$$

However, if the wedge angle A is small and if nearly normal incidence is employed, we may drop the subscripts on the right and by use of eq (10) see that

$$S_{0-1} = (1-R)^2/R \quad . \tag{27}$$

The reflection coefficient R is found by inserting the measured value of S_{0-1} and solving, and then the index of refraction is found by substituting this value of R into eq (8) and solving.

The quantity NA can be determined by measuring the angular separation between two adjacent beams (which, in radians, is 2NA). For this purpose the most convenient pair are -1 and +1. With a laser source, these can make bright spots on a screen, which should be sufficiently distant to allow an accurate measurement of the separation. Finally the value of A can be obtained by dividing the previously determined value of N into this result.

In determining a high order attenuation factor or beam splitter ratio experimentally, it is desirable to make observations of the weak beam with the detector at various distances from the beam splitter because of possible scattering due to dirt and imperfections of the

splitter. The intensity of the true beam is independent of the distance while scattered light varies approximately with the inverse square of the distance. Therefore correction for effects of scattering can be accomplished by extrapolating the observed ratio quadratically to infinite distance.

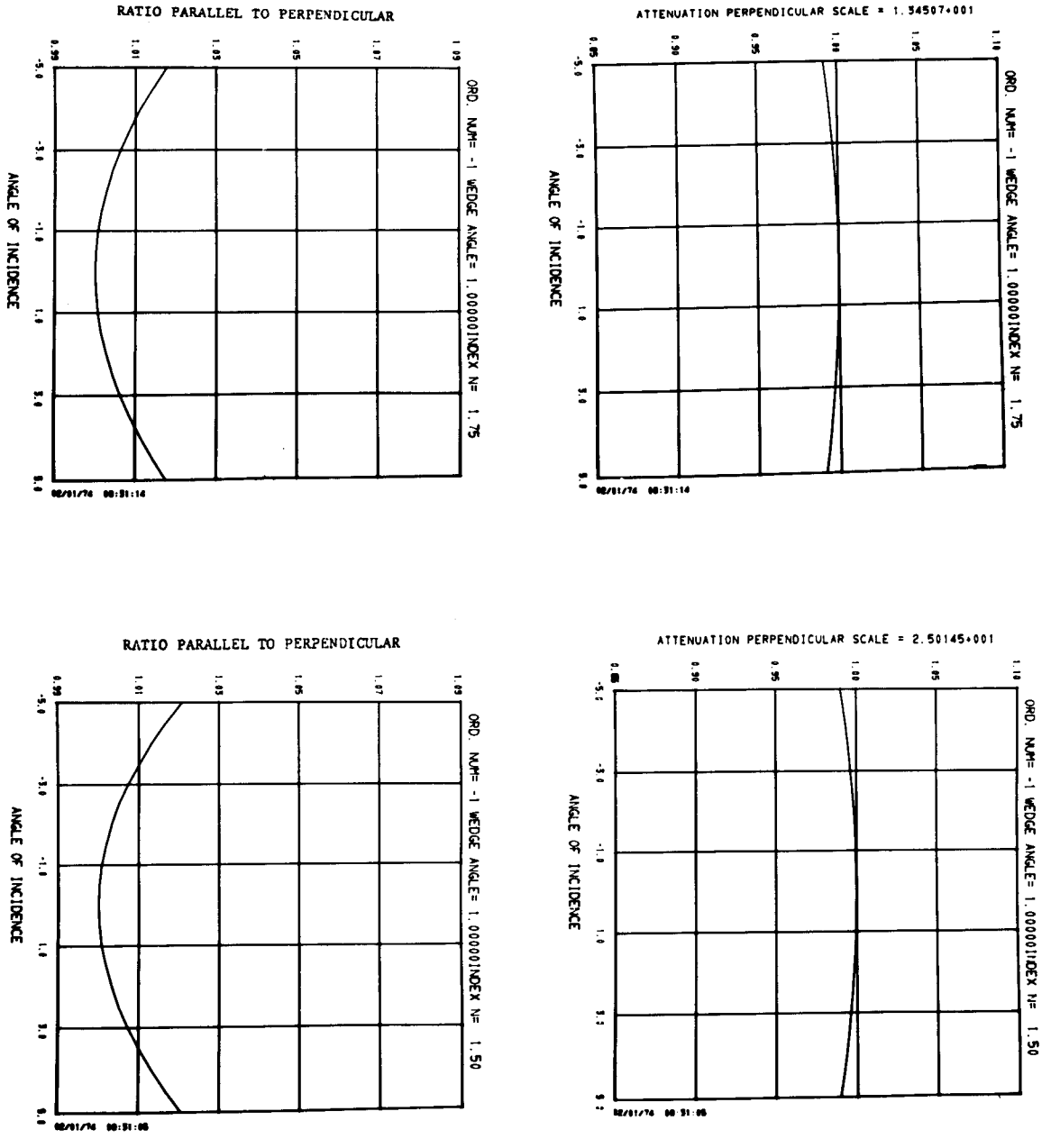
6. NUMERICAL RESULTS AND CONCLUSIONS

The equations that have been presented in the earlier sections have been incorporated in a computer program which, given the wedge angle A, the index of refraction N, the angle of incidence B, and the order of the reflection m, calculates the angular position and the attenuation factor of the emerging beam for both polarization of the incident beam.

Illustrations of attenuation factor calculations are contained in graphical form in figures 5 through 14. In the upper left portion of the first of these, figure 5, is shown the attenuation factor for the perpendicular polarization as a function of the angle of incidence for a range of $\pm 5^{\circ}$ from the maximum for the -1 order, for $A = 1^{\circ}$, and for N = 1.75, the value of the attenuation factor is normalized to the maximum value. The actual value at the maximum is given on the left axis of the graph, and the actual value at any other angle can be found by multiplying it by the ordinate of the graph for that angle.

The graph in the lower left corner gives the ratio of the attenuation factor for the parallel polarization to that for the perpendicular polarization for the same parameters as the graph above. It appears that this has a minimum value at the same angle as the maximum in the graph above. Such a correspondence appears to hold with all of the other pairs of graphs which follow, but the equations are too complicated to make it practical to prove whether these stationary values occur at exactly the same angle or not, but within the accuracy of the graphs they appear to occur at the same angle.

The graph of the ratio of the two attenuation factors is useful for evaluating the limit of error which results if the attenuation factor is measured or calculated for one polarization and then the splitter used with a beam of unknown polarization. (Incidentally, one laser used by our colleagues has a nearly constant power output, but its state of polarization is subject to serious fluctuations.) This error is given by taking the ordinate from the graph and subtracting unity from it. The perpendicular polarization is of greater interest than the parallel one since some of the better engineered lasers have beams


RATIO PARALLEL TO PERPENDICULAR
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ANGLE OF INCIDENCE
ORD. NUM= -1 WEDGE ANGLE= 1.000001INDEX N= 1.75
RATIO PARALLEL TO PERPENDICULAR
ATTENUATION PERPENDICULAR SCALE = 2.50145+001
ANGLE OF INCIDENCE
ORD. NUM= -1 WEDGE ANGLE= 1.000001INDEX N= 1.50

Figure 5.

that are vertically polarized and, if they are used with beam splitters that are oriented to spread the beams out in a horizontal distribution, this is the relevant case. Therefore the attenuation factor for this polarization was chosen for the ordinate in the left hand graph.

The two graphs on the right side of this page (figure 5) are analogous respectively to the two on the left except that the index of refraction had been changed from 1.75 to 1.5.

The graphs on the next page, figure 6, pertain to the same set of conditions (order -1) as the ones in corresponding positions in figure 5 except that the wedge angle A has been changed from 1° to 2°.

The remaining pairs of figures are arranged in the same manner except that succeeding pairs pertain respectively to orders $m = +1, 2, 3$, and 4.

In view of the fact the reflection for the -1 order is independent of A, figures 5 and 6 are identical except for the labeling.

From the examination of these graphs, it is possible to draw several useful conclusions:

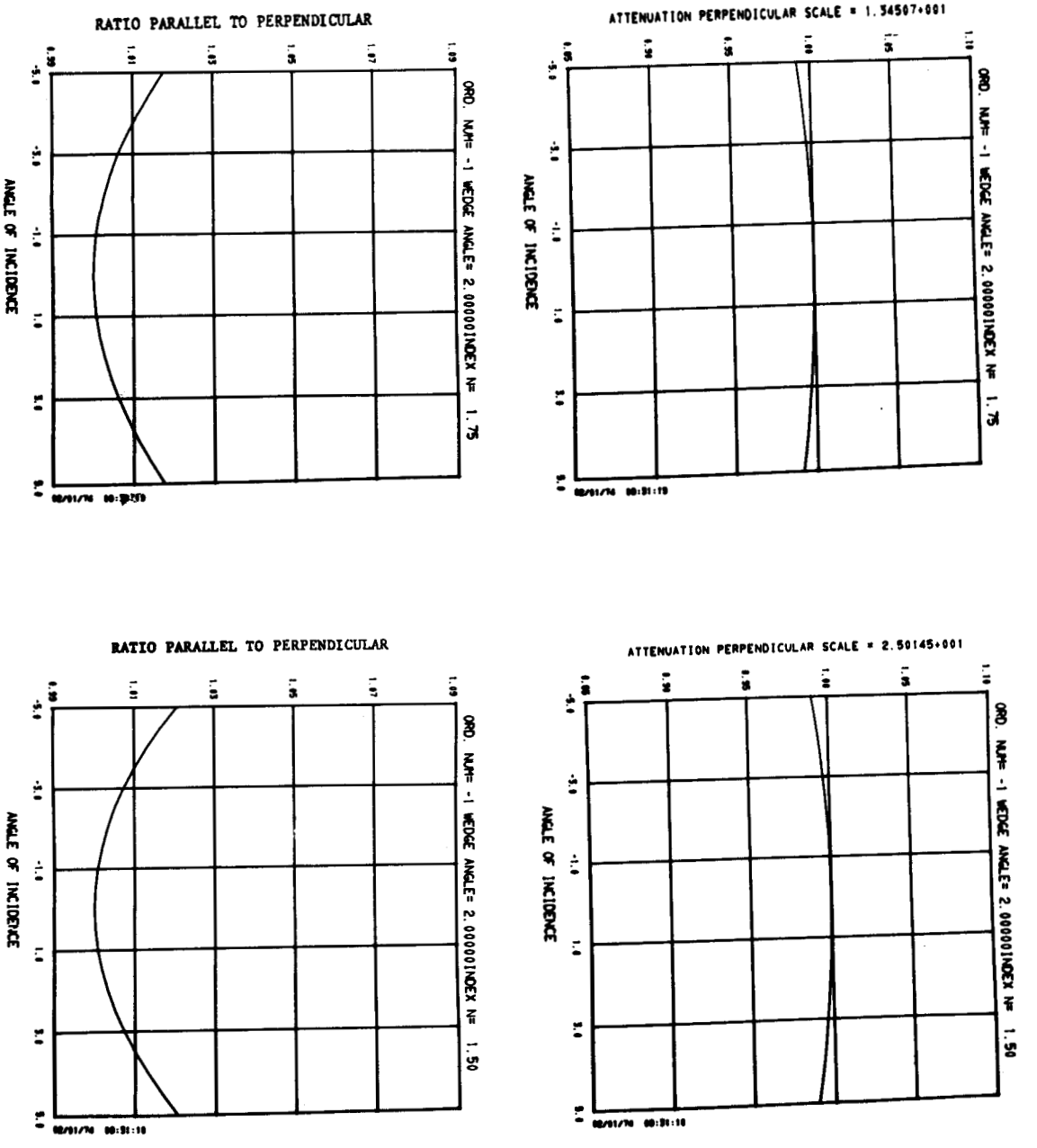
(1) As we have said before, the maximum attenuation factor and minimum polarization effect angle occur at essentially the same angle of incidence. In order to minimize the polarization error and the error due to uncertainties in the angle of incidence, attempt should be made to use this angle of incidence in experimental situations.

(2) As suggested by the previous theory, this optimum angle of incidence is negative in sign and becomes larger as the beam order increases.

(3) The principal effects of increasing the index of refraction N is to lower the attenuation factor for the beam of any given order and to widen the angular separation between adjacent beams.

(4) Increasing the wedge angle A has little effect upon the attenuation factor of any given beam but it makes the polarization effects more serious. Also the optimum angle of incidence is shifted to larger negative value.

(5) By restricting the wedge angle A to 1° or less and by the use of the optimum angle of incidence it is possible to produce attenuation factors of the order of 400,000 (56 db) with polarization errors of just slightly more than one percent. It is doubtful whether such attenuation factors can be measured or calculated to such an accuracy. Therefore, with attenuation factors of this size it seems unnecessary to go to the complication of two symmetrically place beam splitters to cancel out polarization effects if proper care is taken.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ANGLE OF INCIDENCE
ORD. NUM= -1 WEDGE ANGLE= 2.000001INDEX N= 1.75
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ANGLE OF INCIDENCE
ORD. NUM= -1 WEDGE ANGLE= 2.000001INDEX N= 1.50
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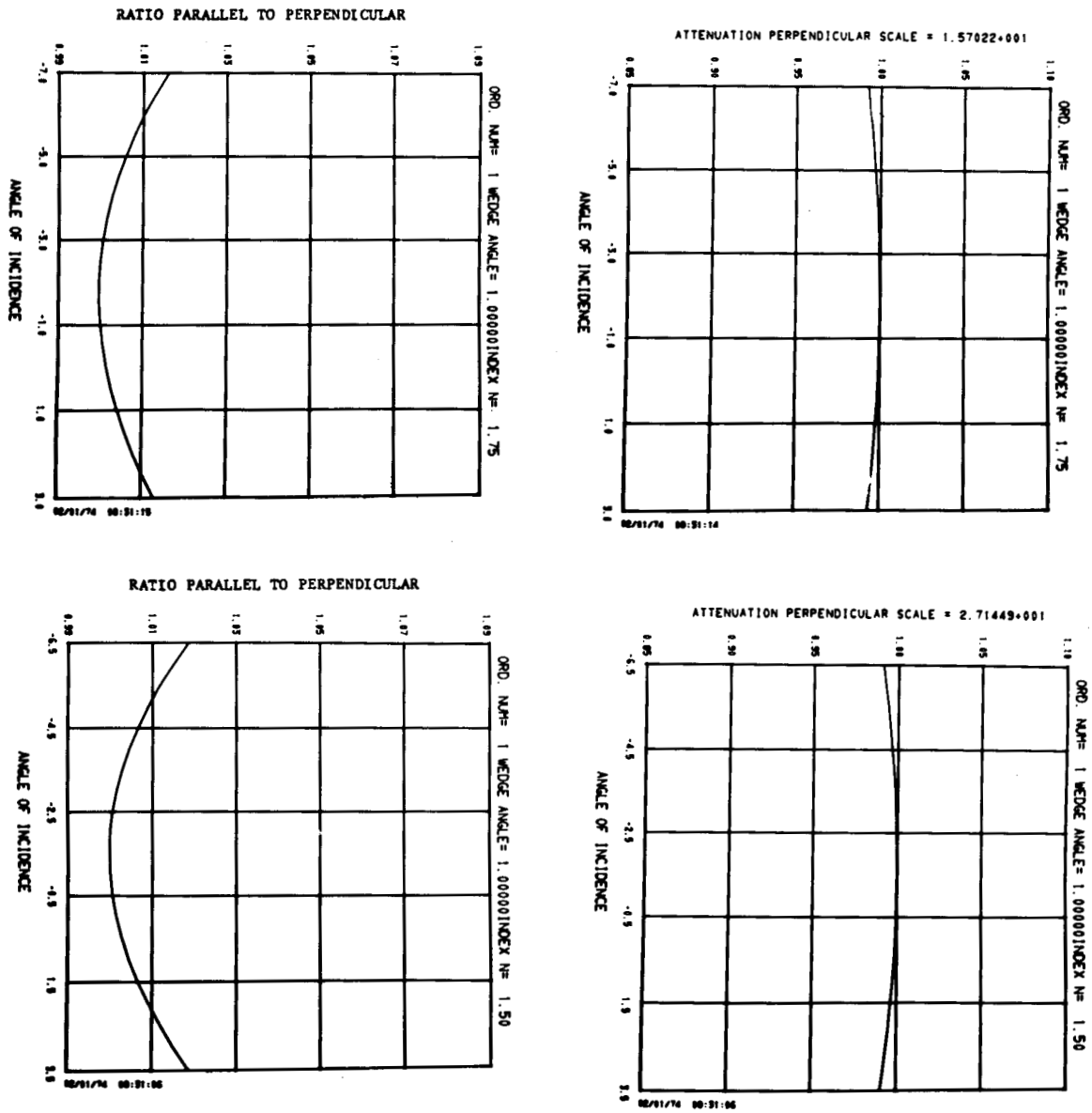
ANGLE OF INCIDENCE
ORD. NUM= -1 WEDGE ANGLE= 2.000001INDEX N= 1.50

Figure 6.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ANGLE OF INCIDENCE
ORD. NUM= 1 WEDGE ANGLE= 1.000001INDEX N= 1.75
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ANGLE OF INCIDENCE
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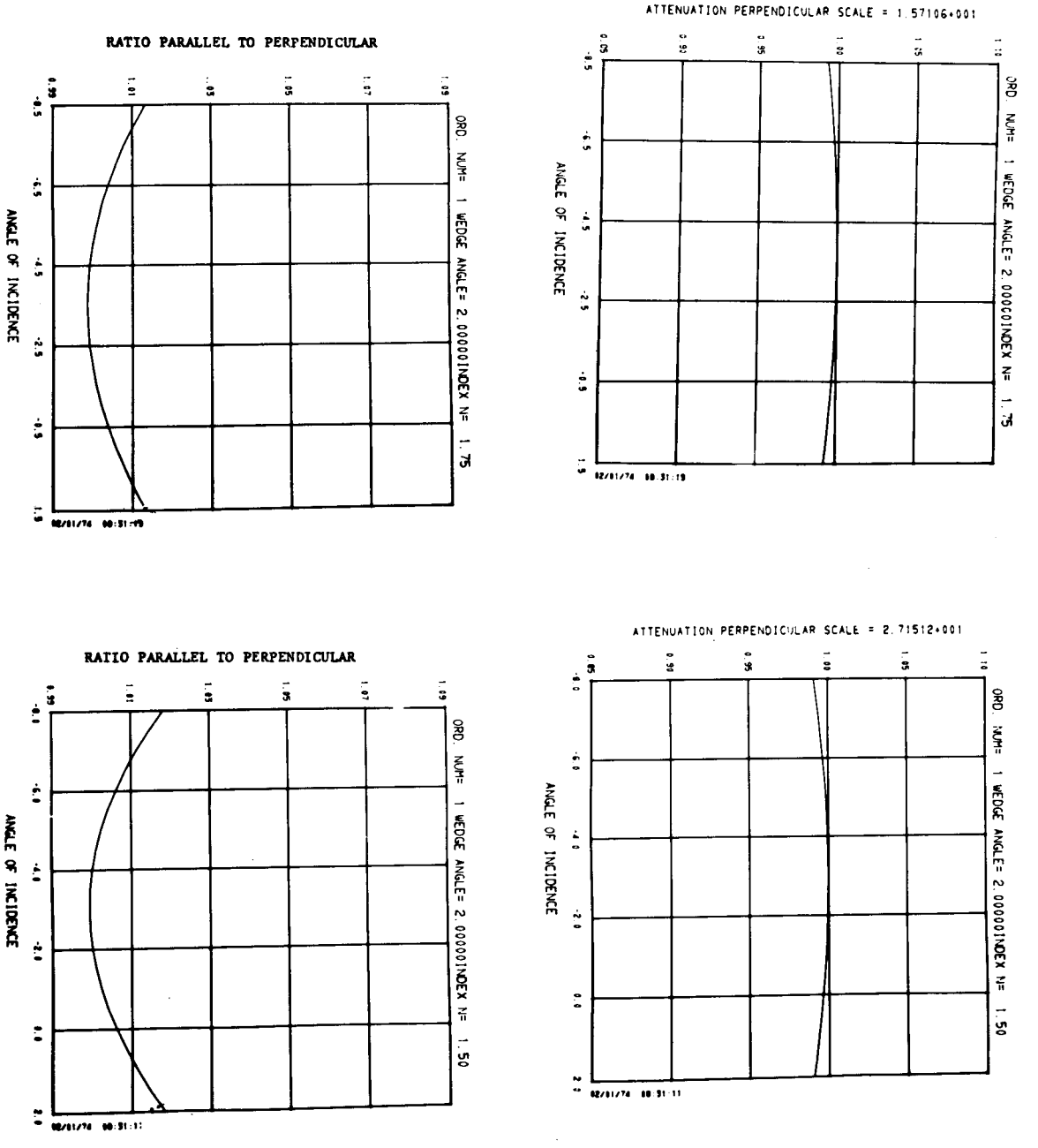
ORD. NUM= 1 WEDGE ANGLE= 1.000001INDEX N= 1.50

Figure 7.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ANGLE OF INCIDENCE
ORD. NUM= 1 WEDGE ANGLE= 2.000001INDEX N= 1.75
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ANGLE OF INCIDENCE
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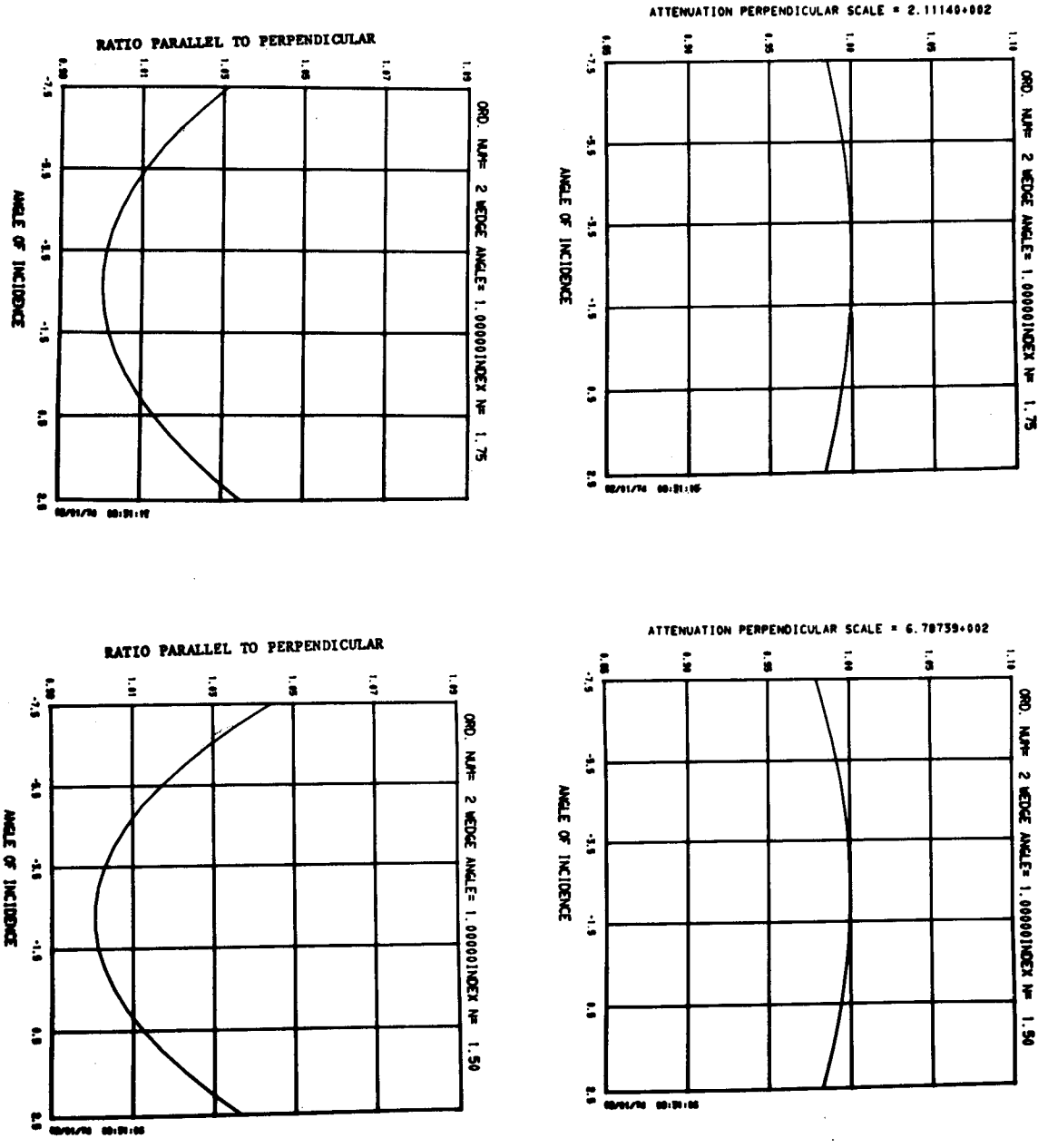
ANGLE OF INCIDENCE
ORD. NUM= 1 WEDGE ANGLE= 2.000001INDEX N= 1.50

Figure 8.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ANGLE OF INCIDENCE
ORD. NUM= 2 WEDGE ANGLE= 1.000001INDEX N= 1.75
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ANGLE OF INCIDENCE
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ANGLE OF INCIDENCE
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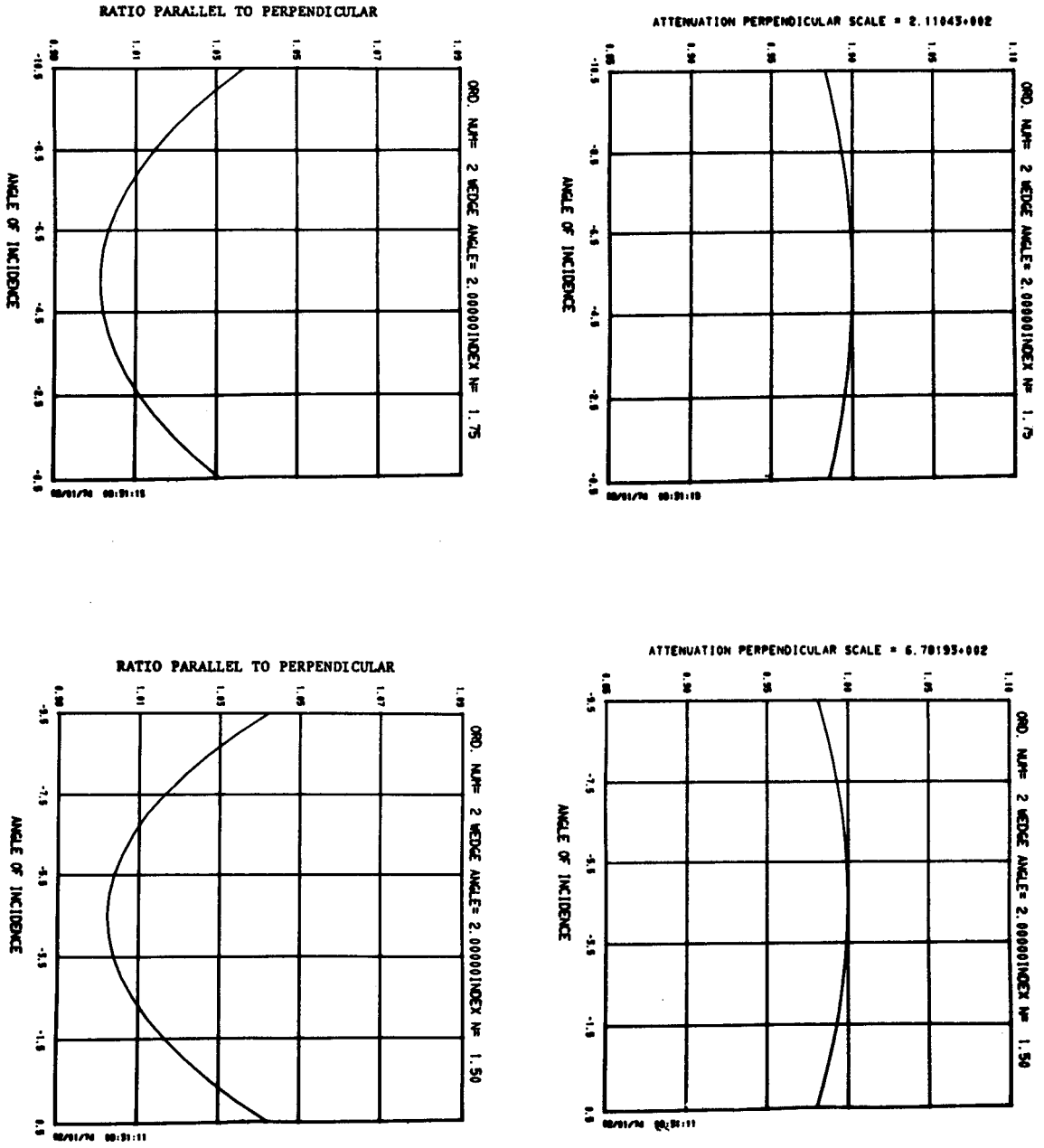
Figure 9.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ANGLE OF INCIDENCE
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ANGLE OF INCIDENCE
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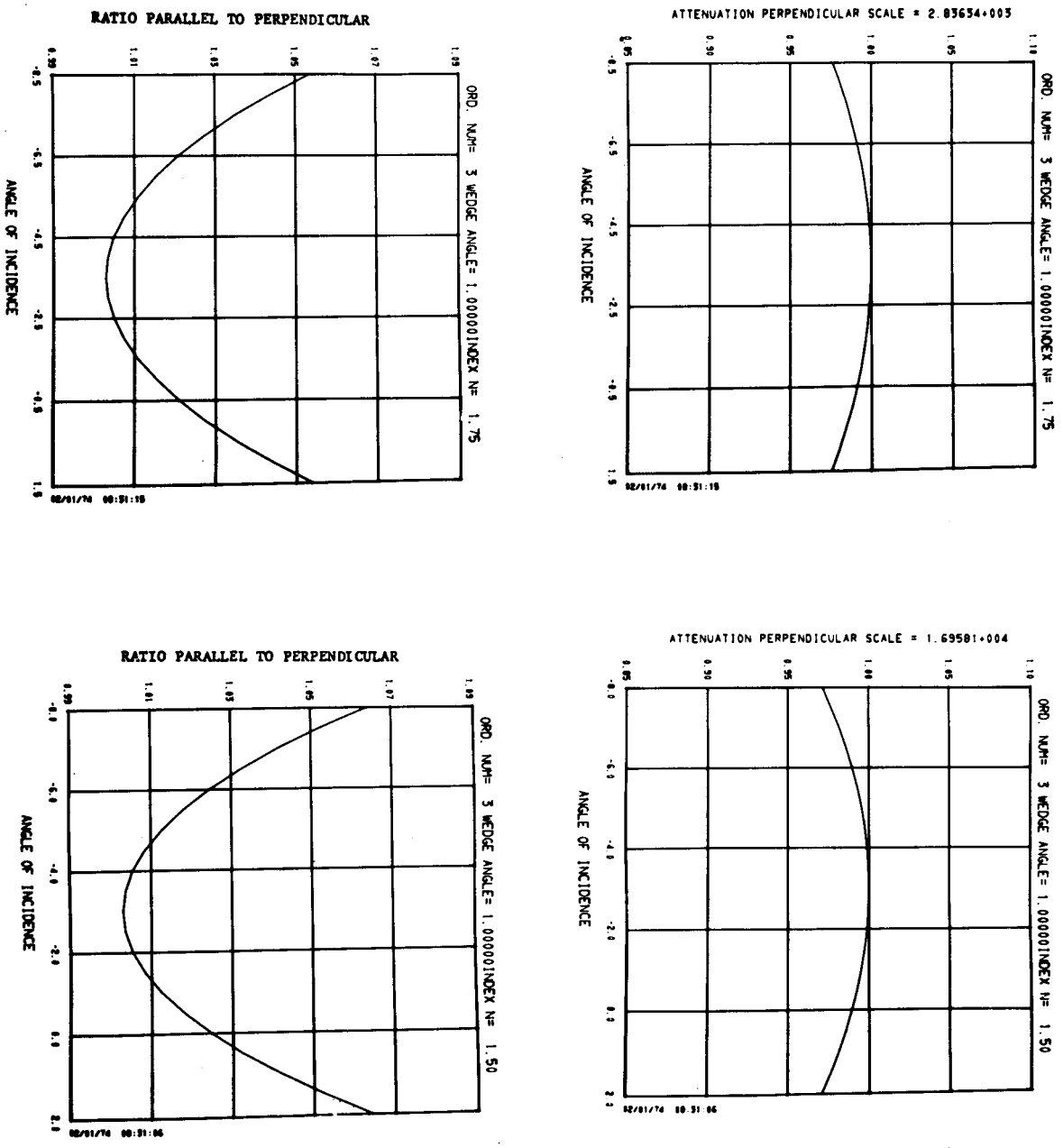
Figure 10.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ANGLE OF INCIDENCE
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ANGLE OF INCIDENCE
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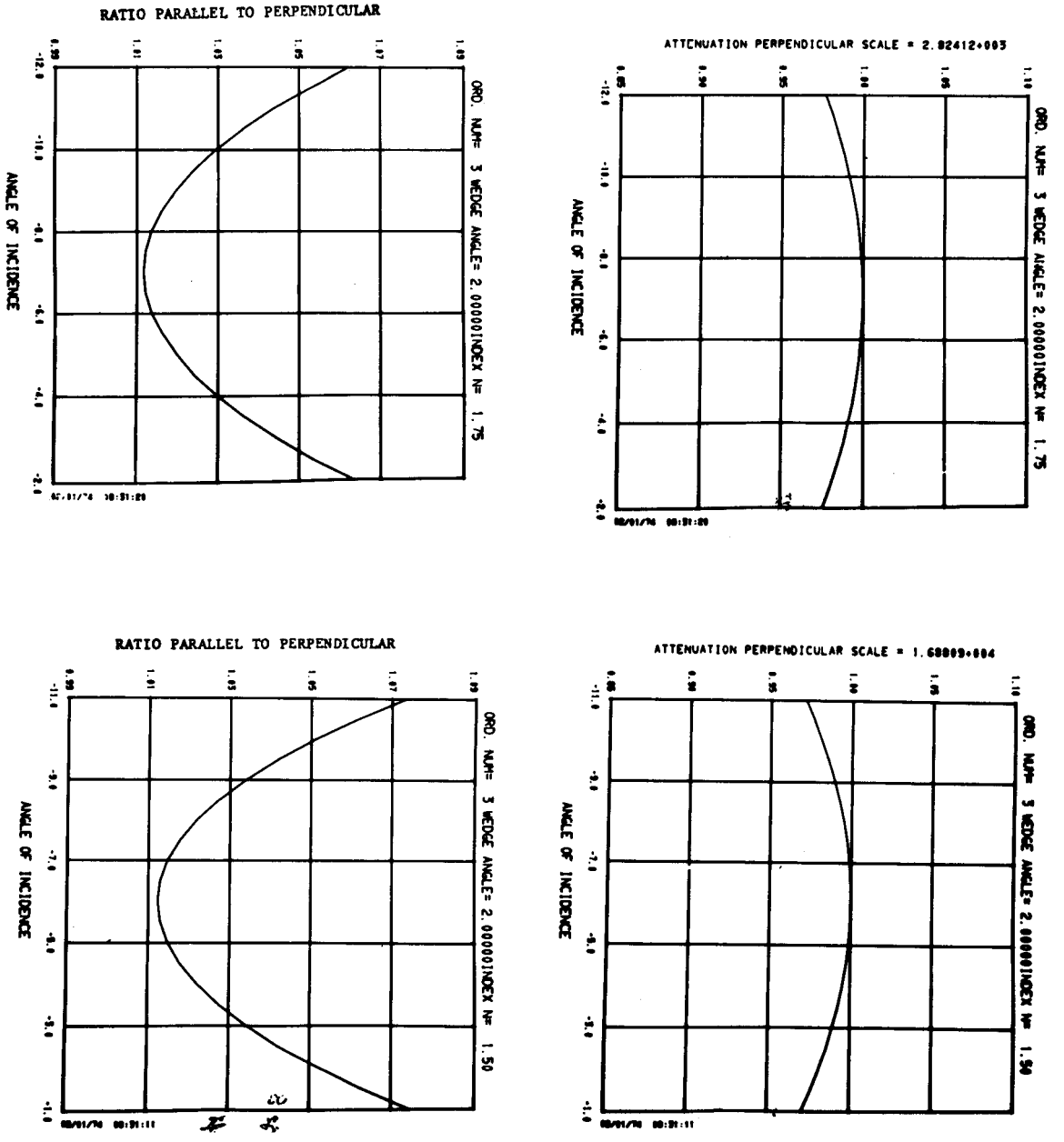
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Figure 12.

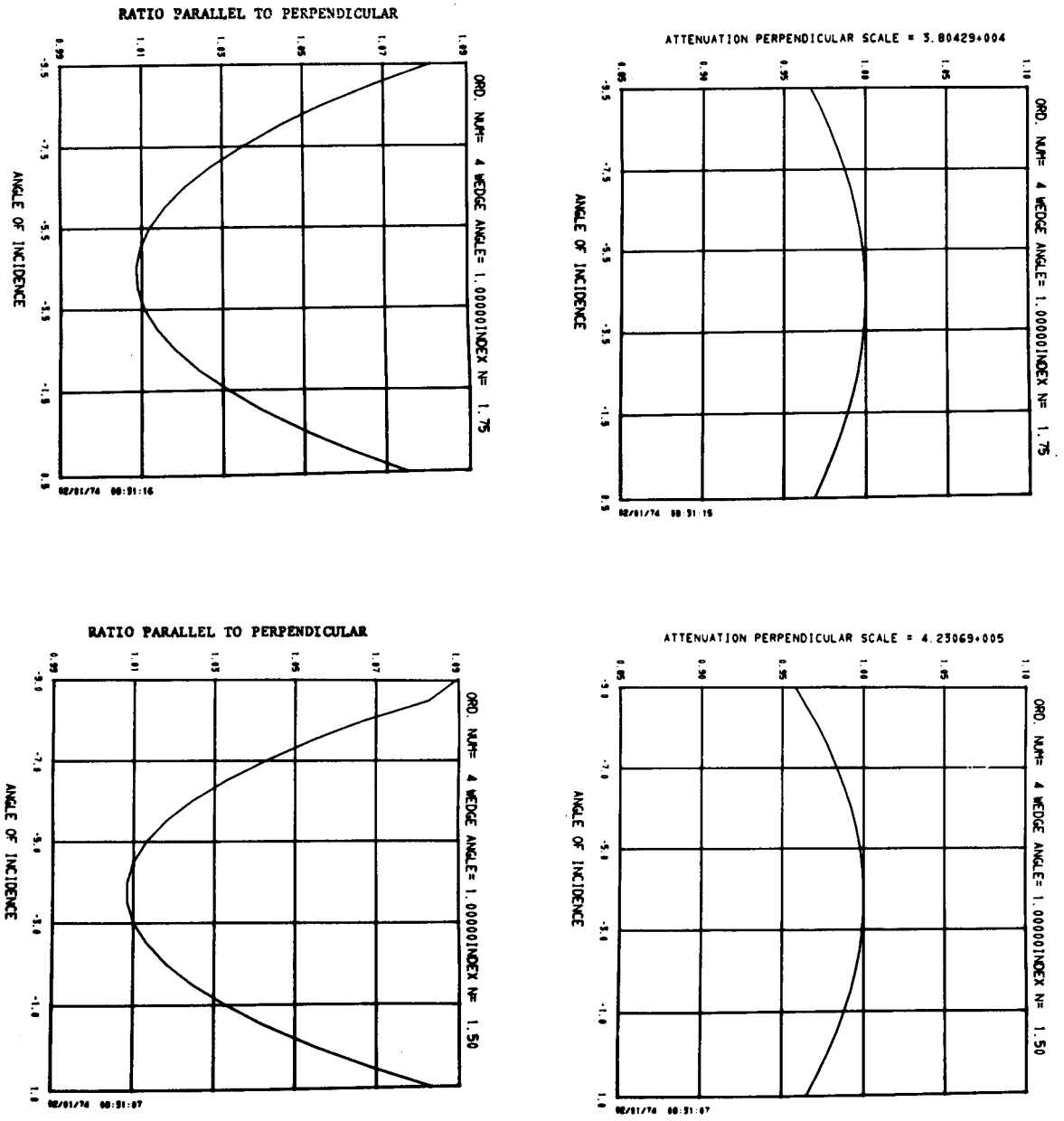
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Figure 13.

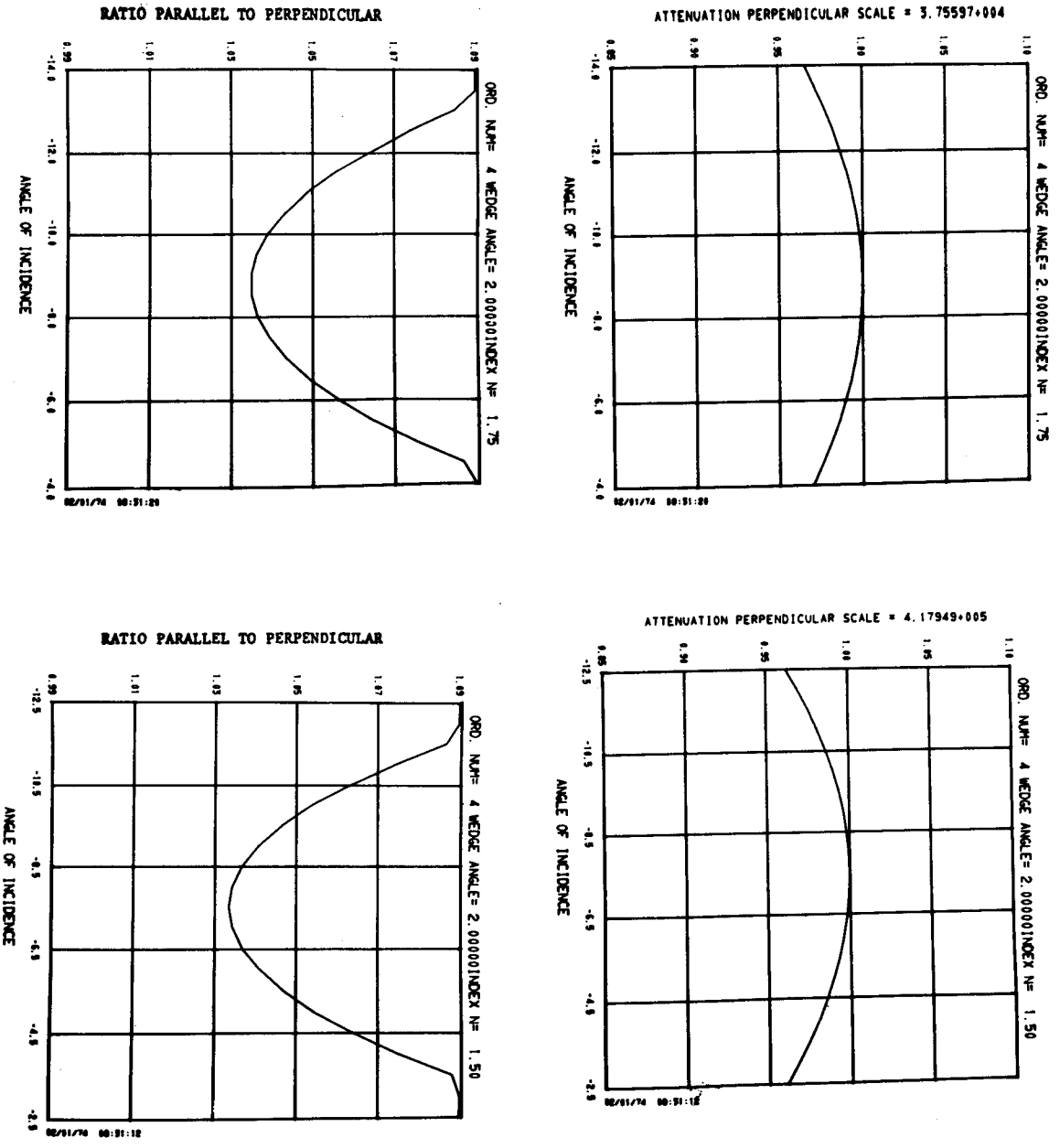

Figure 14.

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7. ACKNOWLEDGMENTS

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3. P. T. Gates, U.S. Patent 3,463,575, "Light Beam Sampling Apparatus", August 26, 1969.
4. See for example, M. Born and E. Wolf, "Principles of Optics", Sec. 1.5.3 3d. Ed., Pergamon Press, N.Y. (1965).

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