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On the Long Term Phase Stability of the 19.8 kc/s Signal Transmitted From Hawaii, and Received at Boulder, Colorado

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The use of VLF signals for intercontinental frequency comparison has become very popular in recent years, and it has been shown by other workers that a precision of about 2 parts in 10^{11} can be achieved with measurements over a 24-hr period. Phase records made at Boulder, Colo., of the NPM, Hawaii, 18.6 kc/s transmission have been studied for several periods of nearly two weeks duration in 1962. Deviations from an assumed linear frequency difference between the transmitter and receiver oscillators show an attainable precision of 2.5 parts in 10^{11} in a 24-hr observation, extending to 3.1 parts in 10^{12} in a 192-hr observation. Without further data on the remaining differences between the oscillators, there is no evidence that the propagation conditions over the path are limiting this precision. It is pointed out that the 50° (peak to peak) semiannual variation in the day to night change in phase, if attributed entirely to one level of reflection, would have an effect on precision of frequency comparison of about 1 part in 10^{12}.

1. Introduction

The use of VLF radio signals for intercontinental frequency comparison has become very popular in recent years since the study of Pierce [1957] on the stability of the 16 kc/s transmission from Rugby, England received at Cambridge, Mass. In that work it was demonstrated that precisions of parts in 10^{10} could be achieved in a few hours, but observations over longer periods did not yield an increase in precision. The extension of measurements to a period of 24 hr was later shown to yield a precision of about 2 parts in 10^{11} over the same path in an experiment comparing cesium oscillators, reported by Pierce, Winkler, and Corke [1960]. It is of interest to study the stability of the lower ionosphere over periods even longer than 24 hr and also to attempt to verify the results of these earlier studies for different paths and frequencies. For this reason a study of the NBA, Panama to Boulder, Colo. (4258 km), 18.0 kc/s transmission was made and reported earlier [Brady and Murphy, 1962], but the difficulties in resolving the effects of the frequent (twice per week) adjustments of that transmitter oscillator have prompted a further study on another path. The NPM, Hawaii, 19.8 kc/s (5374 km) transmission was chosen for this study at the suggestion of Wm. Markowitz (private communication) because of the excellent stability of the transmitter oscillator and its infrequent (once or twice per year) adjustment.

2. Determination of the Long Term Phase Variations

The signal from NPM is compared with a signal derived from a 100 kc/s signal furnished by the NBS Radio Standards Laboratory from a rubidium vapor controlled working frequency standard. The phase is recorded modulo 360 deg on a chart recorder, and after scaling, any 360 deg ambiguities are resolved by an inspection of the diurnal variations plotted on a compressed scale over periods of weeks at a time. Data from the routine observations of NPM made in Boulder during the first six months of 1962 were thus scaled, and out of these data several periods of from 11 to 15 days were chosen where the oscillators involved appeared to be behaving in a consistent fashion, i.e., there were no obvious abrupt changes in the recorded frequency difference. Although selection of data on this basis might bias the results it is believed that such abrupt changes were in instrumental rather than ionospheric.

In order to study variations attributable to the ionosphere, it is necessary to eliminate the effects of frequency changes in the oscillators. Since there was no independent information on the frequency difference between the two oscillators (i.e., independent of the ionosphere), it was assumed that the linear or first order approximation to the apparent frequency difference could be attributed wholly to the oscillators.

This linear difference in the apparent frequency corresponds to a parabolic component in the recorded phase difference, which was removed by fitting a parabola to the hourly values of phase during the daylight hours over the entire two week or so period. In subtracting this parabola, the effects of any possible change in the height of the daytime ionosphere having periods greater than two weeks are also removed. As will be seen later, however, a gradual drift in the daytime height of reflection amounting to several kilometers would be necessary to contribute noticeably to a frequency measurement over such a period. The data periods which were chosen are shown in table 1 along with the coefficients of the least squares fitted parabola, At^2+Bt+C, where A, B, and C yield degrees of phase with time t in hours. The frequency difference observed between the two oscillators will be given by the time derivative of the phase, $2At+B$, and the values of $2A$ and of B are also given in the table in terms of the fractional frequency difference $\Delta f/f$. The (linear) drift rate in the frequency difference is small, being at most a few parts in 10^{12} per day.

TABLE 1.—*Apparent frequency difference between transmitter and receiver oscillators*

Data period	Phase $P=At^2+Bt+C$		Frequency $\Delta f/f=2At+B$	
Dates inclusive 1962	A (deg hr^{-2})	B (deg hr^{-1})	Drift rate $2A$ (parts day^{-1})	Initial offset B (fractional parts)
26 Mar–6 Apr	0.2065×10^{-3}	0.9810	0.4×10^{-12}	4.1×10^{-11}
9 Apr–22 Apr	-1.2690×10^{-3}	1.2061	-2.5×10^{-12}	5.0×10^{-11}
26 Apr–9 May	0.3055×10^{-3}	0.1295	0.6×10^{-12}	0.5×10^{-11}
7 May–20 May	$.9513\times10^{-3}$	$-.5846$	1.8×10^{-12}	-2.4×10^{-11}
21 May–31 May	1.2131×10^{-3}	.1777	2.4×10^{-12}	0.7×10^{-11}

The departures of the observed phase from the parabolic variation will be discussed in terms of *precision* of frequency comparison in a manner essentially that of Pierce [1957]. Thus, the precision obtained in an observation period of duration T will be given by:

$$\sigma(\Delta f)=\frac{1}{T}\sqrt{\frac{\sum_n (P_{i+T}-P_i)^2}{n}}$$

where P_i is the departure of the actual phase from the parabola at time t_i, and the sum ranges over all possible (n) differences between values of P_i separated by period T, the phase values being restricted to daylight hours only or else to nighttime hours only.

Figure 1 shows a typical plot of the precision of measurement, $\frac{\sigma(\Delta f)}{f}$, versus the duration of the observation, T. Only results for $n\geq 10$ are shown. The precision obtained for the daytime is seen to be about twice that for the night, which agrees with the results of Pierce [1957] for GBR received at Cambridge, Mass. The periodic dips in the plot are caused by the presence of the diurnal variation in the data. Its effect can be eliminated by considering only measurements made at multiples of 24 hr, or by subtracting from each day of data the average of the diurnal variations over the 2 week period before making the precision calculation. Figure 2 shows the attainable precision with the same data which has had the average diurnal variation (fig. 3) removed. The effect upon the short term measurements of, say, 1 hr is to increase the attainable precision by more than 20 percent. It has been observed during the last 2 or 3 years that the diurnal variation on any given path is quite reproducible from year to year when considered on a monthly average basis. Considerable work, however, remains to be done before it will be possible to predict exactly what the variation should be on any given path without previous measurements.

The precision results for all periods in table 1 are summarized in table 2. The values attainable after 1 hr and 4 hr of observation are listed for both the cases with the diurnal variation present and with the average variation removed. The remaining periods of observation shown are multiples of 24 hr, for which the results for the two cases are indistinguishable.

It should also be pointed out that a few small solar flares (optical class 3 or less) occurred during the 6-month period, but their effect on the average is negligible. They need to be considered when making individual measurements, but their effect is easily recognized and lasts on the order of only an hour (see Chilton et al. [1963]).

TABLE 2. *Results of precision of measurement NPM to BOULDER, 19.8 kc/s*

Data interval (inclusive, 1962)		Precision attainable in period of							
		1 hr Diurnal variation:		4 hr Diurnal variation:		24 hr	48 hr	96 hr	192 hr
		Present	Removed	Present	Removed				
26 Mar–6 April	Day	4.1×10^{-10}	3.0×10^{-10}	2.1×10^{-10}	1.0×10^{-10}	2.3×10^{-11}	1.2×10^{-11}	6.3×10^{-12}	2.9×10^{-12}
	Night	6.3	5.	2.4	2.1	3.6	2.5	13.8	9.6
9 Apr–22 Apr	Day	4.7×10^{-10}	3.7×10^{-10}	2.6×10^{-10}	1.3×10^{-10}	2.5×10^{-11}	1.4×10^{-11}	5.7×10^{-12}	3.4×10^{-12}
	Night	8.4	5.8	3.4	2.3	3.5	2.0	12.3	10.0
26 Apr–9 May	Day	3.9×10^{-10}	2.7×10^{-10}	2.1×10^{-10}	1.1×10^{-10}	2.7×10^{-11}	1.5×10^{-11}	8.0×10^{-12}	3.1×10^{-12}
	Night	7.0	6.4	2.3	2.0	3.7	1.9	10.3	5.1
7 May–20 May	Day	3.8×10^{-10}	2.6×10^{-10}	2.1×10^{-10}	0.9×10^{-11}	2.6×10^{-11}	1.4×10^{-11}	8.0×10^{-12}	3.2×10^{-12}
	Night	6.5	5.3	4.0	2.4	4.2	2.0	10.0	5.0
21 May–31 May	Day	4.0×10^{-10}	3.1×10^{-10}	2.3×10^{-10}	1.2×10^{-10}	2.2×10^{-11}	1.0×10^{-11}	5.6×10^{-12}	3.0×10^{-12}
	Night	6.4	4.7	4.3	3.0	5.0	3.4	22.1	5.1
Averages	Day	4.1×10^{-10}	3.0×10^{-10}	2.2×10^{-10}	1.1×10^{-10}	2.5×10^{-11}	1.3×10^{-11}	6.7×10^{-12}	3.1×10^{-12}
	Night	6.9	5.5	3.3	2.4	4.0	2.4	13.7	7.0

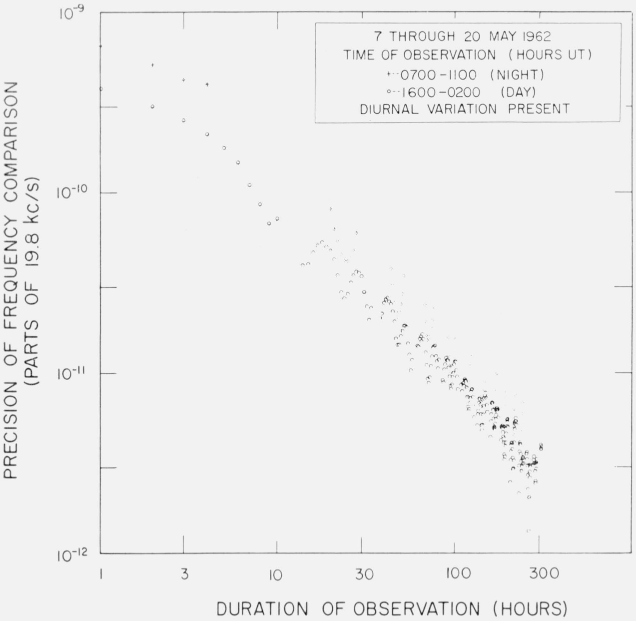

FIGURE 1. *Precision of measurement of frequency versus duration of observation of the 19.8 kc/s transmission of NPM received at Boulder.*

Path length=5374 km.

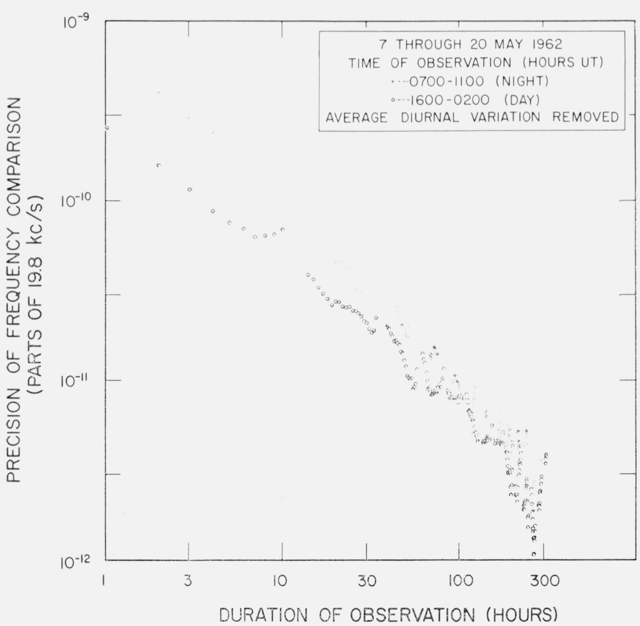

FIGURE 2. *Precision of measurement of frequency versus duration of observation: basic data shown in figure 1 with the average diurnal phase variation removed.*

3. Effect of the Seasonal Variations

It is apparent from these measurements that the precision varies as the reciprocal of the period of observation when it is extended for several days. Any contributions from a vertical drift in the ionosphere must necessarily have a period in excess of two weeks. In the NBA to Boulder study at 18.0 kc/s mentioned earlier [Brady and Murphy, 1962], it was pointed out that a seasonal variation in the depth of the trapezoidal pattern (fig. 4) when interpreted as a change in the level of the nighttime height of reflection relative to the daytime level, might well represent a limit in the precision attainable at VLF.

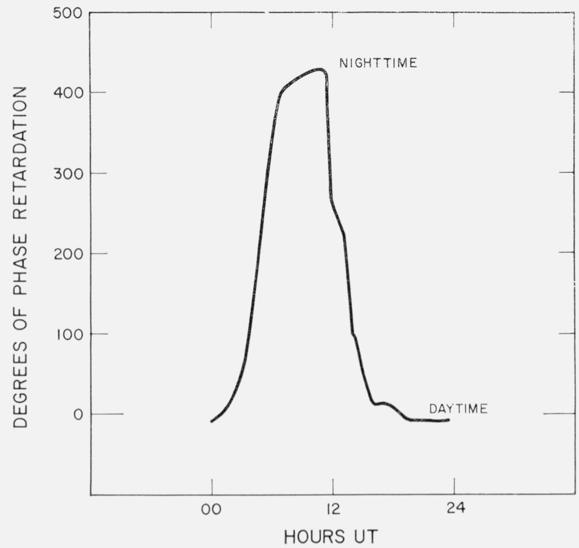

FIGURE 3. *Average diurnal phase variations for the period 7 to 20 May 1962 of NPM received at Boulder.*

Figure 5 shows the monthly averages of diurnal variation of the phase of NPM received at Boulder. Figure 6 shows the seasonal change in the depth of these diurnal patterns along with annual and semiannual harmonic components computed from the 12 points. The annual component is small and not statistically significant, but the nearly 50° $(p-p)$ semiannual variation, if attributed entirely to one level of reflection, would have an effect on precision of frequency comparison of about 1 part in 10^{12}. Such a precision could only be obtained by extending the period of observation of the measurement in figures 1 and 2 to nearly a month. If one were to use the method of continuous measuring and averaging mentioned by Pierce [1957] and discussed by Watt and Plush [1959] so that the precision would vary inversely as $T^{3/2}$, then this might become significant in a much shorter period of time when comparing standards by means of VLF transmissions. While it may be possible to take the seasonal variation into account, little is known about this semiannual seasonal variation at the present. Attempts to correlate the monthly averages (fig. 6) of the diurnal change with geomagnetic variations (which are well known to display a semiannual seasonal variation in activity) have thus far not proved successful.

Since there is no tendency for the precision curve to flatten out in a horizontal direction for a period of two weeks, we can conclude that if there are any

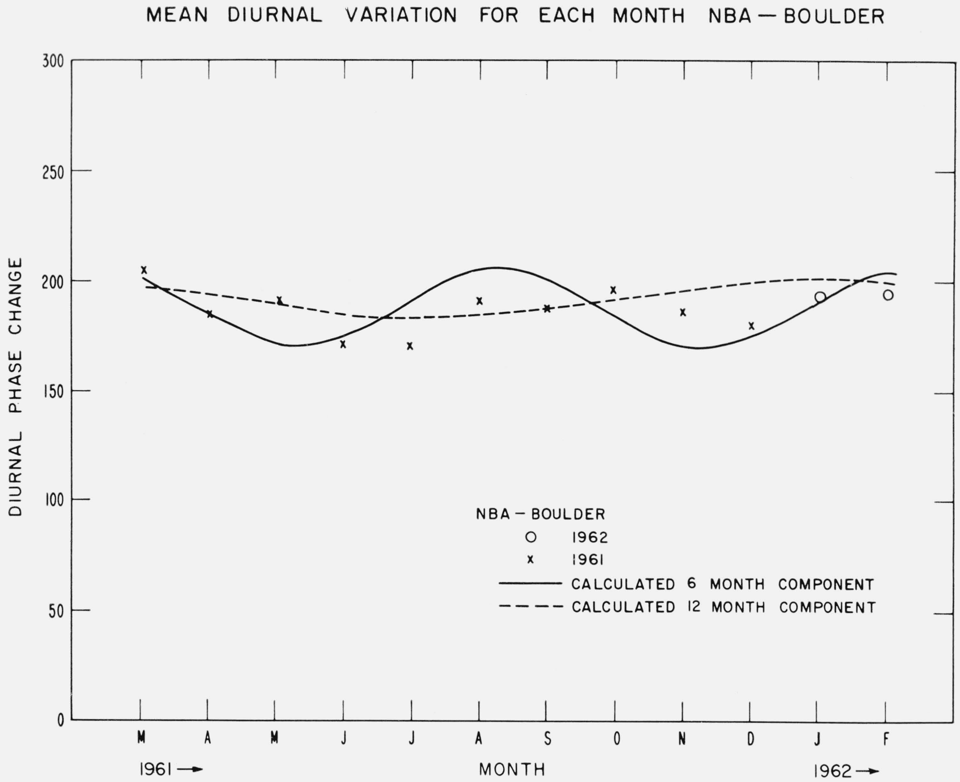

FIGURE 4. *Day to night change in phase of NBA, Panama, 18.0 kc/s, as seen throughout the year at Boulder.*

correlated fluctuations of a period shorter than this, then they are insignificantly small. From the results of the precision study we can make an estimate of the limit of a systematic change in the height of reflection along the path using an approximate relation between the change in average height, Δh, and the observed phase change, ΔP, described by Wait [1959].

$$\Delta h = \frac{\lambda h}{d\left[\frac{h}{2a}+\left(\frac{\lambda}{4h}\right)^2\right]} \cdot \frac{\Delta P}{360^\circ}$$

where λ is the wavelength, h is the average height of reflection, d is the length of the path, and a is the radius of the earth. To effect the precision attainable during the daylight hours in an observing period of 300 hr (fig. 3), it would be necessary to have a systematic variation in phase amounting to about 25 deg. Using the formula above relating this to a change in height of reflection, we obtain a value of approximately 2.5 km for the limit of a gradual change in height of reflection over this period, assuming a daytime value for h of 70 km.

4. Conclusion

Assuming the major portion of long term frequency variation between the transmitter oscillator of NPM and the receiver oscillator at NBS, Boulder to be linear with time and suitably removed, it has been shown (with selected data) that the limitation in precision of measurement of frequency over a 24 hr period using an 18.6 kc/s VLF transmission over a 5370 km path to be 2.5×10^{-11}, which is in close agreement with results of earlier workers [Pierce et al., 1960], at 16.0 kc/s for nearly the same distance. Furthermore, this precision can be improved by extending the observation period to several days without being affected by ionospheric variations, achieving a precision of 3×10^{-12} in 192 hrs of observation. Without further data on the remaining differences between the oscillators, there is no evidence that the propagation conditions over the path are limiting this precision. At any rate, it is clear that the transmission path is not introducing any greater errors for periods less than 2 weeks. It is seen, however, that there may exist seasonal variations which could affect the ultimately attainable precision if they are not taken into account.

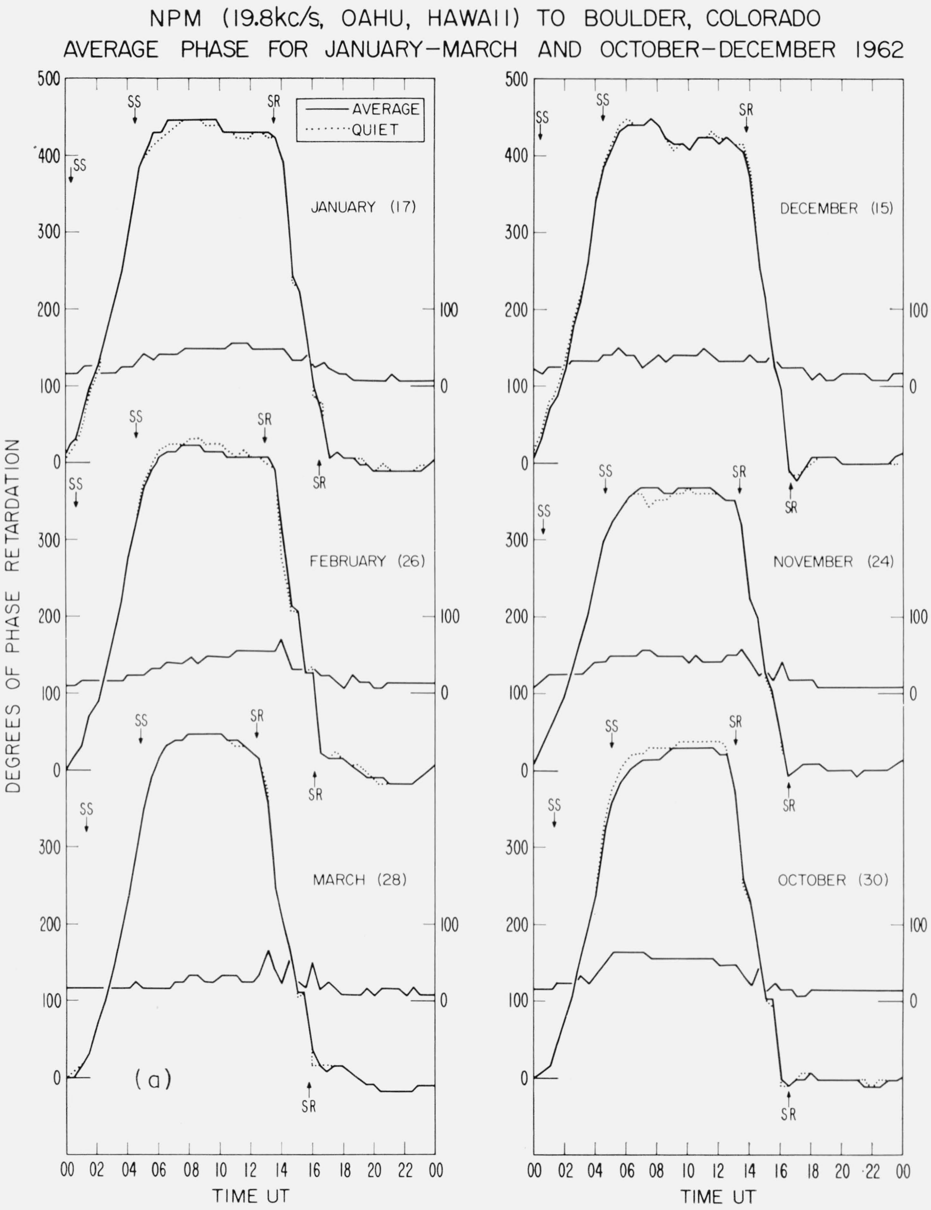

FIGURE 5. *Monthly averages of diurnal variation in phase of NPM, Hawaii, 19.8 kc/s, received at Boulder.*

The "quiet" average is the average of values within one standard deviation.

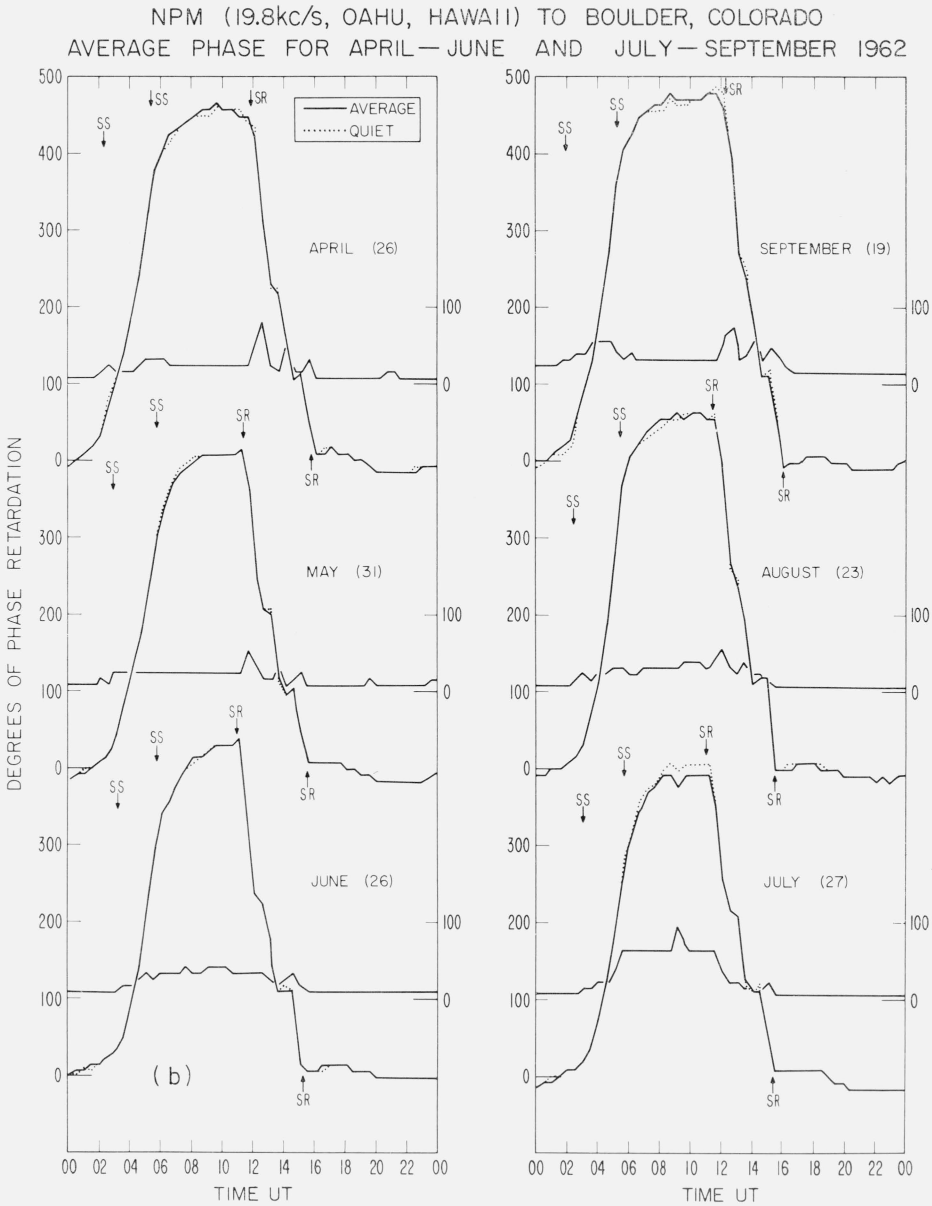

FIGURE 5. *Monthly averages of diurnal variation in phase of NPM, Hawaii, 19.8 kc/s, received at Boulder.*—Continued.

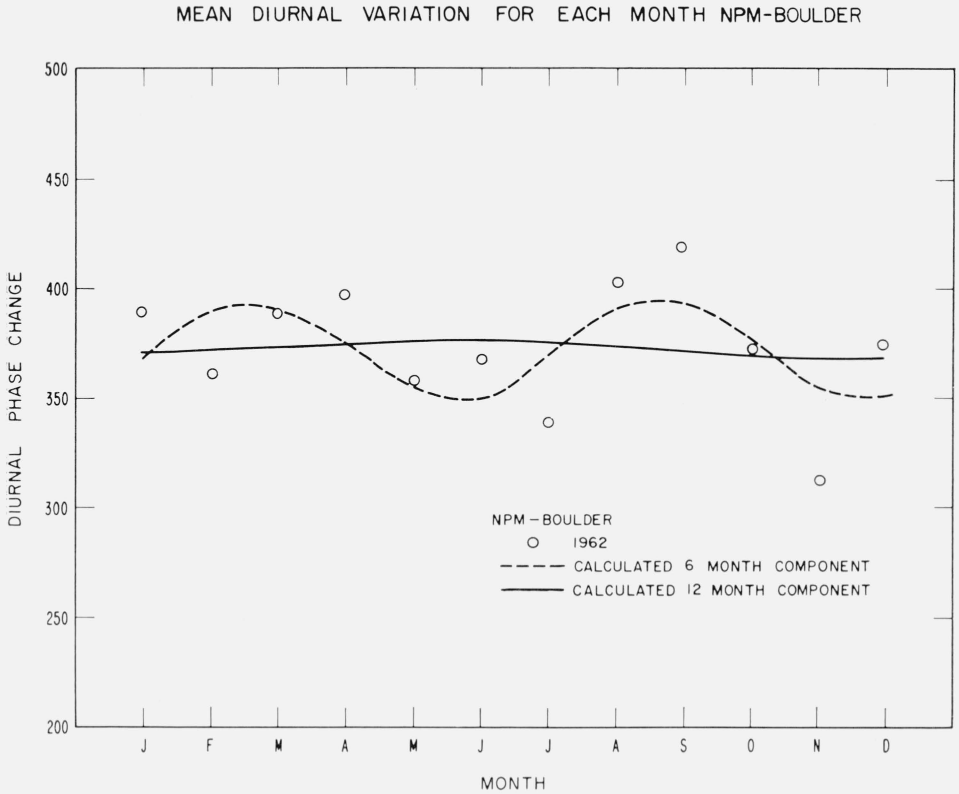

FIGURE 6. *Day to night change in phase of the NPM to Boulder monthly averages showing the seasonal variation.*

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