Update on a Comparison of Cesium Fountain Primary Frequency Standards

Thomas E. Parker
Time and Frequency Division
National Institute of Standards and Technology
Boulder, CO USA
tparker@boulder.nist.gov

***Abstract*—An update to a previous long term frequency comparison of cesium fountain primary frequency standards [1] has been made. This update covers the approximate three year interval from March 2008 to March 2011 and also includes two new standards. Also, simulated data have been used to estimate the biases and standard deviation of the Birge ratio as a function of the degrees of freedom.**

I. INTRODUCTION

In January 2010 the results of a long-term frequency comparison of cesium fountain primary frequency standards were reported in Metrologia covering the interval from August 2000 to June 2009 [1]. The new study presented here gives an update covering ten fountains from seven laboratories over the time frame from approximately March 2008 to March 2011. Included are two new fountains, NPL-CsF2 (at the National Physical Laboratory in the United Kingdom) and PTB-CsF2 (at the Physikalisch Technische Bundesanstalt in Germany), that were not present for the study in [1]. The methodology followed here is substantially the same as that in [1], but with one difference. Rather than using either NIST-F1 (at the National Institute of Standards and Technology in the USA) or SYRTE-FO2 (at the Laboratoire National de Métrologie et d'Essais, Systèmes de Référence Temps Espace in France) as references, only SYRTE-FO2 was used as the main reference. This change was made only because SYRTE-FO2 is almost always present for comparison. In the few instances when NIST-F1 was used as the reference, an adjustment based on the average frequency difference between NIST-F1 and SYRTE-FO2 was applied. Details of the comparison methodology are given in [1]. Also new in this paper are estimates from simulated data for biases and the standard deviation of the Birge ratio as a function of degrees of freedom. This information provides confidence levels for the Birge ratios.

II. FOUNTAIN COMPARISON METHOD

The comparisons were made using fountain data available in the BIPM (Bureau International des Poids et Measures) publication Circular T (www.bipm.org/jsp/en/TimeFtp.jsp?TypePub=publication). Pairs of data points that occurred close in time were used and in most cases there was some overlap. Both TAI (International Atomic Time) and AT1E (a post-processed maser ensemble at NIST) [1] were used as flywheel frequency references, and the average of the two independent results was calculated in order to gain a small reduction in the dead-time uncertainty. In all, 128 comparisons were made.

For a pair of runs all type A uncertainties, u_A, from both standards were combined in quadrature. Also the type B (systematic) uncertainties, u_B, from both standards were combined in quadrature. Over multiple runs the combined u_A averages down, while for multiple runs the weighted mean of the combined u_B was used. Consequently, u_B does not average down over time. u_A and u_B are added in quadrature to get the total uncertainty. It has been assumed that the u_B between standards are uncorrelated and that the u_B for the same standard is highly correlated over time. Neither assumption is strictly true, but the approach is a reasonable compromise.

Table 1 shows an example of a fountain comparison from a data pair. In the table MJD is the Modified Julian Date and y is the fractional frequency difference in units of 10^{-15}. u_{dead} is the uncertainty introduced by the noise in the flywheel due to misalignment of the run times shown in column one. $u_{dead} = 0$ if the start and stop times are identical. u_{TAI} is the transfer noise into TAI [2]. u_l includes dead time internal to the laboratory. u_A, u_l, u_{TAI}, u_{dead} are all type A uncertainties. u_C, u_{CA} and u_{CB} are the combined uncertainties of the comparison. All fountain uncertainties are 1 standard uncertainty (1 sigma).

The top left section in Table 1 shows the frequency of TAI with respect to NIST-F1 over the specified MJDs along with the associated uncertainties. The lower left section shows the same for SYRTE-FO2. The middle right section shows the comparison results, where u_{dead} comes from the lack of perfect overlap. AT1E was also used as a reference fly wheel in a manner similar to that in Table 1, and the two results were averaged to gain a small reduction in the dead-time

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TABLE 1. Example of a fountain comparison data pair and the difference calculation for NIST-F1 and SYRTE-FO2. Fractional frequencies are in units of 10^{-15}.

NIST-F1

MJD	MJD	Duration	y(TAI-F1)	u_A	u_B	u_l	u_{TAI}
54314-54339	54326.5	25d	-3.4	0.3	0.3	0.1	0.4

u_{dead}	y(FO2-F1)	u_C	u_{CA}	u_{CB}	overlap
0.7	0.3	1.2	1.05	0.58	15d

SYRTE-FO2

MJD	MJD	Duration	y(TAI-FO2)	u_A	u_B	u_l	u_{TAI}
54309-54329	54319	20d	-3.7	0.3	0.5	0.1	0.5

uncertainty. The same procedure was used for all of the fountain comparisons.

III. BIRGE RATIO

The Birge ratio, R_B, is a consistency test and should be close to unity if stated uncertainties are correct. It is a test to see if the scatter in the data is consistent with the stated uncertainties. The definition of the Birge ratio is shown in equations (1) and (2) below, where χ_r^2 is the reduced chi-square. R_B can be calculated either for the same pair of standards over time or for the average frequency differences in a group of standards.

$$R_B = \sqrt{\chi_r^2} \qquad (1)$$

$$R_B = \sqrt{\frac{\sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{u_i^2}}{(n-1)}} \qquad (2)$$

The standard chi-square calculator (for example www.fourmilab.ch/rpkp/experiments/analysis/chiCalc.html) is stated to be not accurate when the degrees of freedom, DOF, are less than about ten. Therefore simulated data made up of 81,920 samples were used to confirm the standard deviation and bias of R_B as a function of DOF. The simulated data were generated using a common spread sheet random-number generator and the Box-Muller transformation to provide a Gaussian distribution. The results are shown in Figs. 1 and 2. The standard deviation (for a +/-34 % interval) as a function of DOF provides a confidence interval for the Birge ratio and is essentially the same for the simulated data as from the calculator as shown in Fig. 1. This is also true for the median as seen Fig. 2. The mean and median however are not the same because the distribution is asymmetric. Here the DOF is one less than the number of data points.

IV. FOUNTAIN COMPARISON RESULTS

Table 2 shows the results of the nine fountain comparisons for the interval of approximately March 2008 to March 2011.

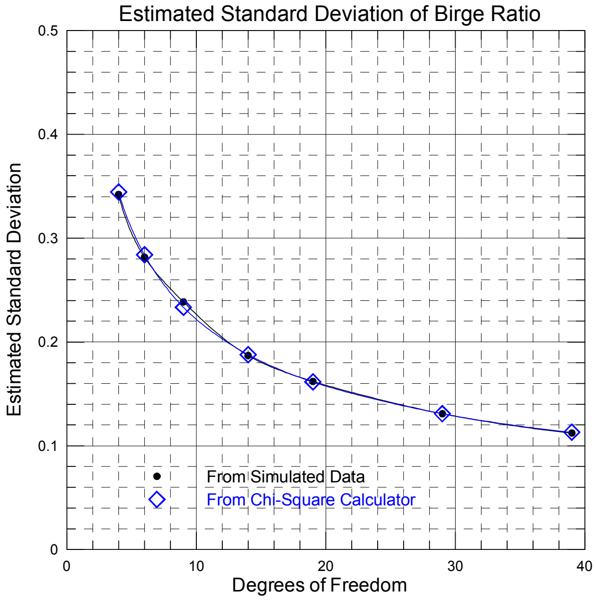

Figure 1. Estimated standard deviation of the Birge ratio from simulated data and from a chi-square calculator.

Each fountain is compared to SYRTE-FO2, and the average frequency difference, y_{avg}, is shown. The type A and type B uncertainties of the comparisons are U_{CA} and U_{CB}, and the combined total comparison uncertainty is U_C. The last column on the right gives the number of data pairs for each comparison. R_{BA} is the Birge ratio using only the type A uncertainties, and R_{BC} is the Birge ratio using the combined uncertainty. Here the Birge ratio is based on the scatter over time. In general one would expect R_{BC} to be less than 1 and R_{BA} to be larger than 1 [1]. There is cause for concern if R_{BA} is smaller than one in a statistically significantly fashion or if R_{BC} is larger than one in a statistically significantly fashion. This would indicate that the uncertainties are either overstated or understated respectively. Only SYRTE-FOM and IT-F1 (operated by the Istituto Nazionale di Ricerca Metrologica, (INRIM) in Italy) exhibit such behavior, with SYRTE-FOM having a small Birge ratio and IT-CSF1 having a large ratio. In both cases the deviation from 1 is more than two standard

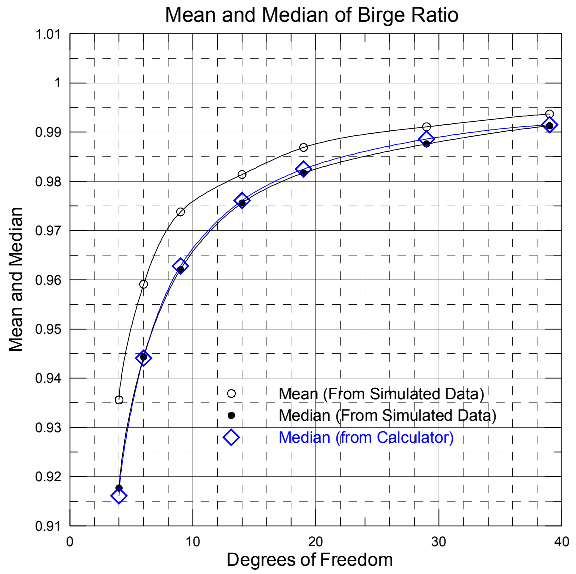

Figure 2. Biases in the Birge ratio at low degrees of freedom.

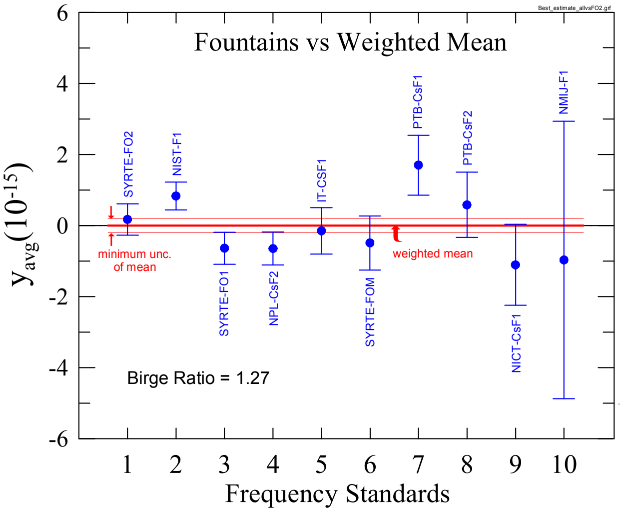

Figure 3. Average fountain frequencies vs the weighted mean.

TABLE 2. Fountain comparisons relative to SYRTE-FO2. All frequencies are in units of 10^{-15}. In this type of comparison an offset of 1.4 sigma or less is considered entirely normal.

Fountains	y_{avg}	U_C	U_{CA}	U_{CB}	R_{BA}	R_{BC}	# pairs
PTB-CsF1 vs SYRTE-FO2	+1.53	0.95	0.27	0.91	1.97	1.09	10
NIST-F1 vs SYRTE-FO2	+0.66	0.59	0.25	0.54	1.16	1.02	18
PTB-CsF2 vs SYRTE-FO2	+0.41	1.02	0.43	0.92	1.22	0.99	8
ITCS-F1 vs SYRTE-FO2	-0.32	0.79	0.31	0.73	1.89	1.56	15
SYRTE-FOM vs SYRTE-FO2	-0.66	0.88	0.24	0.85	0.56	0.43	14
SYRTE-FO1 vs SYRTE-FO2	-0.81	0.63	0.14	0.62	0.83	0.61	21
NPL-CsF2 vs SYRTE-FO2	-0.82	0.64	0.27	0.58	1.08	0.93	15
NMIJ-F1 vs SYRTE-FO2	-1.14	3.93	0.28	3.92	1.68	0.55	21
NICT-CsF1 vs SYRTE-FO2	-1.28	1.22	0.60	1.06	0.71	0.59	6

deviations. Significant errors in total uncertainties may come from sources other than the fountains themselves because substantial uncertainty is introduced by frequency transfer and dead time.

Three fountains in Table 2 show a positive average frequency offset relative to SYRTE-FO2, and six have a negative offset. In five cases the average frequency difference is larger than the total combined uncertainty, but no comparisons with SYRTE-FO2 exceed 2 sigma. In this type of comparison an offset of 1.4 sigma or less is considered entirely normal [1]. NMIJ-F1 is operated by the National Metrology Institute of Japan (NMIJ) and NICT-CsF1 is operated by the National Institute of Information and Communication Technology (NICT) in Japan.

A weighted mean of the offsets from SYRTE-FO2 can be calculated using the data in Table 2, and the average offsets from the mean for all ten fountains are shown in Fig. 3. The Birge ratio for the group of fountains shown in Fig. 3 is calculated using the combined uncertainties from each fountain. At 1.27 it is just over one standard deviation for nine DOF which is about 1.20 (see Figs. 1 and 2). The uncertainty of the weighted mean is about 2×10^{-16} if one assumes that all of the individual fountain uncertainties are uncorrelated. This is probably not true, but accurately quantifying the degree of correlation would be nearly impossible. Six of the ten standards in Fig. 3 are within 1 sigma of the mean and all are within 2 sigma. However, several standards are more than 2 sigma separated from each other. Specifically, NIST-F1 and PTB-CsF1 on the positive side of the mean versus SYRTE-FO1 and NPL-CsF2 on the negative side are separated by nearly 2.5 sigma.

V. CONCLUSIONS

Overall, the fountains are in fairly good agreement, although some deviate from the mean by nearly 2 sigma and from each other by nearly 2.5 sigma. The Birge ratio at 1.27 is just over one standard deviation and therefore does not indicate any serious problems. In the earlier study [1] the Birge ratio was 0.77, indicating less scatter relative to the stated uncertainties, which were generally larger than in the current study. In the current study, the scatter among the fountains is a little larger than one would expect from the stated uncertainties, but not enough to cause serious concerns. The Birge ratio indicates that there is a 90 % probability that, averaged over all the fountains, the uncertainties (including dead time and frequency transfer) have been understated by perhaps 20 %.

ACKNOWLEDGMENTS

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REFERENCES

[1] Thomas E. Parker, “Long-term comparison of caesium fountain primary frequency standards”, *Metrologia*, 47, pp 1-10, 2010.

[2] Gianna Panfilo and Thomas E. Parker, “A theoretical and experimental analysis of frequency tranfer uncertainty including frequency transfer into TAI”, *Metrologia*, 47, pp 552-560, 2010.