2.7: The General Mechanical Model of Random Perturbations which Generate a Noise Spectral Density Law $|f|^{\alpha}$ with Especial Reference to the Flicker Noise Law $1/|f|$

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In the design of a precision measurement or of a standard for measurement, one of the aims is to minimize the random noise of the system. It is often found that the random fluctuations are dominated by the lowest frequency portion of the noise modulation spectrum. The density of this noise is observed to increase without limit as its frequency decreases. A typical behavior is as $|f|^{-1.0}$(flicker noise), but the exponent varies from system to system, and it is not always constant. It is important to realize that a measurement with results which are limited by random noise with a law of $|f|^{-1}$ has a fixed precision which cannot be improved by increasing the length of each run. It is hoped that the model presented here will help in the understanding of flicker noise. The model is general, moreover, and is applicable to any noise law $|f|^{\alpha}$.

A system is considered in which some process of interest is occurring, and in which the process is suffering time-dependent perturbations in a random manner due to some unspecified agent or agents. The uniform portion of the process constitutes the signal, and the random fluctuations are the associated noise. The discussion needs only to be concerned with the mathematical description of the time-dependent perturbations. To avoid the possibility of the mechanical model so generated of being non-physical in principle, only reasonable perturbations are considered. A precise mathematical definition of reasonable perturbations is used.

The analysis shows that a necessary condition for a subclass of reasonable perturbations to generate the noise law $|f|^{\alpha}$, over an arbitrarily large range of f, is that $\alpha_{\infty} \leq \alpha \leq \alpha_{0}$. To generate the noise law $|f|^{\alpha}$ over the arbitrarily large range $(2\pi\zeta)^{-1} << |f| << (2\pi\epsilon)^{-1}$ for α in the more restricted range, $\alpha_{\infty} < \alpha < \alpha_{0}$, it is necessary and sufficient that the subclass satisfy the conditions $P(\tau)A^{2}(\tau) = B\tau^{-\alpha-3}$ over the range $\epsilon \leq \tau \leq \zeta$ and $P(\tau)A^{2}(\tau) \leq B\tau^{-\alpha-3}$ for τ outside the range ϵ to ζ. A class is the set of all perturbations which are identical under some individual independent scaling of amplitude, scaling of time, and translation of time. A subclass is characterized by $P(\tau)$, the lifetime probability density, and by $A^{2}(\tau)$, the mean square effective amplitude of perturbations having lifetime τ. Although α_{∞} and α_{0} depend upon the class (time shape) of the perturbations, any reasonable perturbation has $\alpha_{\infty} \leq -2$ and $\alpha_{0} \geq 0$. Hence the only values of α for which the law

$|f|^{\alpha}$, over an arbitrarily large range of f, can be generated from any class of reasonable perturbations are $-2 \leq \alpha \leq 0$. This range of α is centered on the flicker law, $\alpha = -1$.

With this general mechanical model, any desired noise spectral law, or combination, can be efficiently generated by a simple computer program. The model is currently in use by Fey, Barnes, and Allan at NBS for digital simulation of flicker noise.

From a physical viewpoint, the restrictions which have been placed on reasonable perturbations are not severe. Many random noise processes of physical importance may be represented by reasonable perturbations, and to them the results of this paper are applicable without modification. The present results are also applicable to many random physical processes where the perturbing agent is most conveniently regarded as being of infinite lifetime, provided the principle of ergodicity can be used.

The present results can give insight into the problem of devising models which may explain noise spectra observed in specific physical situations. It is emphasized that to develop a physical model for flicker noise, the crucial problem is to find the physical circumstances which cause the product $P(\tau)A^2(\tau)$ to vary as $\tau^{-\alpha-3}$. The shape (class) of the perturbation is irrelevant when considering flicker noise. The experimental observation, per se, of a noise law of $|f|^{\alpha}$, with $-2<\alpha<0$, gives no information concerning the identity of the class of reasonable perturbations which is operative.