

# An optical frequency comb for infrared spectrograph calibration

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## 1. INTRODUCTION

The search for extrasolar planets is an exciting new field of astronomy. Since detection of a planet orbiting the sun-like star 51 Peg,<sup>1</sup> the field of planet finding has pushed the limits of sensitivity and accuracy in astronomical photometry and spectroscopy. To date 455 exoplanets have been detected\*, of which the radial velocity technique is responsible for nearly 80%.<sup>2</sup> Radial velocity measurements are also an important complement to photometric missions such as *Kepler* and *CoRoT*, which survey vast numbers of stars simultaneously but which require follow up measurements for positive identification of planets.

The chief objective in the search for exoplanets is the identification of habitable Earth-like planets in close proximity to our solar system. Of the currently detected exoplanets, only a few are Earth-like,<sup>3</sup> the vast majority being giants in close orbits. While it is possible that these planets are the most common type, it is likely that an inherent selection bias in planet finding techniques is the cause. Simply, large radial velocity shifts and high contrast occultations are the most detectable by radial velocity spectroscopy and photometry, and so we primarily observe planets capable of inducing them.

An Earth-like planet is a rocky planet existing in the habitable zone of a star—a range of orbital distances where the planet could support the existence of liquid water. Stellar luminosity determines the location of the habitable zone and has significant implications for the detectability of Earth-like planets. The habitable zone of a G star is centered around 1 AU, while a cooler M star's zone is shifted inwards to 0.03-0.1 AU.<sup>4</sup> The stellar radial velocity shift scales as  $1/\sqrt{r}$ ,  $r$  being the planet's orbital radius, and accordingly the radial velocity signal for an Earth-like planet orbiting an M star will be  $\sim 10$  times greater than the same planet in the habitable zone of a G star, making cooler stars excellent targets for radial velocity measurements.

In addition to making attractive targets for radial velocity measurements, M stars are also our most numerous stellar neighbors. A survey of all stars within 10 pc of our solar system indicates that 239 of the 354 such stars are M stars<sup>†</sup>. The stellar flux of an M star peaks at around 1000 nm, making spectroscopy in the IR attractive. The H band atmospheric transmission window from 1400 to 1800 nm is an attractive spectral region for spectroscopy, with a relatively high density of spectral features, allowing for an intrinsic uncertainty of less than 5 m/s.<sup>5</sup> Additionally, this region of the spectrum is a good match for fiber laser technology, as will be discussed in detail.

To achieve the accuracy required for meter per second and better radial velocity measurements, an accurate and highly stable calibration source is required. While thorium-argon lamps and iodine absorption cells can provide meter per second accuracy in the visible,<sup>6,7</sup> in the H band both emission and absorption lines are sparse. Laser frequency combs provide an ideal calibration source for this spectral region.

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†See the RECONS census of nearby stars (<http://www.recons.org>) for an up-to-date listing.

## 2. LASER FREQUENCY COMBS

Mode-locked laser frequency combs are well suited for the calibration of high resolution spectrographs.<sup>8-10</sup> The frequency comb is an emission source, with a spectrum hundreds of nanometers broad and composed of thousands of spectral lines at regular intervals in the frequency or wavenumber domain.

Not only is the comb broad and densely populated, it also provides an absolute calibration, directly traceable to atomic frequency standards. The positions of the comb lines are determined completely by two parameters: the laser repetition rate  $f_{\text{rep}}$ , which sets the line spacing, and the carrier envelope offset frequency  $f_{\text{ceo}}$ . In terms of these two parameters, the locations of the comb lines are described by the simple equation

$$f_n = n \times f_{\text{rep}} + f_{\text{ceo}}, \quad (1)$$

where the mode index  $n$  is a large integer on the order of  $10^6$ . Both  $f_{\text{rep}}$  and  $f_{\text{ceo}}$  can be measured and actively stabilized to an absolute frequency standard traceable to the SI second, allowing spectra recorded years apart or at different facilities to be directly compared.

## 3. FREQUENCY COMB GENERATION AND STABILIZATION

Our frequency comb is built upon a commercial passively mode-locked erbium doped fiber laser.<sup>11</sup> The laser has two primary output ports, each providing 40 mW, with a mode spacing of 250 MHz and a spectral bandwidth of  $\sim 70$  nm. One output port is used for the stabilization of the carrier envelope offset frequency, while the other is used to generate the calibration spectrum.

To stabilize the comb,  $f_{\text{rep}}$  and  $f_{\text{ceo}}$  are locked to global positioning system (GPS) disciplined local oscillator, providing accuracy of 1 part in  $10^{11}$  and sufficient for a cm/s-level calibration. Direct detection and locking of the laser repetition rate is straightforward, as it is well within the bandwidth of inexpensive photodiodes. To detect  $f_{\text{rep}}$ , a small portion of the frequency comb is directed onto a photodiode, and the 1 GHz fourth harmonic is amplified and mixed with a local oscillator signal provided by an RF frequency synthesizer; see Fig. 2. The base band component of the mixed signal is fed into a loop filter, which provides an error signal. Feedback is applied to a piezo-mounted mirror inside the laser cavity, which stabilizes repetition rate.

Detection of the carrier envelope offset frequency is more involved.<sup>12</sup> The carrier envelope offset is an absolute frequency shift, the residual frequency when equation Eq. 1 is extrapolated to  $n = 0$ . Detection of  $f_{\text{ceo}}$  is accomplished with the f-2f optical heterodyne technique. The frequency comb is amplified and broadened to span from 1000 to 2000 nm, and a narrow band of the spectrum around 2000 nm is then frequency doubled in a periodically poled lithium niobate crystal. The resulting frequency doubled light is optically mixed with the 1000 nm portion of the spectrum on a photodiode, resulting in a signal at base band with frequency

$$f_0 = 2(n \times f_{\text{rep}} + f_{\text{ceo}}) - (2n \times f_{\text{rep}} + f_{\text{ceo}}), \quad (2)$$

where the first term is the contribution of the frequency doubled 2000 nm light and the second is the 1000 nm light. The difference frequency  $f_0$  is equal to  $f_{\text{ceo}}$ , and it is divided by 8 and phase locked to a synthesized 17.5 MHz reference frequency. The frequency synthesizers stabilizing  $f_{\text{rep}}$  and  $f_{\text{ceo}}$  are themselves referenced to a GPS disciplined rubidium oscillator with accuracy of 1 part in  $10^{11}$ , sufficient for sub-cm/s radial velocity calibration.

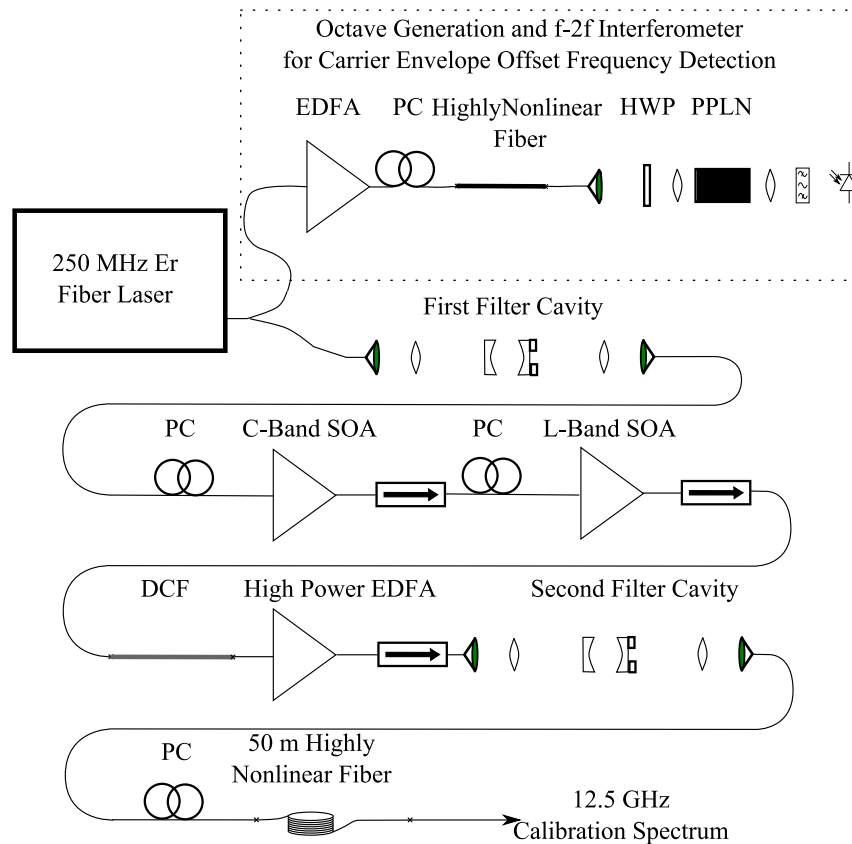
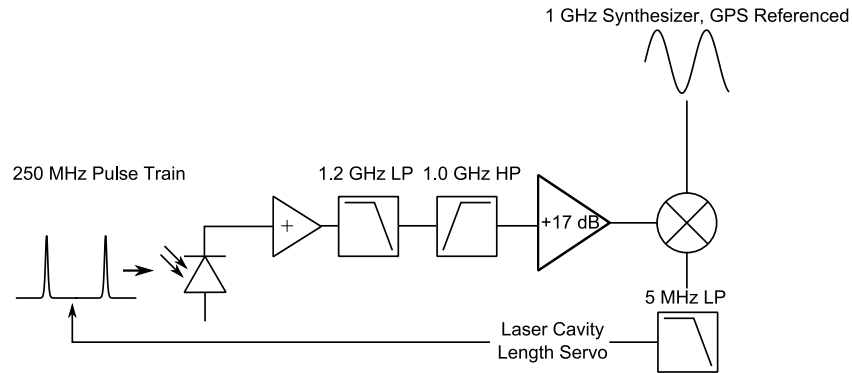
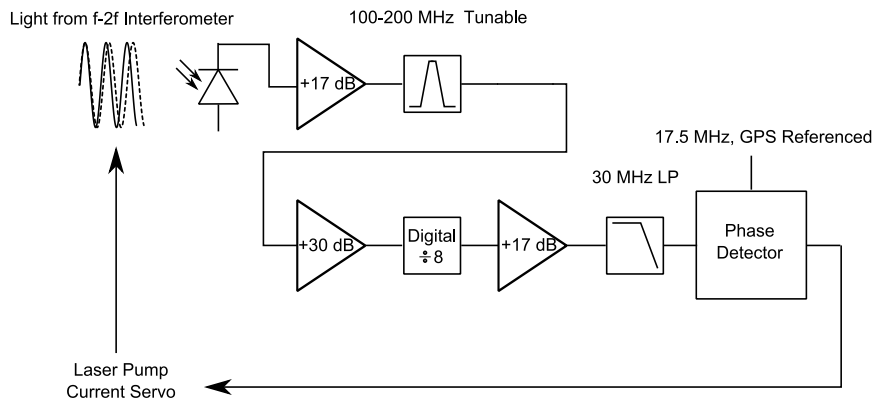


Figure 1: Schematic diagram of frequency comb optics. The upper block broadens the 250 MHz frequency comb to span an octave, from 1000 to 2000 nm, frequency doubles the 2000 nm portion of the spectrum, then photodetects the f-2f beat note (see text.) The optics in the lower block filter the 250 MHz frequency comb by a factor of 50 to 12.5 GHz, then amplify, clean up, and broaden the spectrum for astronomical spectrograph calibration. EDFA: erbium doped fiber amplifier; PC: polarization control; HWP: half wave plate; SOA: semiconductor optical amplifier; DCF: dispersion compensating fiber; PPLN: periodically poled lithium niobate. Unless otherwise specified, optical fiber is SMF-28e.



(a)



(b)

Figure 2: Locking electronics for stabilization of (a)  $f_{\text{rep}}$  and (b)  $f_{\text{ceo}}$ , as described in the text. Feedback to  $f_{\text{rep}}$  is applied by means of a piezo-mounted mirror inside the 250 MHz laser cavity, while  $f_{\text{ceo}}$  is locked by varying the current to the laser's pump diodes.

#### 4. GENERATION OF A CALIBRATION FREQUENCY COMB

A major challenge in developing a frequency comb for calibration of astronomical spectrographs is generating a mode spacing large enough to be well resolved by the spectrograph. For a spectrograph with resolving power  $\lambda/\Delta\lambda = 50,000$  at 1550 nm, one resolution element, spanning 2 to 3 pixels on the detector, corresponds to roughly 3 GHz. An optimal calibration spectrum has one calibration line per 3 to 4 resolution elements,<sup>13</sup> requiring a 12 to 16 GHz mode spacing. While it is possible to build mode-locked lasers with repetition rates of up to 10 GHz,<sup>14</sup> it is difficult and such a laser requires precise alignment. An alternative route to obtaining a high mode spacing is the filtering of a mode-locked laser of lower repetition rate.<sup>8,15–18</sup> This approach makes use of the resonances of Fabry-Pérot cavities, which have fundamental transverse modes at all frequencies where the accumulated round-trip phase is an integer multiple of  $2\pi$ .<sup>19</sup> Neglecting dispersion in air and phase shifts occurring in the dielectric mirrors, the cavity resonance frequencies are

$$f_n = n \frac{2\pi c}{2n_{\text{air}}L}, \quad (3)$$

where  $L$  is the spacing between the mirrors comprising the cavity and  $n_{\text{air}}$  is the index of refraction of air. The frequency  $\pi c/n_{\text{air}}L$  can be set to a multiple  $m$  of the the repetition rate of the mode-locked laser by tuning the cavity length, allowing only every  $m^{\text{th}}$  comb mode to be transmitted. This increases the mode spacing to  $m \times f_{\text{rep}}$ .

To reach a comb line spacing of 12.5 GHz, the spectrum is filtered by a factor of  $m = 50$ . Fabry-Pérot cavities with a finesse of 2000 are used, with two dielectric mirrors with radii of curvature of 50 cm in a confocal configuration.<sup>19</sup> To maintain transmission of every 50<sup>th</sup> mode across 70 nm of bandwidth, the cavity lengths must be kept stable to better than  $\pm 1$  nm. Cavity length control is achieved by mounting one mirror on a piezoelectric ring actuator, which provides a few micrometers of travel. For active locking, two methods were investigated: the Pound Drever Hall (PDH) technique<sup>20</sup> and a dither-lock scheme. For the PDH lock, a fiber-coupled waveguide phase modulator was used to add small 17 MHz offset sidebands to the comb. A circulator was used to direct the light reflected by the cavity onto a photodetector, which was then used to generate an error signal in the standard fashion. The dither technique is a lock-in scheme in which a small,  $\sim 40$  kHz sinusoidal perturbation is imparted upon the cavity length. The intensity of the light transmitted by the cavity is photodetected, and a lock-in amplifier is used to create an error signal. The PDH technique has the advantage of imparting only small amplitude, RF modulation upon the comb, but at the expense of a loss of  $\sim 50\%$  of optical power coupling to the waveguide modulator and a distortion of the pulse shape caused by dispersion in the modulator. The dither-lock has the advantage of adding only a small loss caused by the perturbation of the cavity length, but imparts optical intensity noise at the dither frequency; see Fig 3. Ultimately, the loss and dispersion caused by the PDH lock were too great to outweigh its superior locking performance, and the dither technique is used to lock both cavities.

After filtering the frequency comb, the spectrum spans only 70 nm centered around 1580 nm; to cover the H band, the comb must be broadened. Broadening is accomplished in nonlinear optical fiber,<sup>21</sup> where a large nonlinear coefficient promotes four wave-mixing, self phase modulation, and Raman shifts.<sup>22</sup> In these parametric processes, three light fields interact to produce a fourth. To provide the necessary high peak optical power, the filtered comb light is amplified from  $80 \mu\text{W}$  to 1.4 W. This amplification adds broad bandwidth optical intensity noise, arising from amplified spontaneous emission (ASE) in the gain medium. The presence of optical noise in the four wave mixing process causes optical fields from the comb to mix with noise fields, leading to a loss of coherence in the comb, as in Fig. 5(d). To remove the ASE, the amplified comb light is passed through a second filter cavity identical to the first, which rejects more than 99.9% of optical noise. To compensate for pulse broadening in single-mode fiber, a piece of dispersion compensating fiber (DCF) is inserted before the amplifiers. After optimizing fiber lengths, the system generates a 12.5 GHz train of 400 fs pulses (Fig 4) with an energy per pulse of 40 pJ. The pulses are sent through 50 m of highly nonlinear fiber, where it is broadened to span from 1350 to 1850 nm.

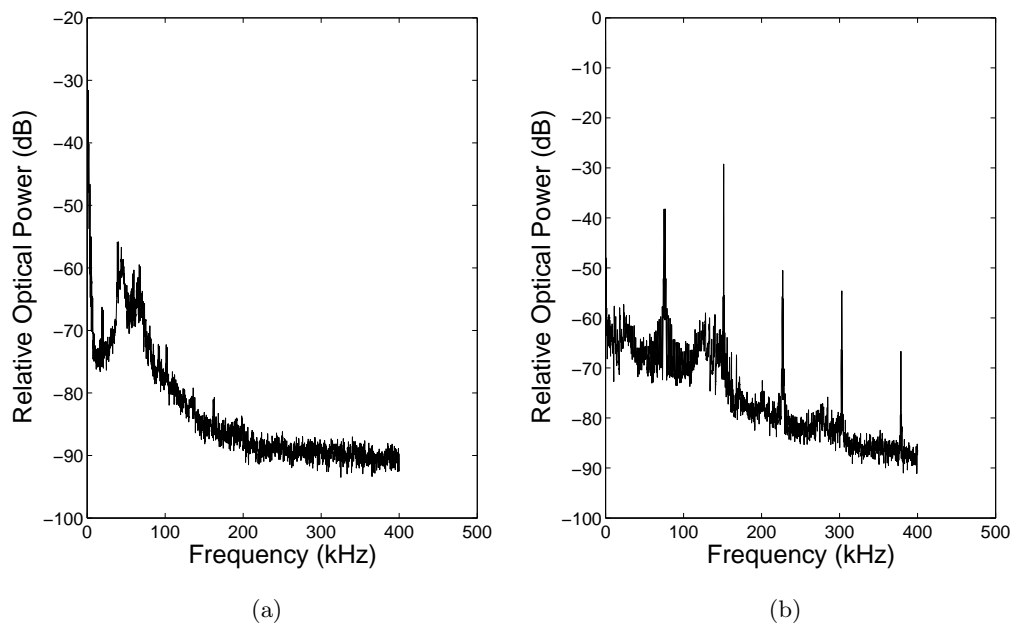


Figure 3: Optical intensity noise after filter cavity for (a) Pound-Driver-Hall and (b) cavity dither locking techniques. Peaks in the dither lock spectrum are harmonics of the dither frequency used to generate the error signal for locking.

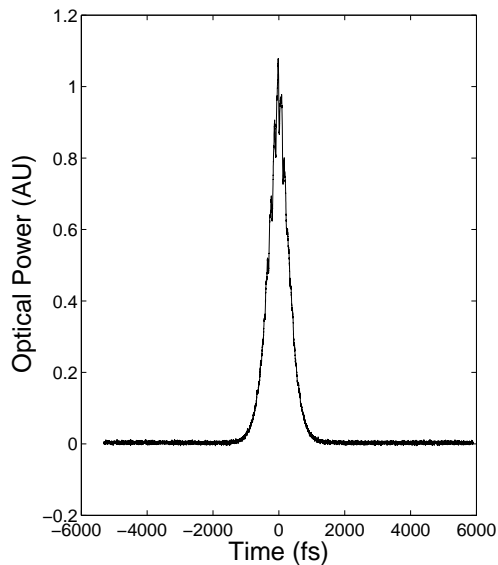


Figure 4: Autocorrelation of pulse entering highly nonlinear fiber. Pulse autocorrelation time is near the transform limit of 600 fs.

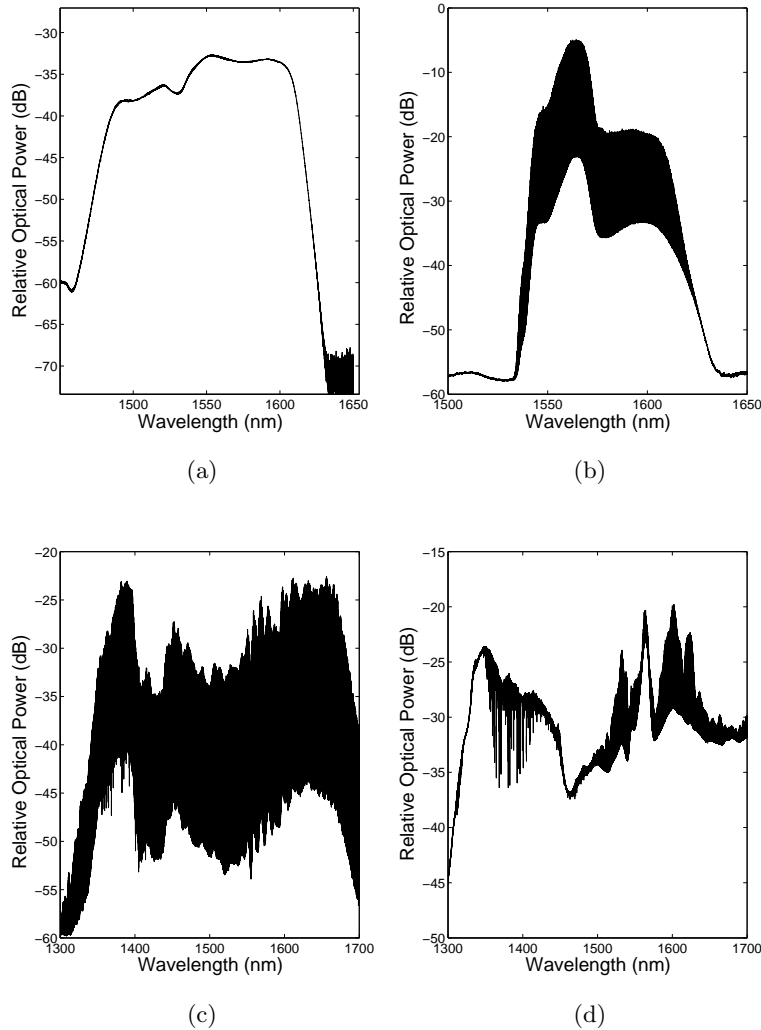


Figure 5: Optical spectra of the frequency comb through the optical system. The 250 MHz frequency comb (a) is unresolved with our  $R=77,000$  optical spectrum analyzer; optical power is 34 mW. After filtering and amplification, the modes are resolvable (b), and the total optical power is 1350 mW. Filtering with a second cavity and nonlinear broadening (c) greatly increases the span of the spectrum, which goes beyond the range of the spectrum analyzer and has a total power of 390 mW. Broadening the spectrum without the second filter cavity results in (d). Without the second filter cavity, mixing of optical noise with the frequency comb during broadening parametrically creates optical noise between comb modes, almost completely erasing the frequency comb.

## 5. MEASUREMENTS OF COMB

Obtaining a calibration spectrum with cm/s accuracy is challenging, even with a frequency comb. Filtering the frequency comb from 250 MHz to 12.5 GHz provides a comb with a spacing precisely 50 times larger, but also introduces ambiguity in the absolute frequency of  $n \times f_{\text{rep}}$  in the remaining comb lines, where  $n$  is a number between 1 and 50. To determine  $n$ , the frequency of a single 12.5 GHz comb mode must be determined with an uncertainty of less than half of  $f_{\text{rep}}$ , or 125 MHz. Such a measurement is performed using a commercial wavemeter, with an accuracy better than 20 MHz and an inexpensive fiber coupled distributed feedback (DFB) laser. The laser is tuned to the proximity of a comb line by adjusting the laser pump current while observing the optical heterodyne beat between the CW and laser comb on a photodiode. Simultaneously, the optical frequency of the CW laser is measured using the wavemeter. Sweeping the CW laser frequency either up or down while observing the beat frequency determines whether the CW laser has a higher or lower frequency than the nearest comb line. By using the CW laser frequency, the difference frequency between the CW laser and the comb, and the sign of the difference frequency, the absolute frequency of the comb mode can be determined to the accuracy of the wavemeter, and  $n$  can be determined.

A more serious challenge is the characterization of the frequency comb after nonlinear broadening. The four-wave mixing process allows a mechanism for revival of the 250 MHz comb modes nominally suppressed by the filter cavities. Because shifting of the apparent centers of the comb lines by only 10 kHz affects the calibration at the cm/s level, an accurate measurement of the side modes after broadening is critical to confidence in the comb. The largest side modes are the nearest neighbors 250 MHz to either side of the 12.5 GHz calibration lines, the magnitude and asymmetry of which can be directly determined by optical heterodyne. A narrow portion of the broadened comb spectrum is selected with a band-pass filter and combined with a reference beam from a tunable CW laser, then photodetected and electronically amplified. An RF spectrum analyzer records the spectrum, which is comprised of both the difference frequency signal between the tunable laser and a single comb mode and harmonics of 250 MHz from the frequency comb alone. By examining the amplitudes of the side modes, as in Fig. 6, side-mode suppression and asymmetry can be measured. A careful series of measurements of the comb<sup>23</sup> at 10 nm intervals across the spectrum from 1350 to 1700 nm indicates that across the broadened spectrum side mode asymmetry is less than 1 dB, or 10 %.

Measuring the spectrum of the entire frequency comb with high resolution is a non-trivial task. Commercial optical spectrum analyzers with resolving power  $\lambda/\Delta\lambda = 77,000$  are capable of resolving the 12.5 GHz comb modes, but are unable to determine line centers to better than a few GHz. A more complete measurement of the spectrum is desirable, as the nonlinear broadening process is complex, and it has been suggested<sup>24</sup> that side modes could grow to the point where they dominate the nominally transmitted 12.5 GHz comb. Accordingly, a measurement of the spectrum was made with the NIST 2 meter Fourier transform spectrometer.<sup>25</sup> The spectrometer has a resolution of 70 MHz, and allows for the determination of the comb modes to better than 125 MHz. A preliminary analysis indicates that the spectrum maintains an even 250 MHz spacing and demonstrates that side modes are suppressed by more than a factor of  $>20$  above 1700 nm. Initial results are shown in Fig. 7.

## 6. CONCLUSION

Laser frequency combs provide an ideal calibration source for high precision astronomical spectrographs. For this purpose, we demonstrate a frequency comb covering the H band, traceable to absolute frequency standards with a mode spacing of 12.5 GHz or  $0.42 \text{ cm}^{-1}$ . Measurements across the spectrum with optical heterodyne and Fourier transform spectroscopy demonstrate adequate suppression of side modes. The comb frequency uncertainty is bounded by  $\pm 30 \text{ kHz}$  (corresponding to a radial velocity of 5 cm/s), limited by the global positioning system disciplined oscillator reference. These results indicate that this comb can readily support radial velocity measurements below 1 meter per second in the near IR.



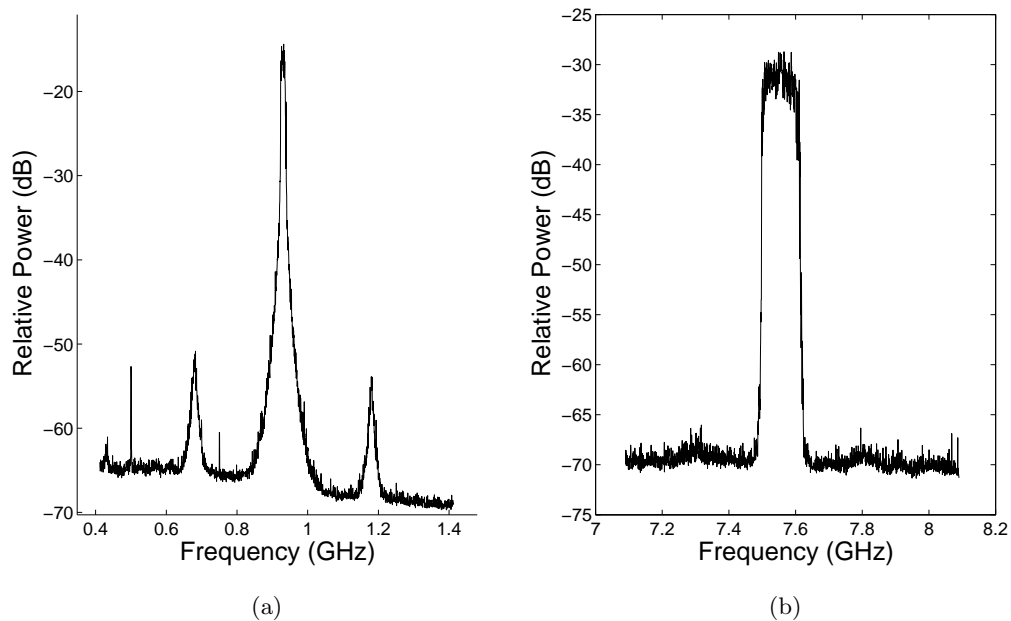


Figure 6: Heterodyne beat signals of individual frequency comb modes after nonlinear broadening at 1400.00 nm (a) and 1628.937 nm (b) with a tunable CW laser, measured with an RF spectrum analyzer. The central peak is the beat of the CW laser with the 12.5 GHz filtered comb, the adjacent peaks are residual modes offset by 250 MHz. The visible asymmetry in (a) is due primarily to frequency dependent gain in the RF amplifier. By averaging measurements made with the tunable laser on either side of the comb line an asymmetry of less than 0.5 dB is measured, shifting the effective line center by less than 6 cm/s. The broad shape of the beat spectra are due to the relatively linewidth of the tunable lasers.

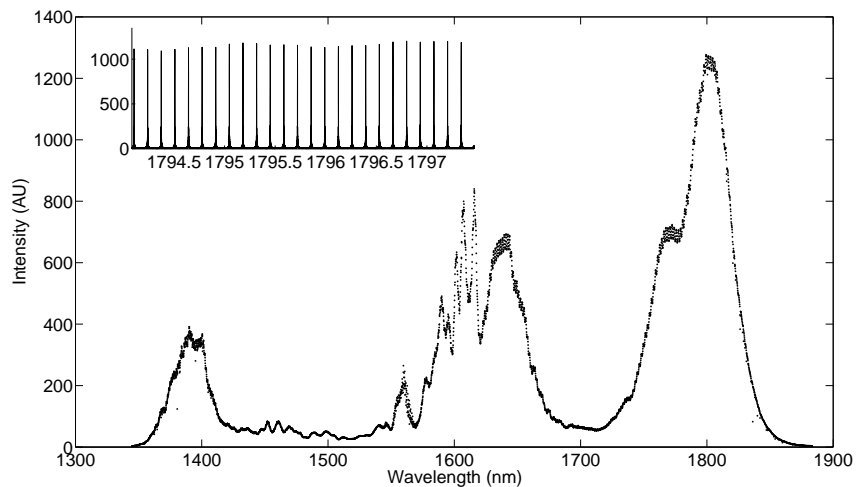


Figure 7: Preliminary analysis of measurement of the frequency comb with the NIST 2 meter Fourier transform spectrometer. The intensities of comb modes are plotted; approximately 5000 of them fill the spectrum. Inset: small section of the Fourier transform spectrogram. Small features around the base of each comb line are due to the instrument response of the FTS.

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