NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

27300-2730900

August 15, 1969

NBS REPORT

9739

STUDIES OF FREQUENCY STABILITY

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$\langle$NBS$\rangle$

U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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1. SUMMARY AND CONCLUSIONS

The primary technical purpose was to attempt to determine the leading source of flicker noise in cesium atomic beam frequency standards. This was based on evidence which indicated most commercial cesium beams display flicker noise at about 3 parts in 10^{13}--perhaps an order of magnitude higher in level than could be reasonably expected. This fact seemed reasonably well documented.

The experiments failed either to display flicker noise at greater than about 1 part in 10^{13} or to reveal marginal components which might contribute flicker noise at levels as high as even a few parts in 10^{14}. Indeed, no flicker noise was observed on the units. From this, I conclude that the source of instabilities is not to be found in the aspects tested. In fact, it is my belief now that these instabilities that have been observed are either: (1) the result of poor environmental control; or (2) older, perhaps not as good, units; (3) poor statistical confidence in the data indicating the higher flicker levels; or (4) inadequate measurement techniques.

A theory was developed and tested to explain frequency pulling resulting from impure r-f signals used to induce atomic transitions. The theory should prove useful in improving error budgets for atomic frequency standards in general.

2. EXPERIMENTS CONDUCTED

2.1 Measurement of Noise in Units

There were a total of three standards (denoted as A, B, and C) which were available for test. Phase data comparing pairs of units were obtained. The longest uninterrupted run was ten days in length which never revealed flicker noise within the confidence of the estimates. The data were consistent with white frequency noise and with figures of merit in the range 1.6 to 1.8.

Analysis of computer simulated white noise revealed that if white noise gives way to flicker noise at some time τ_o then one needs a data length of $8\tau_o$ in length to have about a 70% confidence in the flicker noise diagnosis. Thus, from this data alone, I can say that at a 70% confidence the flicker level is below $\sim 2 \times 10^{-13}$ for the combined measurement or $\sim 1.4 \times 10^{-13}$ per unit.

I requested data giving time difference for about a 3 1/2-month period between two similar standards from NBS Boulder. The data again were consistent with white frequency noise (figure of merit of only ~ 1.25) even at a level of $\sim 1.2 \times 10^{-13}$ or about 8×10^{-14} per unit. These units are in an environmental control chamber controlled in temperature to ~ 10 millidegrees C and with a primary voltage control.

I conclude that, for units in reasonable environments, the flicker level is probably below one part in 10^{13}.

2.2 Stability of Operational Amplifier and Phase Detectors

2.2.1 Zero Error Stability

After following the procedure for zeroing the operational amplifier and the phase detector critically, I recorded the "control voltage" with the a-c amplifier of beam C inoperative (no supply voltage). The output of the amplifier was considered to be the integral of an equivalent input noise and was analyzed using an Allan variance. For the equivalent input noise, the Allan variance was $\langle \sigma_v^2 (2, \tau) \rangle = (0.2 \ \mu v)^2$ and was flicker.

Alignment procedure requires a 1.6 volt output of the phase detector for a 50×10^{-10} frequency error. Thus, the flicker noise FM caused by the operational amplifier and phase detector should be

$$\sigma_y (2, \tau) = (\frac{50 \times 10^{-10}}{1.6v}) (2 \times 10^{-7} v) = 6 \times 10^{-16}$$

where $y \equiv \frac{\delta\nu}{\nu}$.

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2.2.2 Gain Instabilities

It occurred to me that during my experiments the "control voltage" of the cesium beams had been maintained very nearly zero. Since operational amplifiers often have very poor gain stability, one might explain the recent anomalous results by the idea that large control offsets result in higher flicker noise because of gain instabilities.

The two units (B and C) were operated for a 3-day period just below the level to trigger an integrator limit alarm. No flicker noise was observed to a level of $\sim 2 \times 10^{-13}$.

2.3 C-Field Instabilities

2.3.1 18.7-volt Reference Stability

The difference between the two 18.7v references in B and C was recorded. Again an analysis of the data was performed using the Allan variance. The fluctuations in this voltage were consistent with a flicker noise at a level of $\sigma_y(2, \tau) = 1\text{mv}$ or expressed as a fraction $5.5 \times 10^{-3}\%$ of the 18.7v. Since this supplies the current for the C-field and acts as reference for the current, the same percentage change in current or magnetic field is expected. Also,

$$\frac{\partial \nu}{\partial v_{18.7}} \, dv_{18.7} = (2)(427 \text{ Hz/G}^2) \, H \, dH$$

hence,

$$\sigma_y(2, \tau) \approx 2 \times 10^{-14} \text{ per unit}$$

for reference voltage fluctuations affecting the C-field. This result was not reproducible and I suspect some problems with thermal EMF's. Subsequent measurements gave considerably smaller fluctuations in voltage.

2.3.2 Actual Field Fluctuations

In C the internal 12.632...MHz was disconnected from the harmonic generator and replaced by a 12,676,232 Hz signal derived from an internal synthesizer. The recommended adjustment procedure was followed to set signal levels. The instrument was locked onto the field sensitive transition and the phase difference between B and C recorded. A preliminary analysis of the data indicated possible flicker noise FM at a level of $\sim 6 \times 10^{-11}$.

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For the "non-field sensitive" transitions, the effect of the magnetic field fluctuations are reduced by the factor

$$\left(\frac{700,000 \text{ Hz/G}}{(2)(427) \text{ H Hz/G}}\right)^{-1} = (1.37 \times 10^4)^{-1}$$

Thus the actual frequency fluctuations should cause $\sigma_y(2, \tau) \sim 4.5 \times 10^{-15}$ which is about 1/5 of what one expects from the voltage measurements.

The phase data were spectrum analyzed. The resulting spectral density is shown and appears to follow an $f^{-3/2}$ law for low frequencies and a periodicity at $\sim$ 1 cycle/24 minutes on a white background. The white level is not consistent with the shot noise of the beam--it is much higher.

log $S_\nu(f)$
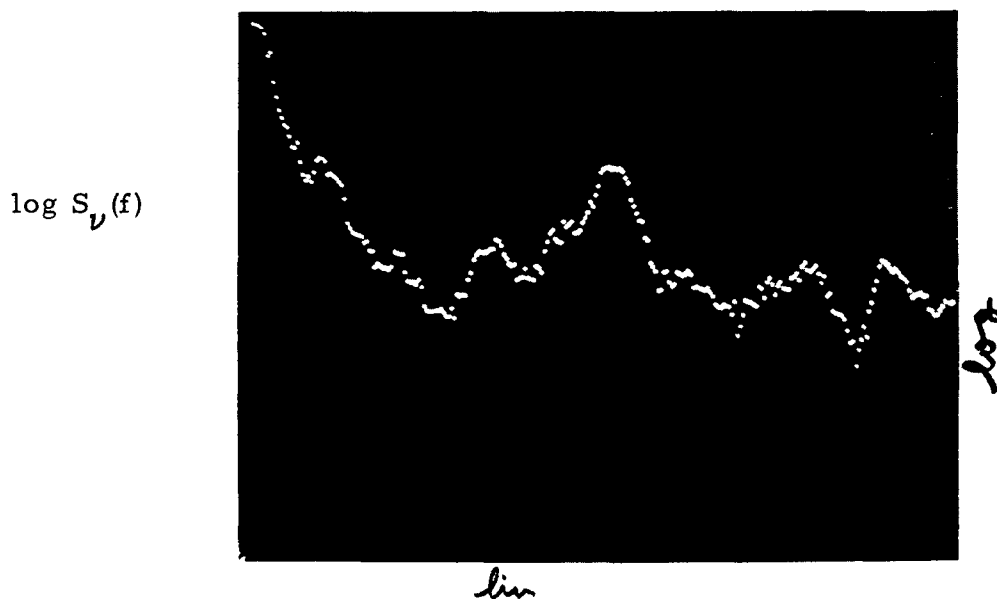
Log Power Spectral Density of Frequency Fluctuations

2.4 Spectrum Effects

2.4.1 Rationale

From papers by J. Shirley, R. Harrach, and others, it is known that asymmetric r-f spectra can cause frequency pulling in a cesium beam. It is therefore reasonable to suspect that a mistuned resonant circuit will cause asymmetries in the noise pedestals of the r-f spectrum and hence cause the frequency to be in error. If the resonant frequency of the circuit changes with time then the output frequency of the standard will change with time.

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Also, I suspect (and checked it experimentally) that with more power in the noise sidebands, the effects of mistuning are greater.

2.4.2 Experiments

I paralleled the output of the 5 MHz oscillator with a voltage variable capacitance. A d-c bias of 1.5 volts was supplied to the diode in addition the broadband ($\sim$ 300 kHz) noise and narrowband ($\sim$ 1 kHz) noise. Levels were chosen to yield approximately a 30% reduction in second harmonic level according to the metering circuits of C. · Also narrowband noise and sinusoids were added to the electrical tuning of the oscillator. In all cases, it was possible to generate frequency offsets, but no flicker noise was observed in the frequency fluctuations to a level of $\sim 8 \times 10^{-13}$ with noise and $\sim 3 \times 10^{-13}$ with sinusoids.

The addition of noise seriously deteriorates the white FM spectrum as one expects but apparently does not significantly increase the flicker level.

With signals applied to the electronic frequency control (EFC), tuning of the crystal filter caused significant ($\sim 10^{-10}$) changes in average frequency. With no signal supplied to the EFC, no changes were observed in average frequency to $\sim 10^{-12}$ with significant changes in filter tuning. It is perhaps reasonable to argue, therefore, that flicker noise caused by filter shifts is less than

$$\frac{1}{100} = (\frac{10^{-12}}{10^{-10}})$$

of the maximum possible flicker level ($\sim 3 \times 10^{-13}$) with poor spectrum or $< 3 \times 10^{-15}$.

The strong effect of signal level on average frequency initiated the studies and considerations of Appendix A.

APPENDIX A

Phenomonological Treatment of
Spectrum Pulling of Resonances

Introduction

There are various means of defining power spectral densities. For convenience assume $\psi_T(t)$ is a function of time with

$$\psi_T(t) = \int_{-\infty}^{\infty} a_T(f)\, e^{i\omega t}\, df\ ,$$

$$a_T(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_T(t)\, e^{-i\omega t}\, dt\ ,$$

and

$$\omega = 2\pi f\ .$$

Then

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_T^*(t)\psi_T(t)\, dt = \int_{-\infty}^{\infty} df \left\{ \int_{-\infty}^{\infty} a_T^*(f')\, a_T(f)\, \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{i(\omega-\omega')t}\, dt\, df \right\}\ .$$

One may define $S_T(f)$ by the ensemble average of the quantity in the curly brackets, i.e.,

$$S_T(f) = E\left[\int_{-\infty}^{\infty} a_T^*(f')\, a_T(f)\, \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{i(\omega-\omega')t}\, dt\, df' \right]\ .$$

In the limit as $T \to \infty$, $S_T(f)$ will approach $S(f)$, the power spectral density of $\psi(t)$.

Consider the real function $\varphi(t)$ and define $\psi(t)$ by the relation

$$\psi(t) = e^{i\varphi(t)}\ .$$

Then,

$$\psi^*(t)\, \frac{d}{dt}\, \psi(t) = i\dot{\varphi}(t)$$

and, taking ensemble and time averages,

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$$\int_{-\infty}^{\infty} i\omega\, S(f)\, df = i2\pi \nu_o$$

or,

$$\nu_o = \int_{-\infty}^{\infty} f\, S(f)\, df$$

where

$$\nu_o \equiv \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1}{2\pi} \dot{\varphi}\, dt \ .$$

Similarly,

$$\left(\frac{d}{dt}\psi^*\right)\left(\frac{d}{dt}\psi\right) = +\dot{\omega}^2 \ ,$$

and, thus,

$$\overline{(\dot{\varphi})^2} = \int_{-\infty}^{\infty} \omega^2\, S(f)\, df \ ,$$

or (using the first moment and zeroth moment)

$$\overline{(\nu(t) - \nu_o)^2} = \int_{-\infty}^{\infty} (f - \nu_o)^2\, S(f)\, df \quad .$$

Again, in a similar fashion,

$$\left(\frac{d^2}{dt^2}\psi^*\right)\left(\frac{d}{dt}\psi\right) - c.c. = -2i\dot{\varphi}^3 \quad .$$

After all averaging,

$$\overline{[\nu(t)]^3} = \int_{-\infty}^{\infty} f^3\, S(f)\, df \ .$$

Making use of the zeroth, first, and second moments,

$$\overline{[\nu(t) - \nu_o]^3} = \int_{-\infty}^{\infty} (f - \nu_o)^3\, S(f)\, df \quad .$$

If $\varphi(t)$ is a monotonically increasing function of t, then $S(f)$ has essentially no power for negative f. Thus a pure FM signal, $V(t)$ may be constructed in the form

$$V(t) = \frac{1}{2} V_o [\psi(t) + \psi^*(t)].$$

Obviously,

$$\int_{-\infty}^{\infty} S'_v(f)\, df = \frac{1}{2} V_o^2$$

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where $S_v'(f)$ is the two-sided spectral density of $V(t)$. Define

$$S_v(f) = \begin{cases} 2\,S_v'(f) & \text{for } f \geq 0 \\ 0 & \text{for } f < 0 \end{cases}$$

where $S_v(f)$ is the one-sided spectral density of $V(t)$. Then,

$$\frac{1}{2}V_o^2 = \int_0^\infty S_v(f)\,df \,,$$

$$\nu_o = \frac{2}{V_o^2} \int_0^\infty f\,S_v(f)\,df \,,$$

$$\overline{(\nu(t) - \nu_o)^2} = \frac{2}{V_o^2} \int_0^\infty (f - \nu_o)^2\,S_v(f)\,df \,,$$

and

$$\overline{(\nu(t) - \nu_o)^3} = \frac{2}{V_o^2} \int_0^\infty (f - \nu_o)^3\,S_v(f)\,df \,,$$

provided $\left| \dfrac{\dot{\nu}(t)}{2\pi\nu_o} \right| << 1$ for all time. Note, $\nu(t)$ is the instantaneous frequency of

$$V(t) = V_o \cos \varphi(t) \,.$$

Phenomonological Postulates

The interaction of the cesium atoms with a complicated rf spectrum is a very complicated problem. For the case of some sinusoidal perturbations to the rf spectrum, see Shirley (1963a, b) or Harrach (1967).

In a more practical sense, the results of these treatments can be obtained approximately by considering the interaction of the rf with a conventional resonance--for example, a single stage LCR resonance. The advantage is that the resulting theory is tractable and gives reasonable results. In effect, the assumptions made are known to be at some variance with reality but often the results are approximately valid. Either experimental measurements or more careful analyses are necessary to vindicate the results.

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To begin with, it will be assumed that the atomic resonance can be represented by the function $|k(f)|^2$. It is assumed that $|k(f)|^2$ is centered at $f = \nu_0$ and is symmetric about $\nu_0 \gg 0$. That is, the Taylor series expansion of $|k(f)|^2$ is of the form

$$|k(f)|^2 = A + B(f-\nu_0)^2 + C(f-\nu_0)^4 + \cdots \cdots \quad .$$

Secondly, it is assumed that the rf signal has a power spectral density $S_v(f-f_0)$ where f_0 is an adjustable frequency. In effect, it is assumed that tuning the rf signal generator moves the spectrum without distorting it. As a matter of convenience, we assume that $S_v(f)$ has appreciable value only in the vicinity of ν_0.

The third and final assumption is that the state of resonance is defined by the maximization of the total interaction,

$$I(f_0) = \int_0^\infty |k(f)|^2 \, S_v(f-f_0) \, df \ .$$

An actual servo system in an atomic frequency standard may approximately maximuze this integral but its detailed operations are at variance with this simplifying assumption.

As stated above, the real justification of the results must rest on experimental verification.

Resonance Equation

If f_0 maximizes $I(f_0)$, then

$$\frac{dI}{df_0} = 0 = \int_0^\infty |k(f)|^2 \, \frac{\partial S_v(f-f_0)}{\partial f_0} \, df \ .$$

But

$$\frac{\partial S_v(f-f_0)}{\partial f_0} = -\frac{\partial S_v(f-f_0)}{\partial f} \quad ,$$

and, therefore, integrating by parts yields

$$\int_0^\infty S_v(f-f_0) \, \frac{d|k(f)|^2}{df} \, df = 0$$

as the condition of resonance.

Making use of the Taylor expansion for $|k(f)|^2$, one obtains

$$2B \int_0^\infty (f-\nu_0) S_v(f-f_0) df \approx 0$$

from the first non-vanishing term. That is,

$$V_0^2 B(\nu_0 + f_0 - \nu_0) = 0$$

or

$$f_0 = 0 .$$

That is, there is no frequency error to a first approximation.

Including the next term yields

$$+ V_0^2 B f_0 + 4 C \int_0^\infty (f-\nu_0)^3 S_v(f-f_0) df \approx 0 ,$$

or

$$f_0 \approx - \frac{4C}{BV_0^2} \int_0^\infty (f + f_0 - \nu_0)^3 S_v(f) df$$

if there is essentially no power for $S_v(f)$ in the range $0 \leq f \leq f_0$. Neglecting terms of order f_0^3, one obtains

$$f_0 [1 - \frac{6C}{B} f_2^2] \approx - \frac{2C}{B} f_3^3 ,$$

where we have written

$$f_2^2 \equiv \frac{2}{V_0^2} \int_0^\infty (f-\nu_0)^2 S_v(f) df$$

and

$$f_3^3 \equiv \frac{2}{V_0^2} \int_0^\infty (f-\nu_0)^3 S_v(f) df .$$

Thus, one obtains

$$f_0 \approx - \frac{\frac{2C}{B} f_3^3}{1 - \frac{6C}{B} f_2^2}$$

which can further be reduced to

$$f_0 \approx - \frac{2C}{B} \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} [\nu(t) - \nu_0]^3 dt$$

if $\frac{6C}{B} f_2^2 \ll 1$.

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It is easy to see that the average frequency of the rf signal is just

$$\frac{2}{V_0^2} \int_0^\infty f\, S_v(f - f_0)\, df = \nu_0 + f_0 \ .$$

Thus the frequency error relative to ν_0 of this device is just f_0. Expressed as a relative error this becomes

$$\frac{f_0}{\nu_0} = - \frac{\dfrac{2C}{B\nu_0} f_3^3}{1 - \dfrac{6C}{B} f_2^2} \ .$$

The Experiment

Two cesium beam frequency standards were used. One was unperturbed and acted as the reference standard for the experiment. For the other instrument, a voltage variable phase shift was accomplished by shunting the 5 MHz line from the oscillator to the multiplier with a voltage variable capacitor (see Fig. 1). This variable capacitance was biased at 1.5 volts and was capable of producing a phase shift of a few milli-radians at 5 MHz with a few tens of millivolts change in bias.
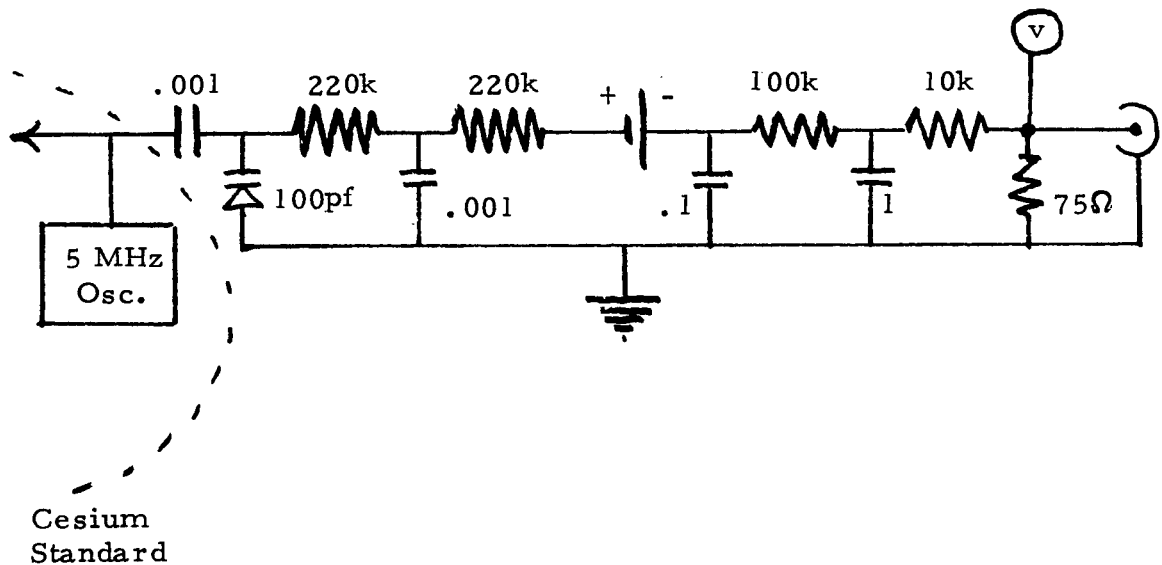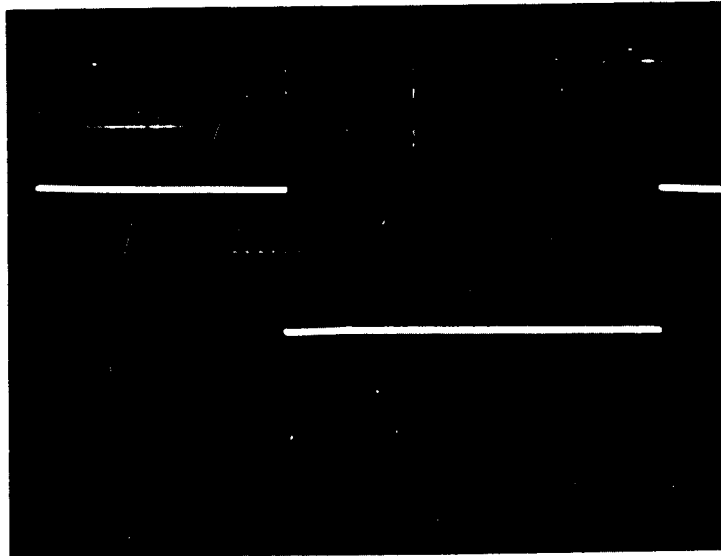

Figure 1

The modulation signal was a square wave (whose relative duty cycle could be adjusted) derived from a 75 Ω source. The actual square wave used is shown in Fig. 2.

Voltage
to
Filter

5 ms/cm

Figure 2

The resonance of the cesium beam will be approximated by

$$|k(f)|^2 \approx A' + B' \cos 2\pi\tau(f - \nu_o)$$

$$\approx A' + B' - \frac{B'(2\pi\tau)^2}{2!}(f-\nu_o)^2 + \frac{B'(2\pi\tau)^4}{4!}(f-\nu_o)^4 + \ldots.$$

with a line width of 529 Hz. That is,

$$2\pi\tau(529) = \pi$$

or

$$\tau = 0.946 \times 10^{-3} \text{ sec.}$$

and

$$\frac{2C}{B} = -5.88 \times 10^{-6} (\text{sec.})^2 .$$

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By comparing the modulated signal with the reference signal in a phase error multiplier it was possible to obtain estimates of the amplitude, and second and third moments of the instantaneous frequency. From a photograph of the frequency analog output of the comparator, it was possible to calculate (graphically) the ratio of f_3 and f_2 to the peak-to-peak frequency fluctuation, f_1. (See Fig. 3) The results are
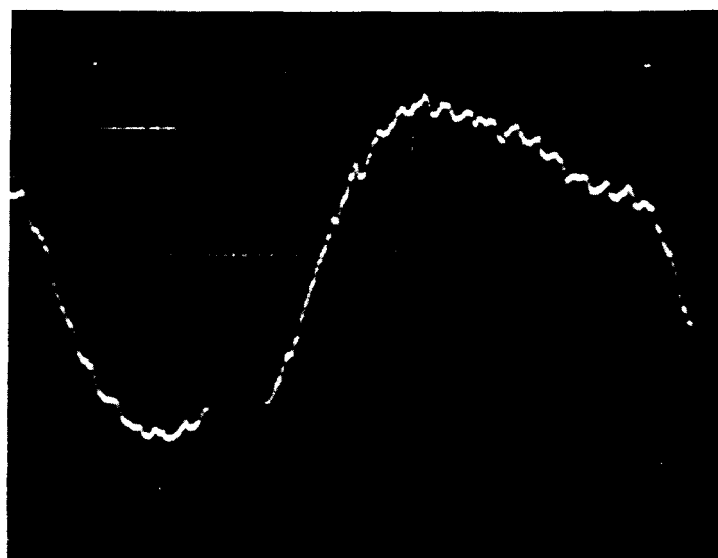
$$f_3 = - (0.257) f_1$$

and

$$f_2 = (0.364) f_1 \quad .$$

The peak-to-peak amplitude was measured relative to the signal generator output voltage by observing the frequency change in the oscillator needed to displace the frequency analog signal on the oscilloscope by its peak-to-peak value. The result of these experiments is summarized by the equation

$$\frac{f_1}{V} = (1769 \pm 60) \text{ Hz} \quad .$$

↑ +
Frequency
Analog
↓ − from C

5 ms/cm

Figure 3

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Thus,

$$\frac{f_2}{V} = 644 \text{ Hz}$$

and

$$\frac{f_3}{V} = -455 \text{ Hz} .$$

Also,

$$1 + \frac{6C}{B} f_2^2 = 1 + 2.44V^2 ,$$

and

$$\frac{2C}{B\nu_o} f_3^3 = + (6.0 \times 10^{-8})V^3 ,$$

or

$$\frac{f_o}{\nu_o} = - \frac{(6 \times 10^{-8})V^3}{1 - 2.44V^2} .$$

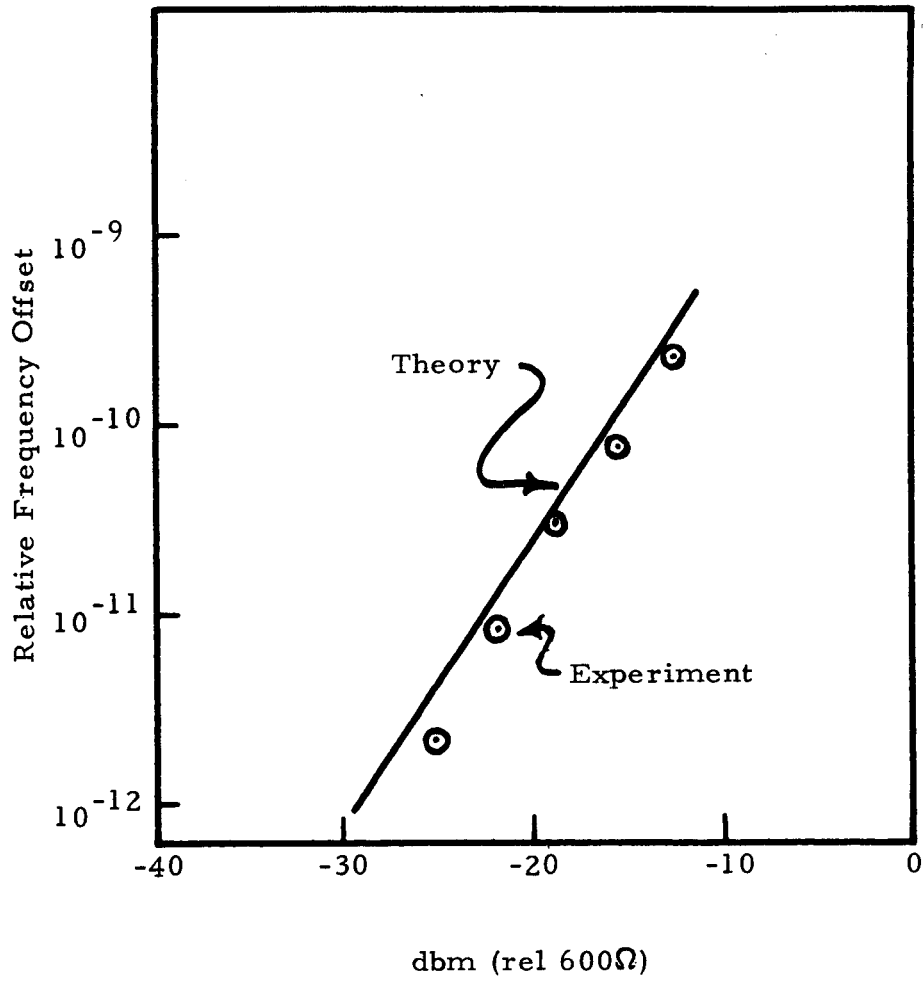The results of the experiment and this equation are plotted in Fig. 4.

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Figure 4

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