

THE MEASUREMENT OF RADIO FREQUENCY RESISTANCE, PHASE DIFFERENCE, AND DECREMENT*

By

J. H. DELLINGER

(ASSOCIATE PHYSICIST, BUREAU OF STANDARDS, WASHINGTON, D. C.)

Various methods of measuring radio (high) frequency resistances, the power factor or phase difference of radio condensers, the sharpness of a resonance curve, and the decrement of a wave have been given. It is proposed in this paper: (1) to show that these quantities all express essentially the same physical magnitude and hence a single process of measurement gives them all; (2) to derive and classify the methods of measurement; (3) to describe improvements in these measurements.

RELATIONS OF THE QUANTITIES

The principal difference between the phenomena of radio (high) and audio (low) frequency is the importance of capacity and inductance at radio frequency as compared with the predominance of resistance in audio frequency phenomena. Nevertheless, resistance is the measure of power consumption, since any dissipation or loss of electrical power is expressible in terms of a resistance. Furthermore, resistances change rapidly with frequency at radio frequencies. The change can be calculated for certain very simple cases, but in most practical cases the resistance can be determined only by measurement. Thus the measurement of resistance at radio frequencies is a necessary and important operation.

Certain related quantities also express power dissipation. The results of a measurement can be expressed in terms of any of these quantities when their relations are clearly established.

In a simple series circuit, Figure 1, when the emf. is a sustained sine wave, a maximum of current is obtained when the inductive reactance is just equal to the capacitive reactance, i. e., when the equation

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

* Received by the Editor, April 17, 1918.

reduces to $I_r = \frac{E}{R}$. This condition of resonance obtains when

$$\omega L - \frac{1}{\omega C} = 0$$

or

$$\omega = \frac{1}{\sqrt{CL}} \tag{1}$$

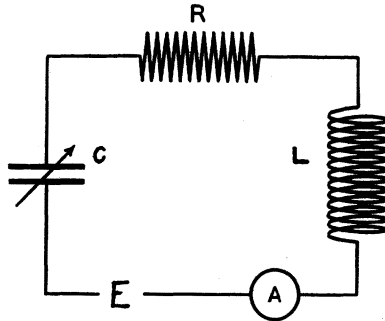


FIGURE 1

For any variation of either ω , L , or C , from this condition, as at A for the curves of Figure 2, the current is smaller than the value for which this relation holds.

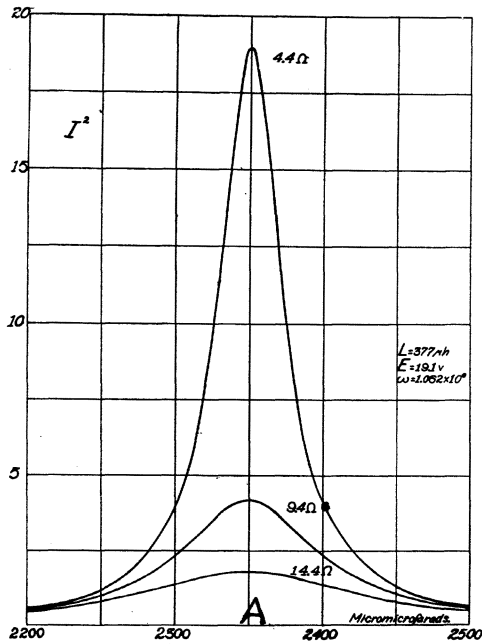


FIGURE 2

The phase angle of the circuit is zero at resonance as shown in Figure 3. The expression, phase angle of the circuit, means the same thing as phase angle of the current in the circuit.

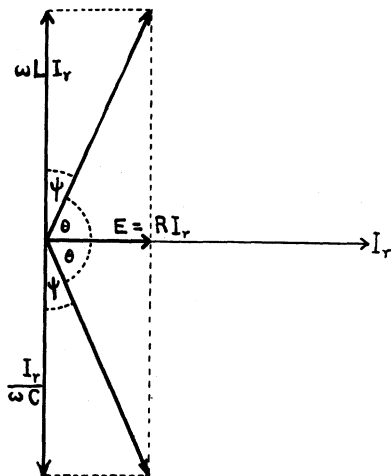


FIGURE 3

The phase angle θ of the coil, considering the resistance R to be associated with the coil L , is equal to the phase angle θ of the condenser, considering the resistance to be associated with the condenser. The complement of the phase angle θ is the phase difference ψ . From Figure 3,

$$\tan \psi = \frac{R}{\omega L} = R \omega C$$

When R and ψ are small, as usually in radio circuits, the tangent equals the angle and

$$\psi = \frac{R}{\omega L} = R \omega C \quad (2)$$

Phase difference is thus a ratio of resistance to reactance. It follows also that $\psi = \sin \psi = \cos \theta$, or phase difference is equal to power factor. The same relation is brought out by multiplying in (2) by I^2 .

$$\psi = \frac{R I^2}{\omega L I^2}$$

or

$$\psi = \frac{R I^2}{I^2} \frac{1}{\omega C}$$

=ratio of power dissipated to power flowing.

Thus

$$\psi = \text{power factor.} \quad (3)$$

Another quantity of importance in connection with the expression of power dissipation is the sharpness of resonance. This is the quantity which measures the fractional change in current for a given fractional change in either the capacitive or inductive reactance from its value at resonance. (Practically the same quantity has been known as persistency, selectivity, and resonance ratio.) It may be defined in mathematical terms for a variation of C by the following ratio

$$S = \frac{\sqrt{I_r^2 - I_1^2}}{\frac{\pm(C_r - C)}{C}}$$

where the subscript “ r ” denotes value at resonance, and I_1 is some value of current corresponding to a capacity C which differs from the resonance value. The numerator of this expression is somewhat arbitrarily taken to be the square root of the fractional change in the current-square instead of taking directly the fractional change of current. This is done partly because of the convenience in actual use of this expression (since the deflections of the usual detecting instruments are proportional to the square of the current), and also because of mathematical convenience. It is easily seen qualitatively that sharpness of resonance is large when phase difference is small; as when R is small compared with ωL , a given fractional change in ωL changes the impedance, and therefore the current, by a relatively large fractional amount. Quantitatively, from the relations,

$$I_1^2 = \frac{E^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \quad I_r^2 = \frac{E^2}{R^2}, \quad \text{and} \quad \omega L = \frac{1}{\omega C_r},$$

it follows at

once that

$$S = \frac{\sqrt{I_r^2 - I_1^2}}{\frac{\pm(C_r - C)}{C}} = \frac{1}{R \omega C_r} = \frac{\omega L}{R} = \frac{1}{\psi} \quad (4)$$

= ratio of power flowing to power dissipated. Thus,

$$\text{Sharpness of resonance} = \text{reciprocal of power factor.} \quad (5)$$

Another related quantity is the logarithmic decrement. For free oscillation in a simple circuit, the value of the decrement

or napierian logarithm of the ratio of two successive current maxima in the same direction is readily shown to be

$$\delta = \pi R \sqrt{\frac{C}{L}} = \frac{\pi R}{\omega L} = \pi R \omega C \quad (6)$$

Comparing with (2) and (4),

$$\delta = \pi \psi = \frac{\pi}{S} \quad (7)$$

That is, altho defined originally in connection with damped oscillations, the decrement is a constant of the circuit and can be dealt with in the same way as resistance and the other energy-determining quantities. It is similarly possible to speak of the decrement of a part of a circuit in the same manner as phase difference and resistance.

The decrement is definable in terms of an energy ratio. Thus

$$\delta = \pi \psi = \frac{\pi R}{\omega L} = \frac{\pi R I^2}{\omega L I^2} = \frac{\frac{\pi R I^2}{L I^2}}{\frac{2 \pi f}{L I^2}} = \frac{1}{2} \frac{f}{L I^2}$$

For sustained sinusoidal oscillations, $\frac{R I^2}{f}$ = energy dissipated per cycle, and $L I^2 = \frac{1}{2} L (2 I^2) = \frac{1}{2} L I_o^2$ = magnetic energy associated with the current at the maximum of the cycle. It follows that the decrement is one-half the ratio of the energy dissipated per cycle to the energy associated with the current at the maximum of the cycle. This relation holding for each cycle, it holds for the average of all the cycles.

That the same definition of decrement applies to the natural damped oscillations of a circuit may be shown as follows. Suppose the circuit to be set oscillating, so that a train of natural oscillations takes place, N times per second, and that the energy of each train is practically all dissipated before the next one begins. N is the group frequency or number of complete trains of oscillations per second. The energy dissipated during a train of waves equals the energy input at the beginning of each train = $\frac{1}{2} L I_o^2$, where I_o is the first maximum of current. The average energy dissipated per cycle must equal the energy dissipated during a train of waves divided by the number of cycles in a train. The number of cycles in a train is the ratio of the frequency of oscillations f to the group frequency N . Therefore, the average energy dissipated per cycle = $\frac{\frac{1}{2} L I_o^2}{\frac{f}{N}} = \frac{N L I_o^2}{2 f}$. The average energy associated with the

current at the maximum of each cycle may be shown to be $L I^2$, where I^2 =root-mean-square current, just as in the case of undamped currents, provided the decrement is not large. Applying the energy-ratio definition of decrement,

$$\delta = \frac{1}{2} \frac{N L I_o^2}{L I^2} = \frac{N I_o^2}{4 f I^2} \quad (8)$$

This checks the familiar equation for the root-mean-square value of natural oscillations,

$$I^2 = \frac{N}{4 f \delta} I_o^2 = \frac{N}{4 a} I_o^2 \quad (9)$$

This definition of decrement, one-half the ratio of the average energy dissipated per cycle to the average energy associated with the current at the maximum of each cycle, is a valuable conception. It has here been shown to apply to both undamped oscillations and to natural damped oscillations.

It is thus evident that resistance, phase difference, sharpness of resonance, and decrement are all constants of a circuit expressing energy dissipation or power factor. Their relations are given in equations (2) to (7). Using these relations, a measurement of any one of them can be made to give all the others. Since resistance is the simplest quantity, specific consideration is given in the following to resistance measurement. Nevertheless in some cases one of the other quantities is the more convenient or more useful one in terms of which to express the results of measurement.

METHODS OF MEASUREMENT

GENERAL—There is considerable difficulty in attaining high accuracy in measurements at radio frequencies. Much of this is due to the fact that the quantities to be measured or upon which the measurement depends are generally small and sometimes not definitely localized in the circuits. Thus the inductances and capacities used in the measuring circuits are so small that the effect upon these quantities of lead wires, indicating instruments, surroundings, etc., must be carefully considered. The capacity of the inductance coil and sometimes even the inductance within the condenser are of importance. In order to minimize these various effects, it is generally best to use measuring circuits and methods which are the least complicated. On this account simple circuits and substitution

methods in which the determination depends upon deflections are usually used in preference to more complicated methods.

In addition to the uncertainty or the distributed character of some of the quantities to be measured, there are other limitations upon the accuracy of radio measurements. The usual ones are the variation with frequency of current distribution, of inductance, resistance, and so on, and the difficulty of supplying radio frequency current of sufficient constancy. The latter limitation is entirely overcome by the use of the electron tubes as a source of current, but is troublesome when a buzzer, spark, or arc is used. As to the other difficulty: the variations of inductance, and so on, with frequency, while these variations have a profound effect, they are generally subject to control; the quantities have definite values at a particular frequency under definite conditions, and their effect can usually be determined by calculation or measurement.

It is not always possible to determine the effects of the capacities of accessory apparatus and surroundings, nor to eliminate them, and thus they remain the principal limitation upon the accuracy of measurements. These stray capacities include the capacities of leads, instrument cases, table tops, walls, and the observer. They may not only be indeterminate but may vary in an irregular manner.

CLASSIFICATION OF METHODS

On account of the requirement of simplicity in radio measurements, the methods available are quite different, and are fewer in number, than in the case of audio-frequency or direct-current measurements.

The methods of measuring radio-frequency resistance may be roughly classed as:

- (1) Calorimeter method.
- (2) Substitution method.
- (3) Resistance-variation method.
- (4) Reactance-variation method.

The fourth has frequently been called the "decrement method," but it is primarily a method of measuring resistance rather than decrement, exactly as the resistance variation method is. Either may be used to measure the decrement of a wave under certain conditions, and in fact the results of resistance measurement by any method may be expressed in terms of decrement.

All four methods may be used with either damped or undamped waves, tho in some of them the calculations are different in the two cases. They are all deflection methods, in the sense of depending upon the deflections of some form of radio-frequency ammeter. In the first and second, however, it is only necessary to adjust two deflections to approximate equality, while in the third and fourth the deflections may have any magnitude.

CALORIMETER METHOD

This method may be used to measure the resistance either of a part or the whole of a circuit. The circuit or coil or other apparatus, the resistance of which is desired, is placed in some form of calorimeter, which may be a simple air chamber, an oil bath, or other suitable form. The current is measured by an accurate radio-frequency ammeter, and the resistance R_x is calculated from the observed current I and the power, or rate of heat production, P

$$P = R_x I^2 \quad (10)$$

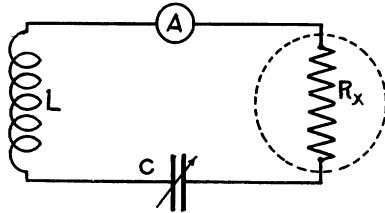


FIGURE 4

While P might be measured calorimetrically, in practice it is always measured electrically by an auxiliary observation in terms of audio-frequency or direct current. Thus it is only necessary to observe the temperature of the calorimeter in any arbitrary units when the radio-frequency current flows, and then cause audio-frequency current to flow in the circuit, adjusting its value until the temperature becomes the same as before. Denoting by the subscript “ o ” the audio-frequency values

$$P_o = R_o I_o^2$$

$$\frac{P}{P_o} = \frac{R_x I^2}{R_o I_o^2}$$

$$\frac{R_x}{R_o} = \frac{P I_o^2}{P_o I^2}$$

$$\text{For } P = P_o, \quad R_x = R_o \frac{I_o^2}{I^2} \quad (11)$$

From the known audio frequency value of the resistance, therefore, and the observed currents, the resistance is obtained.

The radio and audio frequency observations are sometimes made simultaneously, using another resistance of the same magnitude as that of the apparatus, the resistance of which is desired, placed in another calorimeter as nearly identical with the first as possible. High (radio) frequency current is passed thru one, low (audio) frequency thru the other, and the calorimeters kept at equal temperatures by means of some such device as a differential air thermometer or differential thermoelement. To compensate for inequalities in the two sets of apparatus, the radio and audio frequency currents are interchanged. This method may be found more convenient in some circumstances, but the extra complication of apparatus is usually not worth while, and the value of the measurement depends upon the accurate observation of the radio frequency current I , just as the simpler method does.

The calorimeter method, while capable of high accuracy, is slow and less convenient than some of the other methods. It has been used by a number of experimenters to measure the resistance of wires and coils.

SUBSTITUTION METHOD

This method is applicable only to a portion of a circuit. Suppose that in Figure 4 the coil L is loosely coupled to a source of oscillations. The capacity C is varied until resonance is obtained, and the current in the ammeter is read. A resistance standard is then substituted for the apparatus R_x and varied until the same current is indicated at resonance. If the substitution has changed the total inductance or capacity of the circuit, the retuning to resonance introduces no error when undamped or slightly damped electromotive force is supplied, provided the change of condenser setting introduces either a negligible or known resistance change. In the case of a rather highly damped source, however, the method can only be used when the resistance substitution does not change the inductance or capacity of the circuit. The unknown R_x is equal to the standard resistance inserted, *provided* the electromotive force acting in the circuit has not been changed by the substitution of the standard for R ; this condition is discussed below.

The resistance standards usually used are not continuously

variable, and hence the standard used may give a deflection of the ammeter somewhat different from the original deflection. To determine the resistance in this case, three deflections are required, all at resonance. In one application, the apparatus of unknown resistance R_x is inserted and the current I_x observed; then a similar apparatus of known resistance R_n is substituted for it and the current I_n observed; and finally a known resistance R_1 is added and the current I_1 observed. The relations between these quantities and the electromotive force involve the unknown but constant resistance of the remainder of the circuit R , thus,

$$R_x + R = \frac{E}{I_x}$$

$$R_n + R = \frac{E}{I_n}$$

$$R_1 + R_n + R = \frac{E}{I_1}$$

from which

$$R_x - R_n = R_1 \frac{\frac{I_n}{I_x} - 1}{\frac{I_n}{I_1} - 1} \quad (12)$$

This method is closely related to the resistance variation method; see formula (13) below.

The substitution method is very convenient and rapid and is suitable for measurements upon antennas, spark gaps, etc., and for rough measurements of resistances of condensers and coil. In radio laboratory work, however, using delicate instruments and with loose coupling to the source of oscillations, it is found that it is not a highly accurate method, except for measuring small changes in resistance of a circuit. The reason for this is that there are other electromotive forces acting in the circuit than that purposely introduced by the coupling coil, viz., emf.'s electrostatically induced between various parts of the circuit. When the apparatus under measurement is removed from the circuit, these emf.'s are changed, and there is no certainty that when the current is made the same the resistance has its former value. Something of the same difficulty enters into the question of grounding the circuit in the following method, as discussed below.

RESISTANCE VARIATION METHOD

This method measures primarily the effective resistance of the whole circuit, including that due to condenser losses and

to radiation. The principle may be readily understood from the diagram of the simple circuit, Figure 5.

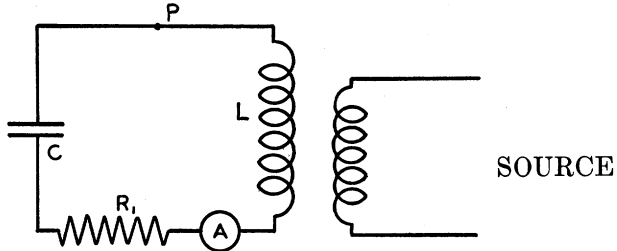


FIGURE 5

If the resistance of some particular piece of apparatus, inserted at P for example, is to be found, the resistance of the circuit is measured with it in circuit and then re-measured in the same way with it removed or replaced by a similar apparatus of known resistance; and the resistance of the apparatus is obtained by simple subtraction.

The results of a measurement may be expressed in terms of ψ , S , or δ by equations (2) to (7) above. The method, however, is particularly convenient where resistance is the actual quantity the value of which is wanted.

The measurement is made by observing the current I in the ammeter A when the resistance R_1 has its zero or minimum value, then inserting some resistance R_1 and observing the current I_1 . Let R denote the resistance of the circuit without added resistance. Suppose that a sine-wave electromotive force E is introduced into the circuit by induction in the coil L from a source of undamped waves, and that the two observations are made at resonance. For the condition of resonance,

$$I = \frac{E}{R}$$

$$I_1 = \frac{E}{R + R_1}$$

from which the resistance of the circuit is given by

$$R = R_1 \frac{I_1}{I - I_1} = \frac{R_1}{\frac{I}{I_1} - 1} \quad (13)$$

The same method can be employed using damped instead

of continuous waves, and can even be used when the current is supplied by impulse excitation, but the equations are different; see (59) and (16) below. When the damping of the supplied emf. is very small, equation (13) applies.

PRECAUTIONS

A limitation on the accuracy of the measurement is the existence of the emf.'s electrostatically induced that were mentioned above. In the deduction of (13) it is assumed that E remains constant. The virtue of this method is that these emf.'s may be kept substantially constant during the measurement of resistance of the circuit. They will invariably be altered by the insertion of the apparatus, the resistance of which is desired, but the resistance of the circuit is measured accurately in the two cases and the difference of the two measurements gives the resistance sought. In order to keep these stray electromotive forces unchanged when R_1 is in and when it is out of circuit, particular attention must be paid to the grounding of the circuit. The shield of the condenser and the ammeter (particularly if it is a thermocouple with galvanometer) have considerable capacity to ground and are near ground potential. A ground wire, if used, must be connected either to the condenser shield or to one side of the ammeter. If connected to the high-potential side of the inductance coil, absurd results will be obtained. The resistance R_1 also must be inserted at a place of low potential, preferably between the condenser and ammeter.

USE OF THERMOCOUPLE

Another necessary precaution is to keep the coupling between source and measuring circuit so loose that there is no reaction. This necessitates the use of a sensitive device for current measurement. As regularly carried out at the Bureau of Standards, in the resistance variation method, a pliotron is used as a source of undamped emf., and current is measured with a thermocouple in series in the measuring circuit. The currents corresponding to given deflections of the thermocouple galvanometer are obtained from a calibration curve, or from the law $d \propto I^2$, where d = deflection, if the instrument follows this law sufficiently closely. When the deflections follow this law, equation (13) becomes

$$R = \frac{R_1}{\sqrt{\frac{d}{d_1}} - 1} \quad (14)$$

Several values of resistance R_1 are usually inserted in the circuit and the corresponding deflections obtained; the resulting values of R are averaged.

When the thermocouple follows the square law accurately, the quarter deflection method may be used, which eliminates all calculation. When the deflection d_1 is $\frac{d}{4}$, equation (14) becomes

$$R = R_1 \tag{15}$$

This method requires a variable resistance standard such that R_1 can be varied continuously in order to make d_1 just equal to $\frac{d}{4}$. Practically the same method is used if the resistance is varied by small steps, as in a resistance box, and interpolating between two settings of R_1 .

USE OF IMPULSE EXCITATION

The procedure for the resistance variation method is the same when the current is damped as when undamped. When the circuit is supplied by impulse excitation, so that free oscillations are produced, the theory of the measurement is very simple. The current being I when the resistance is R , and I_1 when the resistance R_1 is added, the power dissipated in the circuit must be the same in the two cases because the condenser in the circuit is charged to the same voltage by each impulse which is impressed upon it, and there is assumed to be no current in the primary after each impulse.

Therefore

$$R I^2 = (R + R_1) I_1^2$$

whence,

$$R = R_1 \frac{I_1^2}{I^2 - I_1^2} \tag{16}$$

It is difficult to obtain high accuracy by the method in practice because of the difficulty of obtaining pure impulse excitation.

The method is specially convenient when an instrument is used in which the deflection d is proportional to the current squared. Then (16) becomes

$$R = R_1 \frac{d_1}{d - d_1} \tag{17}$$

This is still further simplified if the resistance R_1 is adjustable

so that d_1 can be made equal to one-half d . The equation then reduces to

$$R = R_1 \quad (18)$$

This is commonly known as the half-deflection method.

USE OF DAMPED EXCITATION

The resistance variation method has already been shown to be usable with either undamped or free oscillations. It can also be used when the supplied emf. is damped so that both forced and free oscillations exist in the circuit. The equations (56) to (65) below show how the decrement of a circuit is obtained from such measurements. Resistance is then readily calculated by equation (6). As already stated, when the damping of the supplied emf. is extremely small, equation (13) applies. The decrement of the supplied emf. may itself be obtained by such measurements whether the emf. be due to a nearby circuit or to a wave travelling thru space.

APPLICATION OF METHOD

This method is used in precision measurements upon condensers, coils, wavemeters, etc. The accurate measurement of resistance of a wavemeter circuit is of particular importance because the wavemeter is frequently used to measure the resistance, phase difference, or decrement of other apparatus. It is the calibration of a resistance-measuring standard.

The resistance of a wavemeter is not a single constant value. It varies with frequency and with the detecting or other apparatus connected to the wavemeter circuit. Usually both the resistance and the decrement of the circuit vary with the condenser setting. It is usually desirable to express either resistance, sharpness of resonance, or decrement in the form of curves for the several wavemeter coils, each for a particular detecting apparatus or other condition.

When a pliotron, arc, or other source of undamped wave is used, formula (13) above is used. When the current-measuring device is a current-square meter, thermocouple or crystal detector with galvanometer, or other apparatus which is so calibrated that deflections are accurately proportional to the square of the current, and when in addition a continuously variable resistance standard is used, the quarter-deflection method may be employed eliminating all calculation.

When a buzzer or other source is used, arranged to give impulse excitation, equation (16) above gives the resistance.

When the current indicator is calibrated in terms of the square of the current and the resistance standard is continuously variable, the measurement is conveniently made by the half-deflection method.

REACTANCE VARIATION METHOD

This has been called the decrement method, a name which is no more applicable to this than to the other methods of resistance measurement since all measure decrement in the same sense that this does. That the method primarily measures resistance rather than decrement is seen from the fact that in its simple and most accurate form it utilizes undamped current, which has no decrement.

The method is analogous to the resistance variation method, two observations being taken. The current I_r in the ammeter (Figure 6) is measured at resonance, the reactance is then varied and the new current I_1 is observed. The total resistance of the circuit R (including that due to condenser losses, radiation, etc.) is calculated from these two observations. The re-

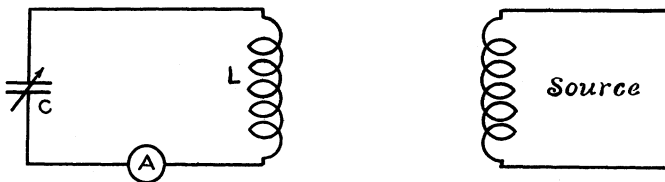


FIGURE 6

actance may be varied by changing either the capacity, the inductance, or the frequency, the emf. being maintained constant. The reactance is zero at resonance and it is changed to some value X_1 for the other observation. With undamped emf. E , the currents are given by

$$I_r^2 = \frac{E^2}{R^2}$$

$$I_1^2 = \frac{E^2}{R^2 + X_1^2}$$

From these it follows that

$$R = X_1 \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (19)$$

This has a similarity to $R = R_1 \frac{I_1}{I - I_1}$, the equation (13) for the resistance-variation method. It is also interesting that when the reactance is varied by such an amount as to make the quantity under the radical sign equal to unity, the equation reduces to

$$R = X_1 \quad (20)$$

This is similar to $R = R_1$, which is the equation for the quarter-deflection and half-deflection resistance-variation methods.

RESISTANCE MEASUREMENT

When the reactance is varied by changing the setting of a variable condenser,

$$X_1 = \pm \left(\frac{1}{\omega C} - \frac{1}{\omega C_r} \right)$$

and the equation (19) becomes

$$R = \frac{\pm (C_r - C)}{\omega C_r C} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (21)$$

For variation of the inductance, (19) becomes

$$R = \pm \omega (L - L_r) \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (22)$$

and for variation of the frequency

$$R = \frac{\pm L (\omega^2 - \omega_r^2)}{\omega} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (23)$$

This equation is equivalent to

$$R = \frac{\pm 6 \pi \times 10^8 L (\lambda_r^2 - \lambda^2)}{\lambda \lambda_r^2} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (24)$$

for λ in meters, R in ohms, and L in henrys.

It must be noted that variation of the frequency or wave length requires some alteration in the source of emf., and the greatest care is necessary to insure that the condition of constant emf. is fulfilled. This is discussed below in connection with equations (30) and (31).

In the use of equation (22), some error is introduced into the measurement if the variable inductor is also used as the coupling to the source, on account of the variation thus introduced into the E supplied. The per cent. error, however, is usually not more than the per cent. variation of L .

A convenient method which differs slightly from those just

described is to observe two values of the reactance both corresponding to the same current I_1 on the two sides of the resonant value I_r . For observation in this manner of two capacity values C_1 and C_2 ,

$$R = \frac{1}{2\omega} \frac{C_2 - C_1}{C_2 C_1} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (25)$$

The simple derivation here given for these formulas is much shorter than the usual treatments, and at the same time is more comprehensive. These formulas are all rigorous, involving no approximations, provided the applied emf. is undamped. They also apply for damped emf. when the damping is negligibly small.

It is customary to reduce the labor of computation by varying the reactance by such an amount that $I_1^2 = \frac{1}{2} I_r^2$, making the quantity under the radical sign equal to unity, so that formulas (21) to (25) are much simplified.

MEASUREMENTS OF PHASE DIFFERENCE, SHARPNESS OF RESONANCE, AND DECUREMENT

Measurements by the reactance variation method are very conveniently expressed in terms of phase difference, sharpness of resonance, and decrement. The formulas are in fact simpler for any of these quantities than for resistance. Thus, utilizing equations (2) to (7) it is readily found that (21) is equivalent to:

$$\psi = \frac{\pm(C_r - C)}{C} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (26)$$

$$S = \frac{C}{\pm(C_r - C)} \sqrt{\frac{I_r^2 - I_1^2}{I_1^2}} \quad (27)$$

$$\delta = \pi \frac{\pm(C_r - C)}{C} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (28)$$

Equation (27) is identical with (4) above, thus suggesting that the definition of sharpness of resonance itself contains inherently this method of measurement. The equations corresponding to (22) to (25) are obtained for ψ , S , and δ , in the same manner as (26) to (28). Those for phase difference, expressed in radians, are

$$\psi = \frac{\pm(L - L_r)}{L_r} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (29)$$

$$\psi = \frac{\pm(\omega^2 - \omega_r^2)}{\omega \omega_r} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (30)$$

$$\psi = \frac{\pm(\lambda_r^2 - \lambda^2)}{\lambda \lambda_r} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (31)$$

$$\psi = \frac{C_2 - C_1}{C_2 + C_1} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (32)$$

Phase difference is a particularly convenient constant in terms of which to express the results of measurements upon condensers, since the phase difference of most condensers is usually a constant with respect to frequency at radio frequencies. These formulas are rigorous provided ψ is small, as it usually is in radio circuits, when the emf. is sustained, and hold also for damped emf. when the damping is negligibly small. The use of the method when the applied emf. has a moderate damping is discussed in the last section of this paper.

A convenient way to utilize the method indicated in (30) and (31) is to vary the wave length by means of a variable condenser or inductor in the source circuit. An incorrect formula has sometimes been given for decrement measurement by this method. The following are rigorous:

$$\psi = \frac{\pm(C_r' - C')}{\sqrt{C_r' C'}} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}}$$

$$\delta = \pi \frac{\pm(C_r' - C')}{\sqrt{C_r' C'}} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}}$$

where the capacities are those of the condenser in the source circuit. This method must be used with great caution because constancy of E , the applied emf., is required. The source circuit is necessarily disturbed by the variation of its condenser setting; when the variation is small, and a pliotron is used as the source, the current I' in the source circuit may not be appreciably changed. It is desirable to use a sensitive indicating instrument and actually observe I' . Constancy of I' , however, would not mean that the emf. acting on the measuring circuit was constant, for $E = \omega M I'$, and thus E varies by the amount of the ω variation. The per cent. error in the resulting value of ψ or δ equals the per cent. change of C' , when the method is made by the familiar procedure which reduces the current ratio under the radical to unity.

DIRECT-READING PHASEMETERS AND DECREMETERS

A phasemeter as used in radio work is a wavemeter conveniently arranged for measurements of phase difference. A decimeter is a wavemeter similarly arranged for measurements

of decrement. While, of course, resistance and sharpness of resonance can be calculated from measured values obtained by either of these instruments, the principal application of a phasemeter is in measurements of phase difference of condensers and of dielectric materials and the principal use of a decremeter is in the measurement of the decrement of a wave. The forms of these instruments usually employed make use of the reactance-variation method. Any such instrument may be used either as a phasemeter or a decremeter by merely changing the instrument scale by a constant factor. While decremeters have been more commonly used, the phasemeter is a somewhat more direct application of the underlying theory. In the development of the theory of the instrument, undamped (sustained) sine-wave emf. is assumed.

DETERMINATION OF THE SCALE OF A PHASEMETER OR DECREMETER

Any wavemeter, the circuit of which includes some form of ammeter, may be fitted with a special scale from which phase difference or decrement may be read directly. The procedure for a wavemeter having any sort of variable condenser is given here.

The usual use of the reactance-variation method is in accordance with equation (32), the currents being adjusted so as to make the quantity under the radical unity. That is, the current-square meter is first observed at resonance, the variable condenser is reset to a value C_1 on one side of resonance such that the current-square is reduced to one-half, and then set to another value C_2 on the other side of resonance giving the same current-square. The phase difference is calculated by

$$\psi = \frac{C_2 - C_1}{C_2 + C_1} \quad (33)$$

A certain value of phase difference, therefore, corresponds to that displacement of the condenser's moving plates which varies the capacity by the amount $(C_2 - C_1)$. The displacement for a given phase difference will, in general, be different for different values of C , the total capacity in the circuit. At each point of the condenser scale, therefore, any displacement of the moving plates which changes the square of current from $\frac{1}{2} I_r^2$ on one side of resonance to the same value on the other side means a certain value of ψ .

A special scale may, therefore, be attached to any variable

condenser, with graduations upon it and so marked that the difference between the two settings on the two sides of resonance is equal to the phase difference. The spacing of the graduations at different parts of the scale depends upon the relation between capacity and displacement of the moving plates. When this relation is known, the scale can be predetermined. A scale may, therefore, be fitted to any condenser, from which phase difference may be read directly, provided the capacity of the circuit is known for all settings of the condenser. The scale may be attached either to the moving plate system or to the fixed condenser top. It is usually convenient to attach it to the unused half of the dial opposite the capacity scale.

The scale for such an instrument is determined as follows. When the change of capacity setting is small, as usually in radio work, (33) may be written

$$\psi = \frac{dC}{2C} \quad (34)$$

Letting s denote readings on the required scale, the value of ψ is the difference of two s readings, or

$$\begin{aligned} \psi &= ds \\ ds &= \frac{dC}{2C} \end{aligned}$$

The readings of the scale are then given by

$$\begin{aligned} s &= \int_c^{C_a} \frac{dC}{2C} \\ s &= \frac{1}{2} (\log_e C_a - \log_e C) \end{aligned} \quad (35)$$

C_a is the arbitrary capacity chosen as the zero point of the scale. Thus the scale can begin anywhere. Such a scale gives ψ in radians.

A wavemeter in which the inductance is variable and the capacity fixed is also convertible into a phasemeter in similar manner. The instrument is operated in just the same way, and the equation, corresponding to (33), is

$$\psi = \frac{L_2 - L_1}{L_2 + L_1} \quad (36)$$

and the direct-reading phasemeter scale is given by

$$s = \frac{1}{2} (\log_e L_a - \log_e L) \quad (37)$$

where L_a is the arbitrary inductance chosen as the zero point of the scale.

A direct-reading decimeter is made in precisely the same

way as a phasemeter. The decrement is π times the phase difference in radians, hence the equations for a decrement scale on a wavemeter with a variable condenser or variable inductor are respectively

$$s = \frac{\pi}{2} (\log_e C_a - \log_e C) \quad (38)$$

$$s = \frac{\pi}{2} (\log_e L_a - \log_e L) \quad (39)$$

The phase difference or decrement measured by such an instrument, using undamped sine-wave emf., is the phase difference or decrement of the measuring circuit itself. Its application to measuring the decrement of a wave is explained in the last section below. When the instrument is used as the measuring circuit with undamped emf., the variable condenser or inductor must be one having zero effective resistance or in which the resistance for each setting and each wave length is accurately known, in order that the resistance or ψ or δ of other apparatus connected in the circuit may be obtained. When a variable condenser is used as the phasemeter it is thus convenient for measurements upon coils, and when a variable inductor is the phasemeter it is a convenient means for measurements upon the R or ψ or δ of any condenser connected to it.

The direct-reading phasemeter or decremeter may also be used in the source circuit, to vary the ω or λ supplied to the measuring circuit, as described in connection with equations (30) and (31) above. In this use it is not necessary to make correction for the resistance of the phasemeter or decremeter itself, whether it be variable condenser or variable inductor, as it has no effect upon the measuring circuit, except insofar as it may effect the value of ω , which would be a second-order effect. This use of the direct-reading phasemeter is now being exhaustively studied by Messrs. G. C. Southworth and J. L. Preston at the Bureau of Standards.

SIMPLE DIRECT-READING PHASEMETER OR DECREMETER

It is particularly easy to make a phasemeter or decremeter out of a condenser with semi-circular plates. Such condensers follow closely the linear law,

$$C = a\beta + C_0 \quad (40)$$

where β is the angle of rotation of the moving plates and a and C_0 are constants. It can be shown that the phase difference scale applicable to such a condenser is one in which the graduations vary as the logarithm of the angle of rotation. Furthermore,

the same scale applies to all condensers of this type. This scale has been calculated and is given in Figure 7 for values of phase difference in degrees.

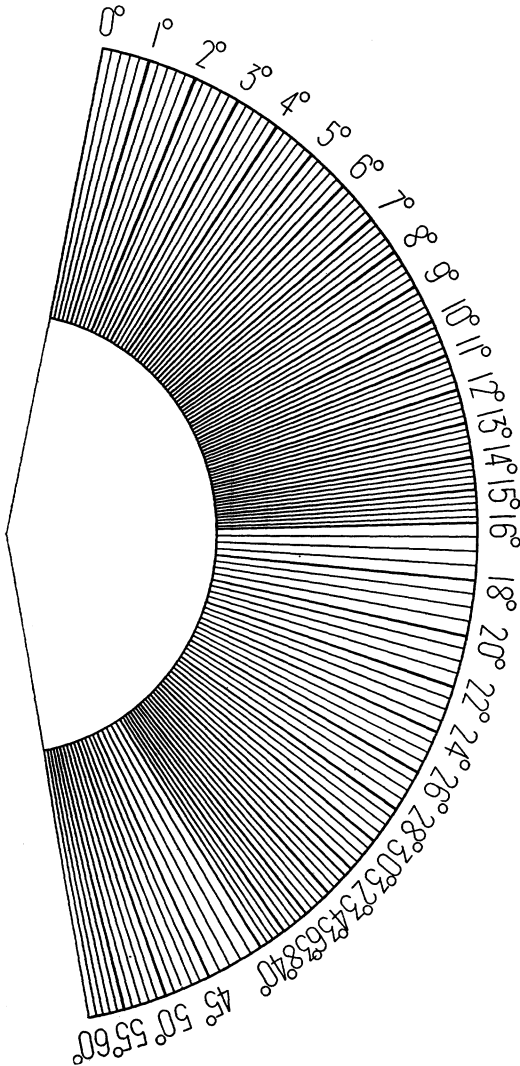


FIGURE 7

The scale was calculated in the following manner. Inserting equation (40) in (35)

$$s = \frac{1}{2} [\log_{\epsilon} (a \beta_a + C_o) - \log_{\epsilon} (a \beta + C_o)] \quad (41)$$

Let $C_o = a \beta_o$,

$$s = \frac{2.303}{2} [\log_{10}(\beta_a + \beta_o) - \log_{10}(\beta + \beta_o)] \quad (42)$$

For $C_o = \beta_o = 0$,

$$s = 1.151 (\log_{10} \beta_a - \log_{10} \beta) \quad (43)$$

$$\log_{10} \beta = \log_{10} \beta_a - \frac{s}{1.151}$$

For s expressed in degrees rather than radians, and for $\beta_a = 180^\circ$, the angular separation in degrees on the ψ scale is given by

$$\log_{10} \beta = \log_{10} 180 - \frac{s}{57.3 (1.151)} \quad (44)$$

This scale may be used as it stands on any variable condenser with semi-circular plates, regardless of the kind of capacity scale on the condenser or even if the condenser has no scale whatever on it. The phase difference scale may, if desired, be cut out and trimmed at such a radius as to fit the dial and then affixed to the condenser, with its zero point approximately in coincidence with the graduation which corresponds to maximum capacity. This usually puts it on the unused half of the dial opposite the capacity scale. If the figures are trimmed off they can be added over the lines in red ink. This scale will then give accurate results if the capacity varies linearly with the setting, a condition which holds closely enough in the ordinary condensers. This same scale may also be affixed to the dial of any variable inductor and used without error if the variation of inductance with setting is linear. Also on either condenser or inductor, the same scale is used either with moving pointer and stationary dial or with moving dial.

A measurement of phase difference is made by first observing the current-square at resonance, then reading the scale at a setting on each side of resonance for which the current-square is one-half its value at resonance. The difference between the two readings on the scale is the value of ψ in degrees. The value of power factor in per cent. may be obtained from the result if desired by multiplying by 1.75.

A similar scale is readily made to read decrements directly. The readings of Figure 7 are all divided by 18.24, or the scale is independently calculated by equation (38). The scale shown in Figure 8 is thus obtained, which may be used on any condenser with semi-circular plates. Since writing this paper, the author has been informed that a scale constructed on this principle was devised for use in a decimeter by Mr. Waterman

of the Marconi Company. This is the only instance known in which the method has been used. It has here been shown to be convenient for measurements of other quantities than decrement, and is worthy of wide application, since it converts any wavemeter into a direct-reading phasemeter or decimeter at no additional cost.

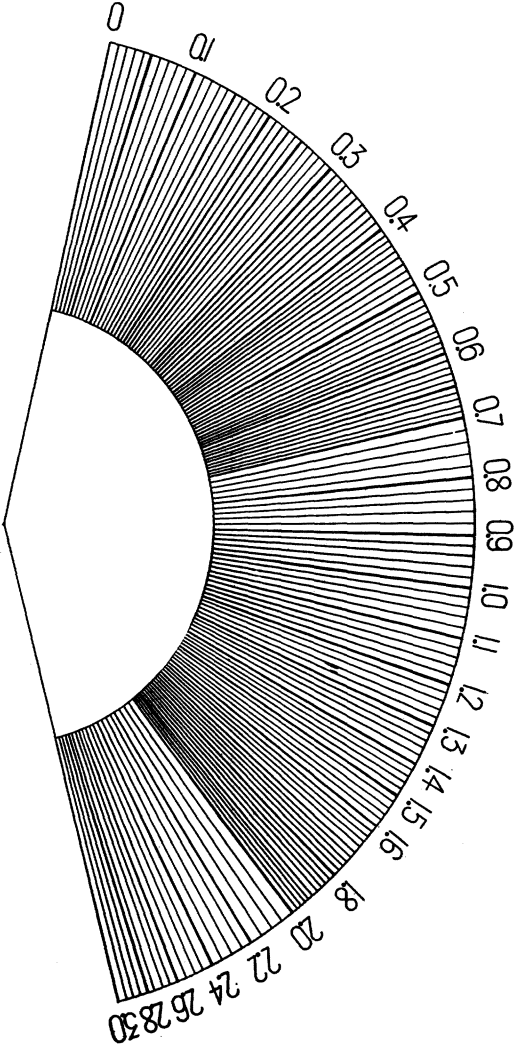


FIGURE 8

LOCATION OF SCALE FOR ACCURATE MEASUREMENTS

The scale gives accurate results only when $C_o = \beta_o = 0$. In many semi-circular plate condensers C_o or β_o has a small positive or negative value. This can always be reduced to zero by shifting the β scale. Thus for a particular position of the scale suppose

$$\begin{aligned} C &= a\beta' + C_o \\ &= a(\beta' + \beta_o) \end{aligned} \tag{45}$$

Define a new β such that

$$\beta = \beta' + \beta_o \tag{46}$$

This reduces (45) to $C = a\beta$, and it is accomplished by shifting the dial toward zero by the amount β_o . The value of β_o is determined by two measurements of capacity. Suppose C_1 and C_2 are the values for β_1' and β_2' . The constant a is the change of capacity per degree and is given by

$$a = \frac{C_2 - C_1}{\beta_2' - \beta_1'} \tag{47}$$

The angle β_o is therefore given by

$$\beta_o = \frac{C_1}{a} - \beta_1' \tag{48}$$

The scale of Figure 7 is accurately placed as follows. It is first placed on the dial by eye, and the capacity in the circuit accurately observed at the two points marked 5 and 36 on the scale. The amount by which the scale is to be shifted toward zero is then the angle in degrees,

$$\beta_o = \frac{100 C_{36}}{C_5 - C_{36}} - 51.2 \tag{49}$$

The capacity concerned is the total capacity in the circuit, which consists mainly of the capacity of condenser and of the inductance coil in parallel with it. Since the coils of a wavemeter do not all have the same capacity, it is desirable to mount the phase difference scale in such a way that its angular position can be varied a few degrees on the dial, to correspond to the different coils that are used. A fiducial mark can be placed on the scale for each coil.

MEASUREMENT OF SMALL PHASE DIFFERENCE OR DECREMENT

These scales permit accurate measurement of fairly large phase differences or decrements, but offer no precision in the

measurement of very small values, particularly at the low-capacity end of the scale. They are thus of particular value in tests upon condensers or other apparatus having fairly large phase differences. The method can, however, be extended to the precise measurement of small values in several ways. One method is to use a gear to open out the scale. The scale can then be in the form of a spiral on the rapid-motion gear shaft, and be spaced by a factor equal to the gear ratio. This device does not have the simplicity of merely attaching a scale to the condenser dial. Another method is to place a condenser of fixed capacity in parallel with the variable and use a different scale on the variable. This narrows the range of capacity variation, but for some kinds of work the method is very satisfactory, and any desired precision of measurement may be obtained. The scale suitable for the use in parallel with the variable of a fixed capacity equal to 10 times that at the middle of the scale of the variable, is obtained as follows. The fixed capacity is C_o in (40), and its value must be equal to 10 times that at the 90° point on the variable condenser (or 19.86 on the scale of Figure 7). Expressing β in degrees, $C_o = a \beta_o = a \times 900$. Equation (41) becomes

$$s = \frac{2.303}{2} [\log_{10} (\beta_a + 900) - \log_{10} (\beta + 900)]$$

For the ψ scale beginning at the upper end of the capacity scale and s expressed in minutes, this becomes

$$s = \frac{60 (57.3) (2.303)}{2} [\log_{10} 1080 - \log_{10} (\beta + 900)]$$

or,

$$\log_{10} (\beta + 900) = 3.033424 - \frac{s}{3957.9} \quad (50)$$

The scale thus calculated is given in Figure 9. It may be used on any linear scale condenser, as in the previous cases, and permits measurements of phase difference to closer than one minute. A similar scale is obtained for decrement by dividing the scale readings by 1094., permitting the measurement of decrement to better than 0.001. These scales have the additional advantage of almost uniform spacing.

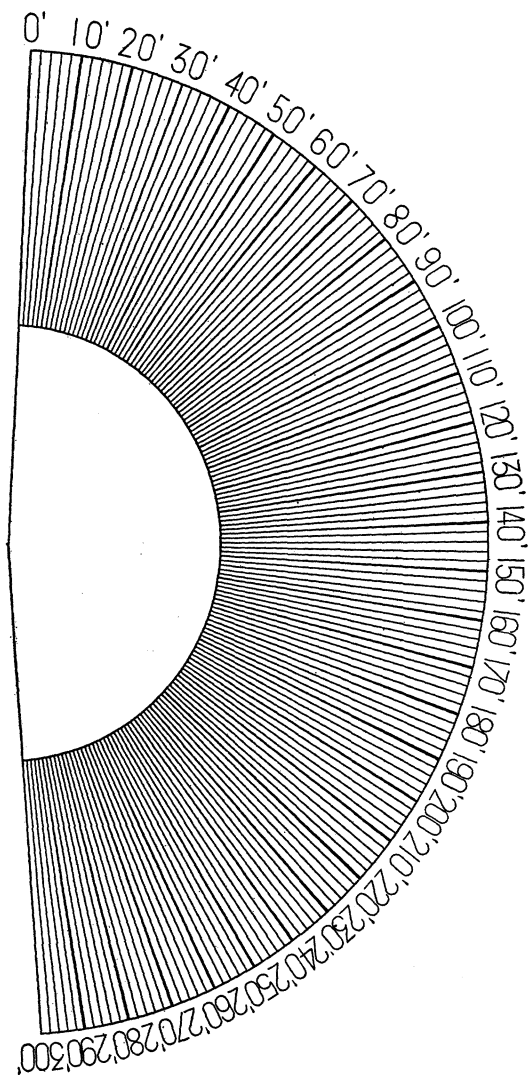


FIGURE 9

DECREMETER OR PHASEMETER WITH UNIFORM SCALE

Just as it is possible to determine a ψ or δ scale to fit a condenser having any sort of law of capacity variation, it is equally possible to design a condenser with capacity varying in such a way as to fit any specified ψ or δ scale. A uniform scale, i. e., one in which the graduations are equally spaced, is particularly convenient, and is the kind used in the Kolster decimeter.

A uniform scale of either ψ or δ requires in accordance with equation (34) that the condenser plates be so shaped that for any small variation of setting the ratio of the change in the capacity to the total capacity is constant. The condenser required to give this uniform scale has its moving plates so shaped that the logarithm of the capacity is proportional to the angle of rotation of the plates.

This decremeter is fully described in "Bulletin of the Bureau of Standards," 11, page 421, 1914, Scientific Paper Number 235.* By the use of a separate shaft geared to the moving plates at a 6-to-1 ratio, the decrement scale is opened out so that very precise measurements may be made. This decremeter is used in the inspection service of the Bureau of Navigation of the Department of Commerce and by radio engineers elsewhere. On account of the uniform scale of decrements its use is more convenient than the instruments with specially shaped scales, but, on the other hand, the adjustment of the instrument to read decrements accurately is more difficult as this requires the adjustment of a small auxiliary condenser in parallel with the variable condenser. It is, of course, a more costly instrument because of the specially shaped condenser plates. It can be made to read phase difference directly in degrees by replacing the decrement scale with another in which the readings are multiplied by 18.24.

USE OF DAMPED OSCILLATIONS

When damped oscillations are used in a measurement of resistance or one of the related quantities, there are two distinct decrements concerned, that of the circuit and that of the emf. supplied to the circuit. To determine either of these, in general two measurements are required. In special cases, however, one measurement only is necessary. For example, when the decrement of the supplied emf. is very small, the measurement of resistance of the circuit is made exactly the same as when the emf. is undamped and the equations are unchanged, in all the methods. Also, when impulse excitation is used for the resistance-variation method, the procedure is the same as with undamped emf.; the equations are different in this case, as shown in equation (16) above. In any of these cases, of course, the results of measurement can be expressed in terms of ψ , s , or δ of the circuit as well as R .

When the emf. supplied to the circuit has a moderate de-

*See also "PROC. INST. RADIO ENGRS.," volume 3, number 1, page 29, 1915.

crement, damped oscillations flow in the circuit. Calculation of the current is very difficult except when the decrement of both the emf. and the circuit are small. The definition of decrement that has been given in terms of an energy ratio furnishes some interesting relations in this connection. Suppose the emf. is produced in the measuring circuit (Figure 5) by coupling to the source circuit so loosely that there is no reaction upon the source. If I' = the root-mean-square current of small decrement in the source circuit and M is the mutual inductance between the two circuits, it may be shown that the r.m.s value of emf. induced in the measuring circuit at resonance is

$$E = \omega M I'$$

and the maximum amplitude is

$$E_o = \omega M I_o'$$

From equation (9), $(I')^2 = \frac{N}{4f\delta'} (I_o')^2$,

therefore

$$E^2 = \frac{N}{4f\delta'} E_o^2$$

where δ' is the decrement of the current in the source circuit and hence of the emf. induced in the measuring circuit. Using $\frac{E^2}{R}$ as a definition of power consumption, the average power dissipated is

$$\frac{E^2}{R} = \frac{N E_o^2}{4f\delta' R}$$

Average energy dissipated per cycle = $\frac{N E_o^2}{4f^2 \delta' R}$.

Average energy associated with current at maxima = LI^2 , provided the decrement is small, as before. Assuming now that decrements are additive, and applying the energy-ratio definition given just after equation (9) to the sum of δ' , the decrement of the applied emf. and δ , the decrement of the circuit,

$$\begin{aligned} \delta' + \delta &= \frac{1}{2} \cdot \frac{N E_o^2}{4f^2 \delta' R L I^2} \\ I^2 &= \frac{N E_o^2}{8f^2 R L \delta' (\delta' + \delta)} \\ I^2 &= \frac{N E_o^2}{16f^3 L^2 \delta' \delta (\delta' + \delta)} \end{aligned} \quad (51)$$

This is the correct relation between I^2 and E_o^2 at resonance,

as obtained from the elaborate rigorous proofs. The short demonstration just given involves the assumption that decrements are additive, which seems reasonable since energies are additive.

RESISTANCE-VARIATION METHOD

Measurements made with damped waves are most conveniently expressed in terms of decrements. Resistances and the other related quantities can then be calculated from the values of decrement.

The equation for the resistance variation method is obtained from (51). Suppose the resistance of the circuit to be increased by an amount R_1 changing δ to $\delta + \delta_1$, and the original resonance current I to some other value I_1 ; then

$$I_1^2 = \frac{NE_o^2}{16f^3L^2\delta'(\delta + \delta_1)(\delta' + \delta + \delta_1)}$$

$$\frac{I^2}{I_1^2} = \frac{(\delta + \delta_1)(\delta' + \delta + \delta_1)}{(\delta' + \delta)} \quad (52)$$

This is the equation for the resistance-variation method of measurement, using damped waves. It applies only when the decrements are small, and when the coupling to the source is so loose that the emf. is not affected by the current in the measuring circuit.

It is possible to solve either for δ' if δ is known or vice versa. When the method is thus used to obtain δ' , the decrement of the applied emf., the result of the measurement really gives the shape of the trains of waves which are acting on the circuit. Decrement measurement may thus accomplish something similar at radio frequencies to what is done at low frequencies by wave analysis.

DETERMINATION OF DECREMENT OF WAVE

The solution for δ' is

$$\delta' = \frac{2\delta\delta_1 + \delta_1^2 - \frac{I^2 - I_1^2}{I_1^2}\delta^2}{\frac{I^2 - I_1^2}{I_1^2}\delta - \delta_1} \quad (53)$$

This may be simplified by choosing the resistance inserted such that $\delta_1 = \delta$; then

$$\delta' = \delta \frac{4I_1^2 - I^2}{I^2 - 2I_1^2} \quad (54)$$

Another convenient simplified procedure is to vary the inserted resistance until the square of the current is reduced to one-half its previous value, then $\frac{I^2 - I_1^2}{I_1^2} = 1$, and

$$\delta' = \frac{2 \delta \delta_1 + \delta_1^2 - \delta^2}{\delta - \delta_1} \quad (55)$$

This equation expresses the method presented by L. Cohen in "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," 2, page 237, 1914.

DETERMINATION OF DECREMENT OF CIRCUIT

When δ' is the known quantity, the direct solution of (52) for δ , the decrement of the measuring circuit, is

$$\delta = \frac{1}{B} \delta_1 - \frac{1}{2} \delta' \pm \frac{1}{2B} \sqrt{(B \delta')^2 + 4 \delta_1^2 + 4B \delta_1^2} \quad (56)$$

where

$$B = \frac{I^2 - I_1^2}{I_1^2}$$

This complicated form of solution is of very little use. Equation (52) is itself a more convenient expression than this explicit solution. The following formula has been found useful in certain cases as discussed below.

$$\delta = \delta_1 \frac{K I_1^2}{I^2 - K I_1^2} \quad (57)$$

where

$$K = 1 + \frac{\delta_1}{\delta' + \delta} \quad (58)$$

It is sometimes advantageous to express this in terms of resistance or the related quantities. Thus the solution for R of the circuit, where R_1 is the inserted resistance, is

$$R = R_1 \frac{K I_1^2}{I^2 - K I_1^2} \quad (59)$$

This is, of course, not an explicit solution for R , since K involves δ and, therefore, R , but gives a ready means for finding R or δ when the sum of the two decrements ($\delta' + \delta$) is known from some other measurement, such as the reactance-variation method described below. Thus a combination of the two methods gives both δ' and δ , or δ' and R .

An interesting special case occurs when δ and δ_1 are both very small compared with δ' . K becomes unity and equation (59) reduces to

$$R = R_1 \frac{I_1^2}{I^2 - I_1^2} \quad (60)$$

This happens to be the same as equation (16) above, the equation for the use of impulse excitation. The proof given here can not, however, be regarded as a deduction of equation for impulse excitation, as it has been by some writers; since equation (51) is involved, which assumes that δ' and δ are both small.

REACTANCE-VARIATION METHOD

The procedure when the supplied emf. is damped is the same as when undamped, two observations of current being taken, one at resonance and the other after varying the reactance. The equations for decrement are only slightly different from those applying to undamped current.

Bjerknes' classical proof shows that the sum of the decrements of the emf. and of the measuring circuit is given by the same expression as that which gives the decrement of the measuring circuit when the emf. is undamped. Thus (28) becomes (61) below, and the equations for decrement corresponding to (21) to (25) become

$$\delta' + \delta = \pi \frac{\pm (C_r - C)}{C} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (61)$$

$$\delta' + \delta = \pi \frac{\pm (L - L_r)}{L_r} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (62)$$

$$\delta' + \delta = \pi \frac{\pm (\omega^2 - \omega_r^2)}{\omega \omega_r} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (63)$$

$$\delta' + \delta = \pi \frac{\pm (\lambda_r^2 - \lambda^2)}{\lambda_r \lambda} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (64)$$

$$\delta' + \delta = \pi \frac{C_2 - C_1}{C_2 + C_1} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \quad (65)$$

These formulas are correct only when: (1) the coupling between the source of emf. and measuring circuit is so loose that the latter does not appreciably affect the former; (2) δ' and δ are both small compared with 2π , and (3) the ratio $\frac{(C_r - C)}{C}$ and the corresponding ratios are small compared with unity. From any of these δ' is obtained if δ is known and vice versa. If a separate measurement is made by the method of equation (57) above, both decrements are obtained.

The appearance of the sum $(\delta + \delta')$ in the equations does not mean that the current flowing in the measuring circuit actually has a decrement equal to $(\delta' + \delta)$. As a matter of fact the actual decrement of the current is a value nearly equal to which-

ever of the two, δ' or δ , is the smaller. For this reason the equations involving $(\delta' + \delta)$ can not be extended to the measurement of the sum of the decrements of two loosely coupled circuits by coupling to one of them a third measuring circuit, as has sometimes been tried.

As mentioned earlier, the reactance-variation method is simplified if the reactance is varied by such an amount as to make $I_1^2 = \frac{1}{2} I_r^2$. This is done very easily when the current measuring instrument is graduated in terms of current squared. The quantity under the square root sign in all the preceding equations becomes unity, greatly simplifying the formulas. Calculation may be entirely eliminated by use of direct-reading decimeters as previously described. Such instruments when thus used with damped waves give directly $(\delta' + \delta)$.

SUMMARY: The methods of measuring resistance and related quantities at radio frequencies are fewer in number and necessarily different from those at low frequencies. The conditions of such measurements and the relations of various methods have not previously been given in comprehensive fashion. This paper shows the relations between resistance, phase difference, sharpness of resonance, and decrement. The methods of measurement are derived and classified. Most of the valuable methods are comprised under the resistance-variation and reactance-variation methods. Special direct-reading methods of measuring phase difference and decrement are presented.*

*Extra copies of the special scales of figures 7, 8, 9 can be obtained from The Institute of Radio Engineers by addressing the Editor, The College of the City of New York.

SYMBOLS USED IN THIS PAPER

- C = capacity of condenser in measuring circuit.
 C' = capacity of condenser in source circuit.
 d = deflection of current measuring instrument.
 d_1 = deflection when known resistance is inserted in circuit.
 E = effective electromotive force.
 E_o = maximum electromotive force.
 f = frequency of alternation.
 I = effective current.
 I_o = maximum current.
 I_r = current at resonance.
 I_1 = current when either resistance or reactance of circuit is increased.
 L = self-inductance of circuit.
 M = mutual inductance.
 N = number of trains of oscillations per second.
 P = average power.
 r = subscript used to denote resonance.
 R = resistance of circuit.
 R_1 = known resistance inserted in circuit.
 R_x = resistance of apparatus under measurement.
 s = scale setting.
 S = sharpness of resonance
 X = reactance.
 X_1 = change of reactance.
 W = average energy.
 α = damping factor.
 δ = logarithmic decrement of circuit.
 δ_1 = increase in decrement caused by adding resistance.
 δ' = decrement of applied emf.
 ϵ = base of napierian logarithms = 2.71828.
 θ = phase angle of C or of L , considering the resistance to be associated with it.
 λ = wave length.
 ψ = phase difference of C or L , considering the resistance to be associated with it.
 $\omega = 2\pi \times$ frequency.