An Intercomparison of Atomic Standards

In early September, 1965, a group of atomically controlled oscillators was assembled at the National Bureau of Standards in Boulder, Colo. The main purpose of the two months of experiments was to obtain intercomparisons of the frequencies of the cesium beam, thallium beam, and hydrogen maser with accuracies substantially better than any previously obtained [1]–[4].

The participants in the experiments were personnel from the Quantum Electronic Devices (Q.E.D.) Division of Varian Associates, the Hewlett-Packard Company, and the Atomic Frequency and Time Standards Section of the National Bureau of Standards, Boulder, Colo. The equipment assembled for the experiments included the United States Frequency Standard NBS-III, a cesium beam; another cesium beam device constructed at Hewlett-Packard Company (incorporating a Varian Associates beam tube); two hydrogen masers constructed at the Q.E.D. Division of Varian Associates; and the NBS-II beam machine recently converted to thallium use. Unfortunately the conversion of NBS-II to thallium was not complete, and no significant numbers are at present available for this system.

The majority of comparisons among the two masers and the two cesium beam devices was obtained by period measurements of the beat frequencies between pairs of 5 MHz signals synthesized from the various controlling atomic transitions. It was recognized that for the precisions and accuracies which are realizable with these devices, 5 MHz is an unfortunately low frequency for comparison (it is worth noting that a slow phase drift of one-half cycle per day at 5 MHz constitutes a frequency offset of about one part in 10^{12}!). Nonetheless sufficient hardware and software existed at the 5 MHz range to make this the most desirable frequency for the present intercomparisons.

There were available three separate data acquisition systems capable of automatically punching data on cards for computer analysis, and thus data reduction was greatly facilitated. Indeed, such volumes of data were obtained during the comparisons that, to date, only a small fraction of the data has received attention. Thus the results reported here are preliminary and subject to a great deal of additional analysis. It is intended that a more comprehensive report of the intercomparisons will be published soon.

In order to have good reliability for the final results, it is desirable to substantiate that the various devices were operating properly and within specifications. Thus it is of value now to discuss briefly some of the experiments performed to establish a realistic and unbiased error budget for each instrument.

In regard to the two Varian hydrogen masers, each is equipped with magnetic shielding and with temperature control of source, cavity, and cavity loading. The magnetic shields of each maser were degaussed at the start of the experiments, and the magnetic fields were calibrated by exciting the Zeeman transitions. Almost daily checks of the cavity tuning of each maser were made with each maser being tuned independently of the other (each maser was used in turn as a stable reference to tune the other but its particular frequency was not considered).

Unfortunately it was not possible in the time available to perform a wall-shift experiment for the hydrogen masers. It was decided to use the values of the wall shift as determined by the Varian group sometime earlier [1].

Preliminary results indicated the frequency fluctuations of one maser relative to the other was within a few parts in 10^{13} from one second to several hundred seconds, and the frequencies of the two masers agreed to within one part in 10^{12} for the entire two-month period. The detailed analysis of the relative fluctuations of the two masers has not been completed, however.

While a realistic error budget for NBS-III has recently been published [5], it was decided that the very stable signals available from the two Varian masers would afford a unique opportunity for redetermining the magnitudes of some of the uncertainties associated with NBS-III. In particular, the oven and detector of NBS-III were interchanged four different times to determine phase-shift effects of the cavity. Also a completely different set of electronics was used to detect possible spectral difficulties or systematic errors in the servo systems.

The fluctuations in the frequency of the 5 MHz signal locked to NBS-III as compared to a hydrogen maser decreased with increasing sample time as $\tau^{-1/2}$ for τ ranging from 100 seconds to five hours. This is in complete agreement with theory [6]. A standard deviation of one part in 10^{13} was obtained for adjacent sample times of two hours. For adequate averaging times ($\tau \geq 200$ seconds), the new total estimated inaccuracy for NBS-III is 1.1×10^{-12} for a *one*-sigma value Beehler, Mockler, and Richardson [5] give a one-sigma value of about 1.9×10^{-12} ($3 \cdot \sigma = 5.6 \times 10^{-12}$).

The comparison of NBS-III with the Hewlett-Packard (H-P) cesium beam indicated a standard deviation for the frequency fluctuations of seven parts in 10^{13} for two-hour samples. If one assumes that the figure of one part in 10^{13} quoted above for the comparison of NBS-III and a hydrogen maser is caused primarily by shot noise modulating the frequency of NBS-III, it is possible to estimate the fluctuations on a shorter machine with a different flux of atoms [6]. Indeed, when this calculation is carried out for the H-P cesium beam, complete agreement with the experimental results of 7×10^{-13} for two-hour samples is obtained. Apparently, the electronic systems of these two independently constructed cesium beam devices function quite comparably.

The H-P cesium beam, being independently aligned, was available for about two weeks of comparisons. Assuming NBS-III as the primary standard, the average frequency of the 5 MHz output of the H-P unit was offset by -149.991 parts in 10^{10}. Since the unit was designed to generate a signal offset by -150 parts in 10^{10}, this indicates a discrepancy in frequency of only 9 parts in 10^{13} which is well within the estimated accuracies for the two beams.

It is thus felt by the authors that the behavior of the cesium beams and the hydrogen masers during this period have proved adequtely reliable to quote a significant frequency value for the hydrogen maser. It should again be emphasized that all data have not been analyzed, and refinements on the results may be expected. A preliminary value for the frequency of the appropriate transition of the hydrogen atom in free space, at zero magnetic field, and zero absolute temperature, is

$$1420{,}405{,}751.7860 \pm 0.0046 \text{ Hz}.$$

The uncertainty of 0.0046 Hz corresponds to 1) an assumed inaccuracy of 3 parts in 10^{12} due to uncertainties in the wall-shift effect in the hydrogen maser, compounded with 2) the inaccuracy of 1.1 parts in 10^{12} for the cesium reference to give a total uncertainty of 3.2 parts in 10^{12}.

Unfortunately, all aspects of wall shifts in the hydrogen maser are not sufficiently well understood to allow one to construct a realistic and objective error budget for the hydrogen maser. While the reproducibility of the wall shift is known to be better than one part in 10^{12}, the magnitude of the effect is not known to this accuracy, and the estimated inaccuracy of three parts in 10^{12} for the wall shift is considered an "outer limit" by the authors. It is, therefore, not on the same objective footing as the one-sigma value of 1.1 parts in 10^{12} for NBS-III.

The following is a list of values for the hydrogen frequency in terms of cesium as published elsewhere and the value given above.

1420,405,751.827 ± 0.02 (Varian-Naval Observatory, 1963) [1]

1420,405,751.800 ± 0.028 (Harvard-Naval Observatory, 1963) [2]

1420,405,751.778 ± 0.016 (Varian-H-P, 1964) [3]

1420,405,751.785 ± 0.016 (Varian-LSRH, 1964) [4]

1420,405,751.781 ± 0.016 (NASA-GSFC, 1965) [7]

1420,405,751.7860 ± 0.0046 (NBS-Varian-HP, 1-65, this letter).

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The state-of-the-art for both cesium beams and hydrogen masers has undergone significant improvement. Absolute accuracy in the vicinity of a few parts in 10^{12} for frequency measurements is thus confirmed.

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Ideal Transformer Realizations with Negative Resistors

The purpose of this letter is to point out that any multiport ideal transformer can be realized at all frequencies with positive and negative resistors only; or, at a single frequency, with positive capacitors and positive inductors only. Thus, the range of possible terminal or port behavior of transformerless networks with positive and negative resistors and other elements is identical with that of the class of networks which contain ideal transformers in addition. For example, the necessary and sufficient conditions on an open-circuit impedance matrix in order that it correspond to an n-port of $\pm R$, C's, and ideal transformers are known [1]; with the present result it follows that these conditions are also appropriate to networks of $\pm R$, C's only. The realization of ideal transformers with positive and negative resistors is also of interest in connection with recent realizability conditions involving ideal transformers with time-varying turns-ratios [2], [3]; such elements can, in fact, be realized by resistive networks with time-varying resistors.

The equivalent network for a common ground two-port ideal transformer of Fig. 1 was, in effect, used by Norton in his work on impedance transforming filters [4], [5]. In the figure, the turns ratio is x and the admittance Y is arbitrary. In Norton's filter application, when $x>1$, the two negative admittances are absorbed by positive admittances in the filter in which the ideal transformer is imbedded by letting Y be the admittance of an inductor. On the other hand, if Y is a real number in Fig. 1, then one obtains a realization of a common ground ideal transformer with two positive and two negative resistors. Also, if $Y=1/j\omega L_0$, the network elements can be obtained at a single frequency using two positive inductors and two positive capacitors.

In order to realize a general multiport ideal transformer, it is necessary and sufficient to realize the terminal behavior of a four-terminal one with an arbitrary turns-ratio. General multiports can then be constructed by interconnecting many two ports, some of whose coils are put in parallel or in series [6]. An appropriate four-terminal network is shown in Fig. 2 where x is the turns-ratio and Y is an arbitrary admittance. This network has the isolation property of the two port in addition to the proper transforming ratio. If Y is a positive real number, the network represents a realization with three negative resistors and four positive resistors for any positive turns-ratio. Also, if $Y=1/j\omega L_0$, then the network elements can be obtained at a single frequency with four positive inductors and three positive capacitors.

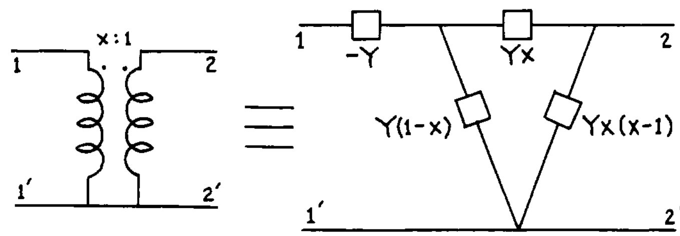

Fig. 1. A common ground two-port ideal transformer with turns-ratio x (Y arbitrary).

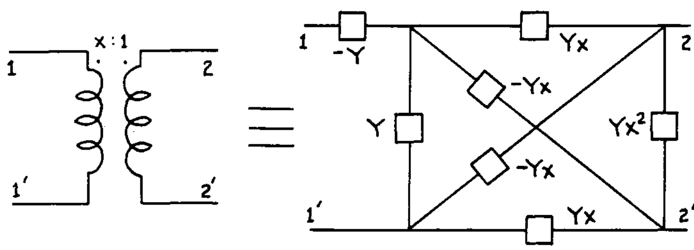

Fig. 2. A two-port ideal transformer including isolation property with turns-ratio x (Y arbitrary).

It can be shown that if positive and negative resistors are used, a common ground two-port ideal transformer can be realized with no fewer than two negative resistors for a turns-ratio different from $+1$. Also, a four-terminal ideal transformer realization which has the isolation property necessarily requires three negative resistors, even for unity turns-ratio.

The networks of Fig. 1 and Fig. 2 have been obtained by using a general synthesis method which will realize any prescribed behavior possible for networks of positive and negative resistors, capacitors, and inductors on any possible port structure. This synthesis technique along with a complete description of the possible port or terminal behavior for this class of networks will appear elsewhere. Using the general methods, one can realize a t-terminal ideal transformer with no more than $t-1$ negative resistors, where the counting is such that k coils in series account for $k+1$ terminals, not $2k$. Thus, fewer negative resistors are used than when multiports are constructed by interconnecting two-ports of three negative resistors each. Also, at a single frequency, a t-terminal ideal transformer can be realized with positive inductors (capacitors) and $t-1$ positive capacitors (inductors).

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