Trapped Atomic Ions and Quantum Information Processing

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Abstract. The basic requirements for quantum computing and quantum simulation (single- and multi-qubit gates, long memory times, etc.) have been demonstrated in separate experiments on trapped ions. Construction of a large-scale information processor will require synthesis of these elements and implementation of high-fidelity operations on a very large number of qubits. This is still well in the future. NIST and other groups are addressing part of the scaling issue by trying to fabricate multi-zone arrays of traps that would allow highly-parallel and scalable processing. In the near term, some simple quantum processing protocols are being used to aid in quantum metrology, such as in atomic clocks. As the number of qubits increases, Schrödinger's cat paradox and the measurement problem in quantum mechanics become more apparent; with luck, trapped ion systems might be able to shed light on these fundamental issues.

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INTRODUCTION

In 1995, Ignacio Cirac and Peter Zoller described how an ensemble of trapped ions might be used to implement quantum information processing (QIP) [1]. Several experimental groups throughout the world have pursued this basic idea, and although a useful device still does not exist, many ion-trappers are optimistic that one can eventually be built. At a modest level, the ion-trap scheme can satisfy the basic requirements for a quantum computer as outlined by DiVincenzo [2]: (1) a scalable system of well defined qubits, (2) a method to reliably initialize the quantum system, (3) long coherence times, (4) existence of gates for universal computation, and (5) an efficient measurement scheme. Most of these requirements have been demonstrated, and straightforward, albeit technically difficult, paths to solving the remaining problems exist. In this paper, we summarize primarily recent trapped-ion QIP experiments carried out at NIST, but similar work is currently being pursued at Aarhus, Barcelona, Garching (MPQ), Griffith University, Innsbruck,

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LANL, London (Imperial), Lucent, Ontario (McMaster), University of Michigan, MIT, NPL, Osaka University, Oxford, Sandia National Laboratory, Siegen, Simon Fraser University, Sussex, University of Ulm, and University of Washington. Most of these groups were represented at ICAP 06, and plenary talks were also given by C. Roos (Innsbruck) and R. Slusher (Lucent).

In this paper we discuss how the trapped-ion system might be scaled up by use of an array of interconnected trap zones and cite some experimental implementations of algorithms that utilize the basic elements of this scheme. We then summarize efforts devoted to construction of traps suitable for large-scale fabrication. We briefly discuss how QIP methods can be used in metrology, and finally suggest how QIP studies might eventually shed light on fundamental issues in quantum mechanics.

UNIVERSAL LOGIC GATES

Universal quantum computation can, in principle, be achieved by combining a series of single-qubit and two-qubit gates [3, 4]. Therefore, for experimentalists the primary goals are to provide high-fidelity one- and two-qubit gates and to implement a large number of them on a large number of ion qubits.

Single-qubit gates (spin rotations) have been implemented in a number of experiments using RF or laser fields. The two-qubit gates that have been implemented so far use the Coulomb interaction between ions stored in the same trap to provide the necessary coupling between ions. Here, a single normal mode of motion acts as a data bus. The original Cirac/Zoller two-qubit gate [1] uses this principle; it has been implemented in [5]. Several groups have also implemented single-step two-qubit gates that were proposed in Refs. [6, 7, 8]. These gates, which can be viewed as geometric phase gates in various bases, are implemented with state-dependent optical dipole forces. They have been demonstrated experimentally in Refs. [9, 10, 11] (x-y basis states) and in Refs. [12, 13] (z basis states). The highest fidelities reported (Bell states with fidelity $F \simeq 0.97$ were produced in [12]) are still considerably below those required for large-scale "fault-tolerant" computation ($F > 0.9999$). Achieving the necessary fidelities will require substantially increased control of technical parameters such as laser intensity and ambient magnetic fields. Currently, the state of the art for number in trapped-ion QIP experiments is represented by experiments that entangle up to eight $^{40}Ca^+$ ions in W-states [14] and six $^9Be^+$ ions in a Schrödinger-cat-type state [15].

SCALING UP WITH TRAP ARRAYS

It is generally thought that large-scale processing time will be dominated by error correction; this will require a large number of ancilla qubits and highly parallel processing. Therefore, we desire methods to multiplex logic operations on large numbers of ions.

So far, large numbers of ions can be cooled and manipulated only in single trapping zones. However, most of the practical multi-qubit gates, and all of those demonstrated so far, require addressing of individual ions and/or single motional modes. Individual ion addressing can be accomplished with focused laser beams as long as the ions aren't too close together (or equivalently, as long as the mode frequencies are not too high). This approach has been very successfully applied by the Innsbruck group. However, as the number of ions increases much beyond what is currently used, and increased gate speeds (proportional to mode frequencies) become more important, such addressing will be more difficult. In addition, mode addressing is usually accomplished by spectrally isolating the frequency of the one mode of interest. Unfortunately, when the number of trapped ions becomes large, the high spectral density of modes renders spectral isolation impractical.

Therefore, many groups are considering multi-zone trap arrays where only a small number of ions are confined in the zones that are used for implementing multi-qubit gates. Sharing of quantum information throughout the array might be accomplished by moving ion qubits between zones [16, 17], by moving an information-carrying "head" ion between zones [18], by coupling separated ions with photons as an intermediary [19], or by probabilistically creating, via light coupling, entangled pairs of separated ions, which then act as a computational resource to be used later [20].

As a first step towards multiplexing using the scheme of Refs. [16, 17], we have employed a six-zone linear array [21]. In this trap we were able to entangle ions in one zone, deliver these ions to separate zones and implement further entangling gates and/or detection. Experiments that used these features included demonstrations of quantum teleportation [22], quantum error correction [23], quantum-dense coding [24], the quantum Fourier transform [25], and entangled state purification [26]. Typical trap dimensions were such that the distance from the ions to the nearest electrode was around 150 μm, and separation times were around 200 μs (for minimal heating). In the future, traps with much smaller internal dimensions should enable shorter separation times with minimal heating [23, 27]. However, with all other parameters held constant, smaller dimensions are expected to aggravate ion heating from stochastic electrode noise [28, 29]. Since the mechanism for this heating is currently not understood, many of the ion trap groups are trying to suppress it by trying different electrode materials and fabrication methods.

For manipulating very large numbers of ions, two-dimensional layouts with simpler methods of construction will be required. Since (two-qubit) gate speed is proportional to the ions' motional frequencies, which are in turn proportional to the inverse square of the electrode dimensions, we desire traps with dimensions smaller than those of previous traps (keeping in mind that we must simultaneously solve the heating problem). For this purpose, it should be possible to take advantage of MEMS fabrication techniques, where significantly smaller structures with better controlled material properties can be fabricated.

At NIST we constructed a two-layer trap for $^{24}Mg^{+}$ ions of the type described in [30], where the electrodes were made of commercially available boron-doped silicon, bonded with an insulating thermal-expansion-matched glass [21]. The University of Michigan group has built a monolithic two-layer trap with GaAs electrodes and AlGaAs insulators [31] and observed trapping of Cd^{+} ions [32]. A three-layer geometry that uses gold plated alumina electrodes has been implemented for Cd^{+} ions [33]. Trap geometries that would optimize the separation of ions into separate zones have been studied in [34]. A two-dimensional "T" junction has been demonstrated in [35]; this trap has been used to exchange the positions of two ions.

A further simplification in fabrication can potentially be obtained with traps based on electrodes located in a surface [36]. These traps would be relatively easy to fabricate on a large scale and might permit on-board electronics beneath the electrode surface [37]. Surface electrode traps have been demonstrated for $^{24}Mg^{+}$ ions in a trap with gold coated fused quartz electrodes [38] and boron-doped silicon electrodes [39]. By use of copper surface electrodes, trapping of $^{88}Sr^{+}$ ions has also been demonstrated [40]. The ion heating rate observed in [38] appears to be small enough to perform simple entangling operations. The Disruptive Technology Office (DTO) has recently funded Lucent and Sandia laboratories to construct ion traps using fabrication techniques compatible with scaling up; R. Slusher reported on the Lucent work at this conference.

In addition to finding a way to construct large-scale trap arrays, a way to multiplex laser beams across many trapping zones must be sought. It might be possible to use miniature steerable mirrors based on MEMS technology for this purpose.[1,2] Miniature, large-solid-angle photon detectors (possibly without optics) located very near trapping zones may be essential for highly parallel detection as required in error correction.

QIP APPLIED TO METROLOGY

QIP might also be used to advantage in metrology, for example, to improve the signal-to-noise ratio in spectroscopy and atomic clocks. As a demonstration, the improvement obtained from "spin-squeezed" states [41, 42, 43], where the operator of the effective mean spin vector is measured, has been demonstrated for two ions [44]. Also demonstrated was the gain in signal-to-noise ratio with certain entangled states in combination with other operators such as the variance and parity [12, 44, 45]. The two-ion phase gate from [12] has been extended to implement a form of Ramsey spectroscopy where the two conventional Ramsey $\pi/2$ pulses are replaced with entangling $\pi/2$ pulses. The gain in signal-to-noise ratio was observed on up to six ions [15, 46]. Other entangled states have been used to achieve independence from Zeeman shifts of the clock transition and to facilitate measurement of the quadrupole shift in optical transitions [47]. This work was discussed by C. Roos at this conference.

QIP can also be used to facilitate detection. For example, transitions in a "clock" ion can be detected in a simultaneously-trapped "logic" ion by mapping the internal state of the clock ion onto the logic ion, where it is efficiently detected [48]. In a more general context, detection sensitivity can be improved by use of elementary entangling operations between the system to be measured and ancilla qubits that are also subsequently measured [24].

QIP AND THE MEASUREMENT PROBLEM

By the measurement problem, we mean the difficulty that arises because we live in a world that gives definite outcomes, whereas quantum mechanics predicts only coherent

[1] O. Blum Spahn, Sandia National Labs, private communication.
[2] J. Kim, Duke University, private communication.

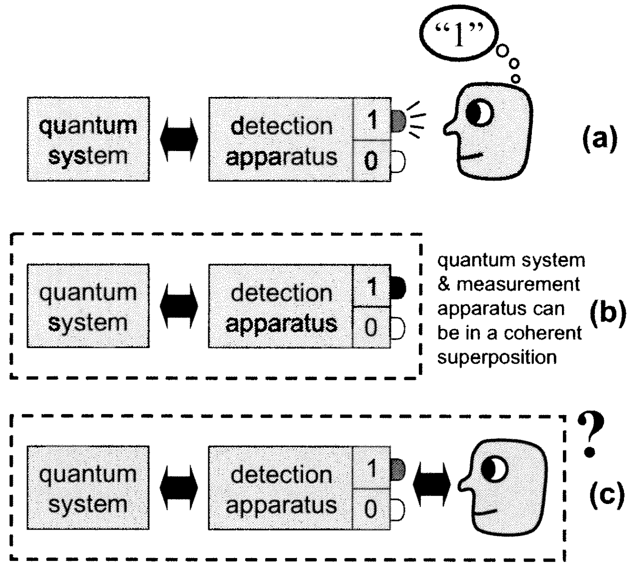

FIGURE 1. One expression of the measurement problem. A way to view an experiment that performs a measurement on a two-level quantum system is depicted in (a). Here, the quantum system is coupled to a detection apparatus or meter, which records information that is monitored by an observer. However, since the meter is really just another quantum system, according to quantum mechanics, the coupled quantum system and meter should coherently evolve as indicated in (b). A simple example of this is given in the text. In principle, the observer is just another (very large) quantum system. Does quantum mechanics predict only coherent evolution of these coupled systems, as might be inferred in (c)?

evolution of (entangled) superpositions. As a simple example, assume an experiment is designed to create a superposition state of a two-level quantum system with eigenstates labeled "0" and "1". We also assume that our quantum system is coupled to a detection apparatus or meter that we subsequently observe, as depicted in Fig. 1a.

We note however that the detection apparatus is also a quantum system (even though it may be a large one) so that when it is coupled to the first quantum system the two should, according to quantum mechanics, evolve to some coherent entangled state. A simple example of this situation was realized in trapped ion experiments [27, 49] where the quantum system under study was the superposition state $\psi_0 = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$, where $|\mathrm{n}\rangle$ denotes a Fock state of a mode of the ion's motion. The coherence of this state was observed by interferometry. The meter in this experiment was composed of two of the ion's internal states, denoted $|\downarrow\rangle$ and $|\uparrow\rangle$, which could be subsequently distinguished by laser-beam-induced state-dependent resonance fluorescence. A coupling was provided between the system and meter that caused coherent oscillations between the separable state $\psi_s = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)|\downarrow\rangle$ and an entangled superposition state $\psi_e = \frac{1}{\sqrt{2}}(|0\rangle|\downarrow\rangle + |1\rangle|\uparrow\rangle)$ in which the system and meter were correlated. If the intensity of the detection laser beam was reduced considerably or turned off, the system/meter combination coherently oscillated between ψ_s and ψ_e. In the separable state condition, the "which path" information in the interferometer experiment was lost and the fringes reappeared.[3] In the context of the discussion here, the system and meter were observed to coherently evolve as depicted in Fig. 1b. Of course, meters in most experiments are much larger, but if size plays a role, one must choose a division between small and large. There does not appear to be any obvious way to do this. To press the point, a human observer is also a (very large) quantum system. Therefore, we can ask: what is the proper description of the case illustrated in Fig. 1c?

Various fixes to the measurement problem have been proposed. In addition to the simple ad hoc collapse postulate, many attempts have been made to resolve it with ideas that include concepts such as "many worlds," decoherence theory, an as-of-yet unseen collapse mechanism, or simply that the theory of quantum mechanics is only a computational tool that allows prediction of classical outcomes - the "shut up and calculate" solution. (For a review of many of these ideas, see for example the paper by Leggett [53]).

However, given the uncomfortable (to many) state of affairs on the measurement problem, it seems interesting to press the issue experimentally; that is, can we realize larger and larger entangled superposition states that begin to approach our more macroscopic world where such states aren't observed? At this stage, since we really don't know what the important parameters are, we might cook up measures that play to the strengths of atomic physics and quantum optics. With atomic ions, we might emphasize the aspects of entanglement and duration. For example, we might take as a figure of merit the product of the number of particles N in a so-called "spin cat state" $\Psi = \frac{1}{\sqrt{2}}(|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N + |\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_N)$ times the duration of the state. A start in this direction is that a six-particle version of this state was observed to last longer than approximately 50 μs [15].[4] Note also that superpositions of the (phase-insensitive) Bell states $\Psi_\pm = \frac{1}{\sqrt{2}}(|\downarrow\rangle_1|\uparrow\rangle_2 \pm |\uparrow\rangle_1|\downarrow\rangle_2)$ have been observed to last for durations approximately 5 s in $^9Be^+$ ions [54] and 20 s for Ca^+ ions [55]. Whatever your favorite measure is, it seems likely that as the quest to make a large-scale QIP machine progresses, states that look more and more like Schrödinger's cat will be produced - or not, if some fundamental source of decoherence is discovered!

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[3] Similar behavior was also observed in the experiments of Refs. [50, 51, 52].

[4] In the future we might expect to see much longer coherence times using other basis states [54].

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