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A Comparison of Direct and Servo Methods for Utilizing Cesium Beam Resonators as Frequency Standards*

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***Summary*—Two systems, in which the frequency of a high quality quartz crystal oscillator can be controlled by a servo system employing as a reference frequency the $(F=4,\ m_F=0)\leftrightarrow(F=3,\ m_F=0)$ transition in the ground electronic state of cesium^{133}, have been in operation for about one year at the National Bureau of Standards. These systems are presently used in conjunction with the United States Frequency Standard, NBS II, and the alternate standard, NBS I, for measuring the frequencies of the United States Working Frequency Standards on a regular basis. The dependability, precision, and accuracy of the servo-derived measurements have been compared with the corresponding figures for the more direct manual method. Although both measurement systems have been found to be highly dependable, the servo method has significant advantages with respect to convenience of operation and measurement precision. These advantages can be utilized with no sacrifice of accuracy. Typical servo measurement precision is 2×10^{-12} for a 30-minute averaging time, while the measurement accuracy for both methods is 1.1×10^{-11}. For longer measurement periods of 12–14 hours, precisions and reproducibilities of 2×10^{-13} have been observed.**

INTRODUCTION

As A RESULT of committee discussions held during the 1957 meeting of the Consultative Committee for the Definition of the Determining the Second, there arose a suggestion that, in studying and developing atomic frequency standards, preference should be given to systems in which the resonance absorption frequency is measured directly, as compared with devices which have an auxilliary oscillator whose frequency is controlled by a servomechanism. At the Boulder Laboratories of the National Bureau of Standards it was initially felt that the simplicity of measuring systems and techniques in which no feedback-type locking circuits are used would merit their exclusive use. In practice, however, the laboratory has come to utilize servo-type circuits to a larger and larger extent in the atomic frequency standards program. The use of these control systems has been expedient to the frequency measurements program primarily by reducing the labor involved in making a frequency measurement so that much more comparison data with an attendant gain in precision can be accumulated in a given amount of time than is possible by means of the more direct measuring technique involving manual determinations of frequency. In addition, the use of servo systems in some cases has resulted in a more useful measure of frequency.

Two systems, in which the frequency of a high quality quartz crystal oscillator can be controlled by a servo system employing as a reference frequency the $(F=4,\ m_F=0)\leftrightarrow(F=3,\ m_F=0)$ transition in the ground electronic state of cesium^{133}, have been in operation for over one year at NBS. These systems are presently used in conjunction with the United States Frequency Standard, NBS II, and the alternate standard, NBS I, for measuring the frequencies of the United States Working Frequency Standards on a regular basis. This paper is primarily concerned with the reliability of these servo-type measurements as compared with the more direct manual type of measurement.

DESCRIPTION OF THE MANUAL AND SERVO MEASUREMENT SYSTEMS

Block diagrams of the manual and servo measuring systems are shown in Figs. 1 and 2, respectively. In the manual system the unknown frequency f_x that is to be measured is multiplied by 1836 to approximately 9180 Mc. A klystron which excites the atomic resonance is then automatically electronically tuned by a servo, so that its beat note with the multiplied unknown is in phase synchronism with a variable frequency source whose frequency f at approximately 12.6 Mc is known to 1×10^{-8}. By changing the frequency f and observing the index of transition probability provided by the atomic beam detector current and displayed on a dc meter, it is possible to tune the klystron first to a frequency f_a about 50 cps above the cesium resonance and then to a frequency f_b equally offset below the cesium resonance. If f_u and f_l are, respectively, the frequencies of the tunable 12.6-Mc source corresponding to f_a and f_b, the unknown frequency f_x can easily be computed from the relation

$$1836 f_x + \frac{f_l + f_u}{2} = 9192631770.0 + \Delta f_0 \qquad (1)$$

where Δf_0 is the known frequency shift in the atomic resonance produced by the uniform magnetic C field, essential to the operation of the beam machine, and f_l and f_u are determined by electronic counter measurements. This method of measurement is referred to as the manual method because the frequencies f_l and f_u

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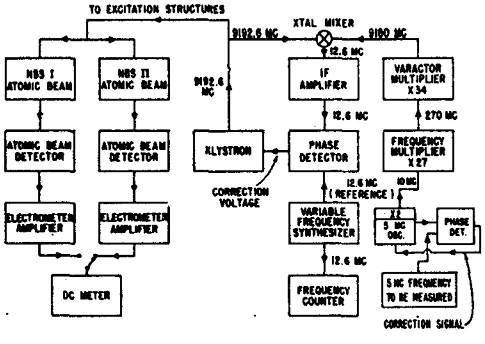

Fig. 1—Block diagram of the manual measurement system.

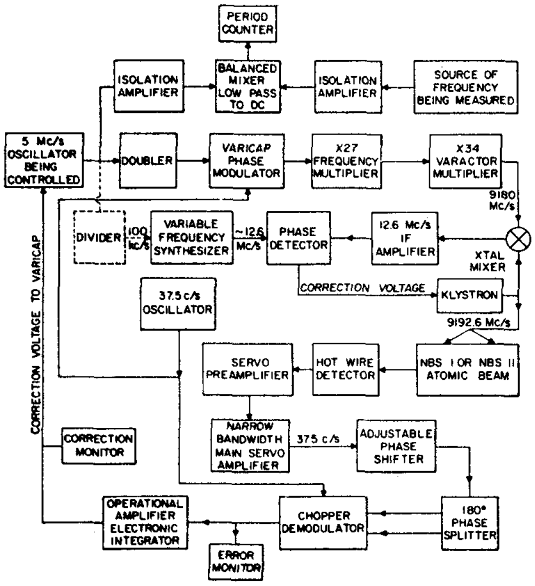

Fig. 2—Block diagram of the servo measurement system.

are alternately set by an operator so that the corresponding sequentially appearing beam currents are judged to be equal.

In the servo method of measurement shown in Fig. 2 the multiplier chain to 9180 Mc and the phase-lock klystron loop, including the source of the variable frequency f, are the same as used in the manual system. In this case, however, a high quality 5-Mc quartz-crystal oscillator, whose frequency f_s is to be controlled by the servo system, is used as the input to the multiplier chain. Phase modulation of a multiple of this frequency at a 37.5-cps rate is accomplished with the aid of a voltage-controlled diode capacitor in the output tank of the 5–10-Mc doubler. As a result of this modulation the beam current varies and the phase of the 37.5-cps component of the beam current depends on whether the average excitation frequency is above or below the cesium resonance frequency. In the postdetection servo electronics, a phase detector is used to develop a signed dc error signal which is integrated to produce a correction signal that is applied to a voltage-variable capacitor in the oscillator to control the frequency f_s at a particular value which depends on the value f of the variable source. With the servo loop closed, the frequency f_s at which the servo oscillator is locked can be computed from the relation

$$1836 f_s + f = 9192631770.0 + \Delta f_0. \tag{2}$$

Under normal operating conditions for the NBS II system, f is selected to be $\approx 12{,}632{,}000.0$ cps and Δf_0 is 1.0 cps, which results in a value for f_s that is about 249×10^{-10} low *relative to the nominal frequency of* 5.0 · · · Mc. It should be noted that when f is derived from f_s through the use of a 5-Mc-to-100-kc divider and synthesizer, its appearance in (2) can be replaced by $(12.632/5) f_s$, where the numerical factor is exact. A measurement of the unknown frequency f_x is effected by measuring, with the servo loop closed, the period of the beat note between f_x and f_s—using an electronic counter. If the measured period is τ, then

$$|f_x - f_s| = \frac{1}{\tau} \tag{3}$$

and f_x is determined since f_s is known from (2). The *appropriate sign for the difference frequency, if not* known, could be determined by shifting f_s in a known direction. The increased convenience of the servo measurement procedure over the manual is apparent, since the use of a digital recorder in conjunction with the period counter permits the accumulation of comparison data over long periods of time without the attention of operating personnel.

Both measurement systems have been used extensively at NBS and found to be highly dependable. The manual technique was used exclusively for a period of nearly two years and in conjunction with servo measurements for the past one year, while servo measurements have been emphasized more in the last year. Neither system has caused the loss of more than a few days of measurement time per year. Occasional problems with the manual system have been confined primarily to the electronic counter, klystron power supply, and components of the phase-lock klystron loop. These particular problems also affect the servo system operation which, in addition, has had minor troubles associated with faulty mechanical choppers and a malfunctioning power supply for the stabilized oscillator. Based on experience to date, then, no preference for one system or the other has been established with respect to dependability of operation.

COMPARISON WITH RESPECT TO MEASUREMENT PRECISION

The term "precision," when used in connection with the NBS standards, refers to the extent to which a measurement of frequency is reproducible. Used in this sense the measure of precision would include contributions from both the standard itself and whatever source of frequency is being measured. The most commonly used measure of precision for the NBS measurements is the standard deviation of the mean associated with the comparison data. For the manual method, the comparison data consists of a group of 20 to 50 determinations of the unknown frequency f_x, obtained by application of (1) to the raw data collected over a 30-minute period. Under normal measurement procedure, f_x is the 5-Mc output of one of the group of working standards maintained at NBS, consisting at present of 2 commercial cesium beam standards and 2 commercial rubidium vapor standards. A typical value of the measurement precision as defined above is about 8×10^{-12}. This precision figure can be reduced to 5×10^{-12} by collecting data over a $1\frac{1}{2}$-hour period. Representative groups of the manual data have been tested by using an appropriate computer program to determine the goodness of fit of a Gaussian distribution to the data. The primary significance of such a test stems from the central limit theorem of statistics. If the measurements did not exhibit a Gaussian distribution, it would be suspected that they were being influenced by at least one noncontrolled fluctuating variable that exerted a relatively large effect on the measurements. If the measurements did possess a Gaussian distribution, it would be suspected that, over the observation time, they vary because of the independent action of many variables which have about equal influence and that a significant improvement of precision could not be accomplished by controlling only a few of the variables that affect the outcome of a frequency measurement. A second reason for investigating the distribution bears on the usual interpretation of the standard deviation of the mean as providing limits within which lie 67 per cent of similarly determined means. This percentage is appropriate only for Gaussian distributions. The χ^2 test was used to give a single numerical measure of the over-all goodness of fit to a Gaussian distribution. In this test exact fit of the normal distribution curve to the experimental data would correspond to $\chi^2=0$, although such a value is extremely unlikely in practice because of statistical fluctuations. One group of 160 manual measurements was tested by dividing the data into 8 subgroups of 20 and computing χ^2 for each subgroup. The values of χ^2 ranged from 4 to 10 with an average of 6.6. One possible interpretation of this value is that because of statistical fluctuations there is a probability of about 60 per cent of finding a χ^2 value of 6.6 or higher even if the parent distribution of manual measurements is perfectly Gaussian. Therefore, one may conclude that a Gaussian distribution is a good fit to the observed distribution of the manual data.

The measure of precision for the servo-type measurements is taken to be the standard deviation of the mean of a set of frequency comparisons obtained from period measurements by using (2) and (3). When the frequency source to be measured is one of the NBS working standards, the precision is typically 2×10^{-12} for a 30-minute averaging period. This type of measurement is made on a daily basis at NBS and serves as the basis for published frequency measurements of the WWV, WWVL, WWVB, GBR, and NBA radio transmissions. Computer analysis of 519 measurements of 3 different working standards made over a period of several months gives an average χ^2 of 6.7—almost exactly the same as for the manual case and again indicative that a Gaussian distribution is a good fit to the data.

Comparisons have also been made between the two NBS frequency standards operating with independent servo systems. Data from two long comparison runs of 27 hours and 12 hours is shown in Figs. 3 and 4, respectively. Each point plotted on the uppermost curve in each figure is an average of 100 periods of the beat frequency between the two controlled oscillators. Since the period was 1.8 seconds, each point represents a 3-minute average followed by a counter display time of equal duration. The other plots in each figure indicate how the relative stability of the standards varies with the amount of time over which the data is averaged. The vertical bar at each point represents the precision of measurement. Analysis of the 27-hour run shows a precision of measurement for 30-minute averaging times of about 1.5×10^{-12}, compared to 2.5×10^{-12} for the servo comparisons with the working standards. If the 27-hour data is split into a 13-hour average and a 14-hour average, which are then compared with the 12-hour average obtained 5 days earlier, the three measurements of the frequency difference between the two standards differ by less than 2×10^{-13}. The value of χ^2 computed from the 267 values of the 27-hour comparison is 6.4, which once again is in close agreement with values obtained in other types of comparisons. From the uppermost plot in Fig. 3 it seems rather apparent that a significant amount of correlation is present among the 6-minute averages. In order to obtain some sort of quantitative measure of this effect the autocorrelation function was computed. The function drops to 49 per cent of the variance after 6 minutes but is still 20 per cent of the variance after 1 hour.

COMPARISON WITH RESPECT TO ACCURACY

The term "accuracy" refers to the degree to which the atomic frequency standard approaches the value f_0, the idealized resonance frequency for the cesium atom in its unperturbed state. This accuracy with respect to f_0 for the manual method of measurement is usually limited primarily by uncertainties associated with the uniform magnetic C field, phase differences between the two oscillating electromagnetic fields producing the atomic transition, the spectral purity of the excitation radiation, and effects of other neighboring atomic transitions.[1] An internal estimate of the accu-

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[1] R. C. Mockler, R. E. Beehler, and C. S. Snider, "Atomic beam frequency standards," IRE TRANS. ON INSTRUMENTATION, vol. 9, pp. 120–132; September, 1960.

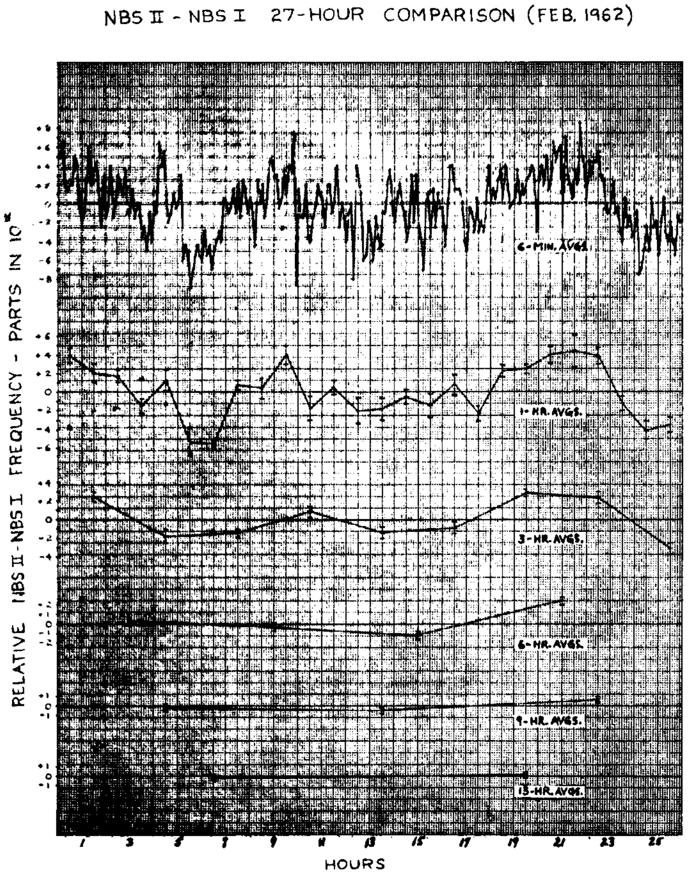

Fig. 3—27-hour frequency comparison of NBS I and II.

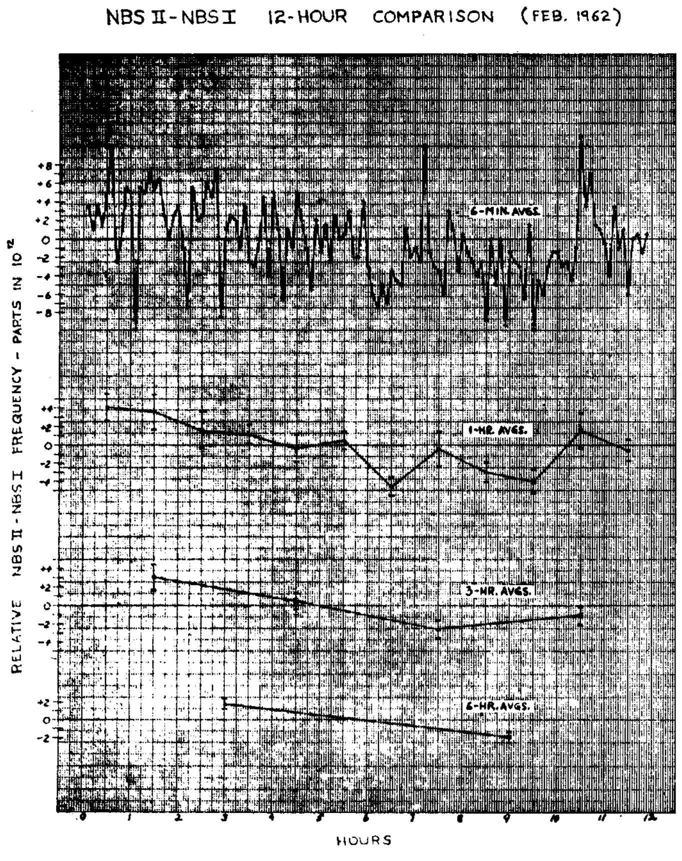

Fig. 4—12-hour frequency comparison of NBS I and II.

racy can be made by combining the estimates of possible frequency shifts due to the above causes.

For the case of NBS II the magnetic C field is produced by passing a current through a rectangular array of four parallel wires located within a triple-layer magnetic shield assembly. The field of about 0.047 oersted is calibrated by frequency measurements of the $(4, 1)\leftrightarrow(3, 1)$ and $(4, 1)\leftrightarrow(3, 0)$ microwave transitions of cesium which are strongly dependent upon the magnitude of the field. The field value for a given current is then determined from a least-squares fit of a straight line to the combined data. The uncertainty associated with this value is the computed standard deviation for a point interpolated from the least-squares line. The corresponding uncertainty in a frequency measurement is 5×10^{-12}, which serves as an estimate of possible inaccuracy due to incomplete knowledge of the C field. The measured nonuniformity of less than ± 0.001 oersted does not contribute significantly to the accuracy figure.

If a phase difference exists between the two separated oscillating fields exciting the resonance in the Ramsey technique, a frequency shift will result. This effect may be observed by rotating the resonant cavity 180° and looking for a resultant frequency shift. For NBS II the measured shift is 4×10^{-12}, which means that one half of this amount or 2×10^{-12} is the actual frequency error produced by the phase difference. Since this amount can be applied with the appropriate sign as a correction to all frequency measurements, the only contribution to inaccuracy from this source is considered to be the uncertainty in the measurement which is about $\pm 2\times10^{-12}$.

Another factor which has been observed to cause large frequency shifts under some conditions is the spectral purity of the radiation exciting the atomic transition. Shifts as large as 32×10^{-10} have been observed by exciting the resonance with a signal containing unsymmetrical sidebands in addition to a carrier frequency. This effect can be eliminated by proper design of the multipliers and, in the event the oscillator being measured is itself at fault, by utilizing an auxiliary oscillator phase-locked to the oscillator being measured with time constants chosen to make use of the long-term stability of the oscillator being measured and the short-term stability of the phase-locked oscillator. The auxiliary oscillator must not have troublesome sidebands and in addition, if the crystal current is high, noise effects will not be troublesome.

Another source of possible errors is the influence of neighboring transitions in the atomic spectrum. Significant frequency shifts may result in measurements made at low C fields, since the separation between transitions is proportional to the magnitude of the field. For measurements made with NBS II at a field of 0.020 oersted, for example, a systematic shift of 3.7×10^{-11} was detected. It has been found possible to eliminate this error by operating at sufficiently high fields (0.047 oersted for NBS II).

A final factor which has produced some error in the past in manual measurements of one particular commercial cesium beam standard involves the necessity for the operator to average by eye the observed beam variations. For this particular frequency source, oscillation in its servo loop produced a corresponding irregular oscillation in the beam current as observed on the dc meter. The manual measurement of this source showed a consistent error of 4×10^{-11} compared to a measurement not requiring the visual averaging of these oscillations. No measurable errors of this type have been found with more recent standards of this type or other frequency sources.

Other possible sources of error which have been investigated and found to make no measurable contribution to the inaccuracy of manual measurements include detuning of the resonant cavity, variations in the microwave power level, changes in the beam geometry and individual operator bias.

The figure of accuracy for a given manual measurement will depend upon the above estimates and the precision of the particular measurement. If we make the reasonable assumption that the above sources of error are independent and use 8×10^{-12} as the precision (typical of a 30-minute measurement), the over-all estimate of accuracy would be the square root of the sum of the squares of the separate estimates or about $\pm1\times10^{-11}$. If the measurement time extends over several days or if the measured source is unusually stable over the measurement period, it has been found possible to achieve a precision of 2×10^{-12} with a corresponding improvement in the accuracy estimate to 6×10^{-12}. An external estimate of the accuracy can be obtained by measuring the actual frequency difference between two similar atomic standards. If 6×10^{-12} is considered to be the accuracy for both NBS I and NBS II, the measured difference would not be expected to be much larger than the standard deviation associated with the difference frequency or 8.5×10^{-12}. The actual measured difference of $(1.6\pm0.4)\times10^{-11}$ or nearly two standard deviations could be expected with only about 5 per cent probability as a result of random sampling. In view of this apparent inconsistency between the internal and external estimates and because of the possible existence of sources of error not yet recognized the larger external estimate of $0.707\times1.6\times10^{-11}$ or 1.1×10^{-11} is considered a more appropriate accuracy figure with respect to f_0 for the manual measurement technique. Considerable confidence in this estimate has been gained during the past 3 years in view of the fact that the measured difference between NBS I and II of 1.6×10^{-11} has remained within a few parts in 10^{12} in spite of major changes in the magnetic shielding and method of producing the C field for both standards and replacement of the end sections of the resonant cavity in NBS I.

Possible sources of measurement error for the servo mode of operation include most of those already discussed for the manual case, plus effects associated with parameters of the servo system electronics. The only error sources characteristic of the manual system which are not applicable to the servo measurements are those of individual operator bias and effects of averaging beam variations by eye. The previously determined estimates of accuracy of 5×10^{-12} and 2×10^{-12} for C-field uncertainties and phase difference, respectively, should apply directly for the servo method also. The problem of the spectral purity of the radiation becomes much more complex for the servo case, since knowledge of the spectrum is generally not sufficient for predicting the existence or magnitude of any frequency shifts. Measurements to check for possible dependence of the frequency on cavity detuning and microwave power level were performed using the servo as well as the manual technique in order to take advantage of the higher precision capability of the servo system. Detuning of the resonant cavity by approximately 1 Mc produced a shift of 1×10^{-12}—well within the measurement precision. Variation of the microwave power level from 0.5 to 2.0 mw produced no significant frequency shift. This data is plotted in Fig. 5. The vertical bars at each point in this graph, as well as in the succeeding ones, represent the measurement precision in the sense defined previously.

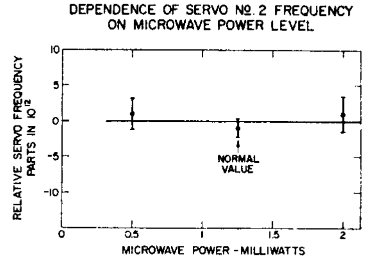

Fig. 5—Dependence of Servo No. 2 frequency on microwave power level.

Over a period of several months detailed investigations were made to determine to what extent 9 parameters of the servo system electronics might affect the frequency at which the controlled oscillator is locked. In each experiment the parameter under investigation was varied over as wide a range as possible while observing changes in the controlled oscillator frequency f_s relative to one of the secondary standard sources. Since the reliability of each experiment was rather strongly dependent upon the extent to which the secondary standard frequency, used as a reference, remained stable over the period of the measurements (usually several hours), great care was taken to select the particular secondary standard whose frequency was the most stable at the particular time as determined in most cases from auxilliary measurements. Both of the rubidium vapor standards and a commercial cesium beam standard were used at various times and, as a result of this careful selection procedure, the reference frequency could usually be depended on to remain within a few parts in 10^{12} over the necessary time interval. In most cases at least one data point was rechecked to guard against erroneous conclusions being formed due to drift of the reference frequency. Results of these experiments are plotted in Figs. 6–17. In each curve the parameter being studied is plotted along the *X* axis and the relative frequency of the controlled oscillator in parts in 10^{12} is plotted along the *Y* axis. The straight lines drawn in are not least-squares fits to the data, but rather attempts to visually fit lines of zero slope to the data that are reasonably consistent with the measurement precisions.

Fig. 6 shows the dependence of the servo frequency on the amount of frequency correction applied to the servo oscillator when the loop is closed. The computed cor-

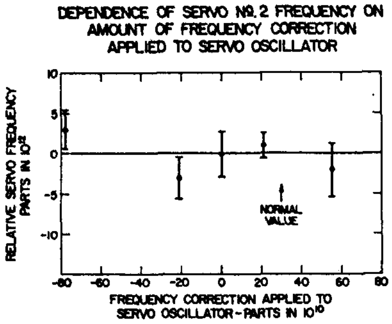

Fig. 6—Dependence of Servo No. 2 frequency on amount of frequency correction applied to servo oscillator.

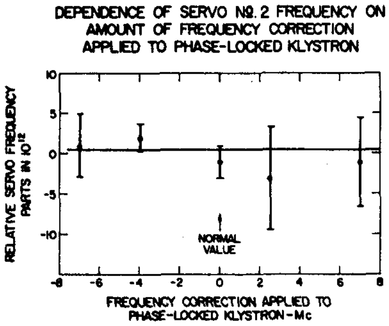

Fig. 7—Dependence of Servo No. 2 frequency on amount of frequency correction applied to phase-locked klystron.

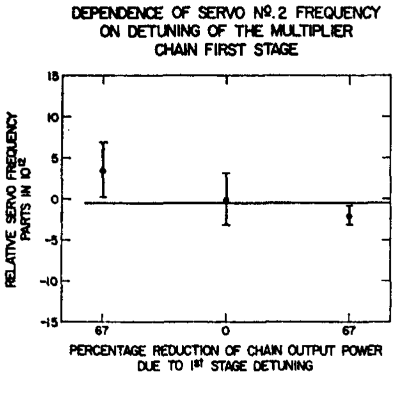

Fig. 8—Dependence of Servo No. 2 frequency on detuning of the multiplier chain first stage.

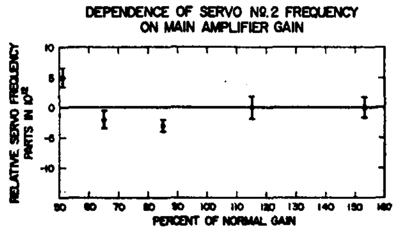

Fig. 9—Dependence of Servo No. 2 frequency on main amplifier chain.

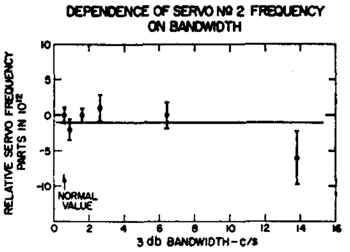

Fig. 10—Dependence of Servo No. 2 frequency on bandwidth.

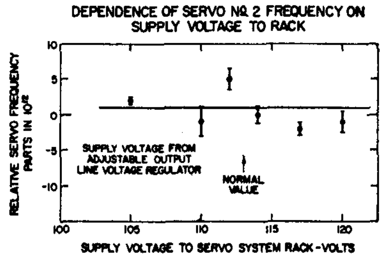

Fig. 11—Dependence of Servo No. 2 frequency on supply voltage to rack.

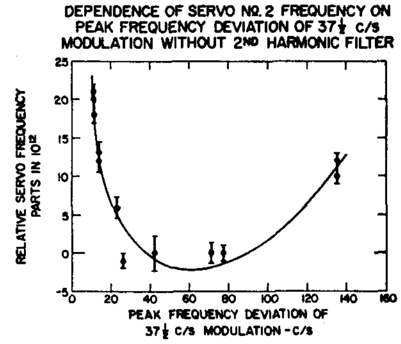

Fig. 12—Dependence of Servo No. 2 frequency on **peak frequency deviation of 37½ cps modulation without 2nd-harmonic filter.**

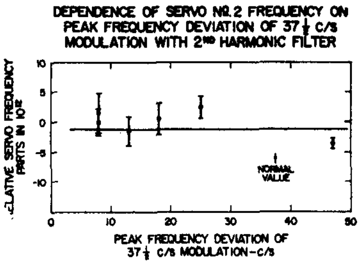

Fig. 13—Dependence of Servo No. 2 frequency on **peak frequency deviation of 37½ cps modulation with 2nd-harmonic filter.**

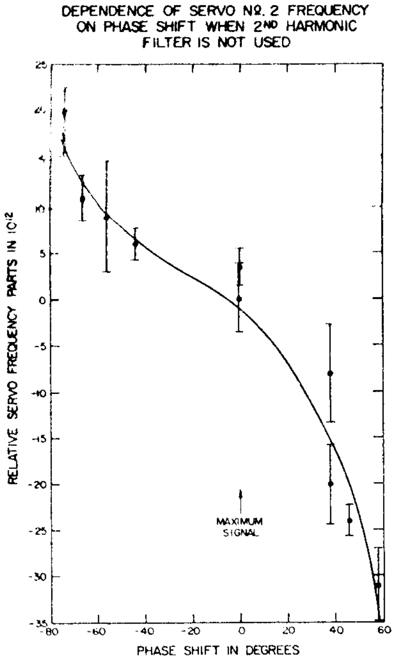

Fig. 14—Dependence of Servo No. 2 frequency on phase shift when 2nd-harmonic filter is not used.

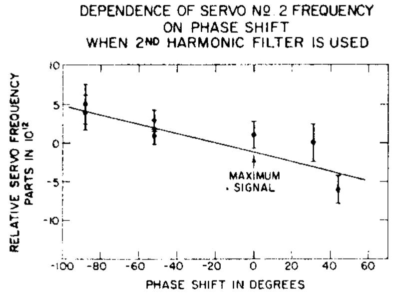

Fig. 15—Dependence of Servo No. 2 frequency on phase shift when 2nd-harmonic filter is used.

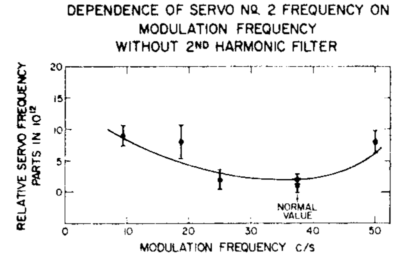

Fig. 16—Dependence of Servo No. 2 frequency on modulation frequency when 2nd-harmonic filter is not used.

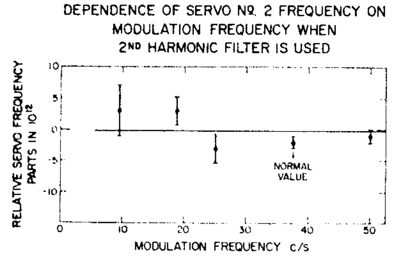

Fig. 17—Dependence of Servo No. 2 frequency on modulation frequency when 2nd-harmonic filter is used.

relation coefficient r for this data is 0.18. Since a value of r this large or larger would be expected in 20 per cent of similar measurements as a result of random sampling from an uncorrelated population, r is not significantly different from zero.

Fig. 7 shows the result of a similar-type experiment involving in this case the amount of frequency correction applied to the phase-locked klystron when its loop is closed. Although the measurement precisions were lower than normal for this experiment, a conclusion of no observable dependence on this parameter seems reasonable.

Fig. 8 presents the results of detuning the input stage of the (10–270)-Mc multiplier chain by equal amounts on each side of the maximum-signal position. Since the input stage is coupled rather directly to the (5–10)-Mc doubler output tank circuit where the phase modulation of f_s takes place, it was felt that, except for the 10-Mc doubler output tank circuit itself, frequency errors due to mistuning would be more severe in this stage than in any other part of the chain. The correlation coefficient is 0.32, a value that would be exceeded in only 15 per cent of such cases. However, when the data was divided rather randomly into 3 groups and r was computed separately for each group, all 3 values of r were not of the same sign. This result, together with the knowledge that the computed r values tend to be too large due both to sampling effects and instability of the reference frequency, lend support to a conclusion of no significant correlation in this case.

The parameter of interest in Fig. 9 is the gain of the main servo system amplifier, which is sharply tuned to the modulation frequency of 37.5 cps. The correlation coefficient of 0.29 for this data again is not unreasonable as a result of random sampling, especially in view of the result that division of the data into 3 subgroups produces 2 very small values of r and 1 larger value. The sign of all three values of r is the same, however, so that this may be considered a borderline case with respect to whether significant correlation exists.

Fig. 10 involves the dependence of the controlled oscillator frequency on the bandwidth of the main amplifier. Considering all the data plotted, the correlation coefficient is 0.40. A significant correlation would then seem to be indicated, since only 4 per cent of such experiments would produce r values this high as a result of the random sampling. However, there is some basis for disregarding the point for a bandwidth of 13.8 cps, because in this case a significant amount of 60-cps signal, which may be detrimental to the system in several respects, is allowed to be amplified. If r is redetermined neglecting this point, a much smaller value of 0.08 is obtained, which is much more consistent with a hypothesis of zero correlation.

The next parameter to be considered is the nominal 115-volt supply voltage to the rack containing the low-frequency portion of the servo system. Included in this rack are the 5-Mc oscillator, (5–10)-Mc doubler with modulator, main servo amplifier, chopper-demodulator and operational amplifier. Results obtained by using an adjustable-output line voltage regulator to vary the supply voltage from 105 to 120 volts are shown in Fig. 11. This again seems to be somewhat of a borderline case, since the *r* value of 0.26 could be expected only 6 per cent of the time, due to random sampling from an uncorrelated population. Efforts to obtain more complete data of this type have been unsuccessful thus far because of higher-than-normal instabilities in the secondary standards.

Fig. 12 shows the significant dependence of the servo frequency on the peak frequency deviation of the 37.5-cps modulation for the case when components at the second harmonic frequency of 75 cps are present in the modulating signal at too high a level. The situation, with respect to second-harmonic shifts, is complicated by the fact that the amount of frequency error produced will in general depend on the phase as well as the amplitude of the 75-cps component. Although second-harmonic components may arise from several different sources in the system, the most likely origin is in distortion of the 37.5-cps modulating signal. The servo circuits do not permit the fundamental peak frequency deviation to be changed without simultaneously affecting the second harmonic frequency deviation; consequently, according to theory, the resultant frequency shift would be expected to depend on the setting of the peak frequency deviation control. Assuming this mechanism to be of major importance, an attenuating filter network tuned to the second harmonic frequency was inserted in the system between the oscillator that modulates the varicap and the phase modulator. The experiment was then rerun with the results shown in Fig. 13. The correlation coefficient for this data is only 0.15 so that the correlation with the second-harmonic filter is no longer significantly different from zero.

Figs. 14 and 15 show a similar effect with respect to dependence of the frequency on the relative phase of the 37.5-cps component from the beam detector. Again in this case the frequency dependence is significantly reduced by the second-harmonic filter, although certainly not eliminated completely. The *r* value of 0.50 for the measurements with the filter would result from chance in less than 0.2 per cent of similar cases. With reasonable assumption about the phase of the second harmonic, calculation shows that a second-harmonic distortion of 0.2 per cent would be sufficient to produce the dependence observed. The normal measurement procedure with the servo system in which the phase shifter is adjusted for maximum observed signal insures that any frequency error produced will not be very large, although it will also almost certainly not equal zero.

If second-harmonic distortion of the modulating audio signal is present due to distortion in the oscillator, then one might also expect resultant frequency shifts to depend on the modulation frequency. This effect with the filter out and in is shown in Figs. 16 and 17, respectively. The effect of the filter is not as pronounced in this case as for the other parameters. The *r* value of 0.19 for the data with filter has a reasonable expectation (26 per cent) of being exceeded as a result of random sampling from an uncorrelated population.

The determination of an over-all internal estimate of accuracy for the servo system based on the data discussed is made difficult by the uncertainties associated with the interpretation of the correlation results. From a general consideration of Figs. 6–17, however, it would seem unlikely that errors of more than a few parts in 10^{12} should arise due to the servo system parameters. Assuming a measurement precision of 1×10^{-12} (typical of 1-hour averages) and combining the various estimates of error as we did for the manual case, the internal estimate of accuracy for servo measurements is about 6×10^{-12}. It should be noted that for measurement times of 1 hour or longer the precision of measurement makes an insignificant contribution to the accuracy estimate.

Two methods are available with the NBS systems for obtaining an external estimate of any inaccuracies due to the servo system parameters alone. First, careful comparisons have been made of the measured frequencies of a stable source as determined by both the manual and servo methods. The measured discrepancy is $(0 \pm 3) \times 10^{-12}$. Second, the difference between two independent servo systems of similar design has been measured (using second-harmonic filters in both systems) and found to be $(4 \pm 3) \times 10^{-12}$. In both of these methods errors due to C-field uncertainties and phase difference in the resonant cavity do not enter in since they will be the same for all measurements. These latter comparisons then appear to substantiate the earlier conclusion that any additional contributions to inaccuracy above those present in the manual technique are less than a few parts in 10^{12}.

Since the accuracy figure for the manual case has been conservatively estimated to be about 1.1×10^{-11}, however, the small additional contribution from the servo parameters is relatively insignificant and for all practical purposes the two methods of measurement may be assumed to be equally accurate.

CONCLUSION

Based on several years experience, both the manual and servo measurement systems have proved highly dependable. Careful analysis of both systems has shown that the very significant advantages of the servo-type measurements in terms of convenience of operation and precision of measurement can be utilized with no significant sacrifice of accuracy. In view of the demonstrated frequency shifts that may occur under certain operating conditions of the servo system, however, it is recommended that any such system to be used with a primary frequency standard should be thoroughly evaluated before being put to routine use, and that periodic comparisons with manual measurements should continue after the system is in regular operation.

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