MASER FREQUENCY STABILITY

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The frequency stability of an ammonia maser is ordinarily quite high for periods of a few minutes. However, to preserve this stability for long periods of time - hours and days - requires rather careful control of the parameters of the system which affect the frequency - in particular the pressure, focuser voltage and cavity temperature. We have had some success with these problems.

Method and Results of Long Term Stability Tests

In order to facilitate frequency comparison and also to gain the greatest possible use of the high stability of the maser, it has proved useful to use the maser as a stabilizing device for a frequency multiplier chain. We then expect that at any point along the chain we may take signals at useful power levels, and these signals will have stabilities closely approaching that of the maser itself.

A block diagram of the maser stabilized chain is shown in figure 1 together with auxiliary apparatus for the purpose of comparing a 5 Mc helium cooled crystal oscillator with the stabilized chain. The servo time constant is about 0. 1 millisecond. Figure 2 shows a comparison of a helium cooled oscillator with the maser stabilized chain; the maximum drift in relative frequency over a two hour period was about 2×10^{-11}. In these experiments the maser cavity temperature was controlled to within .001°C with two proportionally controlled thermostats - one inside the other.

Figure 3 shows some of the best frequency comparisons that we have succeeded in making between two masers. The cavity temperatures were controlled with an ice-water mixture in this recording. The high stability indicated was only observable for periods of several minutes.

A recent comparison between the two masers - both provided with cavity temperature control within .001°C - is shown in figure 4. The fluctuations displayed in this recording are about a factor of 10 worse than those of figure 3. In this preliminary comparison the frequency fluctuations are attributable to corresponding fluctuations in the residual gas pressure. This condition did not exist during the time of the recording of figure 3, and can be remedied. In spite of these fluctuations in the pressure, the frequency variations between the two masers were held within 1×10^{-11} (on the average) over a 6 hour period.

The variation of the maser frequency with the indicated parameters for our machines are shown below. These variations were measured with the maser adjusted for optimum signal:

1. Focuser voltage variations.

With the focuser voltage maintained at 30 kv, a 6v variation in voltage produces a frequency variation in the maser of 1×10^{-12}. The high voltage power supply is stable to about 2 volts after a one hour warm up period. During the first hour the voltage changes about 12v.

2. Cavity temperature variations.

A variation of $1^{\circ}C$ in the cavity temperature changes the frequency by 1×10^{-8}. The cavity thermostats have a measured temperature control of at least $\pm .001^{\circ}C$. Frequency fluctuations resulting from temperature variations should not exceed 1×10^{-11}.

3. Source pressure variations.

For optimum signal our machines have a source pressure of about 0.2 mm Hg. A variation of 1.5% in this pressure produces a change in frequency of 5×10^{-10}. Our devices for measuring the source pressure are not sufficiently sensitive to detect any source pressure changes under normal operation.

4. Residual gas pressure variations.

Variations of 25% in the residual gas pressure (at a pressure of about 4×10^{-6} mm Hg) produce a frequency variation of about 4×10^{-10}. It is this kind of variation that is apparently causing most of the fluctuations in figure 3. Variations in the pressure of about 1% are observable. Any changes in pressure below the observable level contribute fluctuations in frequency no greater than 1×10^{-11}.

These observations of maser stability indicate to us that the maser is quite capable of maintaining a high degree of stability over long periods of time. Our apparatus is suited to observing stability over periods of a few hours but is not convenient for making observations over periods of days, since the small cold traps of our machines require regular tending. A maser designed without traps or with more suitably designed traps together with a "tight" vacuum system would, in our opinion - solve the remaining problems and permit the masers to operate for weeks within narrow frequency limits.

An Application of the Maser's High Short Term Stability

The maser has been used to measure the stability of a crystal oscillator for time intervals as short as 30 milliseconds. The method of comparison is shown in figure 6. A sample of a recording is shown in figure 5. The analysis of the recordings was made by measuring the separation in time of the successive points where the curve passes through zero. Let $\Delta\tau_i$ be the ith such time interval, then $\frac{1}{2\Delta\tau_i} = f_i$ is the mean difference frequency (for the time interval $\Delta\tau_i$) between the 30 Mc reference signal f ref., and the 30 Mc maser-chain difference signal (f chain - f maser). Table I shows the RMS variation in the mean frequency of contiguous time intervals.

The same data can be used to estimate the effect of excitation instability on the resonance signal of an atomic beam machine. We can describe qualitatively how the instability of the excitation (multiplier chain) effects the detected signal in the following way: Consider an atomic beam machine employing Ramsey type excitation* and suppose that the frequency of the excitation changes slightly, due to some sort of instability, during the time that it takes an atom to pass from the first oscillating field to the second. The atom sees a relative phase shift between the two fields as a result of this variation in frequency. Atoms that see no relative phase difference - that is, atoms that see a phase in step with their own precessional motion in both oscillating field regions - have a maximum probability of transition. Atoms that see a phase difference of $\pm\pi$ radians between the two fields have a minimum probability of transition, and in general, a distribution in phase differences will tend to smear out the Ramsey pattern. Larger phase differences can be expected for longer transit times, and under conditions of very long transit times, or very poor frequency stability (or both), the Ramsey pattern will be smeared out so badly that only the broad Rabi line shape will remain.

In order to make a quantitative estimate of the effect of excitation instability on the resonance curve it is necessary to average the transition probability over the appropriate phase distribution (in addition to the usual average over the velocity distribution). If P_{ij} is the probability of transition between states i and j written as a function of the relative phase difference δ, we wish its average over the distribution in phase $\rho(\delta)$. More specifically we require

$$\overline{(P_{ij})}_\delta = \int_{-\infty}^{+\infty} \rho(\delta)\, P_{ij}\, d\delta$$

where δ is the phase difference that - to the atom - appears to exist between the two oscillating fields. As a reasonable choice of the phase distribution function, we assume it to be Gaussian. That is, we let

$$\rho(\delta) = \frac{1}{\sqrt{(2\pi\sigma^2)}}\, e^{\frac{-(\delta - \overline{\delta})^2}{2\sigma^2}}$$

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* N. F. Ramsey, Molecular Beam (Oxford University Press, London, 1956).

P. Kusch and V. W. Hughes, "Atomic and Molecular Beam Spectroscopy," Encyclopedia of Physics, Vol. 37, Springer-Verlag, Berlin 1959.

where $\sigma^2 = \overline{(\delta - \bar{\delta})^2}$ and $\bar{\delta} = 0$ for our particular problem. It is also necessary to average $\overline{(P_{ij})_\delta}$ over the velocity distribution in the beam. We must calculate

$$[\overline{(P_{ij})_\delta}] = 2 \int_0^\infty y^3 e^{-y^2} \overline{(P_{ij})_\delta} \, dy$$

where

$$y = v/\alpha,$$
$$v = \text{velocity},$$
$$\alpha = \text{most probable velocity}.$$

The computed curves for different values of σ are shown in figure 7. Values of σ for different transit times were obtained from the data of the sort indicated in figure 5. The frequency difference $[(f \text{ chain} - f \text{ maser}) - f \text{ ref}]$ was adjusted such that the average of the $\Delta\tau_i$' s, $\overline{\Delta\tau}$, corresponded to the atomic transit time of interest. The progression in phase in the time $\overline{\Delta\tau}$ is $\pi(\frac{1}{\Delta\tau_i})\,\overline{\Delta\tau} = \phi_i$ for the ith time interval. The "discrepancy" in the phase change that occurs in the transit time $\overline{\Delta\tau}$ during the time interval $\Delta\tau_i$ is given by $\delta_i = |\phi_{i-1} - \phi_i|$. The average of the δ_i^2's was used as an estimate for σ^2. Values of σ are tabulated for different values of this transit time in Table II together with the distance of separation of the two oscillating fields for the particular transit time. The values given are representative values for the crystal oscillator and multiplier chain tested.

As the length of a beam machine is increased in an effort to improve its accuracy, the transit time becomes longer, and the frequency instability tends to decrease the amplitude of the Ramsey pattern to a larger degree. It might first be supposed that a correction signal could be derived from the atomic transition and - by means of a servo system - eliminate the fluctuations adequately. This scheme becomes less useful, however, as the transit time increases. The servo requires a modulation of the signal, and obviously the period of the modulation cannot be less than the flight time of an atom between the two oscillating fields. As the flight time is increased the modulation frequency must be reduced, and this necessitates a corresponding increase in the time constant of the servo network. The crystal oscillator will have its free running instability for periods within this time constant.

An obvious solution, if one is wealthy enough to make beams forty feet long or more, is to stabilize the excitation with an ammonia maser. The maser servo time constant is relatively short.

TABLE I

Multiplier Chain Frequency Stability. *

RMS value of the 1st differences in frequency. $\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(f_i-f_{i+1})^2}$, measured in parts in 10^{10}.	Mean time, $\overline{\Delta\tau}$. $\overline{\Delta\tau} = \frac{1}{N}\sum_{i=1}^{N}\Delta\tau_i$. (in seconds).	Total time interval of measurement, $T = \sum_{i=1}^{N}\Delta\tau_i$, (in seconds).	
2.0	.030	85	
1.0	.069	85	
0.83	.092	85	
0.76	.14	85	
0.64	.28	85	

*The values given in Column 1 give an indication of the continuity and smoothness of the frequency variations occurring between contiguous time intervals $\Delta\tau_i$. The standard deviation of the frequency (not the 1st differences) was about 8×10^{-11} for the 85 seconds of measurement.

TABLE II

Standard Deviation, σ, of Phase Differences Experienced by Atoms of the Beam.

σ' measured in radians at 23, 900 Mc.	σ predicted for 9193 Mc $\{\sigma = \frac{9193}{23,000}\sigma'\}$ (in radians).	Transit time, $\overline{\Delta\tau}$ (in seconds).	Corresponding oscillator field separation L (cm). $a = 2 \times 10^4 \frac{cm}{sec}$.	Total time over which σ is averaged, T (in seconds).
0. 51	0. 20	. 030	600	85
0. 54	0. 21	. 069	1380	85
0. 67	0. 26	. 092	1840	85
0. 80	0. 31	. 14	2800	85
1. 4	0. 52	. 28	5600	85

FREQUENCY MULTIPLIER CHAIN STABILIZED BY AN AMMONIA MASER

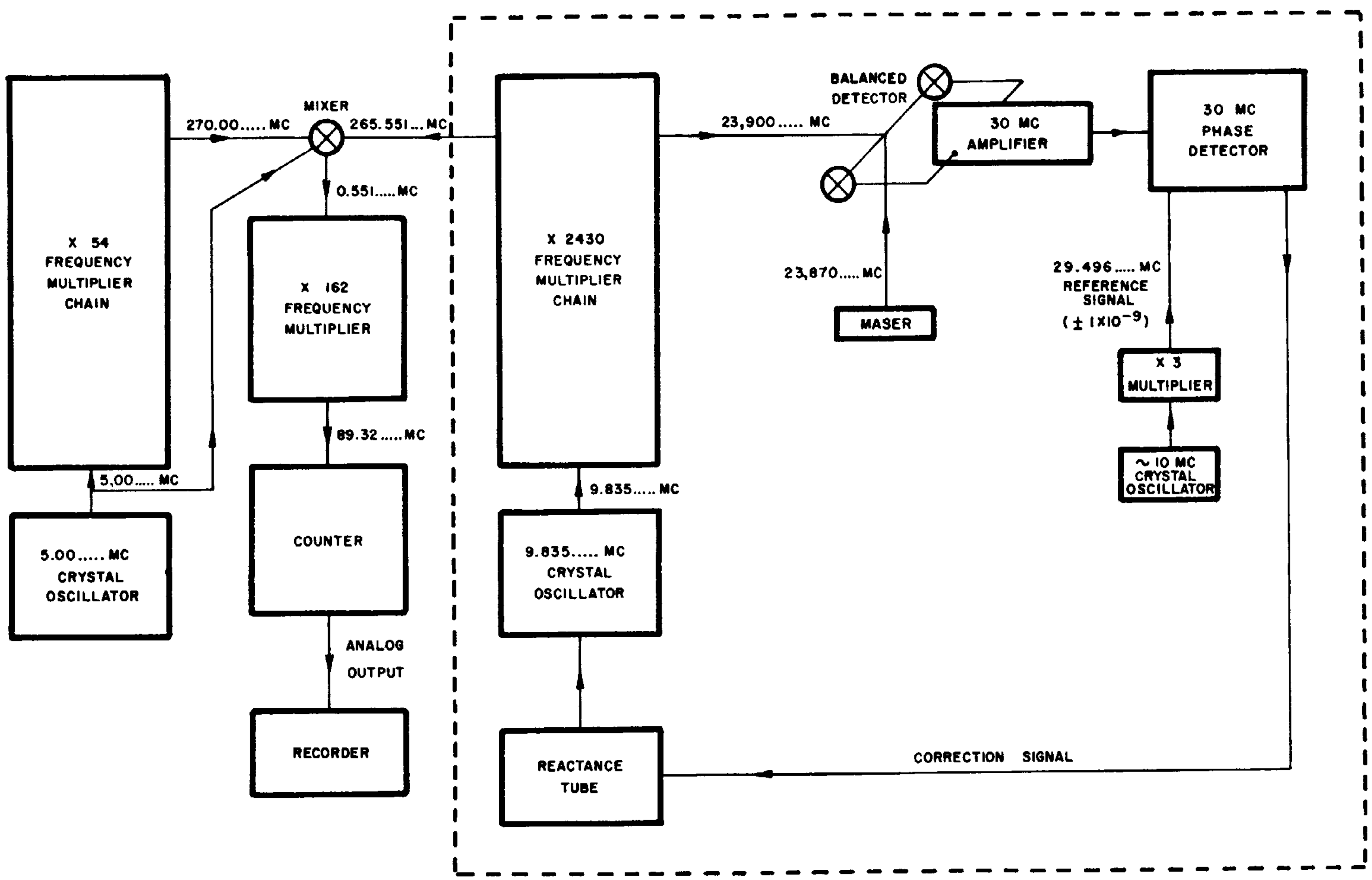
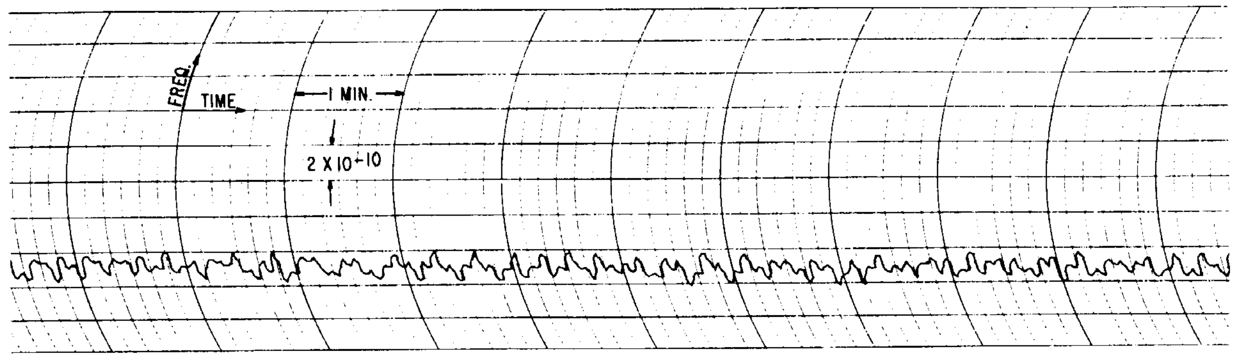
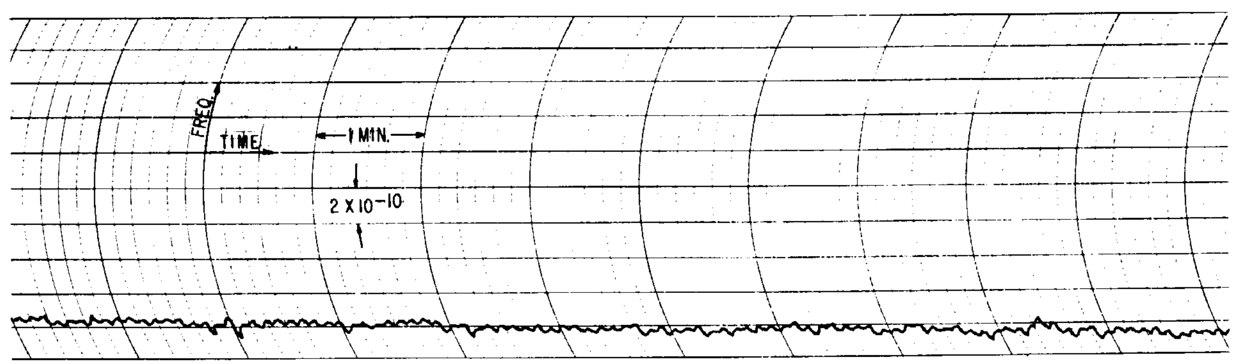


FIG. I THE PORTION OF THE DIAGRAM CONTAINED WITHIN THE DASHED LINE IS THE MASER STABILIZED CHAIN. THE PORTION OF THE DIAGRAM OUTSIDE THE DASHED LINE IS A COMPARISON CHAIN FOR COMPARING THE MASER FREQUENCY WITH AN EXTERNAL SOURCE — IN THIS CASE A 5 MC CRYSTAL OSCILLATOR.

AMMONIA MASER VS HELIUM COOLED CRYSTAL OSCILLATOR (JANUARY 1959)



a. HELIUM WITH PRESSURE REGULATION



b. HELIUM WITHOUT PRESSURE REGULATION

FIG. 2 FIGURE 2a IS A PORTION OF A TWO HOUR COMPARISON BETWEEN A MASER AND A HELIUM COOLED CRYSTAL OSCILLATOR. THE MAXIMUM DRIFT FOR THE TWO HOUR PEROID WAS 2×10^{-11} THE TRACES ARE DERIVED FROM THE ANALOG OUTPUT OF THE COUNTER IN FIGURE 1. THE COUNTER WAS SET FOR 1 SECOND COUNTS IN EACH CASE.

BEAT NOTE BETWEEN TWO AMMONIA MASERS

(APRIL 1958)

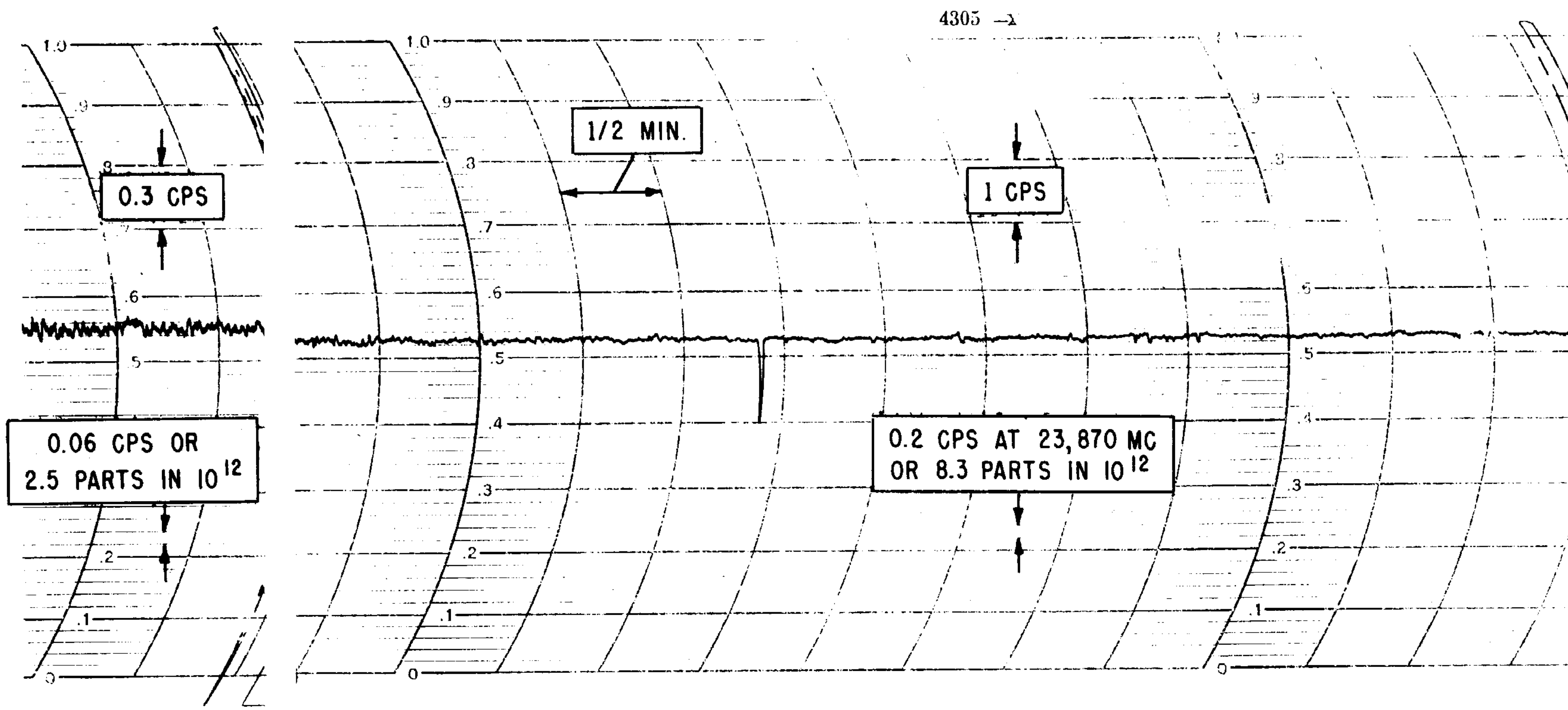


FIG. 3 THIS TRACE IS REPRESENTATIVE OF THE BEST COMPARISONS OF THE NBS MASERS. THE "SPIKE" IN THE CENTER OF THE TRACE IS A RESULT OF VOLTAGE BREAKDOWN OF THE FOCUSER ELECTRODES

BEAT NOTE BETWEEN TWO THERMOSTATED AMMONIA MASERS (APRIL 1959)

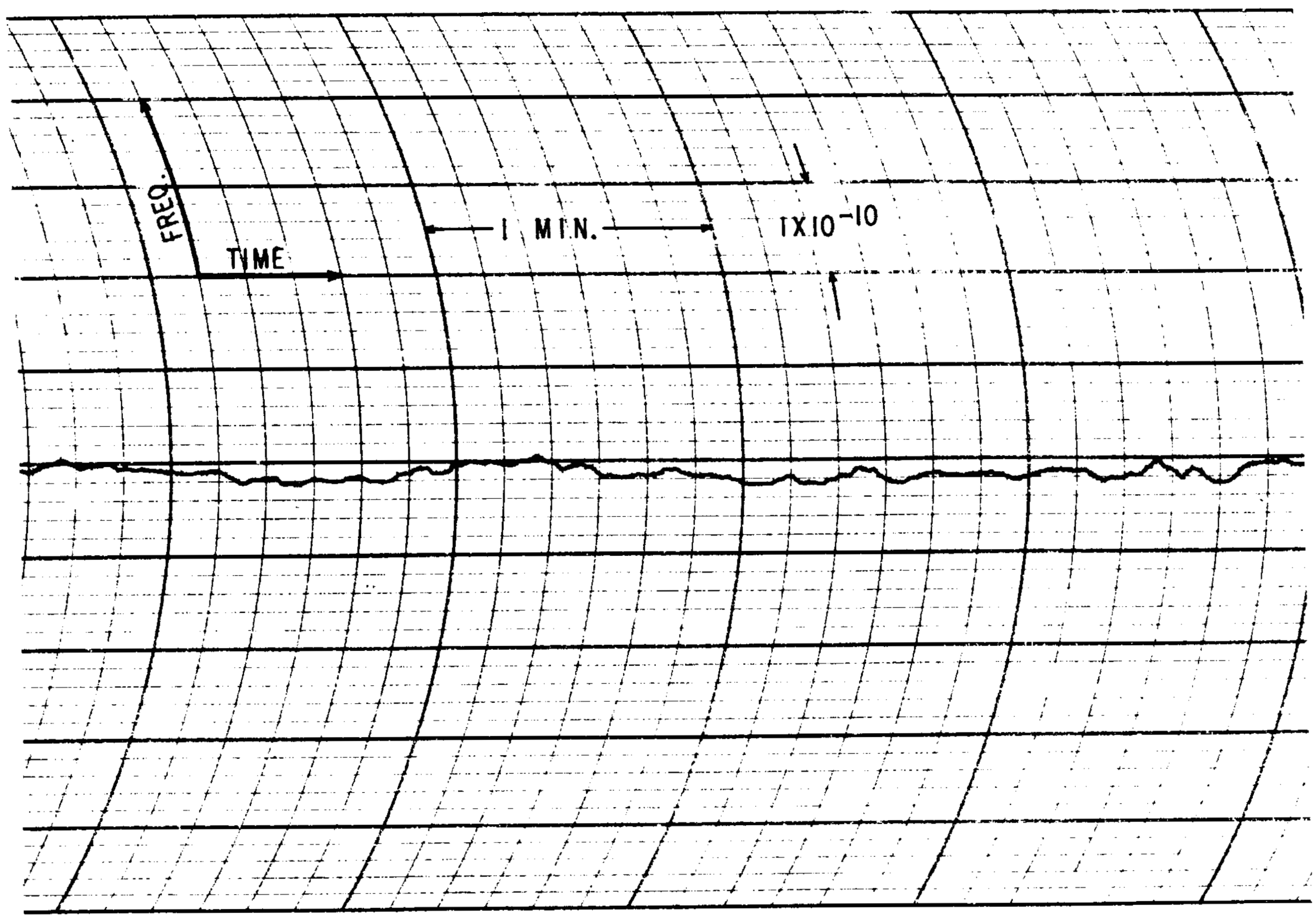


FIG. 4 THE POOR COMPARISON OF THIS CHART WITH THAT OF FIG. 3 IS ATTRIBUTED TO PRESSURE FLUCTUATIONS IN ONE OF THE MASERS

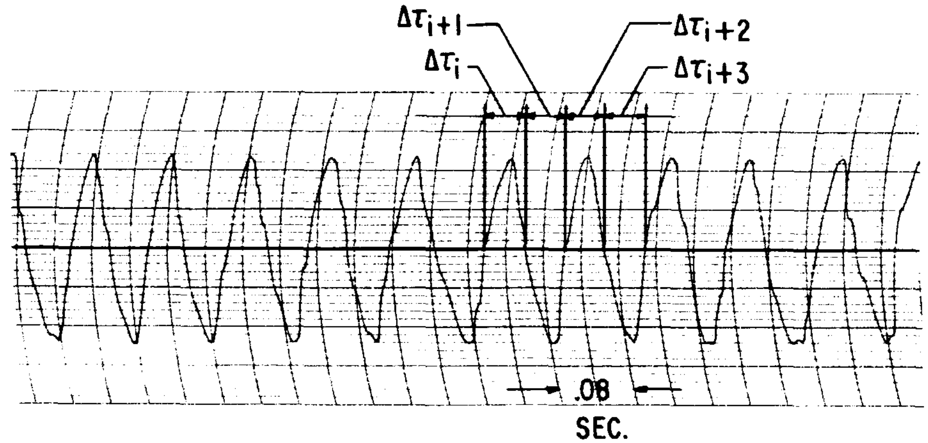


FIG. 5 SAMPLE OF A RECORDING USED TO DETERMINE SHORT TERM STABILITY (20 MILLISECONDS AND GREATER) OF A CRYSTAL OSCILLATOR. THE TRACES WERE OBTAINED USING THE CIRCUIT ARRANGEMENT OF FIG. 6

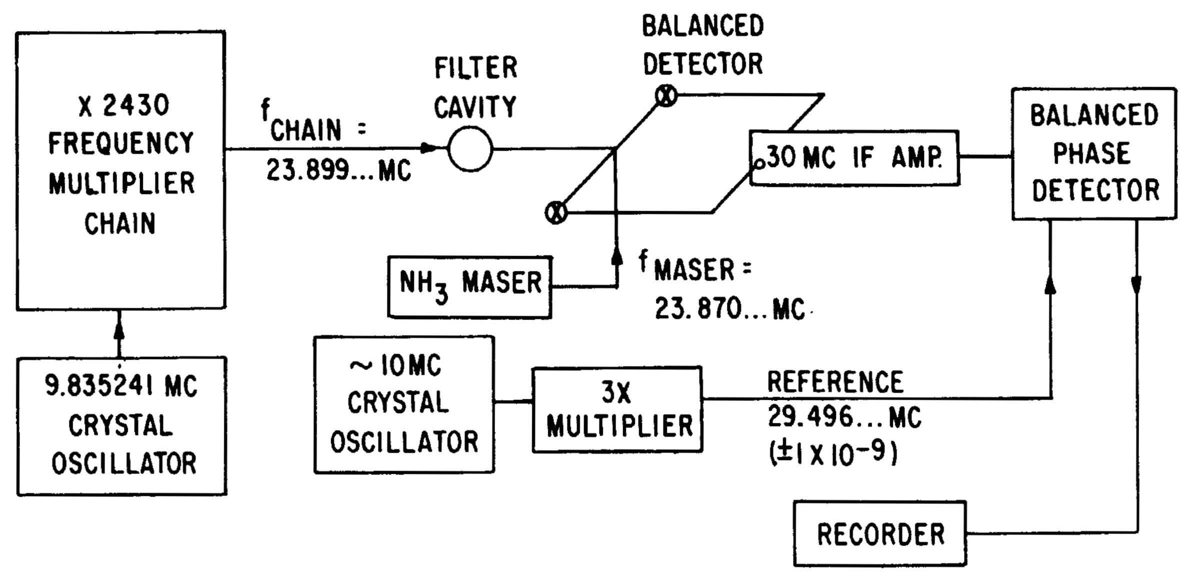


FIG. 6 METHOD OF MEASURING SHORT PERIOD INSTABILITIES OF A CRYSTAL OSCILLATOR.

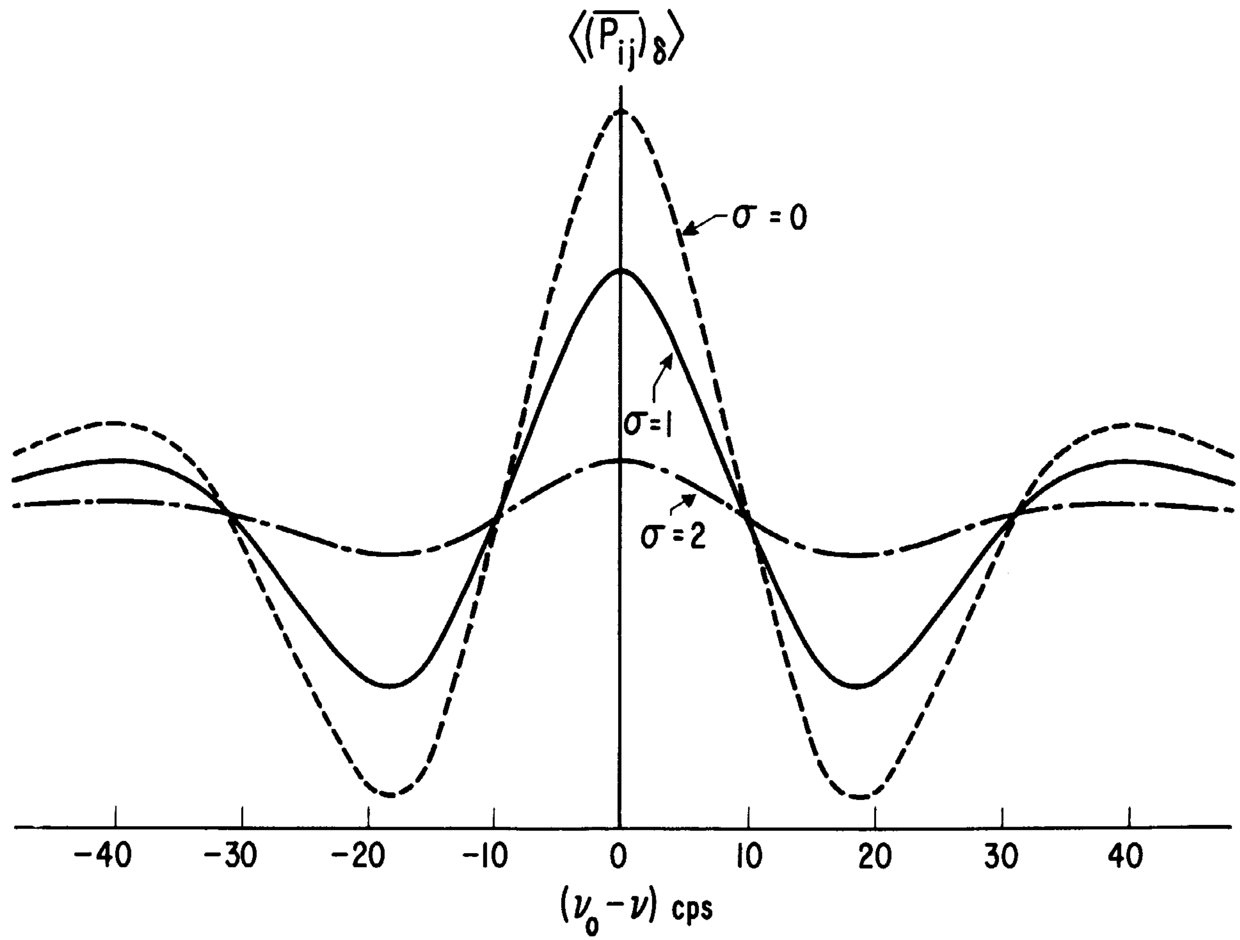


FIG. 7 THE PREDICTED EFFECT OF CRYSTAL OSCILLATOR INSTABILITY ON THE RAMSEY PATTERN OF AN ATOMIC BEAM MACHINE