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Ultimate Noise Limitations of Electron Multipliers and Vacuum Tube Electrometers as Used for the Measurement of Beam Current in a Cesium Clock

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IMPORTANT NOTICE

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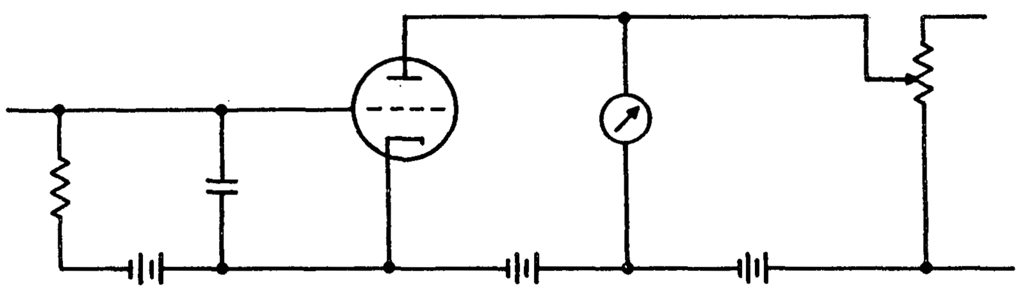
ABSTRACT

It is shown that the ratio of signal current to random noise current is improved by a factor of about 1000 by direct measurement with an electrometer. However, an arrangement involving an electron multiplier connected to an electrometer has the advantage of a much faster time-constant.

I. NOISE IN THE MEASUREMENT OF CESIUM BEAM STANDARDS

We first consider the noise introduced into the measurement of cesium beam currents of the order of 10^{-13} amperes by a vacuum tube electrometer. The shot noise and Brownian motion of the grid current have been treated by the method of Van der Ziel[1] and Van der Ziel and Strutt[2].

In this treatment the electrometer is represented by the model in the figure.



R_g and C_g are the total grid resistance and grid capacitance respectively.

There are three sources of noise inherent in such a system: (1) Brownian motion of electricity in the grid circuit; (2) shot effect of the grid current; and (3) shot effect of the anode current. It can be shown[1] that the third is not significant in any practical case.

The shot effect and Brownian motion of electricity in the grid circuit create a fluctuating grid voltage, thereby producing a fluctuating current through the galvanometer which sets a lower limit to the accuracy of the measurement.

The current flowing to the grid, I_g, is composed of two parts: I_1, electrons emitted from the grid and positive ions arriving at the grid; and I_2, electrons arriving at the grid; so $I_g = I_2 - I_1$. However, since I_1 and I_2 fluctuate independently I_g will fluctuate as a current $I = I_1 + I_2$.

The fluctuation due to shot effect is given by the Schottky Formula:

$$\overline{\Delta i^2} = 2 e I \Delta \nu \tag{1}$$

while the Brownian motion is given by the Nyquist Formula

$$\overline{\Delta i_b^2} = \frac{4KT}{R} \Delta \nu \tag{2}$$

in the frequency range between ν and $\nu + \Delta\nu$, where e is the electronic charge, K is Boltzmann's constant and T is the ambient temperature. The voltage fluctuation in the grid circuit is then

$$\overline{\Delta V^2} = \sum_{\nu=0}^{\infty} \left[\frac{\overline{\Delta i^2} + \overline{\Delta i_b^2}}{\frac{1}{R_g^2} + 4\pi^2 \nu^2 C_g^2} \right]$$

$$= \int_0^\infty \frac{2\left(eI + \frac{2KT}{R}\right)d\nu}{\frac{1}{R_g^2} + 4\pi^2\nu^2 C_g^2}$$

$$= \frac{1}{2}\frac{R_g}{C_g}\left(eI + \frac{2KT}{R}\right). \tag{3}$$

Van der Ziel[1] shows that, so long as the period of the galvanometer, τ, is less then the time constant, $R_g C_g$, of the grid circuit the current fluctuation is

$$\overline{\Delta j^2} = \frac{\overline{\Delta V^2}}{R_g^2} = \frac{\left(\frac{2KT}{R_g} + eI\right)}{2R_g C_g}. \tag{4}$$

But, if the galvanometer responds more slowly then the grid circuit, then

$$\overline{\Delta j^2} = \frac{\left(\frac{2KT}{R_g} + eI\right)}{2R_g C_g} \cdot \frac{\omega^2\left(2n\omega + \frac{1}{R_g C_g}\right)}{2n\omega\left(\omega^2 + \frac{2n\omega}{2R_g C_g} + \frac{1}{R_g^2 C_g^2}\right)} \tag{5}$$

where ω is the mechanical frequency of the undamped galvanometer and n is a measure of the damping. (In practice $n \approx 1$).

If, in addition, $\omega << \frac{1}{R_g C_g}$, then usually $\frac{2KT}{R_g} >> eI$ and under these conditions equation (5) simplifies to

$$\overline{\Delta j^2} = \frac{\pi K T}{n \tau R_g} . \tag{6}$$

II. ANALYSIS OF NOISE

The analysis of noise introduced into the measurement of a small current by an electron multiplier is based on the method of Allen[3].

If we let m be the multiplication per stage (and assume that each stage has the same multiplication); let ΔI_p be the fluctuating component of the primary current; I_p, and let ΔI_s be the fluctuating component of I_s, the output current of a single stage; then

$$\overline{\Delta I_s^2} = m^2 \, \overline{\Delta I_p^2} + 2e \, \overline{I}_s \left(\frac{\overline{m^2} - \overline{m}^2}{m} \right) \Delta \nu . \tag{7}$$

The first term represents the amplified input noise while the second term is the shot noise introduced by the stage.

If there are k stages, let $M = m^k$ be the total multiplication and $\overline{\Delta I_n^2}$ be the mean square noise output. Since the single stage noise of equation (7) is amplified and enhanced by all succeeding stages, $\overline{\Delta I_n^2}$ is given as the sum of a geometric series of k terms.

$$\overline{\Delta I_n^2} = M^2 \, \overline{\Delta I_p^2} + 2e \, \overline{I}_n \left(\frac{M-1}{m-1} \right) \left(\frac{\overline{m^2} - \overline{m}^2}{m} \right) \Delta \nu . \tag{8}$$

If the probability of production of secondary electrons is given by a Poisson distribution (which is approximately the case for a low voltage per stage)

$$\overline{m^2} - \overline{m}^2 = m \tag{9}$$

and equation (8) becomes

$$\overline{\Delta I_n^{\,2}} = 2e\,\overline{I}_n \left(\frac{Mm - 1}{m - 1}\right) \Delta \nu \,. \tag{8a}$$

If the electron multiplier were an ideal amplifier with no fluctuations in the multiplication process $(\overline{m^2} - \overline{m}^2 = 0)$

$$\overline{\Delta I_n^{\,2}} = M^2 \Delta\, \overline{I_p^{\,2}} = 2eM\,\overline{I}_n\,\Delta\nu \,. \tag{8b}$$

Comparing (8a) with (8b), it can be seen that if $m \approx 3$ the electron multiplier increases the noise by about 50% over what would be obtained from an ideal amplifier.

III. TECHNIQUES FOR REDUCING NOISE

In the present cesium beam apparatus a vacuum tube electrometer with $R_g \approx 10^{11}$ ohms and $C_g \approx 20\ \mu\mu f$ is used to measure a beam current of about 10^{-13} amperes.

From equation (4) or (6) it can be seen that in such an arrangement the rms noise current is of the order of 10^{-16} amperes or 10^{-3} of the signal current.

If an electron multiplier were used its output would again be fed to a vacuum tube electrometer. Under these circumstances the effective shot noise seen by the electrometer would be given by equation (8a). Since a typical electron multiplier has a gain of 10^6, the R_g of the electrometer would be reduced by a factor of 10^{-6} from the value when the electrometer alone is used, that is to 10^5 ohms. Under these conditions, the period, τ, of the galvanometer will always be greater than $R_g C_g$ so that the mean square noise is given by equation (5) with the shot noise term replaced by its enhanced value from equation (8). Since $\frac{1}{R_g C_g} >> \omega$, we get

$$\overline{\Delta j^2} = \frac{\pi KT}{n\tau R_g} + \frac{\pi eI}{n\tau}\left(\frac{Mm - 1}{m - 1}\right). \tag{5a}$$

We consider a multiplier of 14 stages with $m = 3$ so that $M = 1.6 \times 10^6$. Then from equation (5a) it can be seen that the rms noise current is of the same order of magnitude as the signal current.

The electron multiplier has the obvious advantage of a much faster RC time constant (2 micro seconds instead of 2 seconds for the electrometer) but this advantage would hardly outweigh the high noise.

A technique for greatly reducing the noise in an electron multiplier is discussed by Milatz and Bloembergen[4]. Their procedure, actually for measuring very low intensity light, involves modulating the source by a mechanical chopper and modulating the detector at the same frequency.

In the cesium beam apparatus, because of the slow time constant of the hot wire detector (due to the long sitting time of cesium atoms on the detector) the beam modulating frequency would have to be of the order of cycles per second. This, and the inconvenience of introducing a mechanical chopper into the vacuum system, would also make this electron multiplier technique impractical.

IV. REFERENCES

1. Physica, 9, 177 (1942)
2. Physica, 9, 513 (1942)
3. Proc. IRE, 38, 346 (1950)
4. Physica 11, 449 (1946)