FREQUENCY UNCERTAINTY ANALYSIS FOR JOSEPHSON VOLTAGE STANDARD

Y.Tang*, M.A.Lombardi**, D.A.Howe**
*National Institute of Standards and Technology, Gaithersburg, MD 20899, USA
** National Institute of Standards and Technology, Boulder, CO 80305, USA

Abstract[+]

The uncertainty in voltage measurement in a Josephson voltage standard (JVS) is proportional to the uncertainty in frequency measurement. The 10 MHz time-base from various commonly used frequency standards for JVS are analyzed using Allan variance to determine the contribution to the uncertainty budget for voltage measurement. The 75 GHz microwave source along with phase-lock electronics and frequency counter for a Josephson junction array is also analyzed for its stability and contribution to JVS uncertainty budget. The results provide realistic estimation for uncertainty contributions of time base and frequency measurements in JVS.

Introduction

A Josephson voltage standard is a frequency to voltage converter based on the Josephson effect which is described by the following equation

$$V = nf / K_{J\text{-}90} \qquad (1)$$

where V is the voltage generated by Josephson junction or junction array, f is the microwave frequency absorbed by the junction or junction array, n is an integer referred as step number, and $K_{J\text{-}90}$ is Josephson constant 483 597.9 GHz/V recommended by Consultative Committee for Electricity (CCE) in 1990. The relative uncertainty $\Delta V/V$ of voltage measurement is therefore proportional to relative uncertainty of frequency measurement $\Delta f/f$ which consists of a time base and phase locked microwave frequency source. Typical high level direct comparisons of JVS systems at 10 V are performed at a level of few parts in 10^{-11} [1]. Therefore, the frequency uncertainty of the 10 MHz reference should meet a requirement of 1×10^{-11} or better to insure proper operation of a JVS. However, in such comparisons, two JVS systems are referenced to a common frequency sources and are not sensitive to any errors in that frequency reference. This work addresses the question of how much additional error could be present if the two JVS systems were referenced to different frequency sources.

In frequency metrology the stability of a signal is typically expressed by Allan deviation [2]. In a typical application of JVS for a voltage measurement the integration time is in a range from a few min up to 10 min. In this paper, we present a measurement set up and analysis method for determining the stability of both 10 MHz reference frequency and 75 GHz microwave frequency used for a JVS standard using the Allan deviation.

Experimental Description of 10 MHz Measurements

We measured the frequency uncertainty of four 10 MHz reference frequency sources. We selected frequency sources that might typically be used by laboratories that operate a JVS. These included a rubidium oscillator, a cesium oscillator, a Global Positioning System (GPS) disciplined oscillator (GPSDO), and a LORAN-C disciplined oscillator (LCDO). The measurements were performed using Universal Time Coordinated or UTC (NIST), the U. S. primary standard for frequency and time interval, as the input reference to a commercially-available dual mixer time difference measurement system [3]. This instrument makes phase measurements using the heterodyne method using a nominal intermediate frequency of approximately 100 Hz when equal frequency oscillators are compared.

Each device under test was continuously measured against UTC (NIST) for 20 000 s, a period deemed to be sufficiently long to provide a high degree of confidence in stability estimates out to 10 min. The collected 1 s phase difference measurements from each source are stored in units of cycles of the input frequency (nominally 10 MHz). They are then analyzed for stability using the Allan deviation for time series data

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(N-2)\tau^2}\sum_{i=1}^{N-2}\left[x_{i+2} - 2x_{i+1} + x_i\right]^2} \qquad (2)$$

where x_i is a set of phase measurements in time units that consists of individual measurements, x_1, x_2, x_3, and so on, N is the number of values in the x_i series, and the data are equally spaced in segments τ seconds long [2].

Results and Discussion

The Allan deviation results from the 10 MHz measurements are summarized in Table 1 for τ values ranging from 1 s to 1024 s.

Table 1. Allan deviation (k = 1) for 10 MHz measurements.

τ (s)	Cesium	Rubidium	GPSDO	LCDO
1	1.18E-11	3.05E-11	1.15E-12	1.32E-12
2	1.19E-11	2.01E-11	8.95E-13	1.86E-12
4	1.04E-11	1.33E-11	7.96E-13	2.56E-12
8	7.72E-12	7.22E-12	7.92E-13	3.91E-12
16	5.24E-12	5.44E-12	8.27E-13	6.70E-12
32	3.56E-12	5.97E-12	8.79E-13	1.09E-11
64	2.55E-12	6.61E-12	9.23E-13	1.25E-11
128	1.94E-12	7.80E-12	9.53E-13	1.02E-11
256	1.37E-12	1.05E-11	9.69E-13	1.07E-11
512	1.00E-12	1.32E-11	1.09E-12	1.19E-11
1024	7.66E-13	1.39E-11	1.36E-12	6.59E-12

[+] This work was performed at NIST in the Quantum Electrical Metrology Division, Electronics and Electrical Engineering Laboratory, and Time and Frequency Division, Physics Laboratory, Technology Administration, U.S. Department of Commerce, not subject to copyright in the United States.

Figure 1 shows the Allan deviation graph for the four devices plotted for all values of τ. Note that all four devices have estimated frequency stability near or below 1×10^{-11} for averaging times ranging from 10 s to 1000 s. This indicates that all four devices have sufficient stability to meet the frequency uncertainty requirements of the JVS. Note that the frequency sources tested in Figure 1 were chosen as "typical" devices that a laboratory is likely to own; they are neither the best nor the worst performing devices of their type. Therefore, the same results cannot be assumed for all devices in these categories, and the frequency stability of the JVS reference should always be verified.

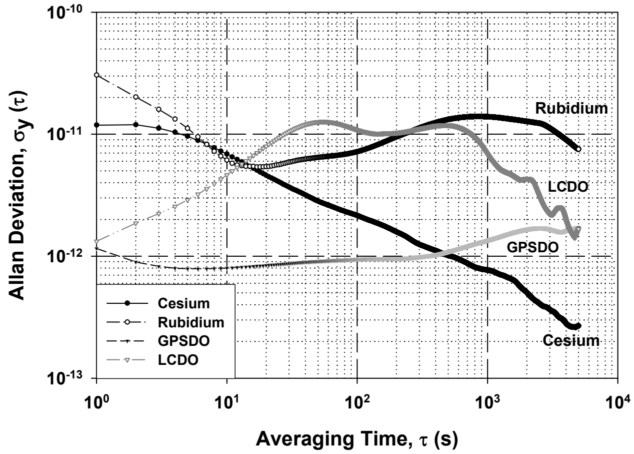

Figure 1. Allan Deviation of 10 MHz signal from four sources.

The Allan deviation represents only a Type A uncertainty, and does not indicate how close a device is to its nominal frequency. The systematic frequency offset of the device from its nominal frequency (sometimes called frequency accuracy) can be classified as a Type B uncertainty. Cesium oscillators have inherent accuracy since cesium provides the basis for the International System (SI) definitions of frequency and time interval. Both a GPSDO and a LCDO compensate for the frequency offset of their local oscillator by continuously disciplining it to agree with an external radio signal referenced to UTC. As a result, all of these devices have a Type B uncertainty of 1×10^{-12} or less, which exceeds the JVS requirement. An undisciplined rubidium oscillator, however, has no inherent accuracy. For example, the rubidium oscillator tested here had a frequency offset near 1×10^{-9}, which would cause an unacceptable error of 10 nV for 10 V measurement. A rubidium oscillator manufacturer might only specify a frequency offset or accuracy of 5×10^{-10} or 5×10^{-11} at shipment. Getting closer to the correct frequency requires calibration or adjustment of the device. This calibration must be repeated periodically to compensate for oscillator aging and frequency drift.

For these reasons, any device chosen as the frequency reference for a JVS must meet two requirements. It must be adjusted to within 1×10^{-11} of its nominal frequency, and it must have the inherent stability to stay on frequency (as estimated by the Allan deviation) for integration times up to 10 min. The first requirement excludes some rubidium oscillators from being an acceptable JVS frequency reference.

75 GHz Frequency Uncertainty Measurement

Usually, the 75 GHz signal is produced by a Gunn diode oscillator whose frequency is controlled by a frequency-lock loop. A commercial V-band frequency counter with a 10 MHz reference signal counts the 75 GHz signal. A voltage is available from the counter that represents the frequency error in the count relative to 75 GHz (or some other preset frequency). We therefore expect the 75 GHz signal to have the same level of accuracy as the 10 MHz reference signal at long averaging times. The problem is that if frequency instability for medium and short averaging times exceeds the long-term accuracy, the actual level of accuracy is worse than the claimed level during these averaging times.

The frequency uncertainty measurement consists of beating the 75 GHz source against a reference signal of known, lower frequency instability. A clean, synthesized reference at about 73 GHz with sufficient power has been generated for this measurement. The basic measurement apparatus has been assembled at NIST [4], and the results will be presented in a later discussion.

Conclusion

A 10 MHz frequency reference used for JVS must meet a frequency uncertainty requirement of 1×10^{-11}. We have measured the Allan deviation and offsets of four 10 MHz devices commonly used as JVS frequency references relative to UTC (NIST). For integration time of up to 10 min, the frequency stability of these four references is near or below 1×10^{-11}. However, the accuracy of a rubidium frequency standard might not satisfy the JVS requirements. If a rubidium reference is used for JVS, it is important to make periodic adjustments or calibrations to assure that it does not become a limiting factor in the uncertainty of the voltage measurements.

Acknowledgement

The authors would like to thank Clark Hamilton of VMetrix for assisting the set up for the 75 GHz stability measurement.

References

[1] D. Reymann, T.J. Witt, G. Eklund, H.Pajander, H.Nilsson, R. Behr, T. Funck, and F. Muller, "A Thee-Way, On-Site Comparison of the 10 V Josephson Voltage Standards of the PTB, the SP, and the BIPM", *IEEE Trans. Instrum. Meas.*, vol.48, pp. 257-261, April, 1999.

[2] D.B. Sullivan, D.W. Allan, D.A. Howe, and F.L. Walls, *Characterization of Clocks and Oscillators*, NIST Tech. Note 1337, Washington, US Government Printing Office, 1990.

[3] L. Sojdr, J. Cermak, and G. Brida, "Comparison of High-Precision Frequency-Stability Measurement Systems", *Proc. of 2003 IEEE Intl. Freq. Control Symp.*, pp. 317-325, May 2003.

[4] D. A. Howe, C. W. Nelson, F. L. Walls, J. F. Garcia Nava, A. Hati, and A. Sen Gupta, "A 100 GHz AM and PM Noise Measurement System: Preliminary Design and Performance", *Proc. of 2002 IEEE Intl. Freq. Control Symp.*, pp. 690-698, May 2002.