Quantum information processing in ion traps II

D. J. Wineland, NIST, Boulder

Lecture 1: Nuts and bolts

- Ion trapology
- Qubits based on ground-state hyperfine levels
- Two-photon stimulated-Raman transitions
    - * Rabi rates, Stark shifts, spontaneous emission

Lecture 2: Quantum computation (QC) and quantum-limited measurement

- Trapped-ion QC and DiVincenzo's criteria
- Gates
- Scaling
- Entanglement-enhanced quantum measurement

Lecture 3: Decoherence

- Memory decoherence
- Decoherence during operations
    - * technical fluctuations
    - * spontaneous emission
    - * scaling
- Decoherence and the measurement problem

Quantum computation and quantum-limited measurement


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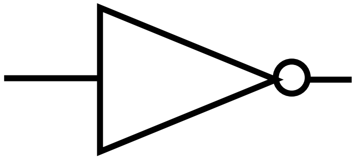
UNIVERSAL LOGIC GATES

- Classical:

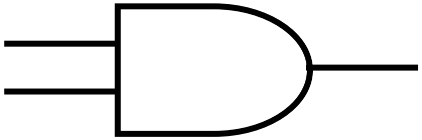
1-bit NOT



$0 \rightarrow 1$

$1 \rightarrow 0$

2-bit AND



$00 \rightarrow 0$

$01 \rightarrow 0$

$10 \rightarrow 0$

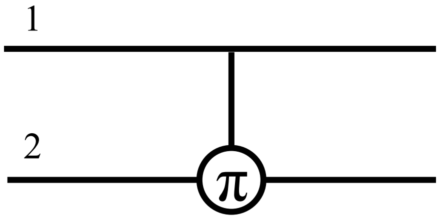
$11 \rightarrow 1$

- Quantum: rotation

$$|0\rangle \rightarrow \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle$$

$$|1\rangle \rightarrow \cos(\theta/2)|1\rangle - e^{-i\varphi}\sin(\theta/2)|0\rangle$$

π-phase gate $U_{1,2}(\pi)$



$$|0\rangle_1|0\rangle_2 \rightarrow +|0\rangle_1|0\rangle_2$$

$$|0\rangle_1|1\rangle_2 \rightarrow +|0\rangle_1|1\rangle_2$$

$$|1\rangle_1|0\rangle_2 \rightarrow +|1\rangle_1|0\rangle_2$$

$$|1\rangle_1|1\rangle_2 \rightarrow -|1\rangle_1|1\rangle_2$$

DiVincenzo, PRA **51**, 1015 ('95)
Barenco *et al*. PRA **52**, 3457 ('95)

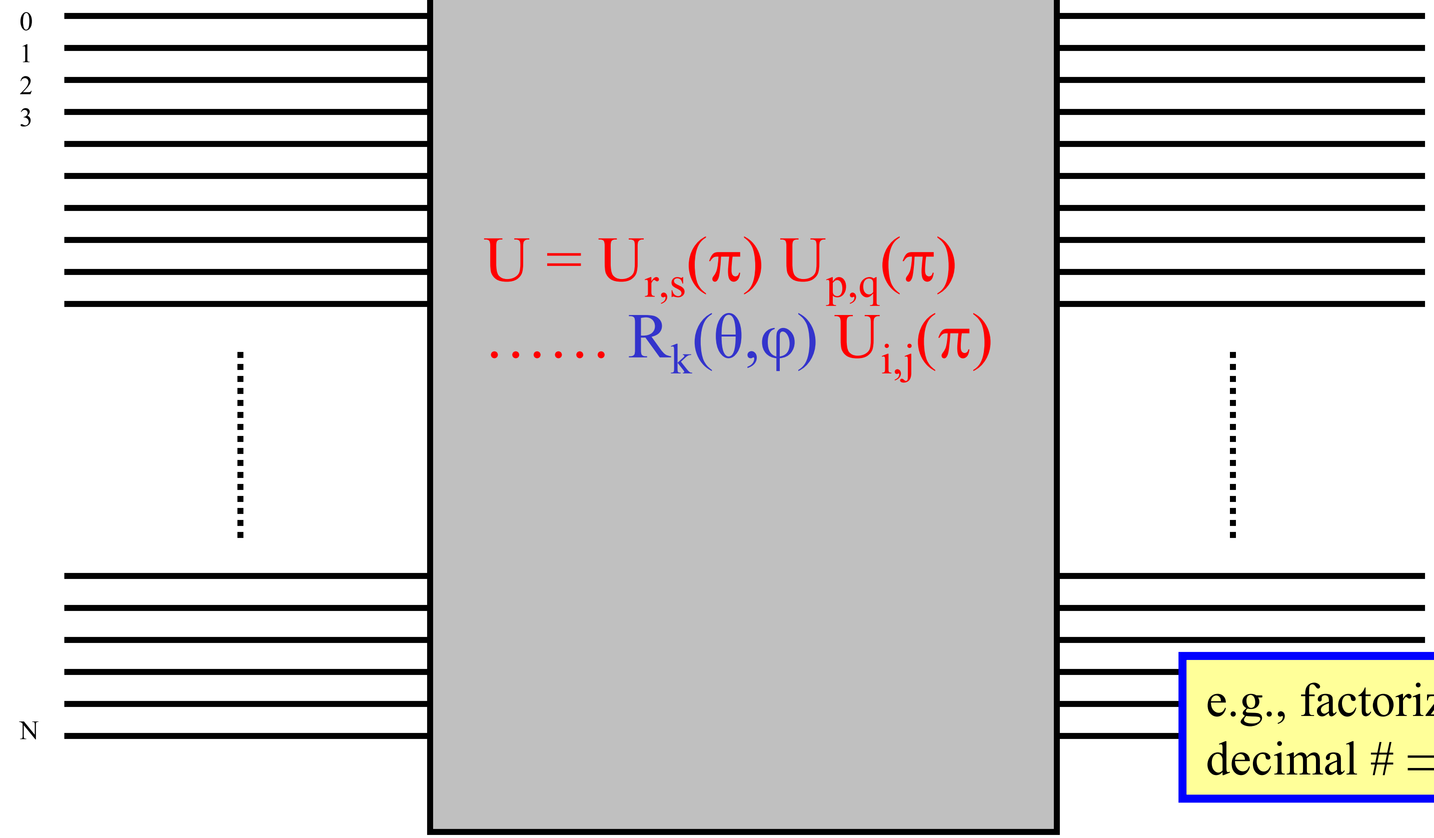
Peter Shor (AT&T, ~1995): efficiently factorize large numbers

$$\psi_{in} = \sum_{i=0}^{2^N-1} C_i |i\rangle$$

$C_i = 2^{-N/2}$ 8 i

Process all possible inputs simultaneously

bit no.



$U = U_{r,s}(\pi)\ U_{p,q}(\pi)$

$\ldots\ldots\ R_k(\theta,\varphi)\ U_{i,j}(\pi)$

$$\psi_{out} = \sum_{i=0}^{2^N-1} C_i |i\rangle$$

$C_i = 0$ for almost all i (quantum interference)

measure qubits

e.g., factorize 150 digit decimal # ⇒ ~ 10^9 ops

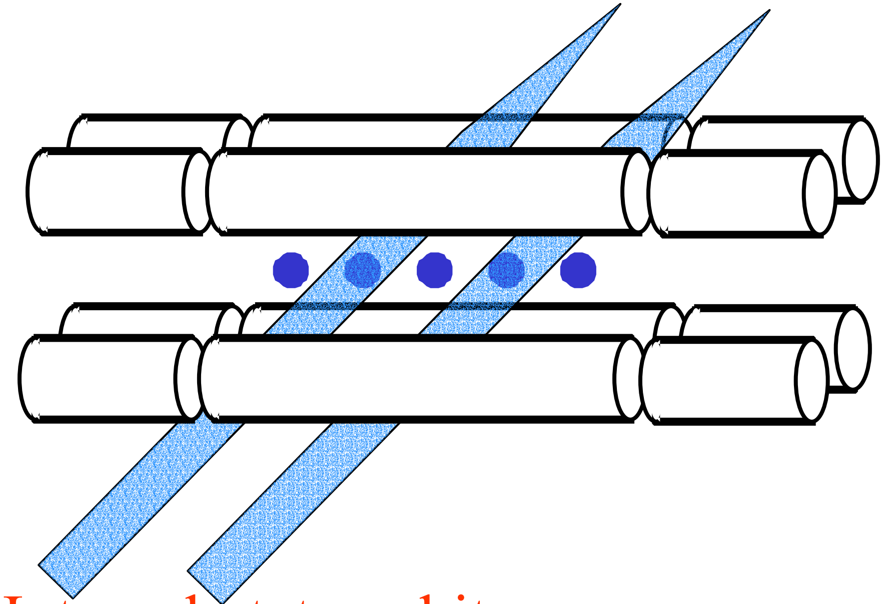
Requirements (David DiVincenzo, IBM)

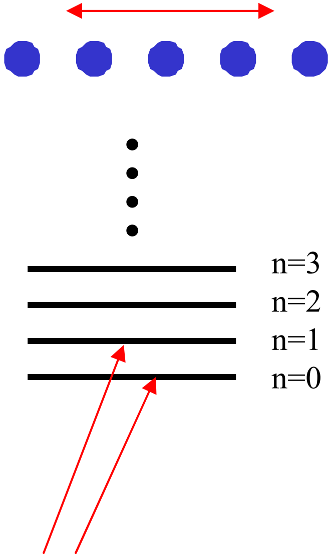
	ions	
▪ well-defined states + state preparation	☑	(efficient optical pumping)
▪ gates	☑	(gates demonstrated; need better fidelity)
▪ efficient read-out	☑	("electron shelving," cycling trans.)
▪ small decoherence (1) memory, (2)during operations	☑	(not at fault-tolerant level yet)
▪ scalable	☑	(schemes outlined; not demonstrated)

Ion Trap QC: Proposal: Cirac and Zoller, '95

Motion "data bus"

(e.g., center-of-mass mode)

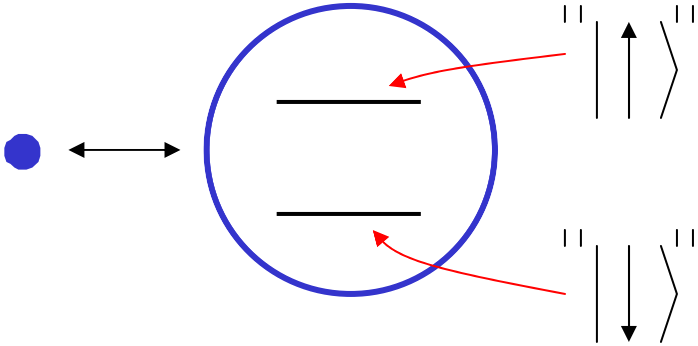




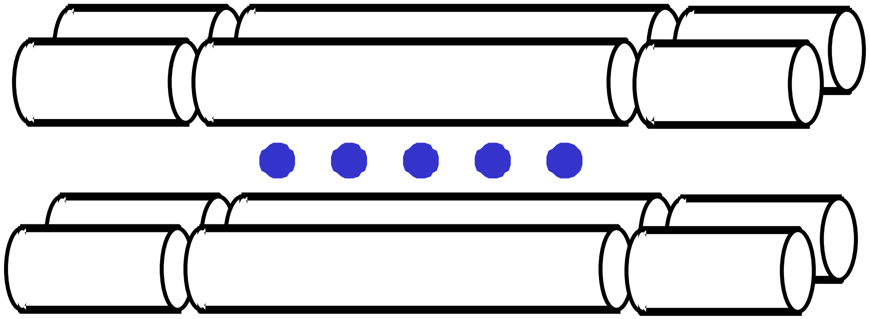
Internal-state qubit ($\tau_{coherence}$ > 30 min)

Motion qubit states

($\tau_{coherence} \sim 0.01 – 100$ ms)

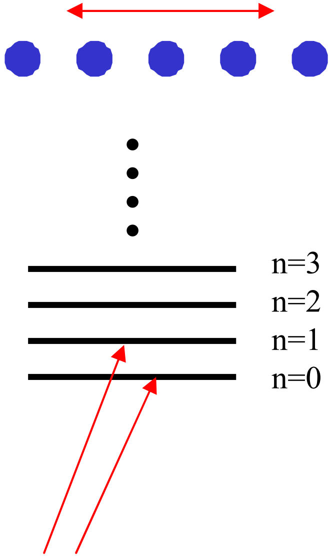


Experiments: Aarhus (Ca$^+$, Mg$^+$); Boulder (Be$^+$, Mg$^+$); Garching (Mg$^+$, In$^+$); Hamburg (Yb$^+$); Innsbruck(Ca$^+$); LANL(Sr$^+$); McMaster (Mg$^+$) Michigan (Cd$^+$); Oxford(Ca$^+$); Teddington (Sr$^+$)



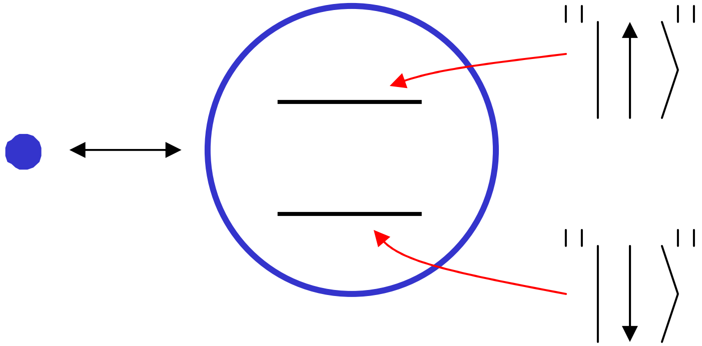
Motion “data bus”

(e.g., center-of-mass mode)



Motion qubit states

Internal-state qubit

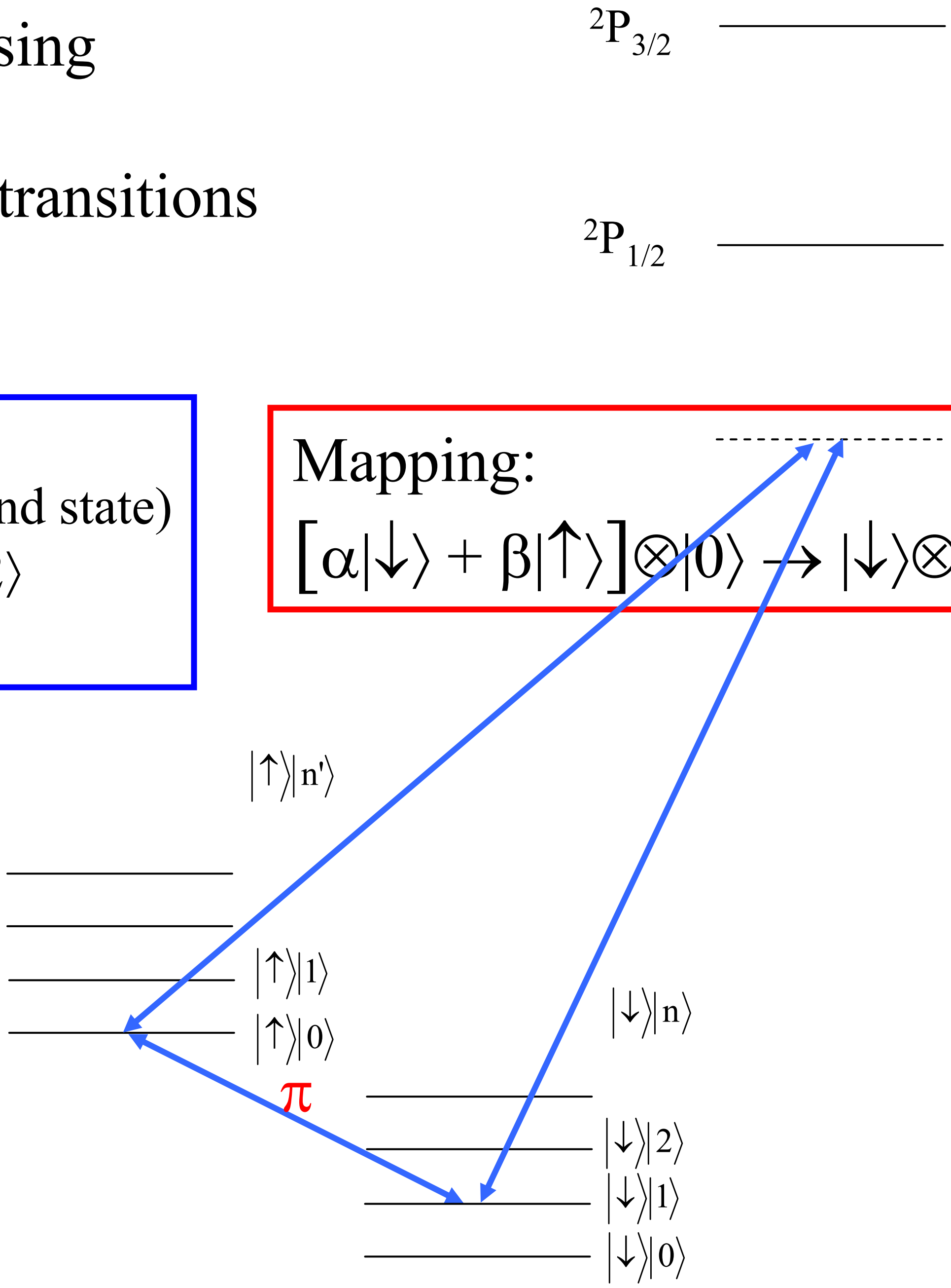


original Cirac/Zoller gate realized by Innsbruck group: F. Schmidt-Kaler *et al.*, Nature **422**, 408-411 (2003)

Logic operations using two-photon stimulated-Raman transitions

e.g., $^9\mathrm{Be}^+$
($^2S_{1/2}$ electronic ground state)
$|\downarrow\rangle \equiv |F = 2, m_F = -2\rangle$
$|\uparrow\rangle \equiv |F = 1, m_F = -1\rangle$

Mapping:
$$[\alpha|\downarrow\rangle + \beta|\uparrow\rangle]\otimes|0\rangle \rightarrow |\downarrow\rangle\otimes[\alpha|0\rangle + \beta|1\rangle]$$

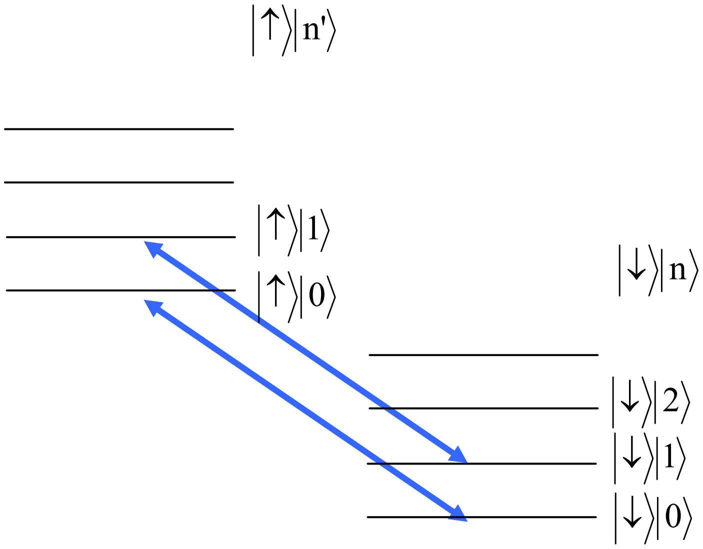


Rotations:

$R(\theta,\phi)$:

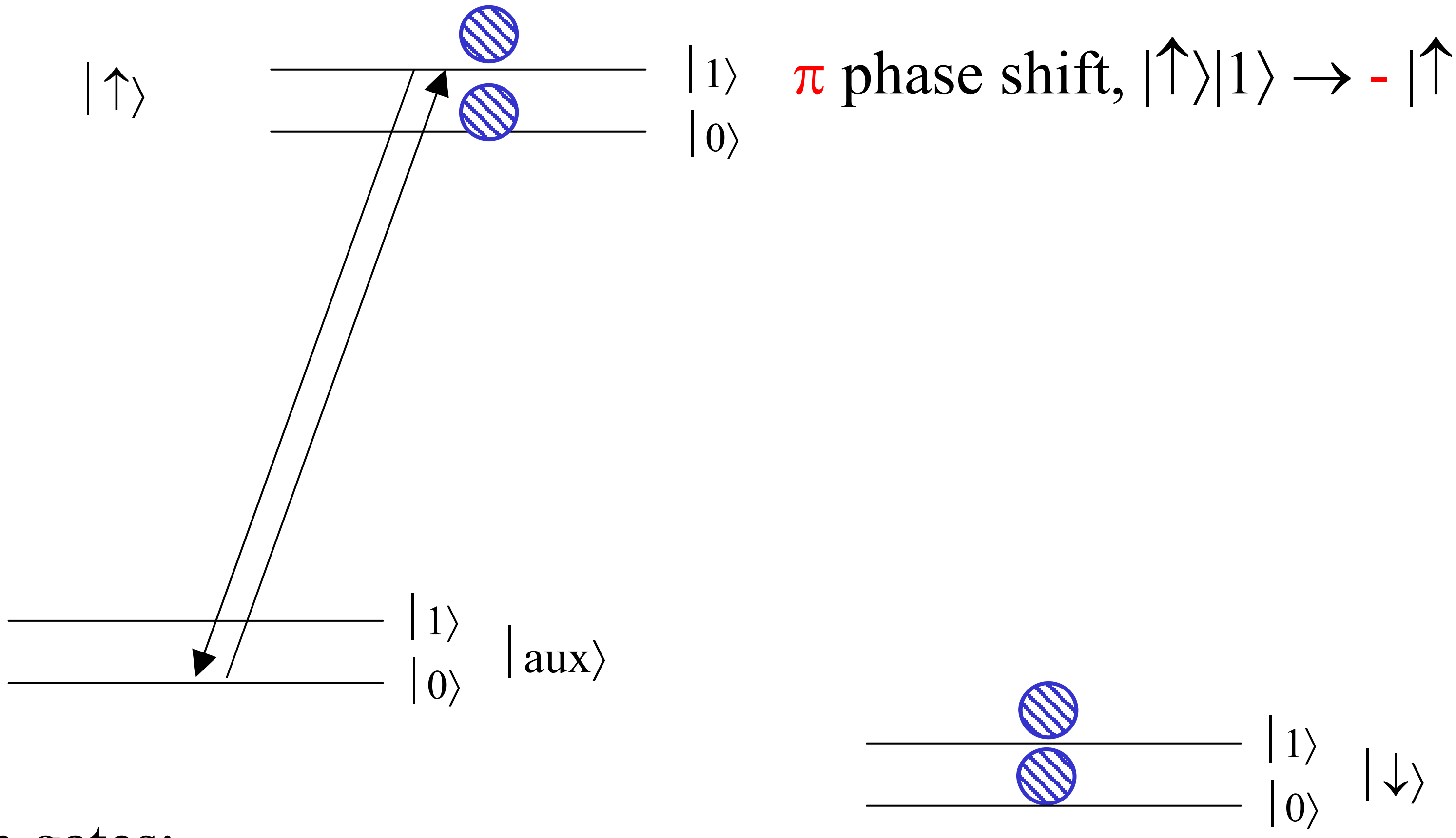
$$|\downarrow\rangle|n\rangle \rightarrow \quad \cos(\theta/2)|\downarrow\rangle|n\rangle + e^{i\phi}\sin(\theta/2)|\uparrow\rangle|n\rangle$$
$$|\uparrow\rangle|n\rangle \rightarrow -e^{-i\phi}\sin(\theta/2)|\downarrow\rangle|n\rangle + \cos(\theta/2)|\uparrow\rangle|n\rangle$$

for $\vec{k}_r$ parallel to $\vec{k}_b$, independent of n



• Quantum logic

conditional dynamics:
$\Rightarrow$ gates!



π phase shift, $|\uparrow\rangle|1\rangle \rightarrow - |\uparrow\rangle|1\rangle$

Ion gates:

<u>Motion/spin gates</u>:

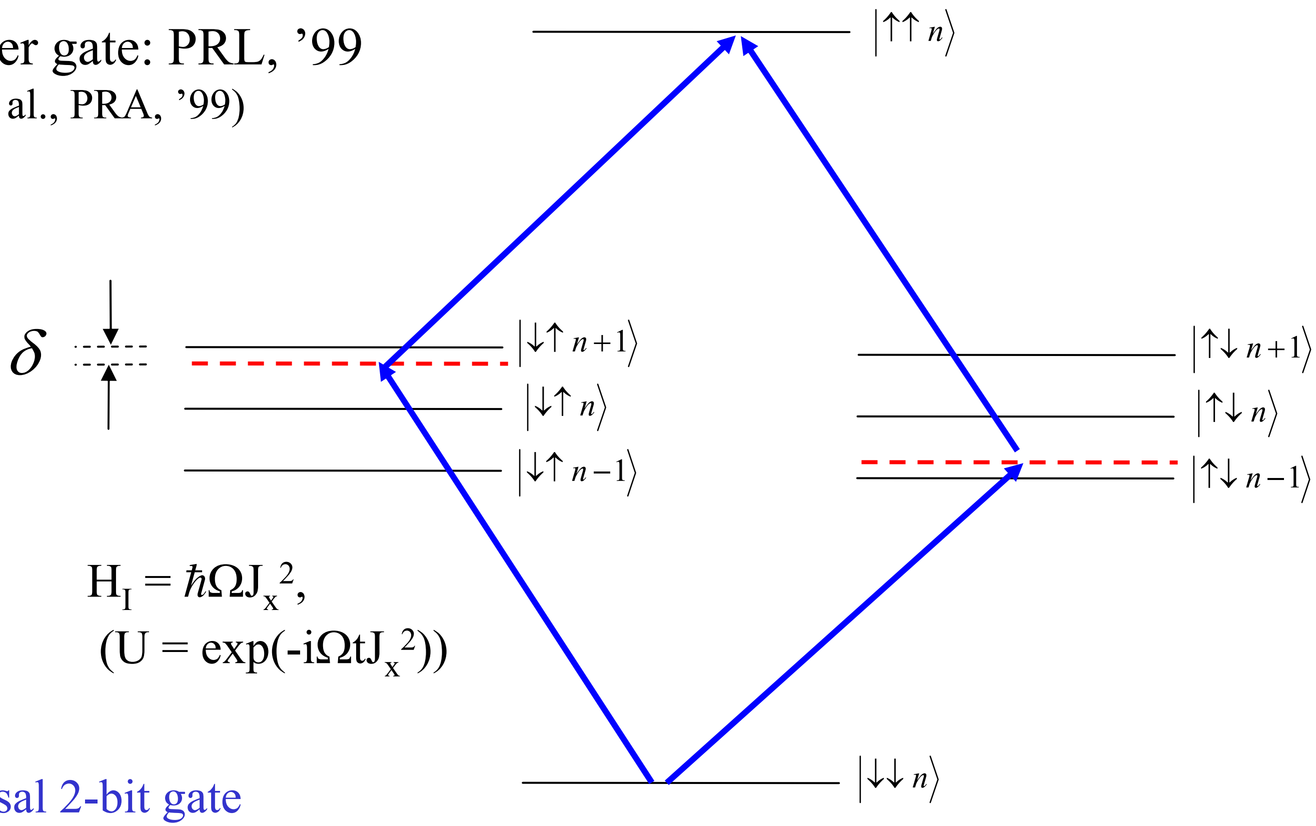
Monroe *et al.* ’95 (NIST); DeMarco *et al.* ’02 (NIST); Gulde *et al.* ’03 (Innsbruck)

<u>≥ 2 spin gates</u>:

Sackett *et al.* ’01 (NIST); Leibfried *et al.* ’03 (NIST); Schmidt-Kaler *et al.* ’03 (Innsbruck)

Sørensen & Mølmer gate: PRL, ’99

(also, Solano, et al., PRA, ’99)



$H_I = \hbar\Omega J_x^{\,2}$,

$(U = \exp(-i\Omega t J_x^{\,2}))$

$\Omega t = \pi/2 \Rightarrow$ universal 2-bit gate

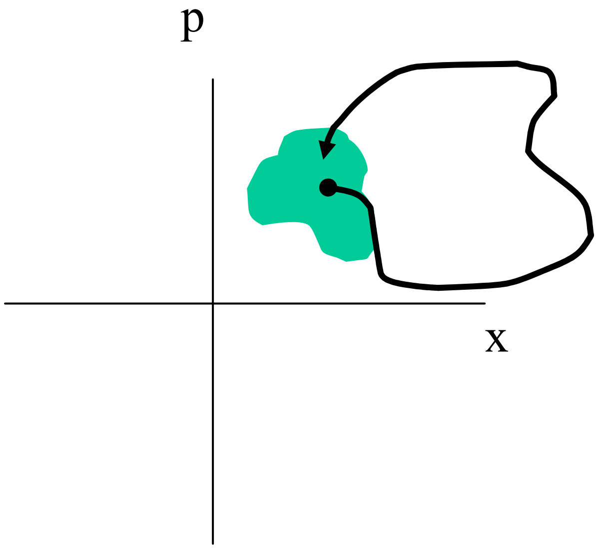
$$|\downarrow\downarrow\rangle \rightarrow \tfrac{1}{\sqrt{2}}\left[|\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle\right]$$
$$|\uparrow\uparrow\rangle \rightarrow \tfrac{1}{\sqrt{2}}\left[|\uparrow\uparrow\rangle + i|\downarrow\downarrow\rangle\right]$$
$$|\downarrow\uparrow\rangle \rightarrow \tfrac{1}{\sqrt{2}}\left[|\downarrow\uparrow\rangle + i|\uparrow\downarrow\rangle\right]$$
$$|\uparrow\downarrow\rangle \rightarrow \tfrac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle + i|\downarrow\uparrow\rangle\right]$$

Experiment: Cass Sackett *et al.*, *Nature*, ‘01

- **one step process**
- **auxiliary internal state not needed**
- **do not need individual-ion laser addressing**
- **motion eigenstates not needed (for motion << λ)**
- **extendable: e.g., $|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle$**
  $\rightarrow |\downarrow\rangle|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle$

Geometrical phase gate: (Didi Leibfried *et al.*)

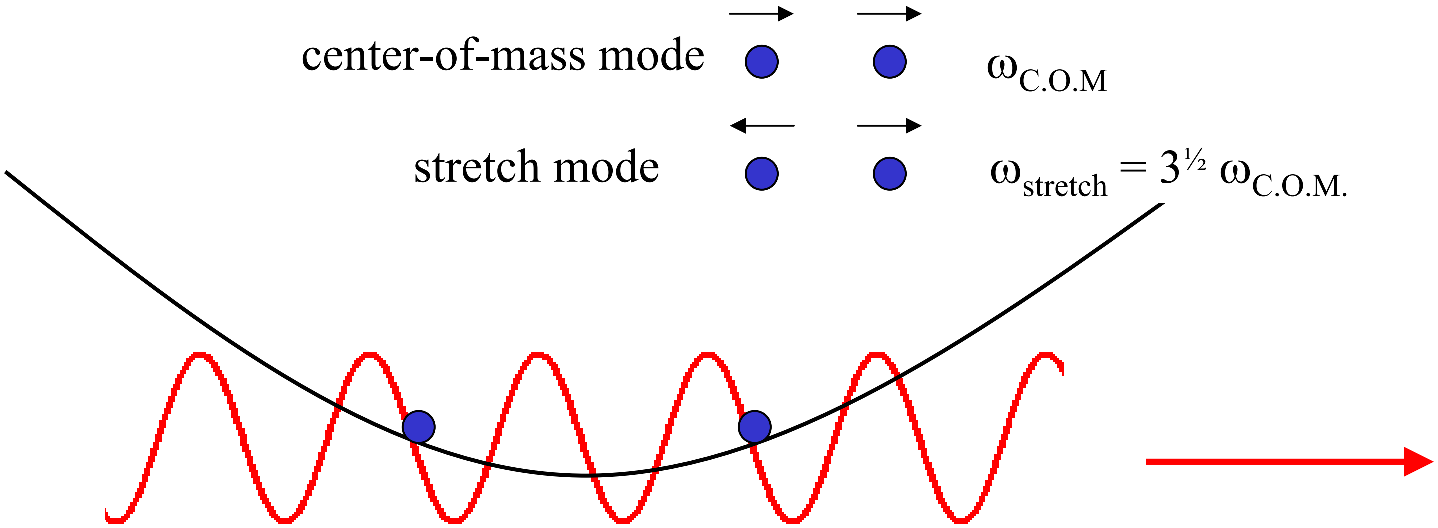
phase-space diagram for (axial) motion



$\Psi \rightarrow e^{i\phi}\,\Psi$

$\phi \propto$ enclosed area

special case of more general formalism by:
Milburn, Schneider, James (1999)
Sørensen & Mølmer (1999,2000)



center-of-mass mode $\omega_{C.O.M}$

stretch mode $\omega_{stretch} = 3^{½}\,\omega_{C.O.M.}$

“walking” standing wave

$$\vec{E} = \vec{E}_1 \sin(kx - \omega t) + \vec{E}_2 \sin(-kx - (\omega - \omega_{diff})t)$$

Stark shifts. Assume:

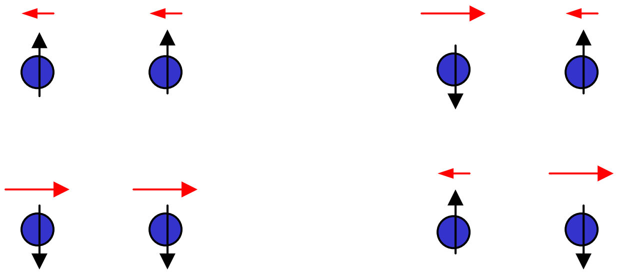
1. $\langle \Delta_{S\downarrow}(t) \rangle_t = \langle \Delta_{S\uparrow}(t) \rangle_t$

<u>but</u>, $\Delta_{S\downarrow}(t) \neq \Delta_{S\uparrow}(t)$

(Chris Myatt *et al.*, *Nature*, 2000)

2. $\omega_{diff} \cong \omega_{stretch}$

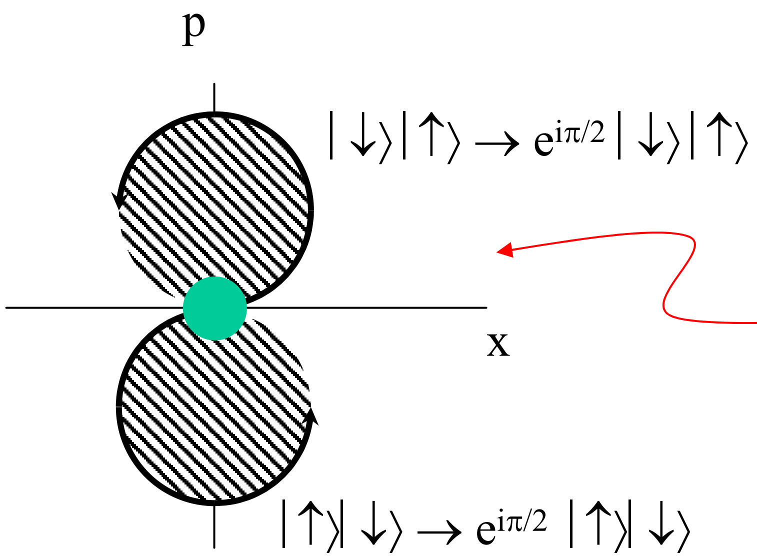
Optical-dipole force



AC version of neutral-atom displacement gates

$|\downarrow\rangle|\downarrow\rangle, |\uparrow\rangle|\uparrow\rangle$ no displacement

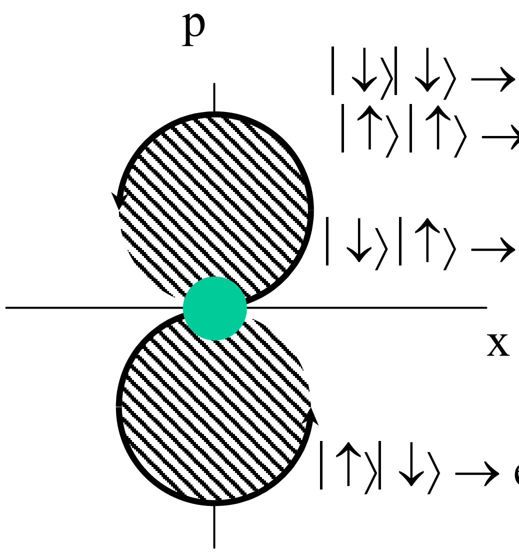
$$|\downarrow\rangle|\downarrow\rangle \rightarrow |\downarrow\rangle|\downarrow\rangle, \quad |\uparrow\rangle|\uparrow\rangle \rightarrow |\uparrow\rangle|\uparrow\rangle$$



$$|\downarrow\rangle|\uparrow\rangle \rightarrow e^{i\pi/2}|\downarrow\rangle|\uparrow\rangle$$

$$|\uparrow\rangle|\downarrow\rangle \rightarrow e^{i\pi/2}|\uparrow\rangle|\downarrow\rangle$$

Phase space for two-ion stretch mode



$|\downarrow\rangle|\downarrow\rangle \rightarrow |\downarrow\rangle|\downarrow\rangle$

$|\uparrow\rangle|\uparrow\rangle \rightarrow |\uparrow\rangle|\uparrow\rangle$

$|\downarrow\rangle|\uparrow\rangle \rightarrow e^{i\pi/2}|\downarrow\rangle|\uparrow\rangle$

$|\uparrow\rangle|\downarrow\rangle \rightarrow e^{i\pi/2}|\uparrow\rangle|\downarrow\rangle$

- one step
- input eigenstates not required (for Lamb-Dicke limit)
- individual addressing not required
- auxiliary internal states not needed

(advantages shared with Sørensen/Mølmer gate (experiment Sackett *et al*. 2001)

PLUS:

- decoupled from spin dynamics
- equal ion coupling not needed

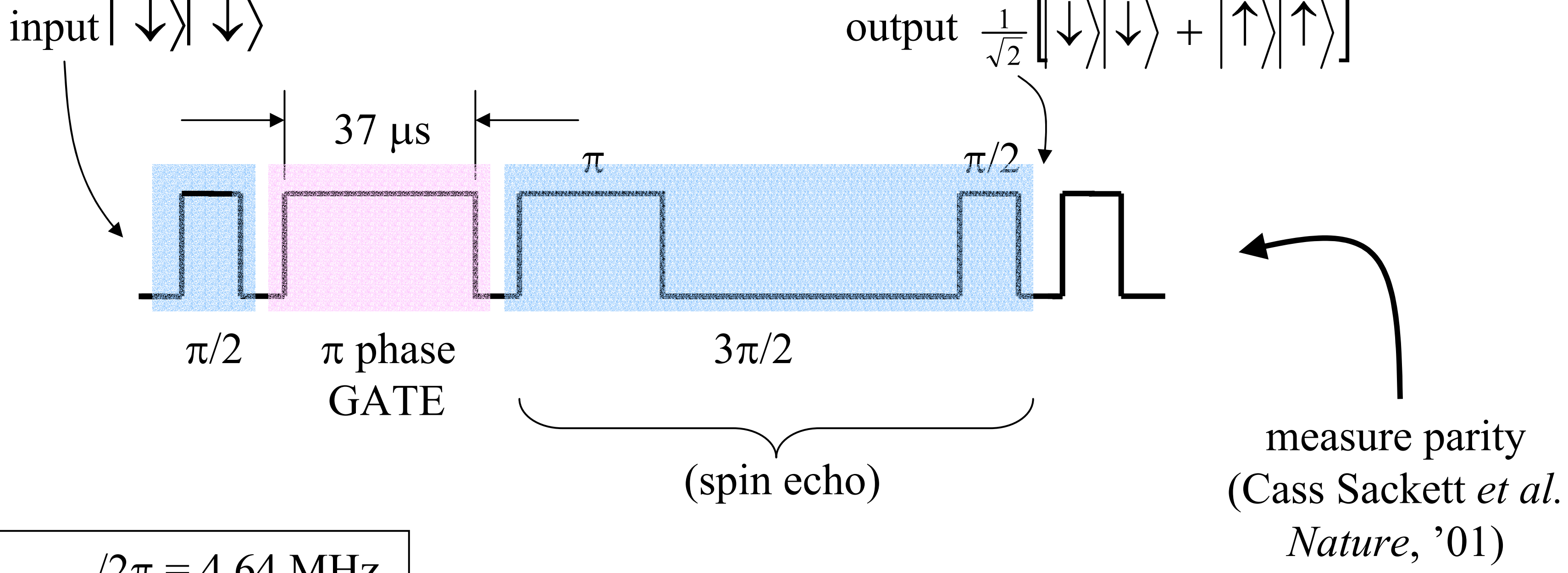
state vector

$$\begin{pmatrix} C_{\downarrow\downarrow} \\ C_{\downarrow\uparrow} \\ C_{\uparrow\downarrow} \\ C_{\uparrow\uparrow} \end{pmatrix}$$

rotation π phase gate

$$U = \begin{pmatrix} 1 & & & \\ & e^{i\pi/2} & & \\ & & e^{i\pi/2} & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & e^{i\pi/2} & & \\ & & e^{i\pi/2} & \\ & & & e^{i\pi} \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & e^{-i\pi} \end{pmatrix}$$

$\pi/2$, $3\pi/2$ Ramsey interferometer



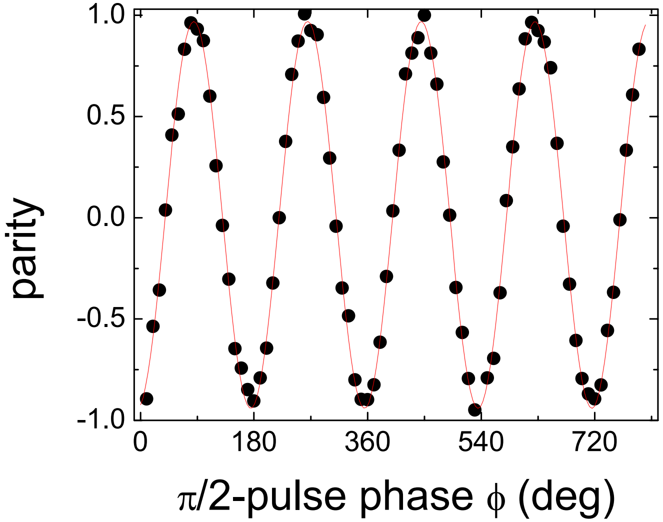
input $|\downarrow\rangle|\downarrow\rangle$

output $\frac{1}{\sqrt{2}}\left[|\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle\right]$

measure parity
(Cass Sackett *et al.*
Nature, '01)

$\omega_{\text{C.O.M.}}/2\pi = 4.64$ MHz

Contrast: 0.955(8)



Fidelity

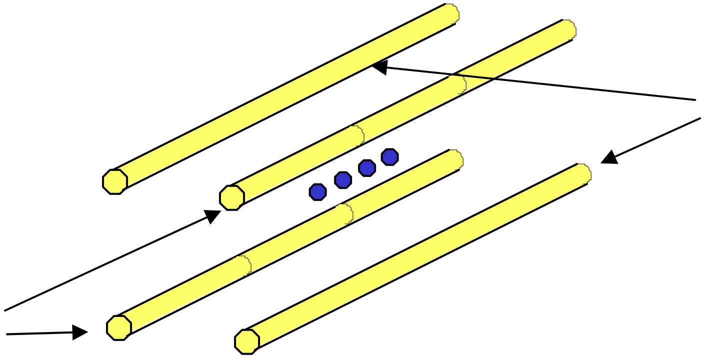
$$\mathbf{F} \equiv \tfrac{1}{2}\left\{\langle\downarrow|\langle\downarrow| + \langle\uparrow|\langle\uparrow|\right\}\rho\left\{|\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle\right\}$$

$$\cong 0.97$$

Didi Leibfried *et al.*, *Nature* **422**, 412 (2003)

Idealized trap:

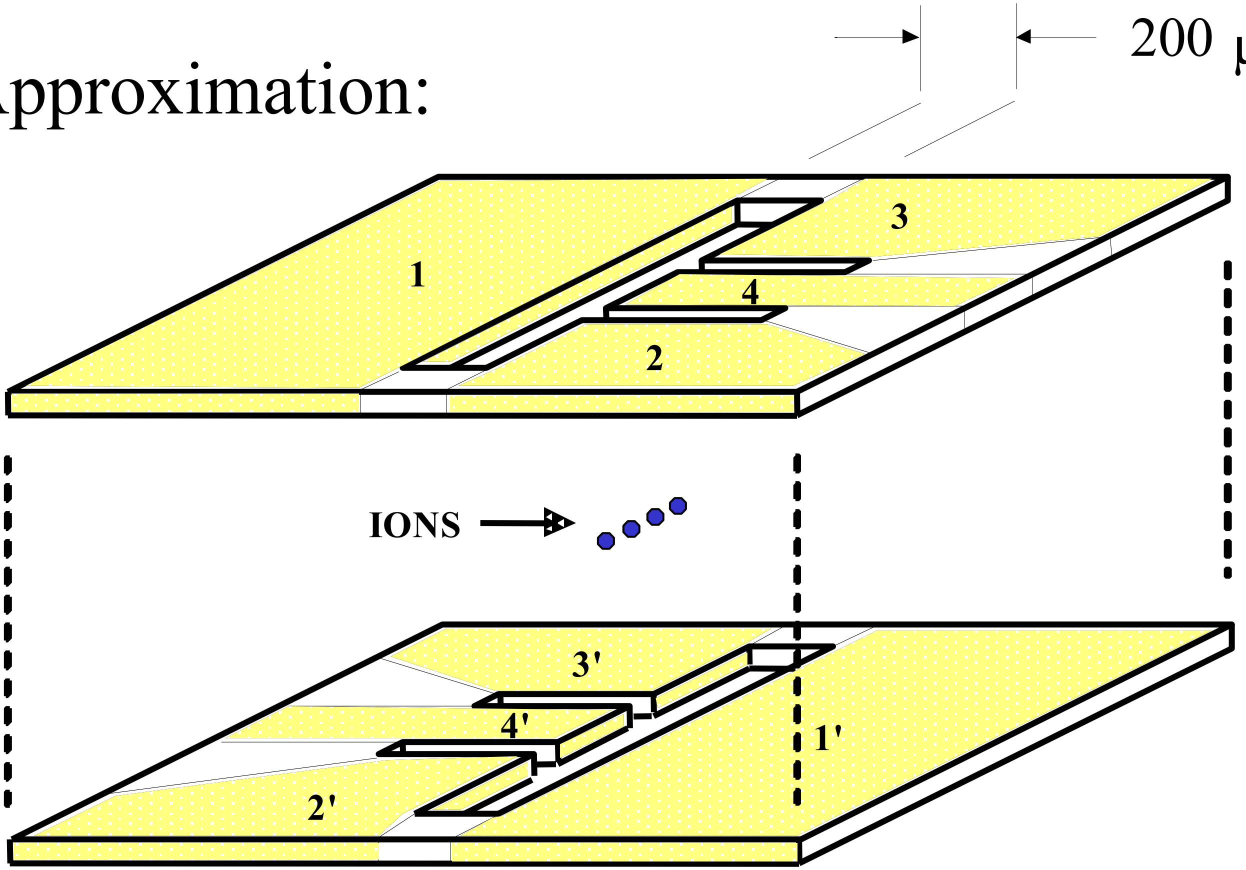
(RF quadrupole mass analyzer with trapping zones)



RF electrodes

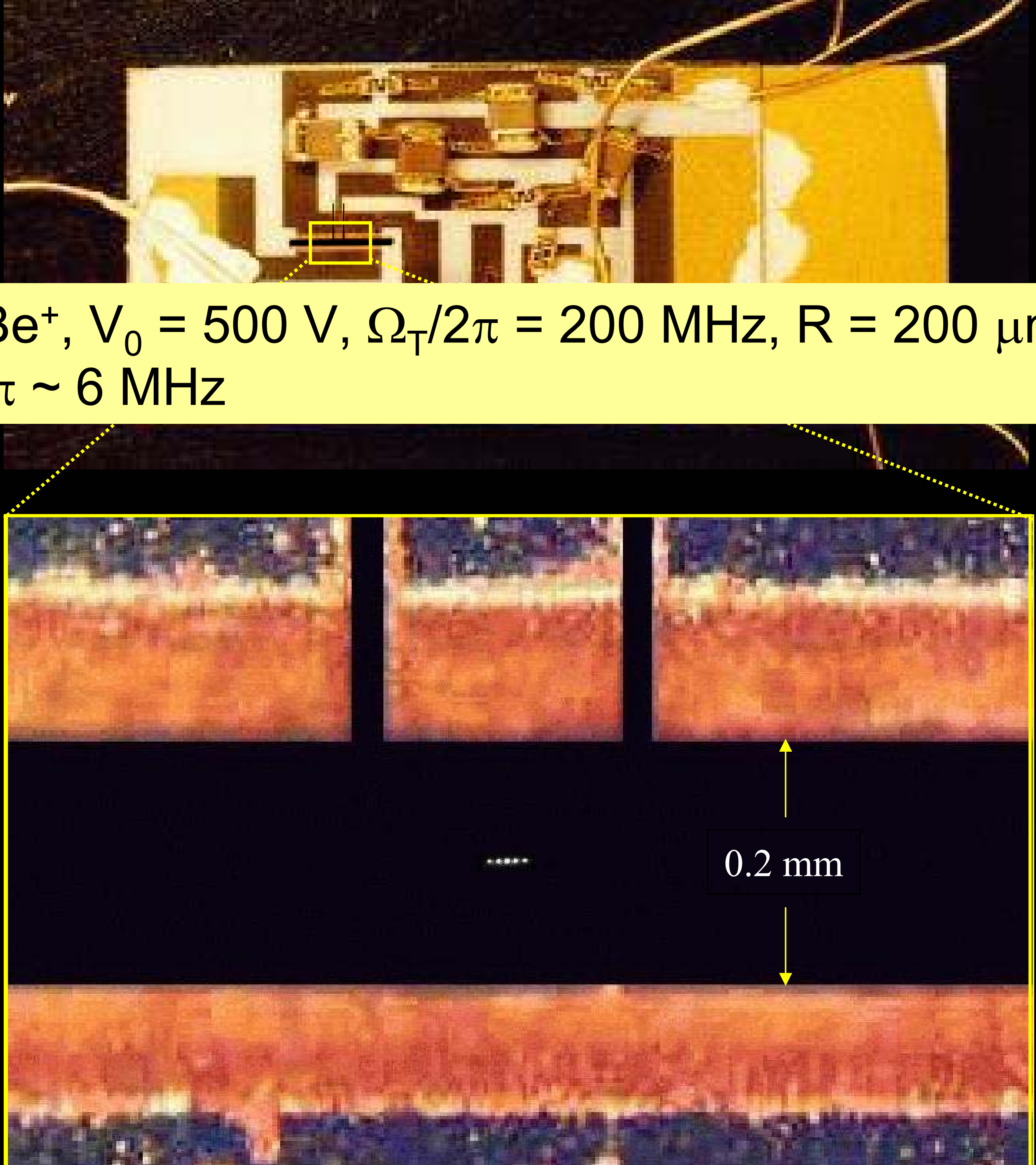
control electrodes

Approximation:



200 µm

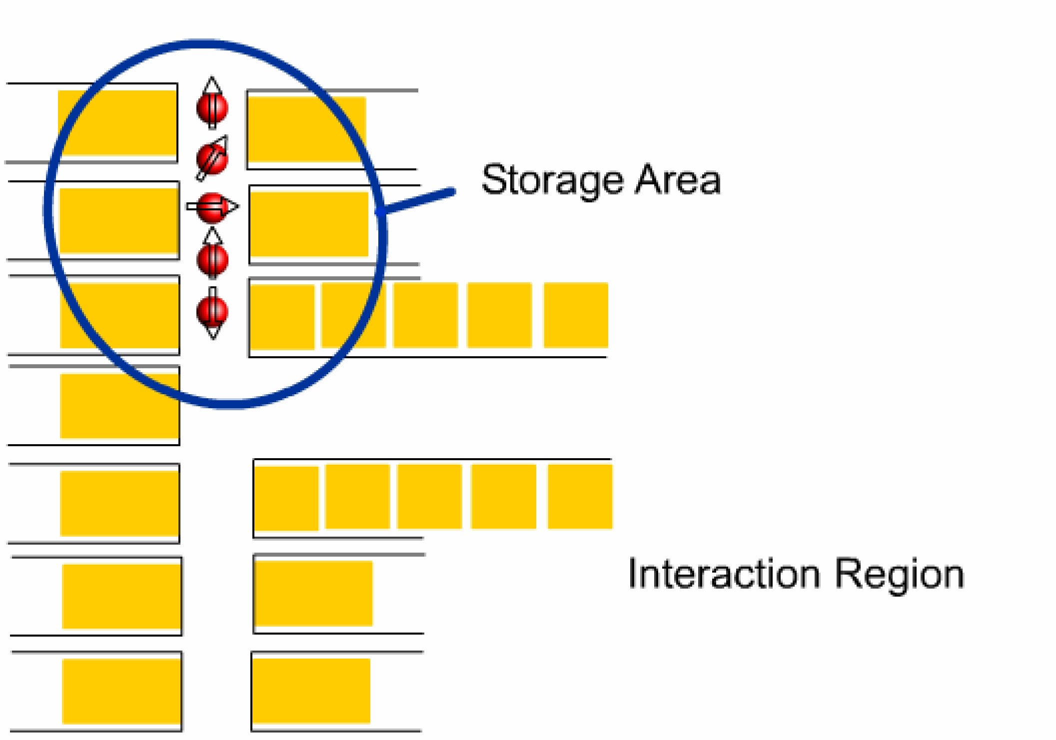
gold-coated alumina wafers



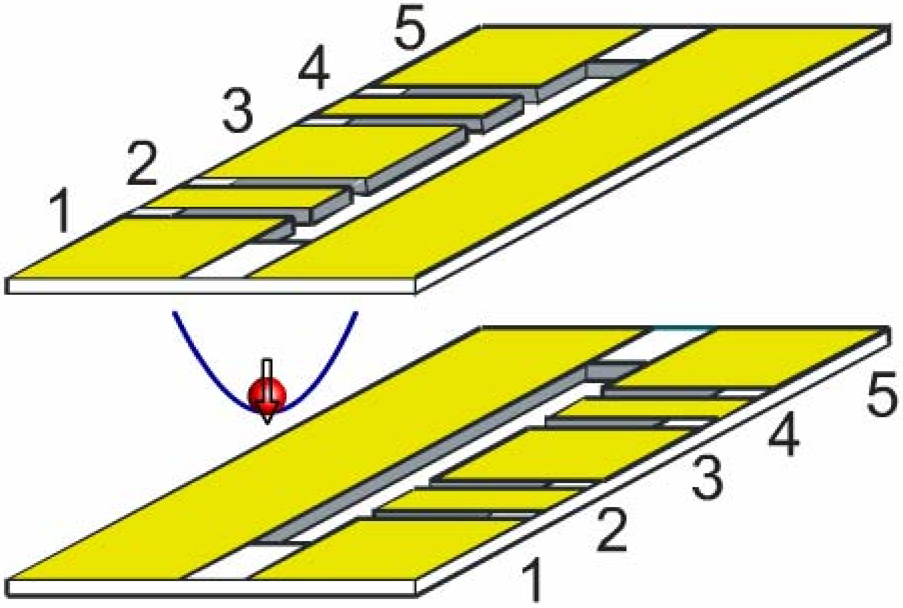
For $^{9}\mathrm{Be}^{+}$, $V_0 = 500$ V, $\Omega_T/2\pi = 200$ MHz, $R = 200\ \mu\mathrm{m}$
$\omega_{x,y}/2\pi \sim 6$ MHz

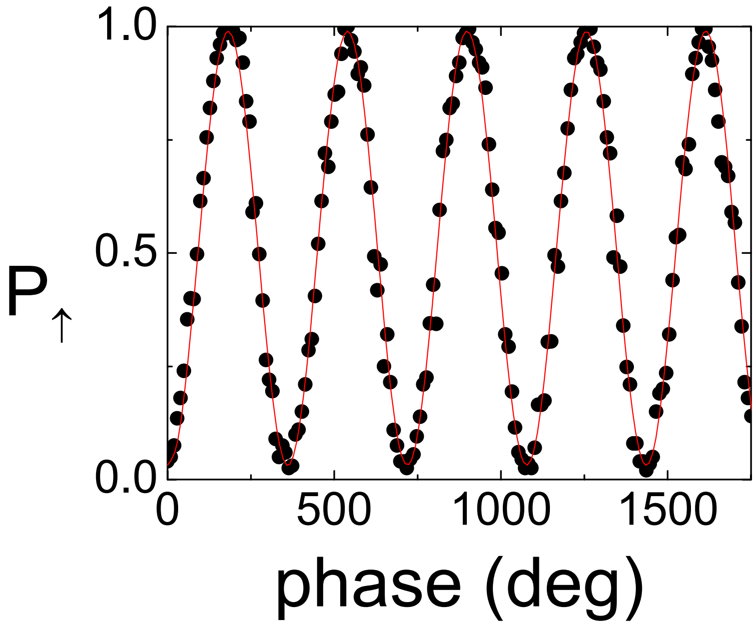
Multiplexing scheme

(DJW *et al.*, NIST J. Res., ’98; Dave Kielpinski *et al. Nature*, ’02)



- Move qubits
- Separate qubits
- Logic Gates
- Sympathetic Cooling



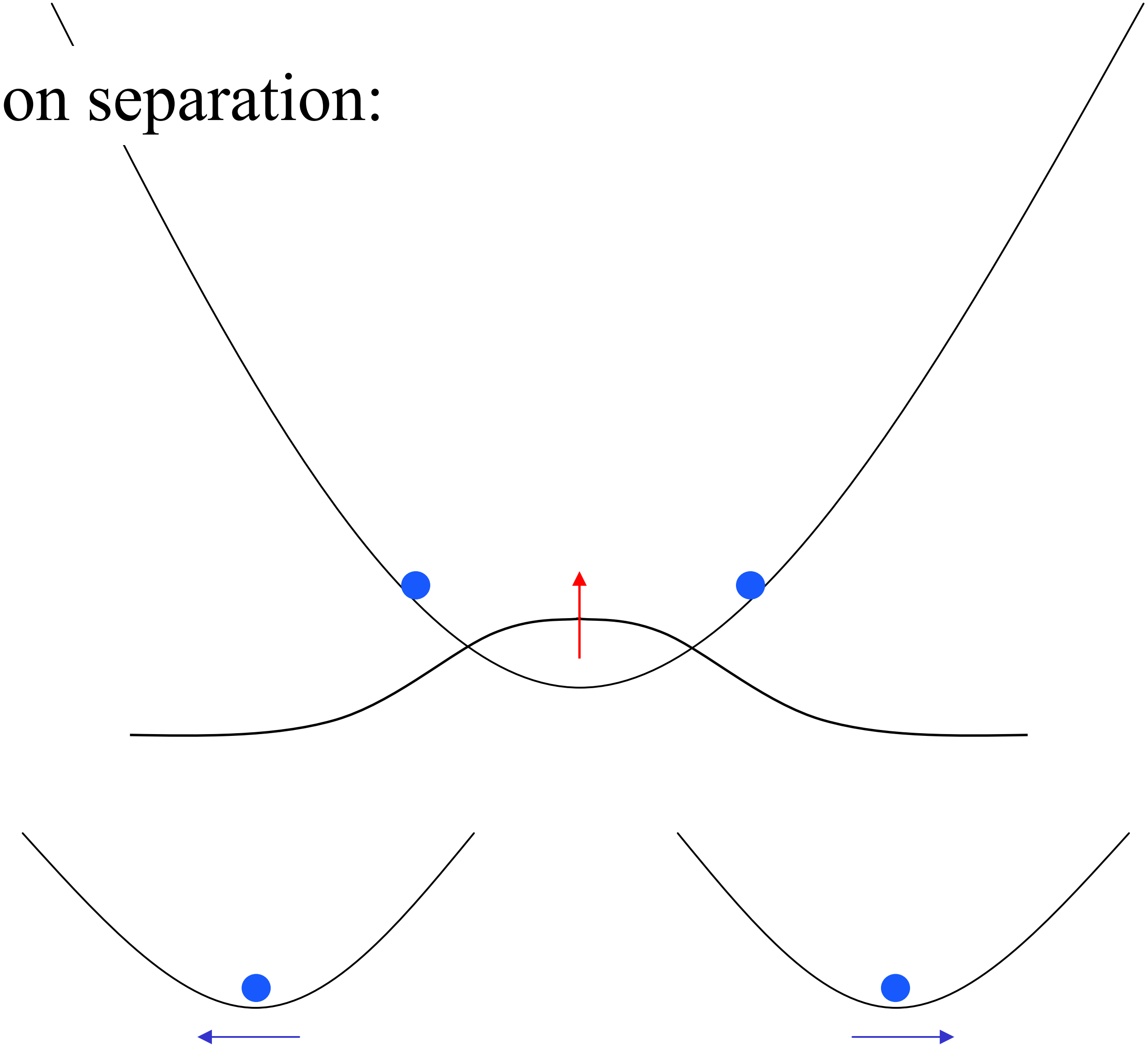


Initial results

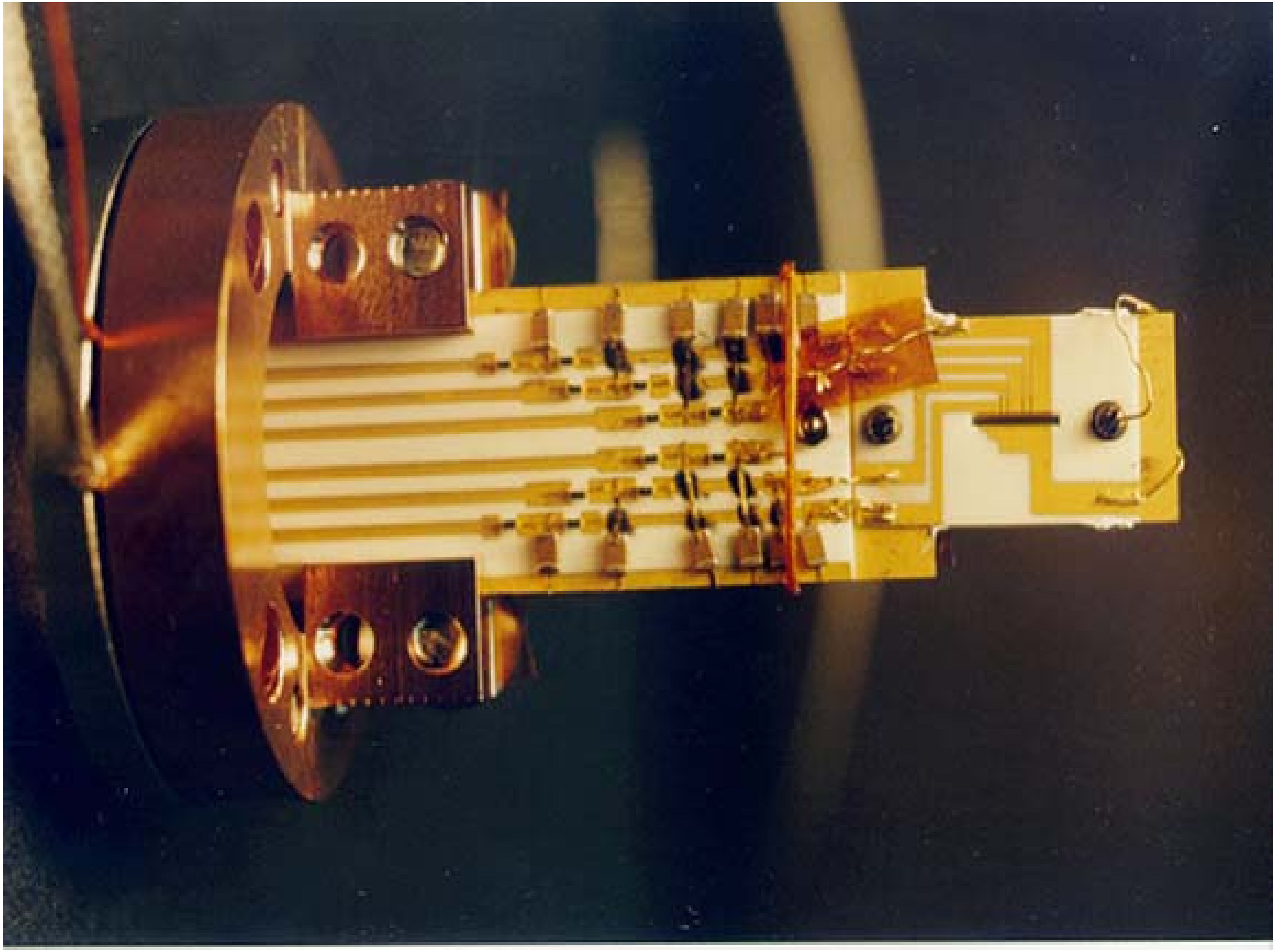
- τ(transfer) ≅ 25 μs (motion heating < 1 quantum)
- qubit coherence preserved during transfer (0.5 % measurement accuracy)
- robust (no loss observed from transfer; > 10^6 consecutive transfers typical)
- two ions “split” to separate traps

M.A. Rowe *et al*., Quant. Info. Compt., **4**, 257 (2002).

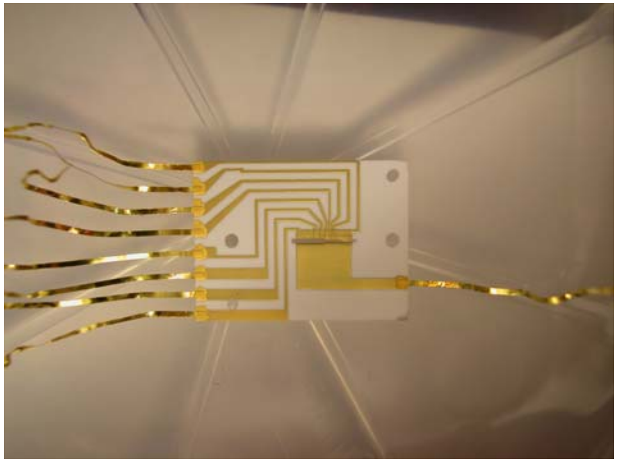
Ion separation:

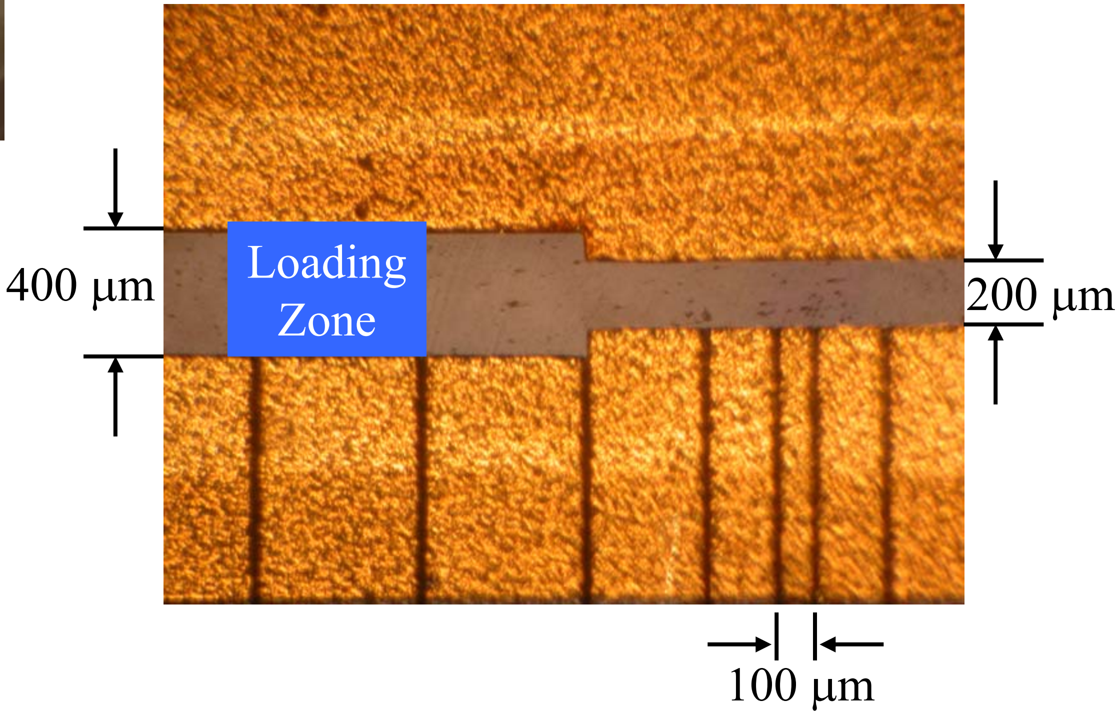


3-zone trap (Mary Rowe *et al.*)



6-zone trap, $^{9}\mathrm{Be}^{+}$ & $^{24}\mathrm{Mg}^{+}$ ions





- Separate loading zone
- Smaller features
- New coatings to be tested

(Murray Barrett, Tobias Schaetz)

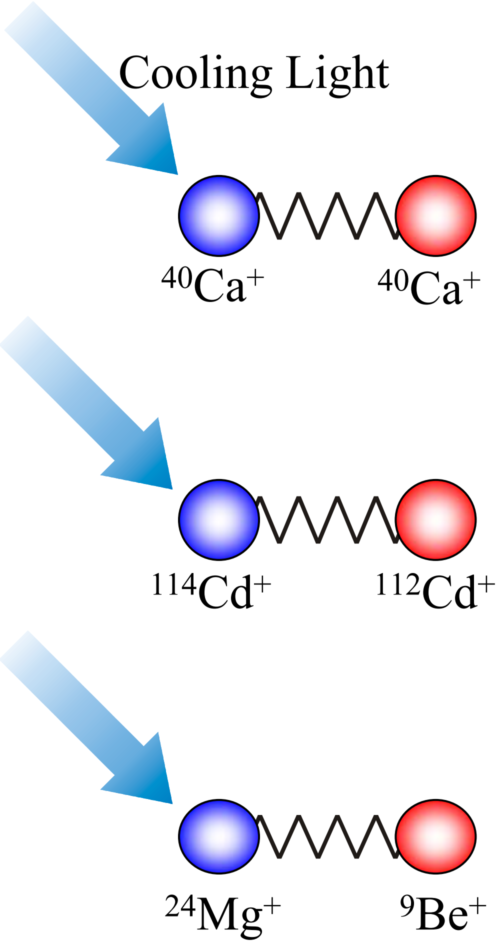
Sympathetic Cooling

Approaches:

Cooling with same species
Innsbruck group: Rhode, *et al.*,
J. Opt. B **3**, S34 (2001)

Cooling with different isotopes
Michigan group: Blinov, *et al.*,
PRA **65**, 040304 (2002)

Cooling with different ion species
NIST, Murray Barrett *et al.*



Cooling Light

$^{40}Ca^+$ $^{40}Ca^+$

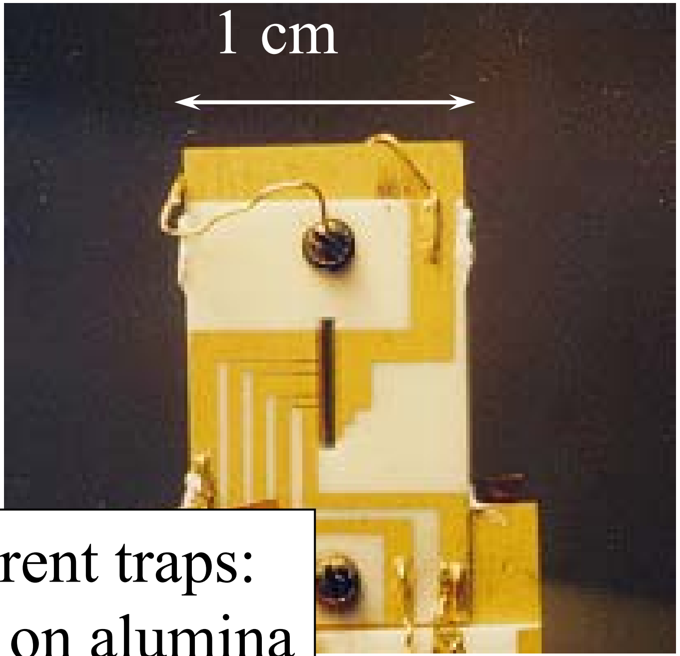
$^{114}Cd^+$ $^{112}Cd^+$

$^{24}Mg^+$ $^{9}Be^+$

Trapology:

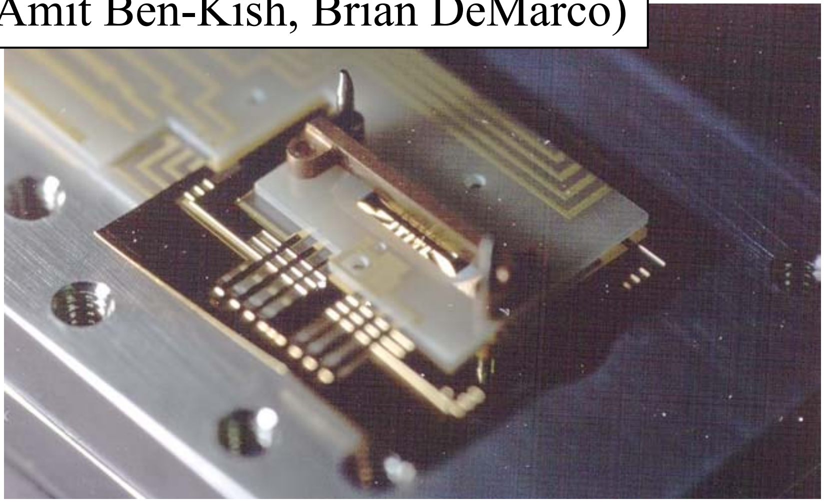
Requirements:

- small (~ 100 μm electrode separations)
- no RF breakdown (~ 500 V, ~ 100 MHz)
- no RF loss
- high-vacuum (~ 10^{-11} Torr)
- bakeable (~ 350° C)
- CLEAN electrodes



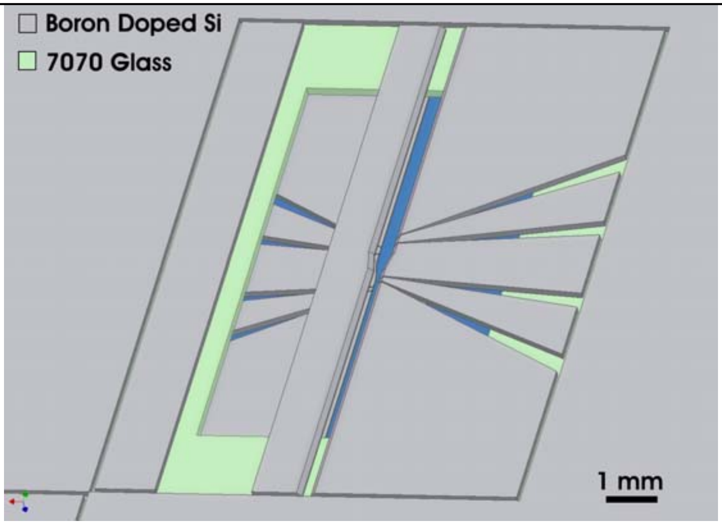
current traps:
gold on alumina

“gold leaf” trap
(Amit Ben-Kish, Brian DeMarco)



MEMS (John Chiaverini)

silicon-based (Joe Britton, Dave Kielpinski)



Simple applications of quantum processing ideas?

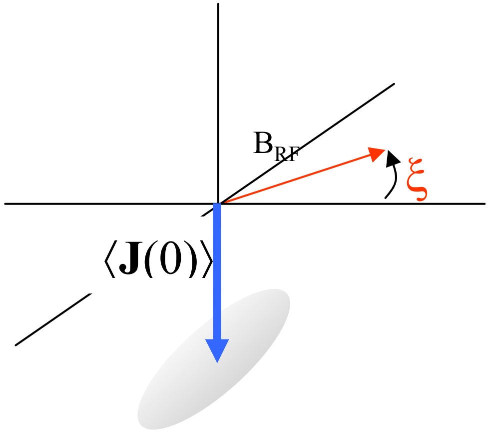
Spin-squeezing ($\mathbf{J} = \Sigma_i \mathbf{S}_i$, S = ½)

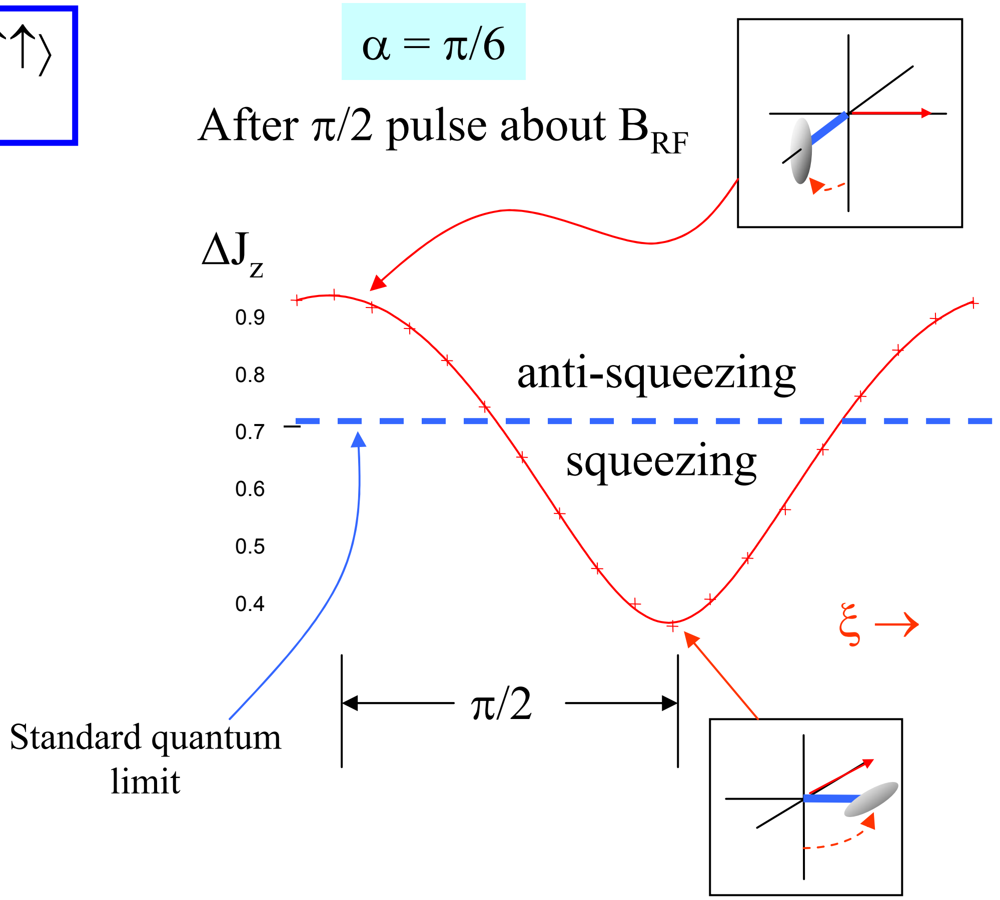
⇒ improved rotation angle estimation

for 2 spins: $|\downarrow\downarrow\rangle \rightarrow \cos\alpha|\downarrow\downarrow\rangle + \sin\alpha|\uparrow\uparrow\rangle$

$H_{int} \propto J_x^2$ (Sørensen & Mølmer, '00)

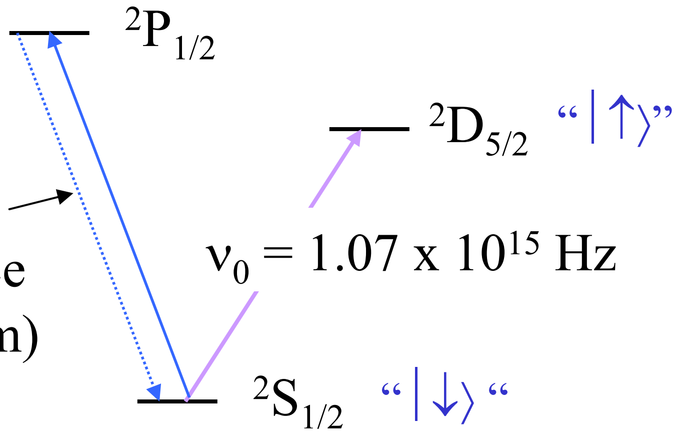
Experiment (N = 2):
(Volker Meyer *et al.*, '01





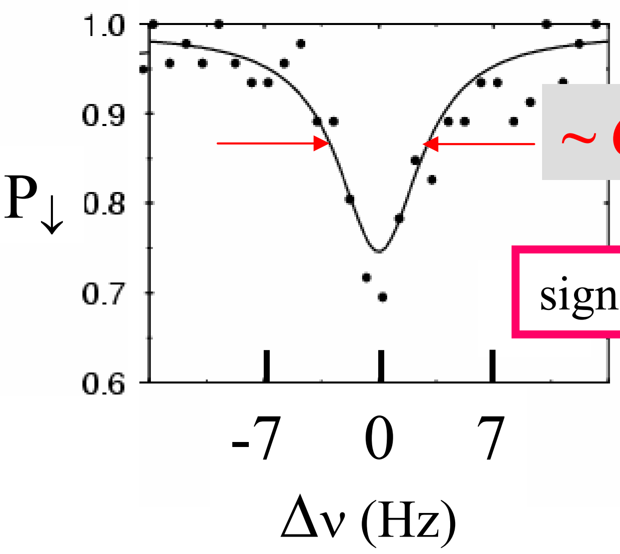
- applied to spectroscopy
- simulation of (photon) dual-Fock state interferometer (a la Holland & Burnett, Kasevich, …)

single $^{199}Hg^+$-ion optical frequency standard (Jim Bergquist *et al.*)



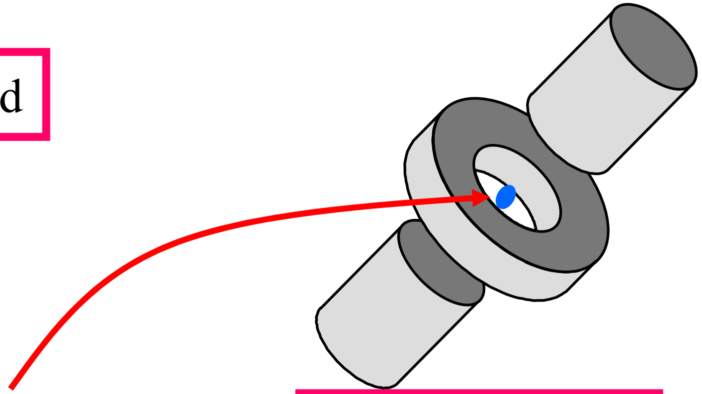
Observe fluorescence (λ = 194 nm)

$\nu_0 = 1.07 \times 10^{15}$ Hz



~ 6.5 Hz, Q = 1.6×10^{14}

signal is lifetime limited



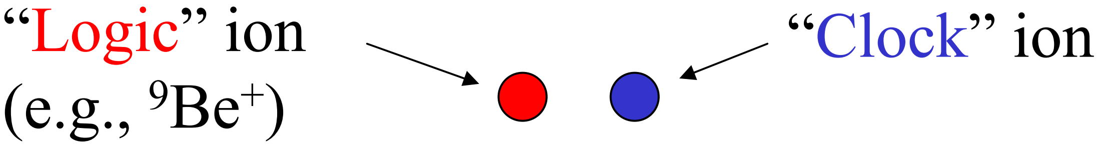
$^2D_{5/2}$ state has quadrupole moment.

quadrupole shift

Measure ν_0 along three B-field directions (Wayne Itano)

Quantum information processing and clocks

Basic idea (2 trapped ions): (re: Dan Heinzen & D.J.W., PRA**42**, 2977 (1990))



"Logic" ion
(e.g., $^9Be^+$)

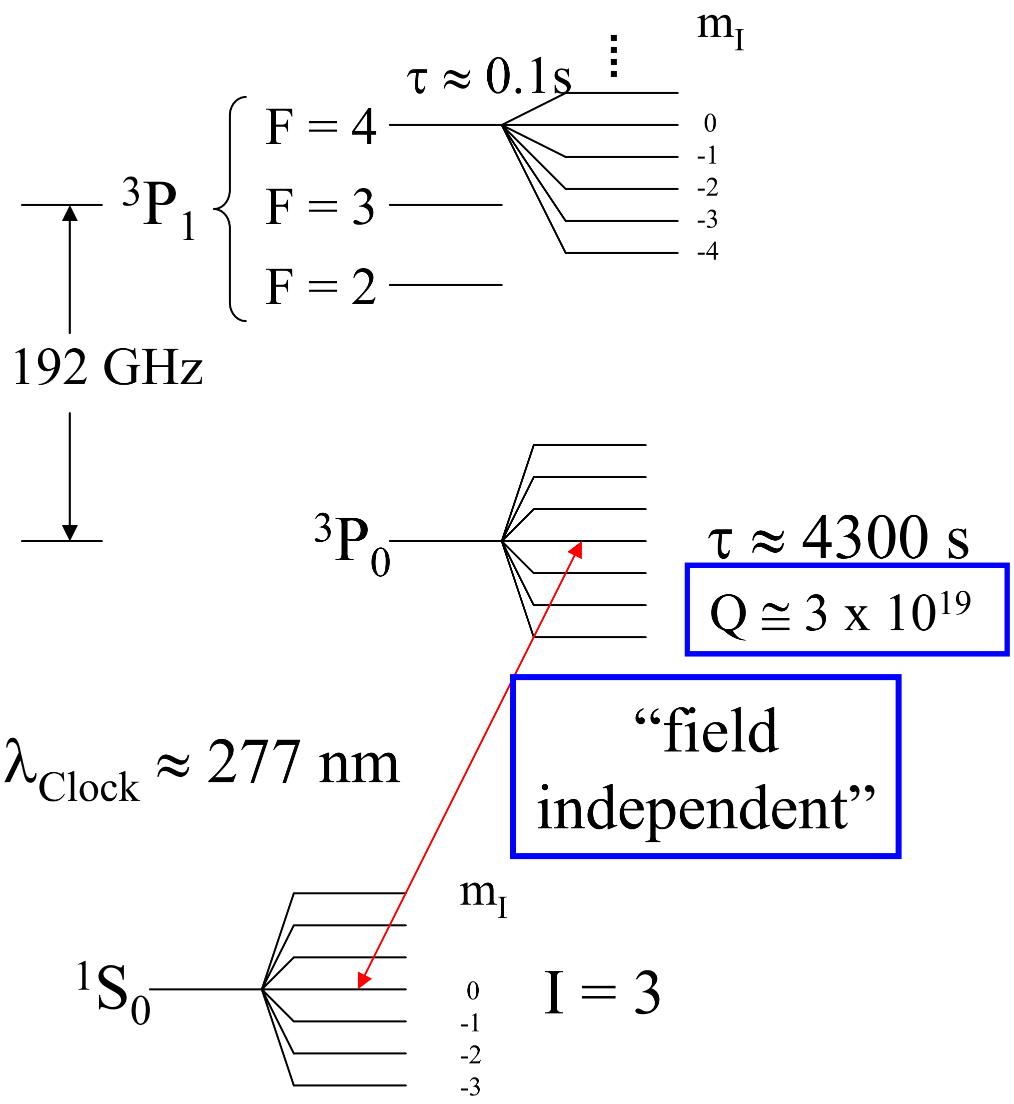
"Clock" ion

- (Sympathetically) cool and detect Clock ion with Logic ion

Example:

$^{10}B^+ \quad {}^1S_0 \rightarrow {}^3P_0$

- no quadrupole shift
- lots of headroom on lifetime



$\tau \approx 4300$ s

$Q \cong 3 \times 10^{19}$

“field independent”

$\lambda_{Clock} \approx 277$ nm

I = 3

Future:

- multiplexed traps: ion separation, sympathetic re-cooling, more qubits
- multi-ion experiments: need to assemble all steps:
    - * repetitive error correction, …
- applications: e.g., atomic clocks, “spin squeezing”, ….
- fundamental: decoherence, measurement problem, …

Other “recent” work:

- Decoherence-free subspace (DFS) qubit encoding (Dave Kielpinski *et al.*, Science, ’01)
- Bell’s inequalities (two ^{9}Be$^+$ ions); “detection loophole” closed (Mary Rowe *et al.*, *Nature*, ’01)
- “Spin-squeezing” and application to spectroscopy (Volker Meyer *et al.*, PRL, ’01)
- quantum simulation: nonlinear Mach-Zehnder interferometers (PRL, Dec., ‘02)
- Controlled-NOT “wave packet” gate (PRL, Dec., ‘02)
- demonstration of Law/Eberly (PRL, ’96) arbitrary state generation technique (PRL, Jan., ‘03)
- high-fidelity π phase gate (*Nature,* March, ‘03)
- sympathetic ground-state cooling, ^{9}Be$^+$ + ^{24}Mg$^+$ (submitted for publication)

Quantum information processing in ion traps II

D. J. Wineland, NIST, Boulder

Lecture 1: Nuts and bolts

- Ion trapology
- Qubits based on ground-state hyperfine levels
- Two-photon stimulated-Raman transitions
    - * Rabi rates, Stark shifts, spontaneous emission

Lecture 2: Quantum computation (QC) and quantum-limited measurement

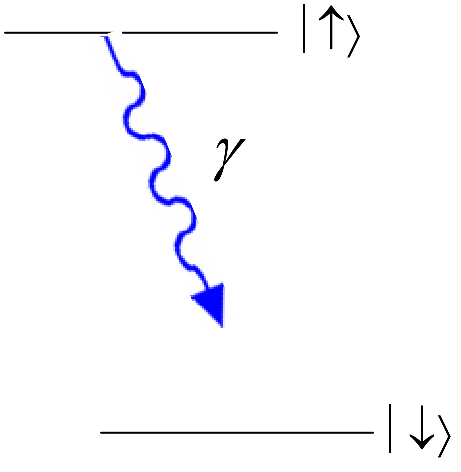
- Trapped-ion QC and DiVincenzo's criteria
- Gates
- Scaling
- Entanglement-enhanced quantum measurement

Lecture 3: Decoherence

- Memory decoherence
- Decoherence during operations
    - * technical fluctuations
    - * spontaneous emission
    - * scaling
- Decoherence and the measurement problem

Memory coherence (motion factors out before and after gates):

fundamental limit
spontaneous emission (τ_1, τ_2):



allowed electric-dipole transition:

$$\gamma = \frac{4e^2\omega_0^3}{(2J_\uparrow + 1)3\hbar c^3} |\langle J_\downarrow || r^{(1)} || J_\uparrow \rangle|^2$$

^{9}Be$^+$ (first optical transition):
ω_0(optical) /$2\pi \cong 0.96\times10^{15}$ Hz, $\tau = 1/\gamma = 8.2$ ns

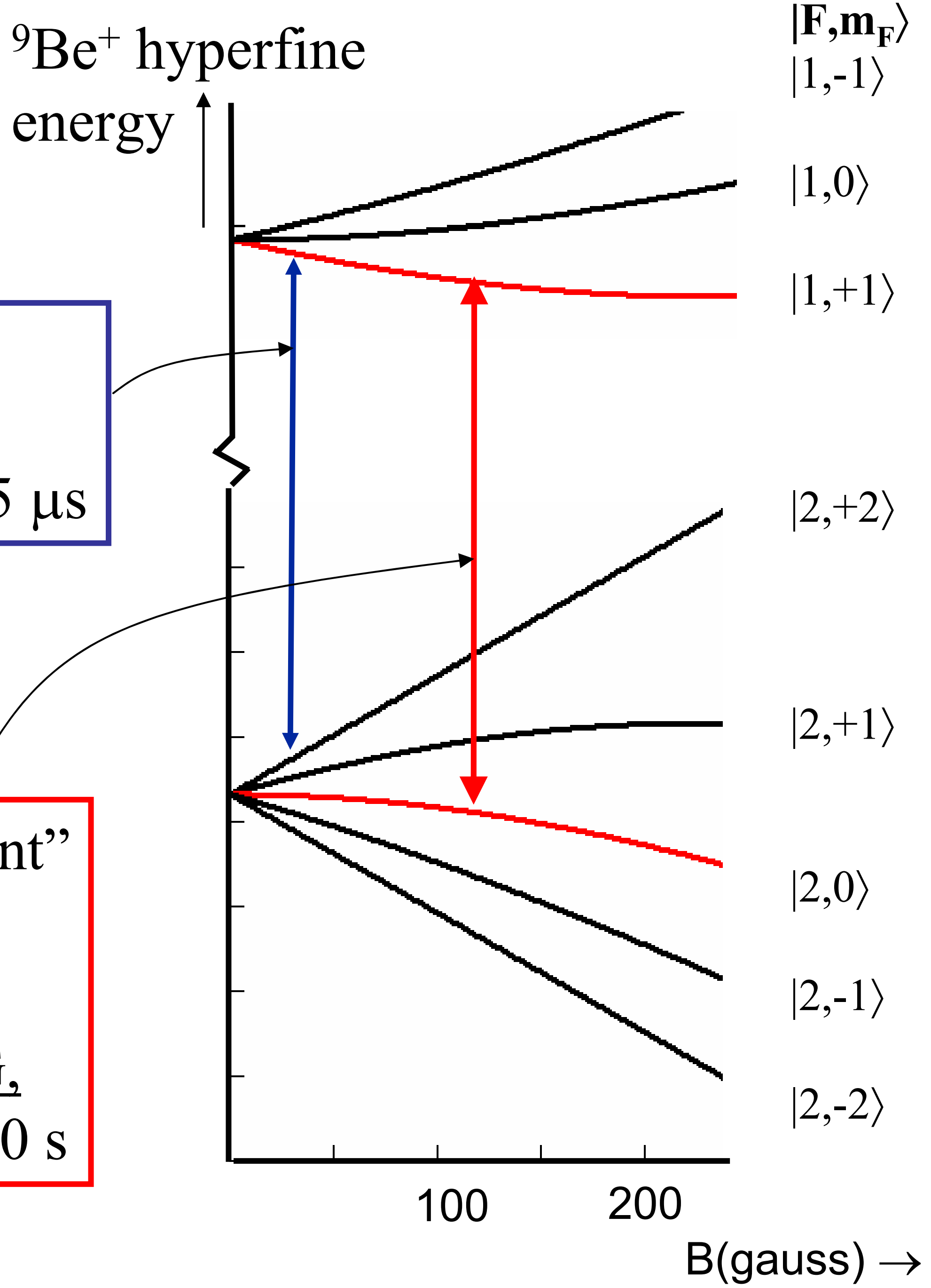
^{9}Be$^+$ (ground-state hyperfine transition):
$\omega_0/2\pi$ (hyperfine) = 1.25 GHz

$$\tau(hyperfine) \cong \tau(optical)\left[\frac{\omega_0(optical)}{\omega_0(hyperfine)}\right]^3 \frac{1}{\alpha^2}$$

fine structure
constant

for ^{9}Be$^+$, τ(hyperfine) $\cong 7\times10^{13}$ s
$\cong 2\times10^6$ yr

Dephasing (τ_2): magnetic-field fluctuations



for $\delta B = 0.001$ G

$|2,2\rangle \leftrightarrow |1,1\rangle$

$\tau(\delta\phi = 1 \text{ rad}) \cong 75\ \mu s$

“field-independent” at 119.5 gauss

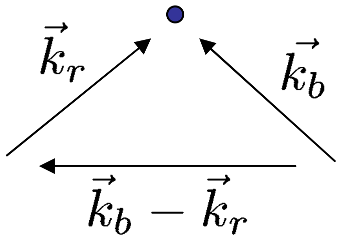
for $\delta B = 0.001$ G,

$\tau(\delta\phi = 1 \text{ rad}) \cong 50$ s

Dephasing during gates:

$$\Omega_{n,n'} \equiv \Omega\langle n|e^{-i(\vec{k}_b-\vec{k}_r)\cdot\vec{X}}|n'\rangle = \Omega\langle n|e^{-i\eta(a+a^\dagger)}|n'\rangle = \Omega_{n',n}$$

motional-state fluctuations (e.g., heating)



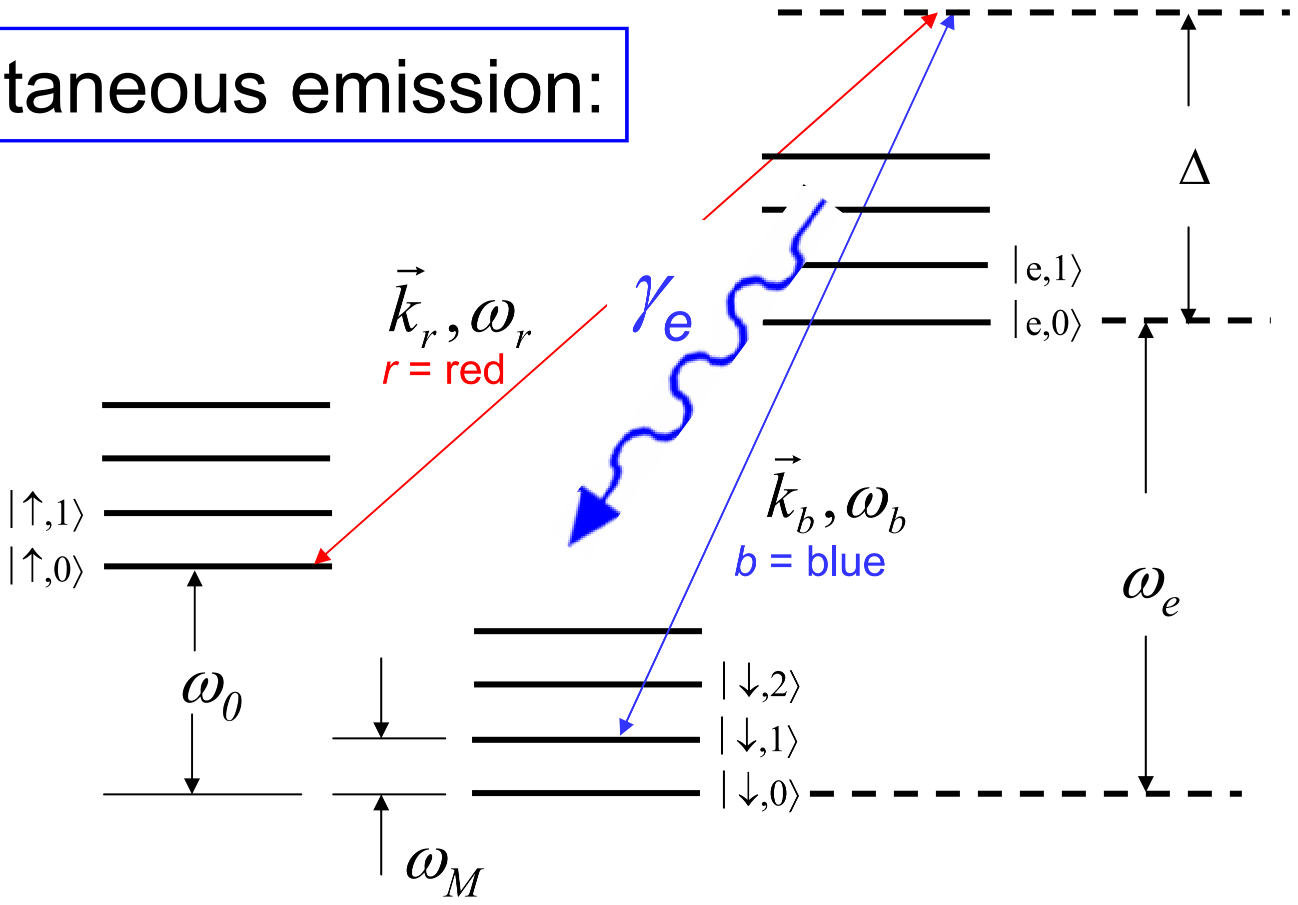
intensity
fluctuations

$$\Omega \equiv g_b g_r^*/\Delta = \langle\downarrow|\hat{\epsilon}_b\cdot\vec{r}\,|e\rangle\langle e|\hat{\epsilon}_r\cdot\vec{r}\,|\uparrow\rangle\frac{e^2E_{b0}E_{r0}}{4\hbar^2\Delta}e^{i(\phi_r-\phi_b)}$$

polarization
fluctuations

phase fluctuations
e.g., path length
fluctuations between
Raman beams

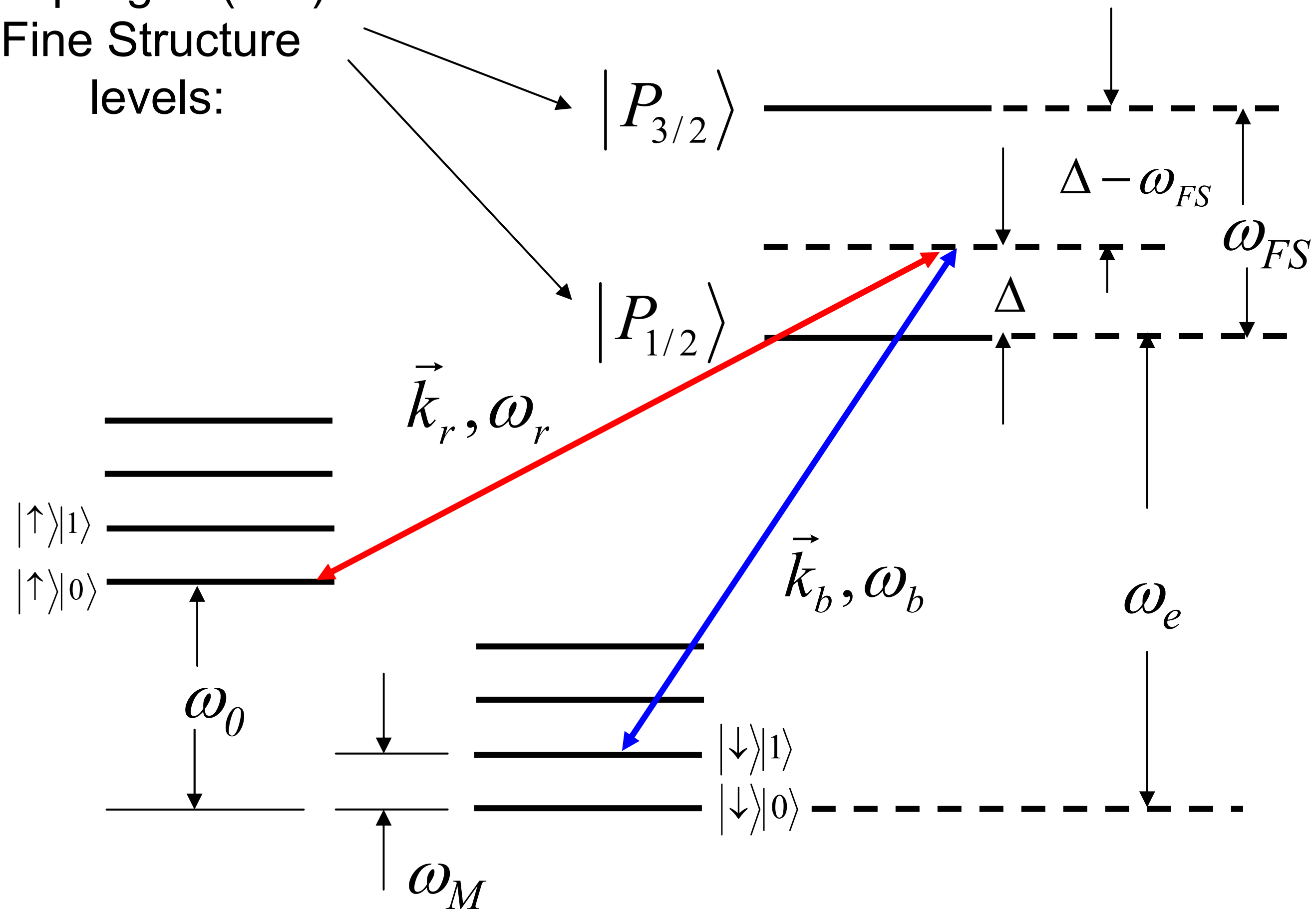
Spontaneous emission:



$$R_{SE} = \gamma_e \sum_{m=0}^{\infty} |C_{e,m}|^2$$

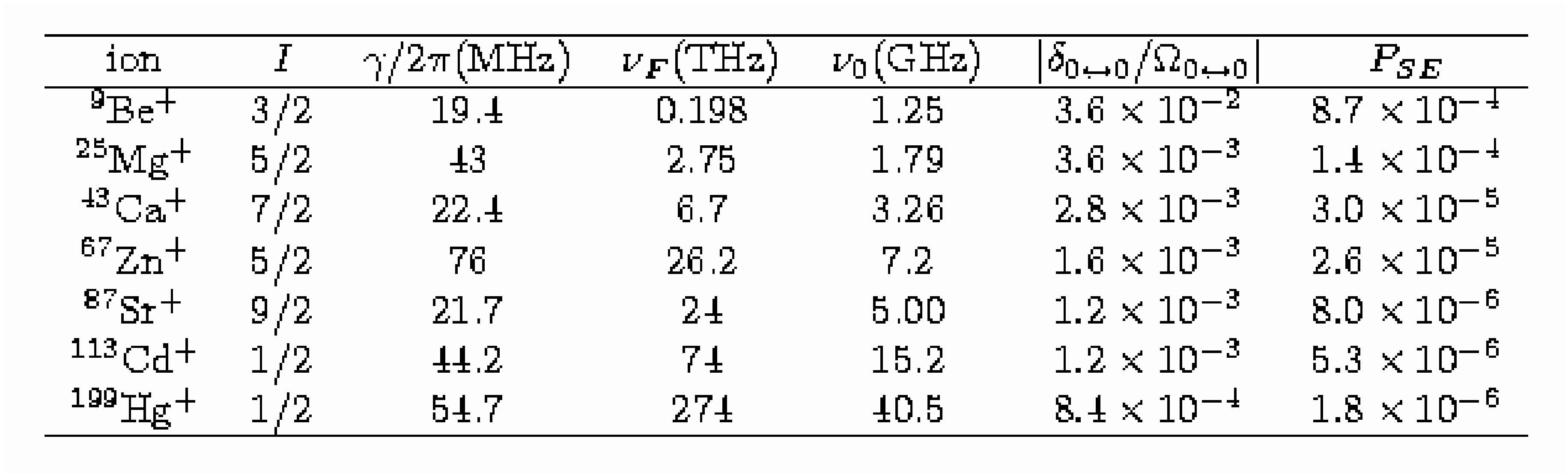
$$\simeq \gamma_e \sum_{n=0}^{\infty} \left[|C_{\downarrow,n}|^2 \left(\frac{|g_b|^2}{\Delta^2} + \frac{|g_{\downarrow,e,r}|^2}{(\Delta - \omega_0)^2} \right) + |C_{\uparrow,n}|^2 \left(\frac{|g_r|^2}{\Delta^2} + \frac{|g_{\uparrow,e,b}|^2}{(\Delta - \omega_0)^2} \right) \right]$$

For most ions,
coupling to (two)
Fine Structure
levels:



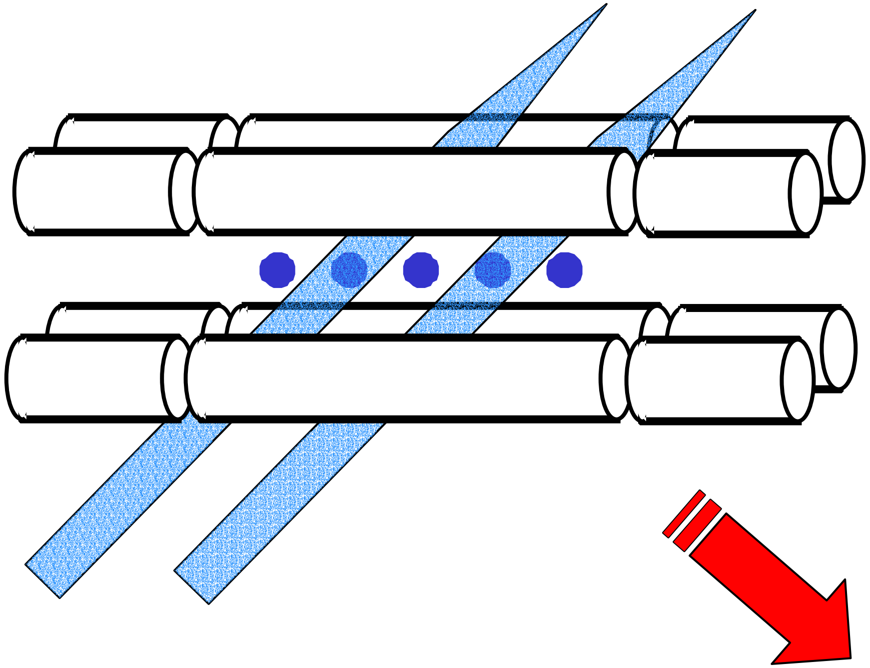
Spontaneous emission during π pulse on carrier of $|F = I - ½, m_F = 0\rangle \rightarrow |F = I + ½, m_F = 0\rangle$ transitions

I = nuclear spin

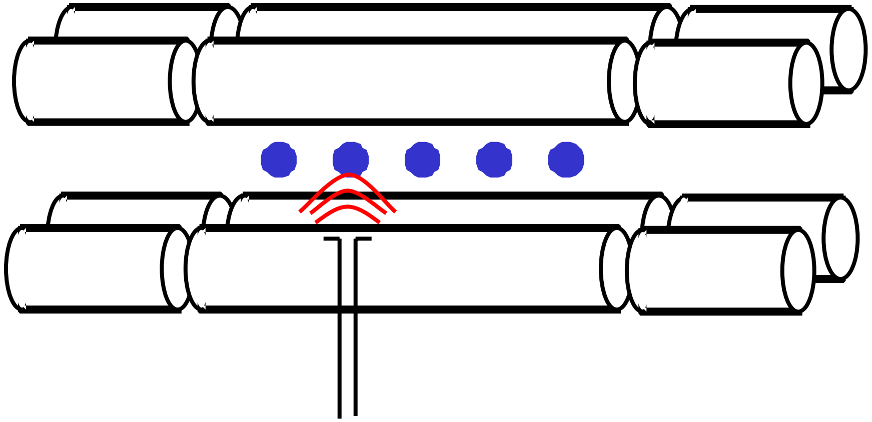


ion	I	$\gamma/2\pi$(MHz)	ν_F(THz)	ν_0(GHz)	$\lvert\delta_{0\leftrightarrow 0}/\Omega_{0\leftrightarrow 0}\rvert$	P_{SE}
^{9}Be$^+$	3/2	19.4	0.198	1.25	3.6×10^{-2}	8.7×10^{-4}
^{25}Mg$^+$	5/2	43	2.75	1.79	3.6×10^{-3}	1.4×10^{-4}
^{43}Ca$^+$	7/2	22.4	6.7	3.26	2.8×10^{-3}	3.0×10^{-5}
^{67}Zn$^+$	5/2	76	26.2	7.2	1.6×10^{-3}	2.6×10^{-5}
^{87}Sr$^+$	9/2	21.7	24	5.00	1.2×10^{-3}	8.0×10^{-6}
^{113}Cd$^+$	1/2	44.2	74	15.2	1.2×10^{-3}	5.3×10^{-6}
^{199}Hg$^+$	1/2	54.7	274	40.5	8.4×10^{-4}	1.8×10^{-6}

D.J.W. *et al.,* Phil. Trans. R. Soc. Lond. A**361**, 1349 (2003)



difficult to obtain
required field gradients

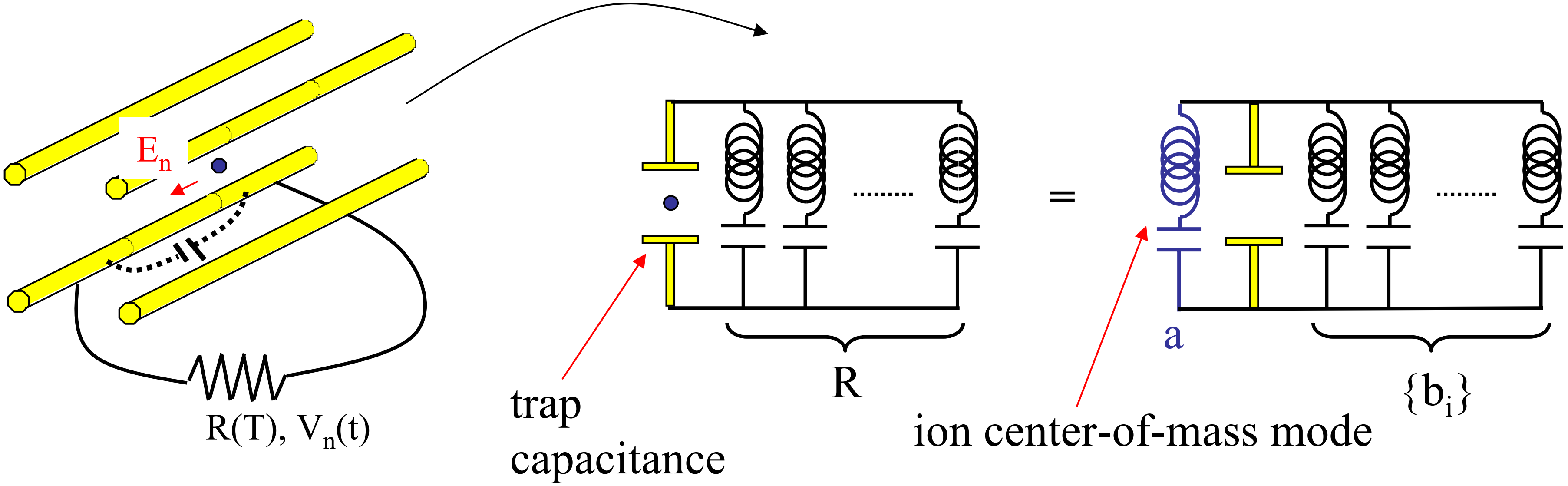


Replace lasers with RF?
- D.J.W. *et al.* PRA, ’92
- Mintert & Wunderlich, PRL, ’01
- Ciaramicoli, Marzoli,
  Tombesi, PRL, ‘03

no spontaneous emission!

Motional decoherence (heating):

thermal electronic noise: Black body radiation, Johnson noise, …



trap capacitance

ion center-of-mass mode

<u>“amplitude reservoir”</u>

$$H_I \propto \Sigma_i \{\Gamma_i (a b_i^\dagger + a^\dagger b_i)\}$$

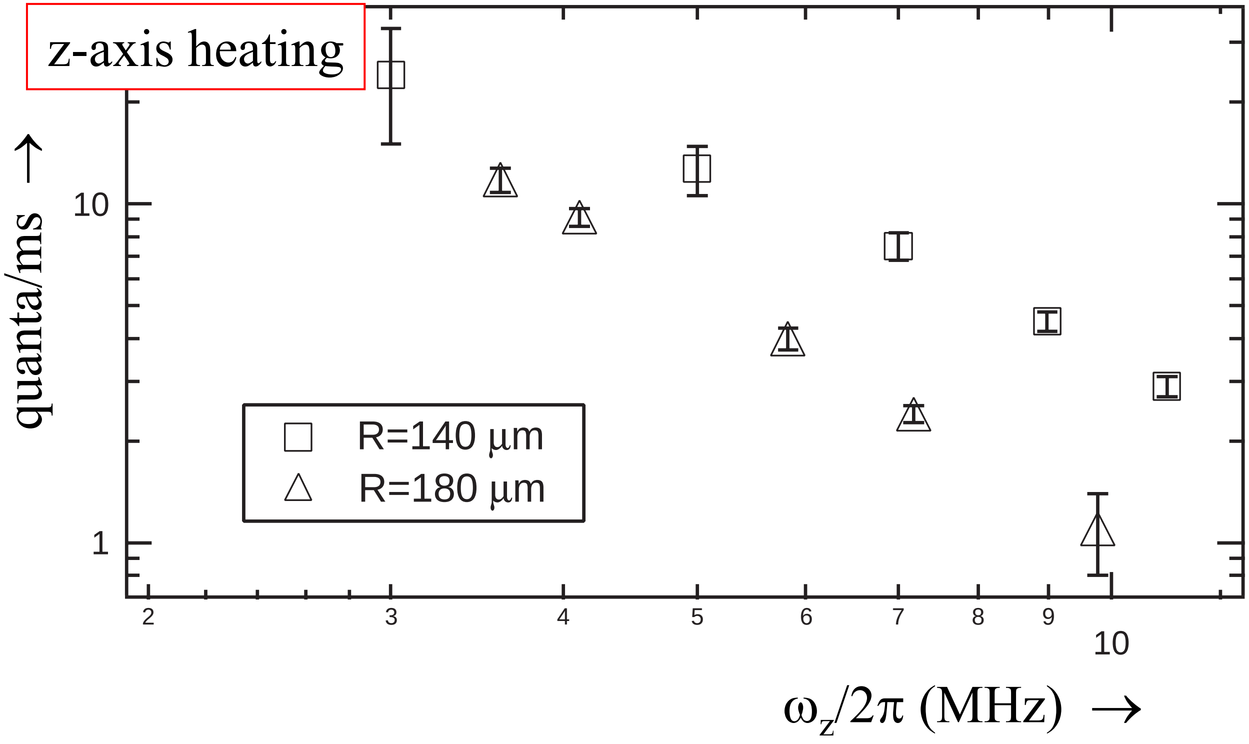
ion oscillator

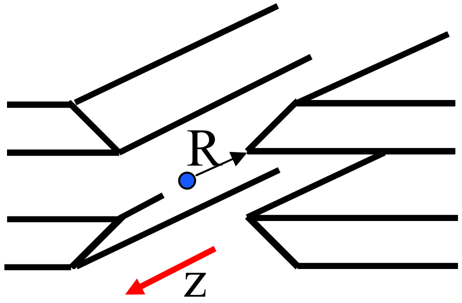
environment oscillators (R)

Heating in linear traps

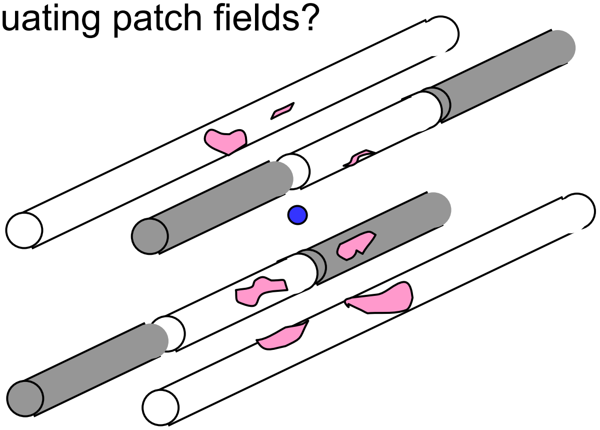
(not thermal electronic noise)

Turchette *et al.*, PRA **61**, 063418 (2000)



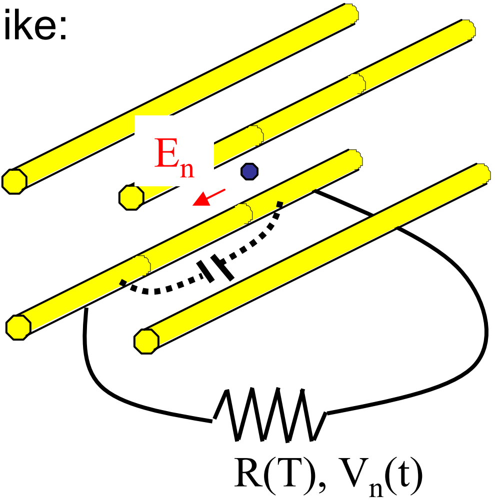


fluctuating patch fields?



R = 270 µm, shielded electrodes
(M. Rowe *et al.*, '01)

If thermal relaxation, $\tau \approx$ 4 hours looks like:



$R \approx 1\ \Omega$, $T >> 10^6$ K !

to study:

With T >> 300 K, could, in principle, measure noise and correct for it

or:

Apply noisy potentials

Decoherence formalism:

(overview: W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003))

System: harmonic oscillator: e.g. superpositions $|\psi_{osc}\rangle = \Sigma_n c_n |n\rangle$ coupled to environment $|\phi_e\rangle$

$$|\psi_0\rangle = |\psi_{osc}\rangle \otimes |\phi_e\rangle = (\alpha|\psi_1\rangle + \beta|\psi_2\rangle) \otimes |\phi_e\rangle \rightarrow \alpha|\psi_1'\rangle|\phi_{e1}\rangle + \beta|\psi_2'\rangle|\phi_{e2}\rangle$$

if $\langle\phi_{e1}|\phi_{e2}\rangle = 0$, and if $|\phi_{ei}\rangle$ unmeasured or unmeasurable,

$$\rho_0 = |\psi_{osc}\rangle\langle\psi_{osc}| \rightarrow |\alpha|^2|\psi_1\rangle\langle\psi_1| + |\beta|^2|\psi_2\rangle\langle\psi_2| \quad (|\psi_1'\rangle = |\psi_1\rangle,\ |\psi_2'\rangle = |\psi_2\rangle)$$

Incude quantum “meter:”

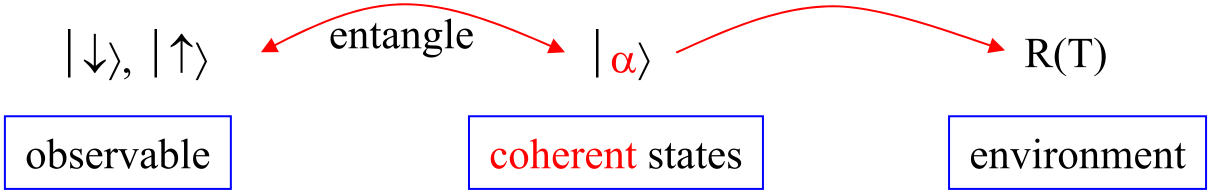
$$(\alpha|\psi_1\rangle + \beta|\psi_2\rangle)\otimes|\psi_M\rangle \otimes |\phi_e\rangle \rightarrow (\alpha|\psi_1'\rangle|\psi_{M1}\rangle + \beta|\psi_2'\rangle|\psi_{M2}\rangle) \otimes |\phi_e\rangle$$
$$\rightarrow \alpha|\psi_1''\rangle|\psi_{M1}'\rangle|\phi_{e1}\rangle + \beta|\psi_2''\rangle|\psi_{M2}'\rangle|\phi_{e2}\rangle$$

- if $\langle\phi_{e1}|\phi_{e2}\rangle = 0$, and $|\phi_{ei}\rangle$ unmeasured or unmeasurable,

$$\rho_0 \rightarrow |\alpha|^2|\psi_1'\rangle\langle\psi_1'|\,|\psi_{M1}\rangle\langle\psi_{M1}| + |\beta|^2|\psi_2'\rangle\langle\psi_2'|\,|\psi_{M2}\rangle\langle\psi_{M2}|$$

- or, if $|\phi_{e2}\rangle \cong |\phi_{e1}\rangle \cong |\phi_e\rangle$, but if $|\phi_{ei}\rangle$ unmeasured or unmeasurable, average over $\{|\phi_{ei}\rangle\}$

ion experiments: $|\psi_{osc}\rangle$ = mode of ion motion, $|\psi_M\rangle$ = spin (internal state)



$|\downarrow\rangle, |\uparrow\rangle$ entangle $|\alpha\rangle$ $R(T)$

observable

coherent states

environment

$$(\alpha|\psi_1'\rangle|\psi_{M1}\rangle + \beta|\psi_2'\rangle|\psi_{M2}\rangle) \otimes |\phi_e\rangle$$
$$\to \alpha|\psi_1''\rangle|\psi_{M1}\rangle|\phi_{e1}\rangle + \beta|\psi_2''\rangle|\psi_{M2}\rangle\,|\phi_{e2}\rangle$$

Simulate $V_n(t)$ with applied noisy potentials

$\Rightarrow$ small R, high temperature

$|\phi_{e2}\rangle \cong |\phi_{e1}\rangle \cong |\phi_e\rangle$, but if $|\phi_{ei}\rangle$ unmeasured or unmeasurable, average over $\{|\phi_{ei}\rangle\}$

To see, construct (Ramsey) interferometer:

Amplitude reservoir / coherent states |α⟩ (single ion)

🔴 $= |\downarrow\rangle$ 🟢 $= |\uparrow\rangle$

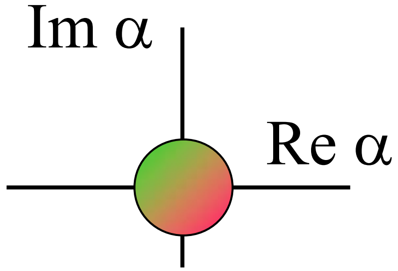
$$\Psi = |\downarrow\rangle \otimes |\alpha = 0\rangle$$

$$\rightarrow \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle + |\downarrow\rangle\right\} \otimes |\alpha = 0\rangle$$

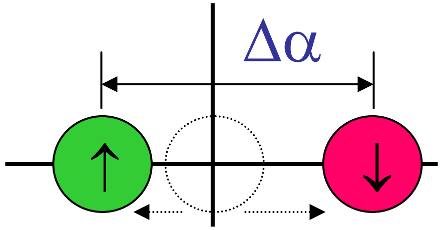
$$\Psi = \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle|\alpha\rangle + |\downarrow\rangle|\alpha'\rangle\right\}$$

"Schrödinger cat"

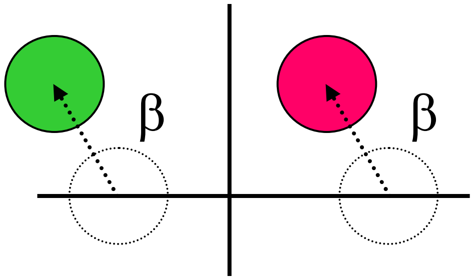
1) π/2 on spin



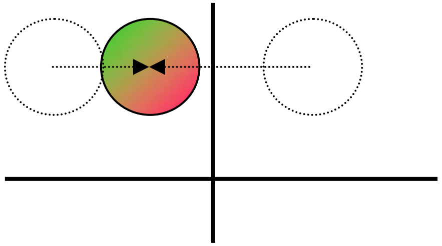
2) displacement



3) noise



4) recombine



5) Final π/2 Ramsey pulse on spin, relative phase φ_R

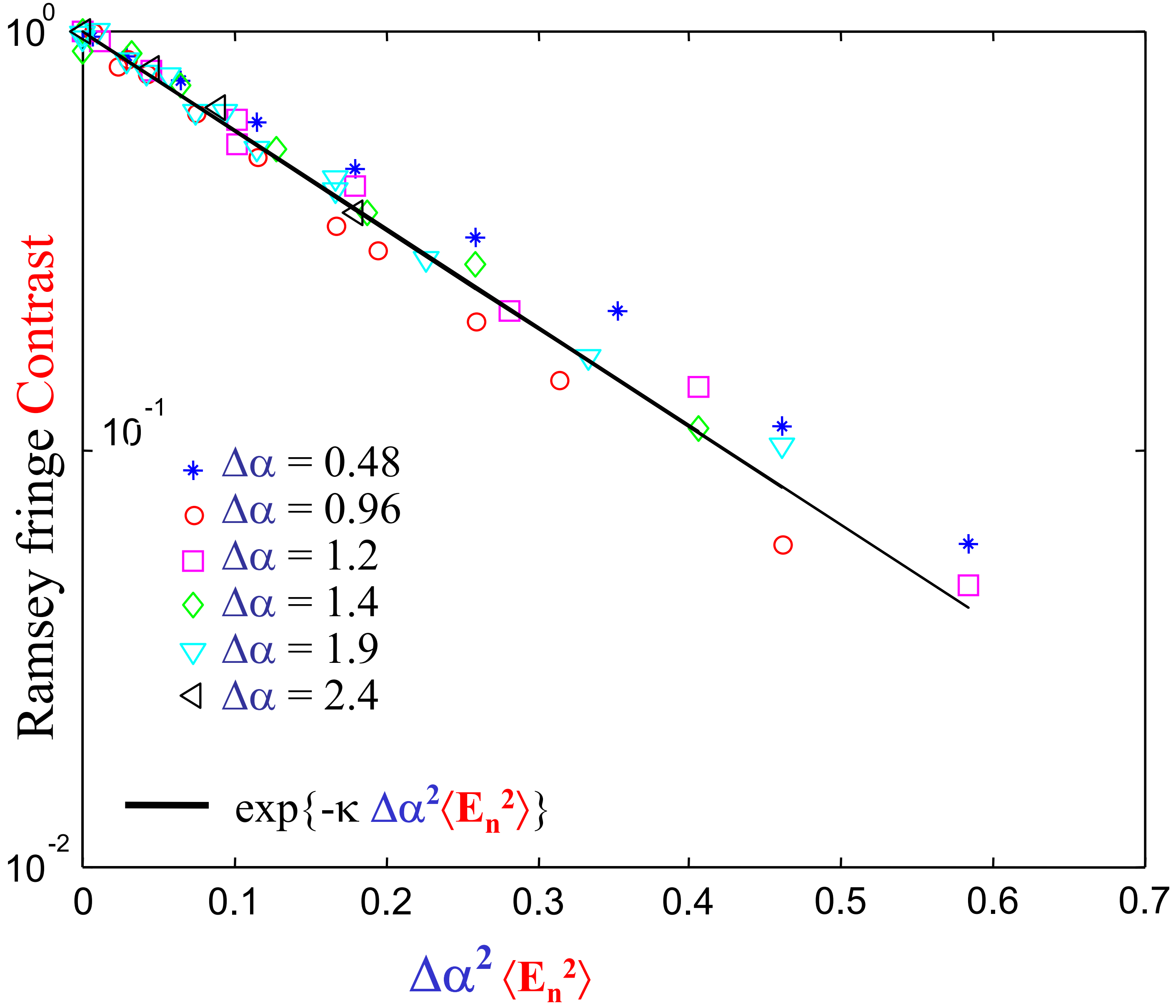
$$P_\downarrow = \frac{1}{2}\left[1 + \cos\left(\varphi_R + 2\,\mathrm{Im}\,\beta^*\Delta\alpha\right)\right]$$

controlled phase shift

random (from resistor)

Amplitude Reservoir / Coherent States

C. Myatt *et al.*, *Nature* **403**, 269 (2000); Q. Turchette *et al*. PRA**62**, 053807 (2000).



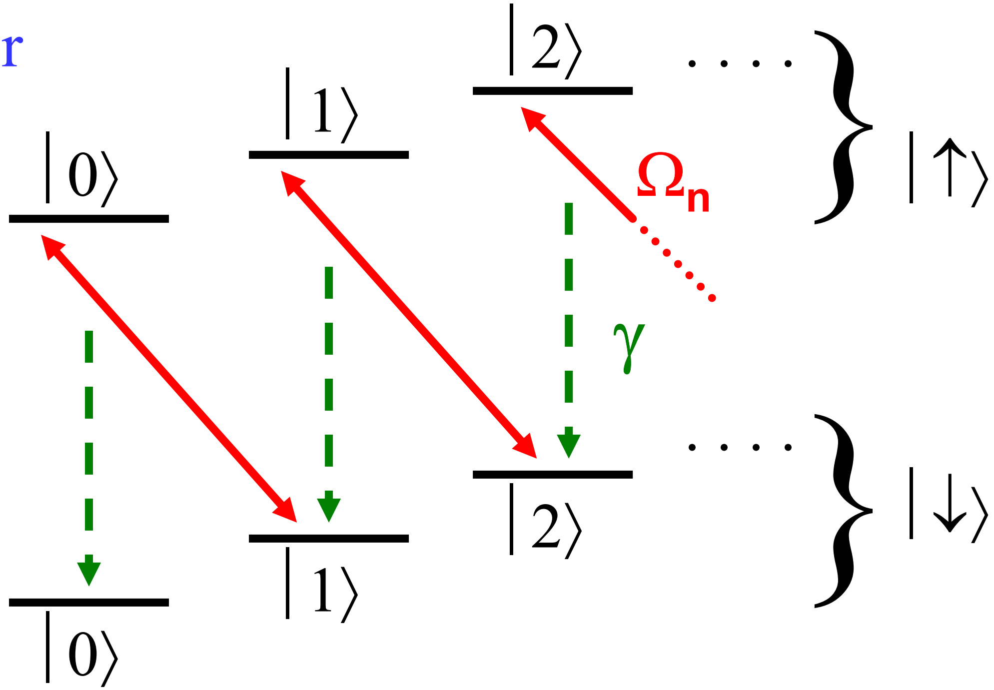
$T \approx 0$ case?

Cavity-QED: Maître *et al*., PRL **79**, 769(1997);
Brune *et al*., PRL **77**, 4887 (1996)

Ions: $\omega_{trap}/2\pi \cong 5$ MHz $\Rightarrow$ want $T_{Reservoir} << 0.2$ mK. Technically hard.
Proposal: “Engineered” reservoirs:
Poyatos, Cirac, & Zoller PRL **77**, 4728 (1996)

“Engineered” T=0 reservoir

(laser cooling)



Red sideband (coherent): $|n, \downarrow\rangle \Leftrightarrow |n\text{-}1, \uparrow\rangle$ (rate Ω_n)

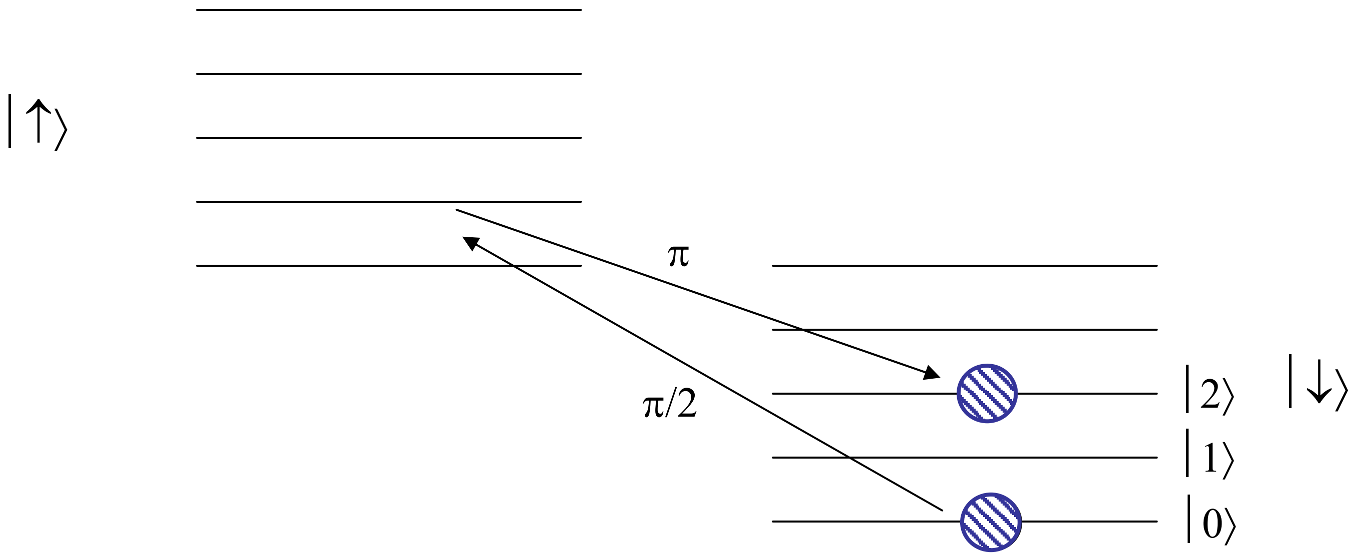
Optical pumping (incoherent): $|n, \uparrow\rangle \Rightarrow |n, \downarrow\rangle$ (rate γ)

For $P_\downarrow \approx 1$ ($\gamma >> \Omega_n$) $\Rightarrow$ T $\approx$ 0 amplitude reservoir for motional states

To see, do Ramsey spectroscopy on motion superpositions

$\alpha|\mathbf{n}\rangle + \beta|\mathbf{n'}\rangle$ entangle $\underbrace{|\downarrow\rangle, |\uparrow\rangle, + \text{spont. em.}}_{\text{environment}}$

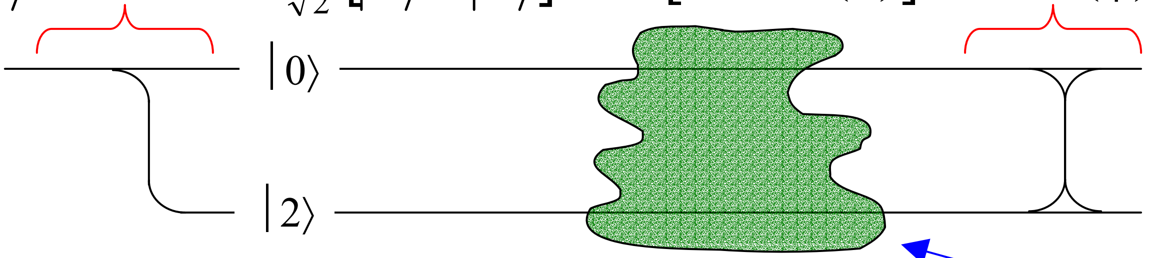
Fock-state superpositions



$$\boxed{|\downarrow\rangle|0\rangle \rightarrow \tfrac{1}{\sqrt{2}}|\downarrow\rangle\left\{\,|0\rangle+|2\rangle\right\}}$$

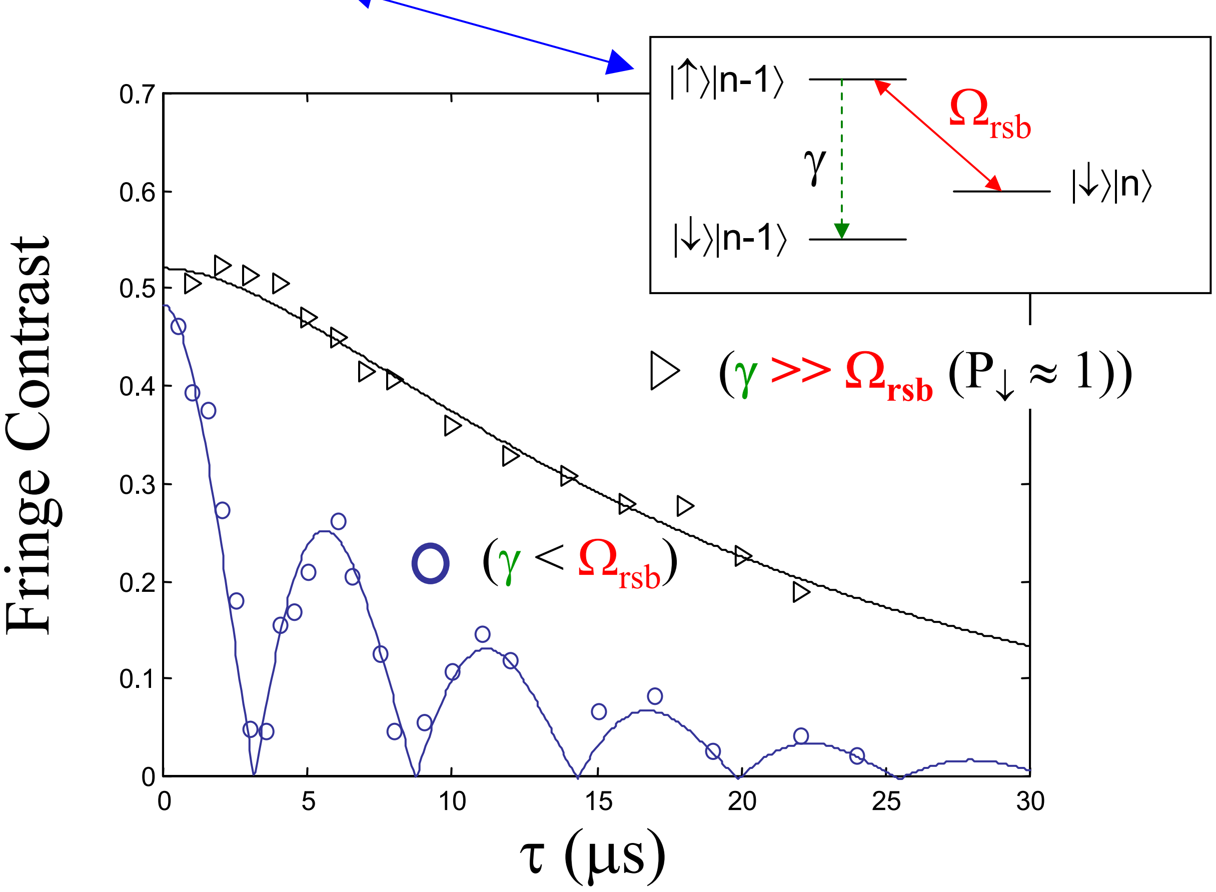
Ramsey interferometer on motion state superpositions

$$|0\rangle \;-\pi/2\rightarrow\; \frac{1}{\sqrt{2}}\big[|0\rangle+|2\rangle\big] \;\rightarrow\; \big[\mathrm{decoh}\,(\tau)\big] \rightarrow \pi/2(\varphi)$$

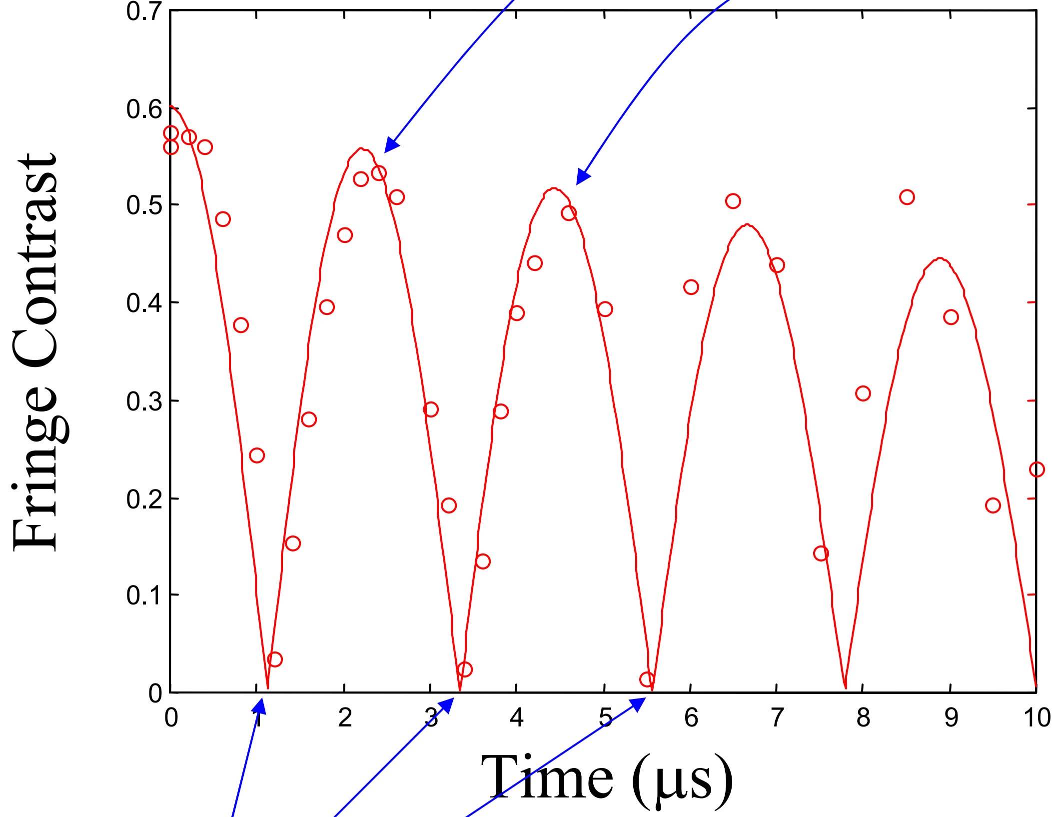


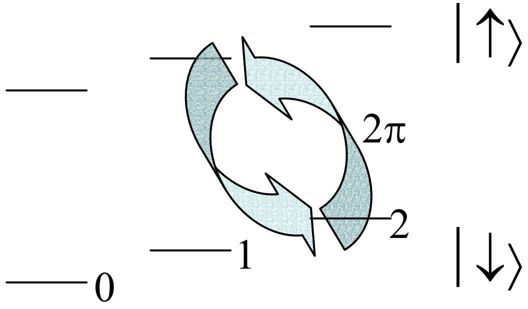
fringe contrast

(∝ $|\rho_{02}|$) vs. time τ



$\boxed{\gamma \to 0}$





$$\frac{1}{\sqrt{2}}\left[|0\rangle + |2\rangle\right] \otimes |\downarrow\rangle$$

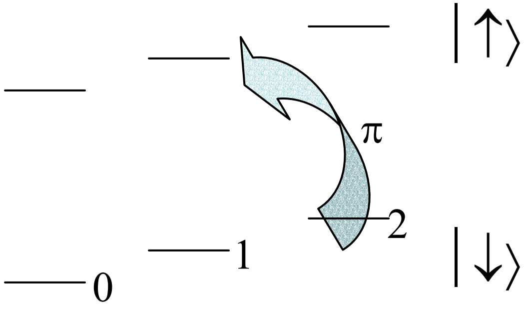
$$\boxed{\langle \psi_e' | \psi_e'' \rangle \cong 1}$$

erasing
which path information

$$\frac{1}{\sqrt{2}}\left[|0\rangle|\downarrow\rangle + |1\rangle|\uparrow\rangle\right]$$

"which path"
information

$$\boxed{\langle \psi_e' | \psi_e'' \rangle \cong 0}$$



$$\underbrace{(\alpha|\psi_1\rangle + \beta|\psi_2\rangle)}_{\boxed{\text{quantum system}}} \otimes \;\vdots\; \underbrace{|\psi_M\rangle \otimes |\phi_e\rangle}_{\boxed{\text{environment}}}$$

or:

$$\underbrace{(\alpha|\psi_1\rangle + \beta|\psi_2\rangle) \otimes |\psi_M\rangle}_{\boxed{\text{quantum system}}} \;\vdots\; \otimes \underbrace{|\phi_e\rangle}_{\boxed{\text{environment}}}$$

quantum/
classical
boundary ?

Related complementarity/quantum-erasing experiments:

<u>Photons:</u>

- P. Kwiat, A. Steinberg, R. Chiao, PRA **45**, 7729 (1992)
- T. Herzog, P. Kwiat, H. Weinfurter, A. Zeilinger, PRL **75**, 3034 (1995)

<u>Atoms:</u>

- M. Chapman, T. Hammond, A. Lenef, J. Schmiedmayer, R. Rubenstein, E. Smith, and D. Pritchard , PRL **75**, 3783 (1996)
- S. Dürr, T. Nonn, and G. Rempe, PRL **81**, 5705 (1998)
- P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. Raimond, and S. Haroche, *Nature*, **411**, 170 (2001)

$$\underbrace{(\alpha|\psi_1\rangle + \beta|\psi_2\rangle)}_{\boxed{\text{quantum system}}}\otimes \;\vdots\; \underbrace{|\psi_M\rangle \otimes |\phi_e\rangle}_{\boxed{\text{environment}}}$$

or:

$$\underbrace{(\alpha|\psi_1\rangle + \beta|\psi_2\rangle)\otimes|\psi_M\rangle}_{\boxed{\text{quantum system}}} \;\vdots\; \otimes \underbrace{|\phi_e\rangle}_{\boxed{\text{environment}}}$$

but, $|\phi_e\rangle$ is another quantum system ⇒

$$(\alpha|\psi_1\rangle + \beta|\psi_2\rangle)\otimes|\psi_M\rangle \otimes |\phi_e\rangle \;\vdots$$

Where’s the measurement?

Perspective:

- Problems technical (not fundamental) – but hard!

  ⇒ quantum computers someday

- **or**: fundamental decoherence not seen yet!