

# An Intercomparison of Hydrogen and Cesium Frequency Standards

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**Abstract**—Intercomparisons of average frequency and of frequency stability were made among one Hewlett-Packard 5060A cesium beam, two Varian Associates H-10 atomic hydrogen masers, and the National Bureau of Standards NBS III cesium beam designated as the United States Frequency Standard. Each of the standards displayed a white noise frequency fluctuation behavior with a transition into an approximate flicker of frequency fluctuation behavior for longer time intervals. The rms fractional frequency fluctuation between adjacent samples,  $\sigma(\tau, N=2)$ , was  $6 \times 10^{-11} \tau^{-1/2}$  down to a flicker level of about  $3 \times 10^{-13}$  for the hp 5060A cesium beam ( $10^2 \leq \tau \leq 10^4$ s),  $1 \times 10^{-11} \tau^{-1/2}$  down to a flicker level of less than  $1 \times 10^{-13}$  for NBS III cesium beam ( $10^2 \leq \tau \leq 10^4$ s), and  $5 \times 10^{-13} \tau^{-1/2}$  down to a flicker level of about  $1 \times 10^{-14}$  for the H-10 hydrogen masers ( $1 \leq \tau \leq 10^4$ s). The accuracy capabilities of NBS III and H-10 #4 are now  $1.1 \times 10^{-12}$  and  $0.47 \times 10^{-12}$ , respectively ( $1\sigma$  estimate).

A discrepancy of only 1.1 parts in  $10^{12}$  was observed between the average frequencies of the hp 5060A cesium beam and the NBS III cesium beam, with the former being higher in frequency.

In terms of the frequency of the  $\text{Cs}^{133}$  hyperfine transition ( $F=4, m_F=0 \leftrightarrow F=3, m_F=0$ ), defined as 9 192 631 770.0000 Hertz, the measured frequency of the  $H^1$  hyperfine transition ( $F=1, m_F=0 \leftrightarrow F=0, m_F=0$ ) was  $\nu_H = 1\,420\,405\,751.7864 \pm 0.0017$  Hertz. This is believed to be the most accurate and precise measurement of any physical quantity.

## I. INTRODUCTION

A SERIES of measurements involving hydrogen and cesium frequency standards [1] was made at the National Bureau of Standards (NBS), Boulder, Colo., during September and October 1965. The object of the measurements was to make intercomparisons of frequency between standards that then represented the state-of-the-art; to measure the frequency of the hyperfine separation of hydrogen in terms of the provisional international standard of frequency, the hyperfine separation of cesium 133, with higher accuracy than ever before; and to obtain statistical information that would give an insight into the nature of the perturbations that cause changes in frequency.

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Participating in the experiments were personnel from the NBS Atomic Frequency and Time Standards Section, the Hewlett-Packard Co., and the Quantum Electronics Division of Varian Associates. The equipment involved in the measurements were the NBS III cesium beam frequency standard used as the United States Frequency Standard located at Boulder, a Hewlett-Packard model hp 5060A cesium beam standard, and two atomic hydrogen masers model H-10 built by Varian Associates. A preliminary report of some of this work has been published [2].

Measurements involving time intervals ranging from 1 second to a maximum of  $4 \times 10^6$  seconds were made between these standards by various techniques that are described in Section II of this paper. An effort was made to obtain statistically significant quantities of data to allow analysis of the spectral density of the phase or frequency fluctuations for sampling time intervals ranging from seconds to hours. For time intervals of greater duration, the number of samples available obviously diminishes and the relative magnitude of the confidence limits that apply to any given statistically determined parameter increases. Hence, it is difficult to make precise conclusions concerning the spectral density of the very low frequency fluctuations. Therefore, the long term data are presented in the form of relative frequency vs. the time of measurement, as well as in the form of  $\sigma(\tau)$  vs.  $\tau$  plots. The plots of frequency vs. running time can also help one to detect systematic behavior of the frequency fluctuations. These two forms of data presentation supplement each other.

Emphasis was placed on making measurements at what would normally be considered as the useful output frequency, 5 MHz, in order to include the effects due to the electronic circuits and frequency synthesizers that normally form part of the frequency control system. A great deal of the data was taken with a 5 MHz system developed at NBS using period measurement (Section II.A). This system was especially useful for measurements over continuous time intervals of  $10^2$  seconds to  $10^4$  seconds. For measurements between masers involving shorter time intervals, a different period measuring system (Section II.D) was used that referred the comparison to a common 5 MHz frequency and, by frequency multiplication back up to 1420 MHz, allowed greater resolution over the shorter time intervals. Relative stability measurements between masers

over intervals of days were made with this high resolution system by means of automatically printed period measurements and strip chart recordings of relative phase at 1420 MHz.

An important result from these experiments is a more accurate value of the hydrogen hyperfine frequency (Section IV) in terms of the cesium 133 hyperfine frequency assumed to be given by 9 192 631 770.0000 Hz. Due to improved understanding of the perturbations affecting the frequencies of the hydrogen and cesium devices, new error budgets (Section III) have been made for each which are more complete than those previously made. The frequency of the hyperfine separation of hydrogen is found to be  $1\,420\,405\,751.7864 \pm 0.0017$  Hz. The uncertainty of  $\pm 0.0017$  Hz is a  $1\sigma$  (68 percent confidence) measure and corresponds to 1.2 parts in  $10^{12}$ .

Very closely related to the problem of intercomparisons is the question of intrinsic frequency reproducibility between instruments and particularly between instruments made by different people. In the case of the two cesium devices having enormously different size, beam flux, and resonance linewidth, the data giving the average frequency over a run of several days duration indicated that the average frequencies agreed well within the accuracy limits assigned to each standard (Section II.D). The frequency fluctuations measured over a large range of time intervals  $\tau$  are random and vary as  $\tau^{-1/2}$ , the behavior of a source which is frequency modulated by white noise (Section II.B).

Frequency reproducibility measurements between hydrogen masers constructed in different laboratories have been made [3]. However, since there are relatively few well stabilized H masers and since they are located in fixed installations, it is not yet possible to make definitive conclusions. It is possible to determine the frequency of the hyperfine separation of hydrogen for the idealized conditions of zero wall collision effects, zero magnetic field, zero absolute temperature, and zero electromagnetic pulling by applying suitable corrections. Though the reproducibility of *operating* frequency from one maser to another for masers made in a single laboratory is considerably better than the absolute reproducibility specified in the idealized manner previously mentioned, the reproducibility of hydrogen frequency given here is in terms of those ideal conditions.

## II. MEASUREMENTS AND TECHNIQUES

### A. Measurements with 5 MHz Comparison Systems

Long-term frequency comparisons among the cesium beam devices and hydrogen masers were made by measuring the period of the beat frequency between pairs of synthesized 5 MHz signals controlled by the various atomic transitions. The system is shown in block form in Fig. 1 and consists of two input channels at 5 MHz, each consisting of an isolator amplifier leading to a mixer. The output signal from the mixer, with peak-to-peak

amplitude of 8 volts and having a frequency typically ranging from 0.001 Hz to 10 Hz, is led through a low pass filter to a period measuring counter connected to a card punch. Period data were automatically punched on cards.

As many as three such systems were used simultaneously during a run lasting 49 hours, during which the Hewlett-Packard cesium beam, the NBS cesium beam, and a Varian H maser were compared. Plots of these data are shown in Fig. 2 and some explanation of the circumstances should be given. Frequency offsets of a few tenths of a Hertz (cycle per second) from 5 MHz were arbitrarily chosen in order to obtain frequency differences suitable for making period measurements at 5 MHz. The Hewlett-Packard instrument was supplied with a  $-150 \times 10^{-10}$  offset and was operated according to factory procedure, the NBS III instrument was operated at a nominal  $-249 \times 10^{-10}$  offset, and the H maser digital synthesizing phase lock system was adjusted to the nearest 0.01 Hz at 1420 MHz so as to have a nominal zero offset of its 5 MHz output frequency. The data shown give the hour-by-hour variation between pairs of instruments. Each point is flagged with error bars representing the estimated standard deviations of the mean for each one-hour segment of the run as determined by computer analysis of the punched cards. On the average, during each hour there were 17 measurements for the Hewlett-Packard 5060A vs. NBS III, 25 measurements for the Hewlett-Packard 5060A vs. Varian H-10 #4, and 44 measurements for the H-10 #4 vs. NBS III.

At the beginning of the run a slow drift was observed in the H-10 #4 vs. NBS III data, due to the stabilizing of the maser cavity temperature control system, since a few hours prior to the beginning of this record both masers had been moved from a room having temperature of about 90°F to a room with ambient temperature of about 70°F. At 1500 hours September 14 local time the maser cavities were tuned and data taking was begun. At 2200 hours the same day the masers were checked and the cavity of H-10 #4 was found to have drifted high in frequency; it was retuned, the correction being  $21 \times 10^{-13}$  in  $\Delta f/f$ .

The tuning was checked at intervals during the run by observing the change in the beat frequency of H-10 #4 vs. H-10 #3 induced by a change in beam flux in H-10 #4. No interruption of the data resulted from this procedure; the offset introduced temporarily as a result of this process is of the order of a few parts in  $10^{+13}$  in  $\Delta f/f$  and is negligible. This tuning procedure is described in Section II.E and Section III.A.

There is evidence that the temperature control had not completely stabilized before the retuning at 2200 hours September 14. At 1500 hours September 15 it was found that H-10 #4 was  $4.0 \times 10^{-13}$  high, at 0200 on September 16 H-10 #4 was  $3.8 \times 10^{-13}$  high, and at 1800 September 16 it was found to be  $1.2 \times 10^{-13}$  high. During this run the instruments were not adjusted in any way except for the change in tuning of H-10 #4 at 2200 Sep-

tember 14, noted above. In the H-10 #4 vs. NBS III plot given in Fig. 2, the open circles represent the frequency offset observed under the conditions of tuning given above. The solid black points represent the frequency offset which would have been observed if H-10 #4 had been properly tuned.

Further data were taken for 50 days between Varian H-10 #4 and NBS III. The fractional differences of the synthesized frequencies, nominally at 5 MHz, are shown plotted against measurement number and date of measurement in Fig. 3. Further data between the hp 5060A and NBS III were taken for twelve days and are shown in Fig. 4. The hourly averages made during these two runs are shown as points with vertical flags indicating the estimated  $1\sigma$  error of the averages that typically ranges from 1 to  $3 \times 10^{-13}$  in the case of the NBS III vs. H-10 #4 and from 5 to  $9 \times 10^{-13}$  in the case of comparisons involving the Hewlett-Packard instrument.

In addition to the 49 hour and 50 day runs just described, two more runs were made between H-10 #4 and NBS III using the 5 MHz comparison system. An 11 hour run was made on September 17 and a 52 hour run was made on September 23 through 25. The fractional frequency offsets are plotted against running time in Fig. 5. The data have been corrected for H maser cavity pulling and for magnetic field, cavity phase, power, and second-order Doppler shifts of NBS III. These corrections are discussed in Section III.

The circumstances of the 11 hour run were such that the noise level on the signals was very much lower than for the three other runs. This could be due to the fact that the runs were made during a quiet period on a Friday evening. As seen in Fig. 5, the 11 hour run data, taken in hourly groups of approximately 44 measurements, have a standard deviation for each group of about  $1 \times 10^{-13}$  in  $\Delta f/f$ , as compared to about 3 to  $5 \times 10^{-13}$  for the data of the 52 hour run. The 11 hour run was used for statistical analysis of  $\sigma$  as a function of sampling time  $\tau$  in order to determine the frequency dependence of the spectral density of the relative instabilities of the signals. The analysis was performed using the method of Allan [4], which is outlined in the Appendix. A plot of  $\sigma(\tau)$ , the root mean square fractional relative frequency deviation for sampling time  $\tau$ , vs. the sampling time, is given in Fig. 6.

Similar runs were performed using the Hewlett-Packard 5060A and the Varian H-10 #4 instrument. The  $\sigma$  vs.  $\tau$  plots, given in Figs. 7 and 8 for two separate runs, show that the data are reproducible and follow closely the  $\tau^{-1/2}$  law described in Section II.B.

#### B. White Frequency Noise of Cesium Beam Standards: Short Term Frequency Stability

A comparison of the  $\sigma$  vs.  $\tau$  plot for the NBS III instrument in Fig. 6, and that for the Hewlett-Packard 5060A instrument given in Figs. 7 and 8, indicates that the ratio of  $\sigma_{HP}$  to  $\sigma_{NBS}$  is about 5 to 1. This can be compared with the ratio predicted [5] from measured

values of 1) the resonance linewidth, and 2) the signal-to-noise ratio at the output of the beam detector preamplifier (Fig. 9).

In notation consistent with the present paper,  $\sigma(\tau)$  is taken as the fractional rms frequency deviation for a sample time interval  $\tau$ , and

$$\sigma(\tau) = 0.387 \frac{\Delta f_l}{f_0} \frac{I_n}{I_m} \tau^{-1/2}, \quad (1)$$

where

$\Delta f_l$  is the width of the atomic resonance,

$f_0$  is the center frequency of the atomic resonance,

$I_n$  is the rms noise current from the detector preamplifier in a 1 Hz bandwidth centered at the modulation frequency, and

$I_m$  is the peak signal current from the preamplifier.

In the case of the Hewlett-Packard instrument

$$\begin{aligned} \Delta f_l &= 550 \text{ Hz,} \\ I_n/I_m &= 1.92 \times 10^{-3}, \text{ and} \\ \sigma(\tau) &= 4.45 \times 10^{-11} \tau^{-1/2}. \end{aligned}$$

For the NBS III instrument

$$\begin{aligned} \Delta f_l &= 45 \text{ Hz,} \\ I_n/I_m &= 1.95 \times 10^{-3}, \text{ and} \\ \sigma(\tau) &= 3.7 \times 10^{-12} \tau^{-1/2}. \end{aligned}$$

The calculated ratio  $\sigma_{HP}/\sigma_{NBS}$  is 12 to 1 and should be compared with the measured ratio of 5 to 1. This discrepancy is largely due to the fact that in NBS III the frequency of the modulation is a large fraction of the resonance linewidth and results in a loss of recovered signal at the detector as compared to that obtained with slower modulation. The discrepancy in the ratio also may be due to the fact that only the noise contributions from the beam and the preamplifier are included in the calculation. The frequency lock system noise is not included, nor is the noise in the frequency comparison equipment. Over the complete range of time intervals, two minutes to four hours, of Figs. 6, 7, and 8, the data for both instruments show no evidence of systematic variations.

#### C. NBS III vs. Hewlett-Packard 5060A: Long Term Frequency Stability and Average Frequencies

The 12 day run (Fig. 4) was used to compare the average frequencies of the hp 5060A and NBS III cesium beam standards. To compare the standards in a way so as to yield the best average frequency relation, it is necessary to include corrections discussed in Section III.B. In Fig. 4 the data are shown plotted in the corrected form and the date of the measurement is indicated below the axis designated as "measurement number." The Hewlett-Packard instrument was operated at a design frequency offset of  $-150.00$  parts in  $10^{10}$  from the 9 192 631 770.0000 Hz frequency defined as the

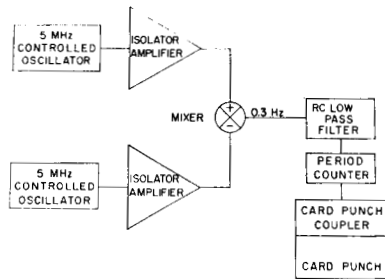


Fig. 1. NBS 5 MHz comparison system.

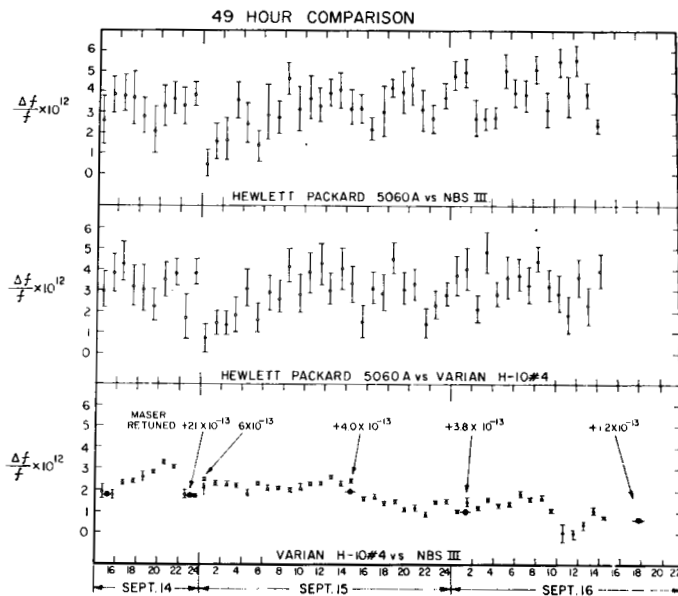


Fig. 2. Simultaneous measurements of frequency stability among NBS III, Varian H-10 #4, and Hewlett-Packard 5060A; 49 hour comparison.

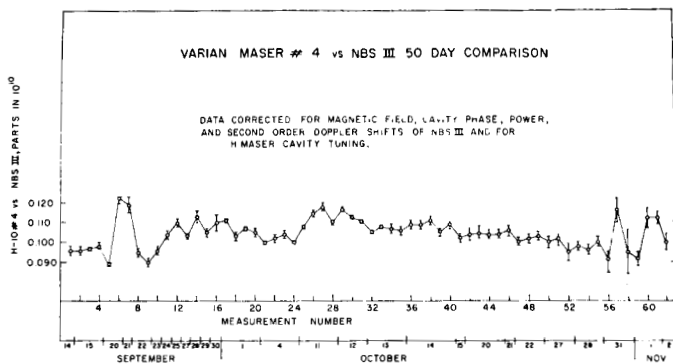


Fig. 3. 50 day plot of data between Varian H-10 #4 and NBS III.

frequency of the  $Cs^{133}$  hyperfine transition under ideal conditions of zero magnetic field, infinitesimal radiation field intensity, zero second order Doppler shift, and zero phase error between the cavities.

There were 102 measurements of length ( $\tau$ ) of one hour, and the dead time between measurements ( $T-\tau$ ) ranged from a few seconds to greater than a day. A  $\sigma(\tau)$  vs.  $\tau$  plot of these measurements is in good agreement with the data of Figs. 7 and 8, with an indication of a flicker of frequency level [8], [4], [5] of 2 to 3 parts

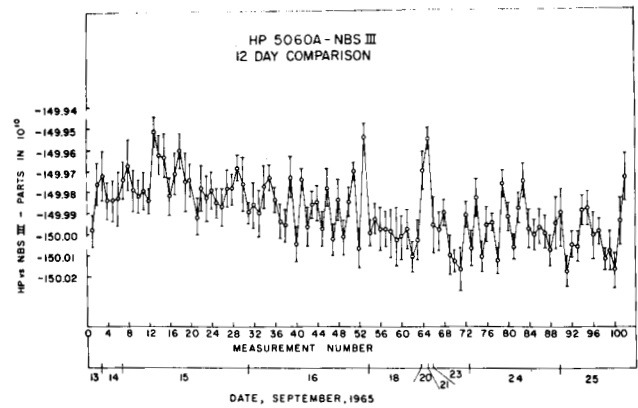


Fig. 4. 12 day plot of frequency offset between Hewlett-Packard 5060A and NBS III.

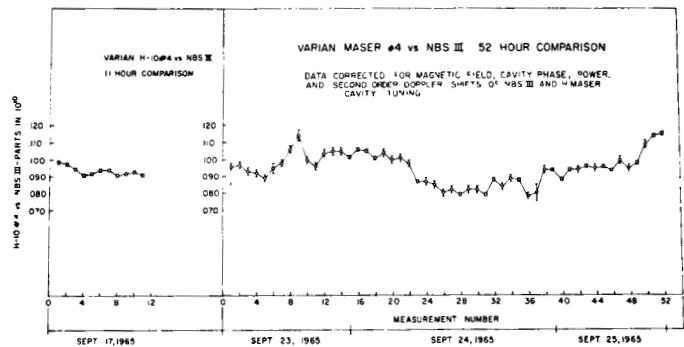


Fig. 5. 11 hour and 52 hour plots of data between Varian H-10 #4 and NBS III.

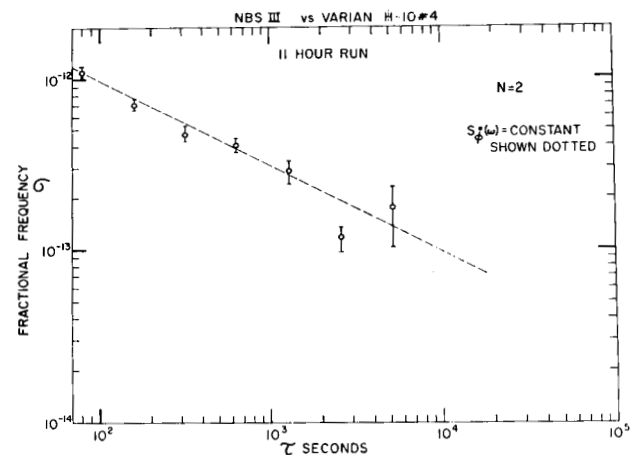


Fig. 6. Standard deviation of fractional frequency fluctuations as a function of sampling time, NBS III vs. Varian H-10 #4; 11 hour run.

in  $10^{13}$ . The flicker of frequency level for NBS III is known to be less than 1 part in  $10^{13}$  for adjacent samples ( $N=2$ ). The overall average of the 102 measurements was  $-149.989$  parts in  $10^{10}$  with a standard deviation  $\sigma$  of a measurement of 1.5 parts in  $10^{12}$ . It is worth noting that the hp 5060A was calibrated in an internally consistent manner by adjusting the magnetic field to a predetermined level found by monitoring the field dependent hyperfine transitions at frequencies above and below the frequency of the  $(F=4, m_F=0) \leftrightarrow (F=3,$

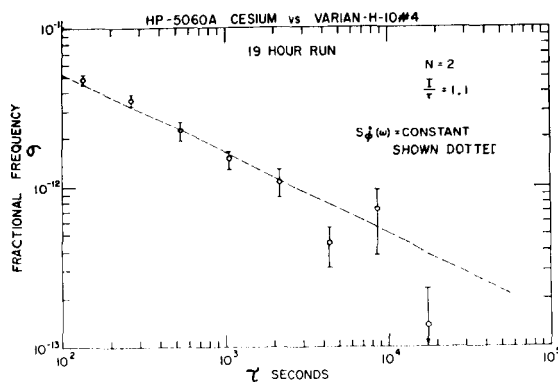


Fig. 7. Standard deviation of fractional frequency fluctuations as a function of sampling time, Hewlett-Packard 5060A vs. Varian H-10 #4; 19 hour run.

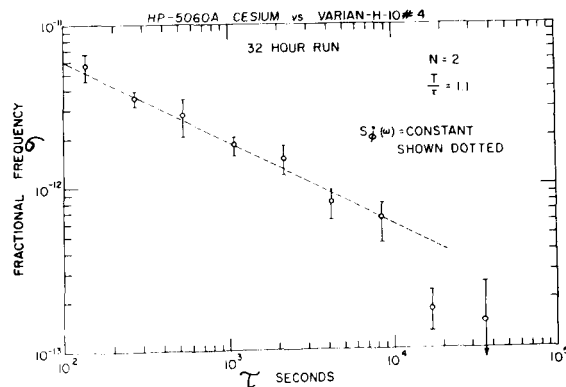


Fig. 8. Standard deviation of fractional frequency fluctuations as a function of sampling time, Hewlett-Packard 5060A vs. Varian H-10 #4; 32 hour run.

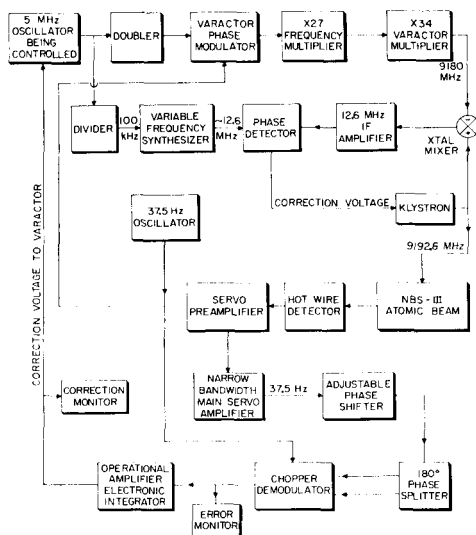


Fig. 9. Frequency lock system of NBS III cesium beam.

$m_F = 0$ ) transition. The NBS III instrument was operated in the manner described in reference [6]. A schematic of the NBS III frequency lock system is shown in Fig. 9. The discrepancy between the hp 5060A and NBS III is  $1.1 \times 10^{-12}$ , with the former being higher in frequency. The accuracy claimed for the hp 5060A by the manufacturer is  $\pm 2 \times 10^{-11}$ .

### D. H Maser vs. H Maser: Short Term Frequency Stability

Measurements of short term relative frequency stability between masers were made using the system shown in Fig. 10. In this system a 5 MHz crystal oscillator is phase locked to Maser A and serves as a driver for a 1400 MHz multiplier serving as local oscillator for Maser B. The intermediate frequency signal  $f$  at 20.405 MHz is then compared with an offset signal from a digital synthesizer operating at  $f_2$ . The beat frequency  $f_2 - f$  can be adjusted and allowed beat period measurements to be made over a range determined both by the beat rate  $f_2 - f$  and by the number of periods chosen to represent the interval. As an example, if  $f_2 - f = 1$  Hz, one can obtain 1 second averages by single-beat period measurements, 10 second averages by 10-beat period measurements, etc.

This method was used for runs of data typically involving several hundred measurements over a total time interval of one hour. An important parameter for each of these runs is the setting of the low pass filter which determines the amount of noise sideband power that accompanies the signal at the beat frequency. The behavior of the output fluctuations is a strong function of the filter setting for cases where additive white phase noise is present with the signal. The effect of this noise for fixed bandwidth  $B$  is given [5] by

$$\sigma = \frac{1}{2\pi f \tau} \sqrt{\frac{FkT2B}{P}}, \quad (2)$$

where  $FkT/P$  is the ratio of the additive noise power spectral density to the carrier power, and  $f$  is the frequency of the carrier. Low pass filtering of the beat signal has approximately the same effect on white phase noise as does band pass filtering of both carriers.

Under these conditions, one expects  $\sigma$  to vary as  $1/\tau$  if the bandwidth is kept fixed. The presence of other spectral types of frequency fluctuation can be determined from the dependence of  $\sigma$  on observation time with different bandwidths or by relating the bandwidth to the reciprocal of the observation time and plotting the data in terms of  $B\tau$  [7].

Data were taken by printing the period for a fixed number of beats and later transferring the data to card form for processing by a computer. In general, the number of samples was chosen to be some integral power of two for compatibility with the method used by Allan [4] and described in the Appendix. Plots of  $\sigma(\tau)$  vs. sample time  $\tau$  are given in Fig. 11 for two different bandwidths of the RC low pass filter. It is observed that the data are bandwidth dependent and are very much above the level predicted by the effect of known additive white noise of the microwave mixer. For the conditions  $F = 10$ ,  $kT \approx 4 \times 10^{-21}$  Joules,  $P \approx 5 \times 10^{-13}$  watts, and a 2 Hz bandwidth low pass filter at the beat frequency, the value of  $\sigma$  given by the above equation for

white phase noise is

$$\sigma \approx 6 \times 10^{-14} \tau^{-1}.$$

It is clear from Fig. 11 that the measured instability is not due to the effect of added white noise of the microwave mixer. The tau dependence of  $\sigma(\tau)$  in Fig. 11 is as  $\tau^{-1/2}$  from 2 seconds to 120 seconds (i.e. white frequency noise) and is not the  $\tau^{-1}$  dependence (i.e. white phase noise) described in (2). In addition, the  $\sigma$  values in Fig. 11 are greater than the values expected due to additive white noise. Other sources must be found to account for the instabilities observed. These may involve the frequency synthesis systems or interference pickup in the measurement system. Further study of the instabilities due to these components is in progress.

Figure 12 is a plot of relative fractional frequency vs. running time, as obtained from printed period data obtained from the system of Fig. 10. A constant frequency offset has been subtracted to emphasize the nature of the frequency fluctuations. It is seen that there is a reasonably regular frequency modulation with a peak-to-peak fractional frequency excursion of about  $8 \times 10^{-14}$  and having a modulation period of approximately one half-hour. The explanation of this modulation is still not available. However, there are several possible explanations including a cycling temperature-servo affecting the tuning of the maser cavity.

The presence of this periodic frequency modulation will have a distinct effect upon a  $\sigma(\tau)$  vs.  $\tau$  plot of the experimental data. If the chosen time interval  $\tau$  is equal to one half of the period of the frequency modulation, then the value of  $\sigma(\tau)$  will be a local maximum. If the chosen time interval is equal to one period, then the value of  $\sigma(\tau)$  will be a local minimum. The same data as were presented in Fig. 12 are given in Fig. 13 in the form of a  $\sigma(\tau)$  vs.  $\tau$  plot. A local maximum value of  $\sigma(\tau)$  is observed at  $\tau \approx 800$  seconds, and a local minimum value is observed at  $\tau \approx 1800$  seconds. This behavior of  $\sigma(\tau)$  is due to the periodic frequency modulation evident in Fig. 12.

In general, the data beyond  $\tau \approx 2000$  seconds exhibit a flicker of frequency level [8], [4], [5] of about one part in  $10^{14}$ . This leveling-off of  $\sigma(\tau)$  for  $\tau > 2000$  seconds is not due to the periodic frequency modulation shown in Fig. 12. Data beyond  $2.5 \times 10^4$  seconds have a very large uncertainty indicated by the arrows.

An additional method of obtaining data was used in order to avoid the effects of having in the measuring loop the quartz crystal oscillator, the two multiplier chains, and the two digital synthesizers which were shown in Fig. 10. A technique is described in Fig. 14 where both maser signals are separately converted to 20.405 MHz using a common local oscillator signal which is fed through 60 dB isolators to two separate mixers. Each maser signal is fed through a total of 120 dB of isolation using four 30 dB ferrite circulators optimized to operate at 1420 MHz and connected in series. The data are taken by obtaining the beat fre-

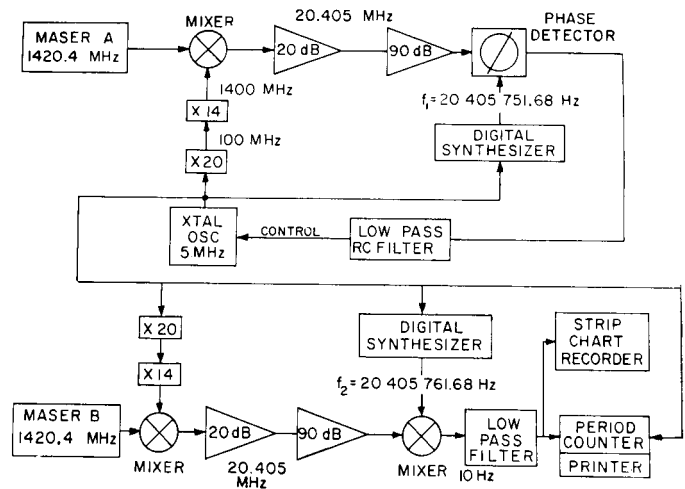


Fig. 10. Phase lock system for frequency comparison of  $H$  masers.

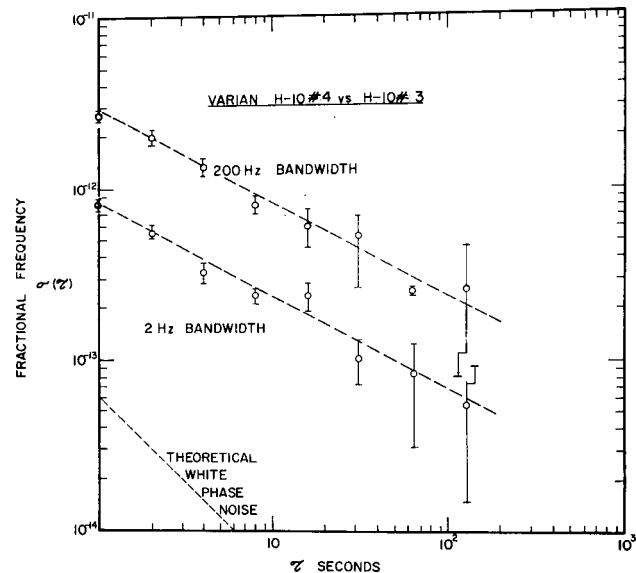


Fig. 11. Standard deviation of fractional frequency fluctuations between  $H$  masers as a function of sampling time and bandwidth, from automatically printed data.

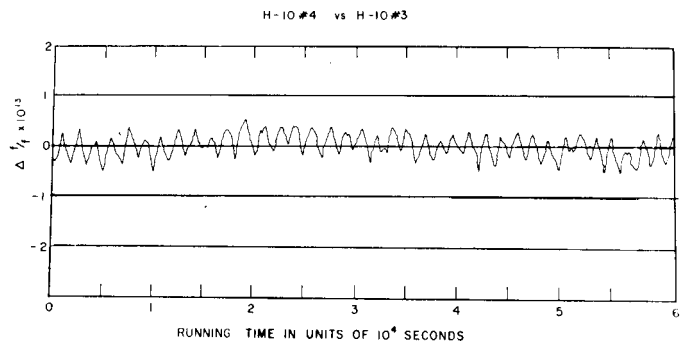


Fig. 12. Plot of relative fractional frequency fluctuations as a function of running time, Varian H-10 #4 vs. H-10 #3. A constant frequency offset has been subtracted to emphasize the nature of the frequency fluctuations.

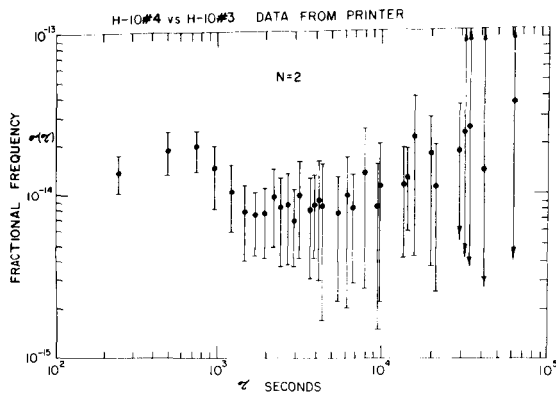


Fig. 13. Standard deviation of fractional frequency fluctuations between  $H$  masers as a function of sampling time, from automatically printed data.

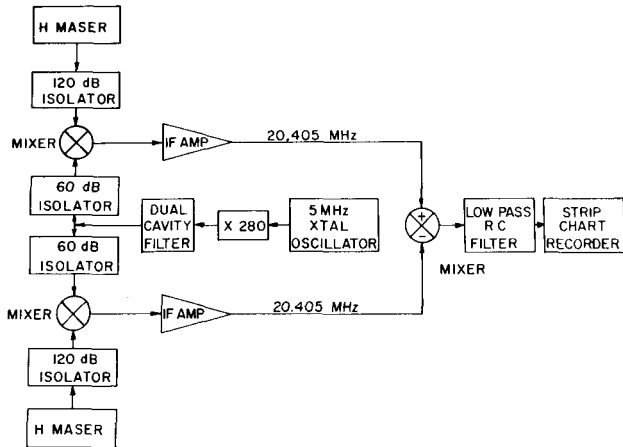


Fig. 14. An alternate system for comparison of  $H$  masers which avoids the use of phase lock electronics.

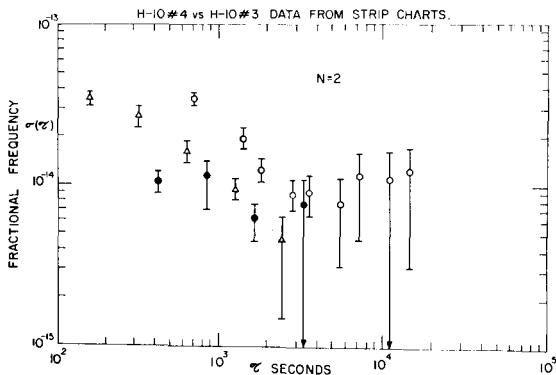


Fig. 15. Standard deviation of fractional frequency fluctuations between  $H$  masers as a function of sampling time, from strip chart data. The solid black points were obtained by the method of Fig. 14. All other points were obtained by the method of Fig. 10.

quency between the two 20.405 MHz intermediate frequency signals in a mixer and tracing the low-frequency beat directly on a strip chart recorder.

In Fig. 15 are  $\sigma$  vs.  $\tau$  plots of chart data from one run (solid black points) using the method of Fig. 14 and of chart data from two runs (triangles and circles) using the method of Fig. 10. Analysis of the period data taken from charts was performed in the same manner as for the automatically printed data by using cards which

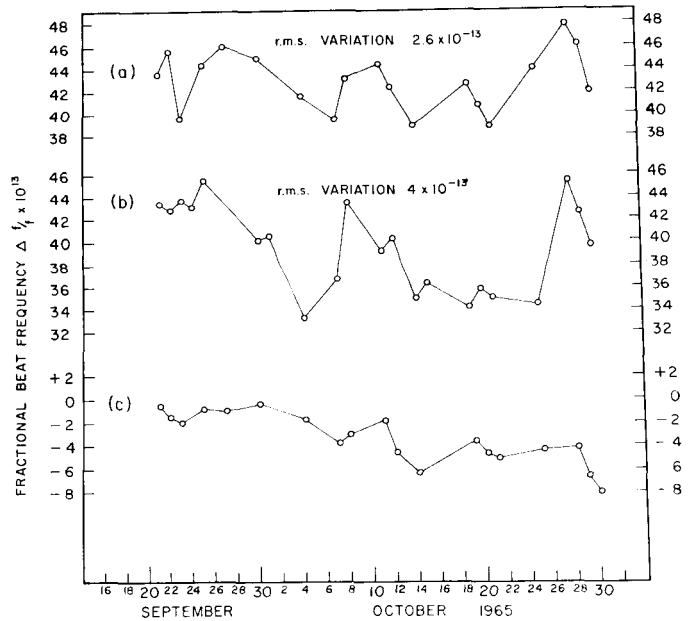


Fig. 16. (a) Fractional frequency difference of  $H$  masers as a function of time, corrected for cavity tuning effects in both masers. (b) Uncorrected fractional frequency difference of  $H$  masers as a function of time. (c) Cavity tuning effects in H-10 #4 as a function of time.

were manually punched using the same format as for the previously mentioned runs. By comparing Figs. 13 and 15, it is seen that the data taken with the method of Fig. 10 show the same general level of instability, whether from strip chart recordings or from automatically printed period measurements. In addition, it is seen that the data taken by the method of Fig. 14 show the same general level of instability as shown by the data taken by the method of Fig. 10. In Fig. 15 the white phase noise behavior evident between  $10^2$  and  $10^3$  seconds is due to random error in the estimation of phase from the strip chart recordings.

#### E. $H$ Maser vs. $H$ Maser: Long-Term Frequency Stability

Comparisons were made between the  $H$  masers for a period of five and one-half weeks. The masers were checked for cavity mistuning by varying the atomic hydrogen beam flux and monitoring the beat between the two masers. As the concentration of atomic hydrogen in the storage bulb is increased, the atomic resonance linewidth is broadened due to spin-spin exchange collisions between pairs of atoms in the maser storage bulb, and the magnitude of the cavity pulling, if any, is increased. When the cavity frequency is such that a variation in beam flux does not affect the output frequency, the maser is considered to be properly tuned. The tuning process is more fully described in Section III.A and in references [9]–[11].

Figure 16(b) shows, as a function of time, the hypothetical fractional beat frequency which would have been observed between the two masers if neither maser had been retuned during the run. The rms fractional frequency variation is  $4.0 \times 10^{-13}$ . Figure 16(a) shows the

corrected beat frequency which would have been measured had both masers been tuned. Here the rms variation is  $2.6 \times 10^{-13}$ , and the variations are due to unexplained processes which probably involve cavity pulling (see Section III.A).

Figure 16(c) shows a plot of the cavity pulling effect on H-10 #4. This was obtained by determining the frequency correction, using the tuning method described above, and plotting the correction in the output frequency as a function of time. There is evidence of a slow drift in the cavity tuning resulting in an output frequency drift of  $7 \times 10^{-13}$  in 50 days. This is probably due to a slow temperature variation, possibly due to a thermistor that had not been previously stabilized by aging in a temperature bath.

For these measurements there was an applied magnetic field offset in H-10 #3 of  $37 \times 10^{-13}$ . The average frequency of H-10 #3 was found to be  $7 \pm 2$  parts in  $10^{13}$  higher than the frequency of H-10 #4, after corrections for all known effects had been made. This discrepancy has since been reduced to less than  $\pm 2$  parts in  $10^{13}$  by renewal of the wall coating in the storage bulb of H-10 #3.

### III. ACCURACY OF THE FREQUENCY STANDARDS

#### A. Error Budget for Varian H-10 #4 Hydrogen Maser

Corrections are necessary to relate the output frequency of the hydrogen maser to the frequency of the hyperfine separation of the hydrogen atom under the least perturbed conditions. These conditions are [12]

- 1) zero collision rate,
- 2) zero absolute temperature,
- 3) zero magnetic field, and
- 4) zero cavity mistuning effect.

The effects from collisions are twofold, as there are collisions among the atoms as well as collisions with the walls of the storage bulb. In the latter case, while there is some question as to the actual mechanism of the frequency perturbation due to the wall collisions and also uncertainty as to the exact nature of the surface, it has been found that the Teflon<sup>1</sup> surfaces currently in use yield reproducible effects on the frequency of a maser. The wall shift is assumed to be proportional to the wall collision rate. By varying the wall collision rate through the use of different bulb dimensions, the value of the shift can be determined by extrapolation of the data to infinite bulb dimensions [12]. In the course of such measurements, a temperature dependence of the wall shift has been found. Recent measurements made by Varian Associates [3] indicate that for a  $5\frac{1}{2}$  inch (14 cm) diameter spherical bulb, in the temperature range 27 to 45°C, the shift is  $+2 \times 10^{-4}$  Hz per degree with an uncertainty of about 25 percent in the measurement. A semi-empirical relation to give the wall shift in the region of about 40°C can be written as

$$\delta f_w = K[1 + a(t - 40)] \frac{1}{D}, \quad (3)$$

where  $D$  is the inside diameter of the bulb and  $t$  is the temperature of the bulb in degrees Celsius. The value of  $K$  determined by Varian Associates [13] from wall shift measurements at various temperatures is  $-0.208 \pm 0.002$  Hz inch ( $-0.528 \pm 0.005$  Hz cm), and the value of  $a$  is  $-5 \times 10^{-3}$  per degree. The wall shift of the 6.87 inch (17.46 cm) diameter bulb of H-10 #4 is calculated using the above relation to be

$$\delta f_w(6.87 \text{ inch}, 45^\circ\text{C}) = -0.0295 \pm 0.0006 \text{ Hz.}$$

The effect of collisions of pairs of hydrogen atoms has been studied by Bender [14], Crampton [15], Vanier [11], and others. A small frequency shift of about  $\frac{1}{2}$  the spin exchange linewidth has been calculated for atomic hydrogen [14]. However, in the case of the atomic hydrogen maser, the collision-induced spin-spin exchange frequency shift depends upon the atomic resonance width in the same fashion as does the cavity pulling [16]; the tuning procedure involving the variation of the linewidth includes and cancels the effect of these collision shifts. The pertinent relation is [10]<sup>2</sup>

$$f - f_H = \left[ \left( \frac{f_c - f_H}{f_c} \right) Q - \left( \frac{13\sqrt{2} \bar{v} \hbar a_0^3}{32Q\mu_0^2 \eta'} \right) \right] \Delta f_i, \quad (4)$$

where

- $f$  is the output frequency of the maser,
- $f_H$  is the output frequency of the maser when tuned,
- $\Delta f_i$  is the linewidth of the atomic resonance,
- $f_c$  is the cavity resonance frequency,
- $Q$  is the loaded cavity resonance quality factor,
- $\bar{v}$  is the mean speed of the H atoms in the storage bulb,
- $\hbar$  is Planck's constant divided by  $2\pi$ ,
- $a_0$  is the first Bohr orbit radius of hydrogen,
- $\mu_0$  is the Bohr magneton, and
- $\eta'$  is

$$\frac{\left( \int_b H_z dV_b \right)^2}{V_b \int_c H^2 dV_c},$$

where

- $V_c$  is the cavity volume,
- $V_b$  is the bulb volume,
- $H_z$  is the  $z$  component of the RF magnetic field in the cavity, and
- $H$  is the RF magnetic field in the cavity.

When the expression in the brackets is made zero (which is the "tuned" condition), the output frequency is independent of hydrogen pressure and is equal to  $f_H$ .

Second-order Doppler effect in the hydrogen maser due to thermal motion of the atoms is given by

<sup>1</sup> Du Pont's polytetrafluoroethylene.

<sup>2</sup> The second term of (4) as written in reference [10] contains an error of a factor of 2.



$$\Delta f_T = -f_H \frac{3kT}{2mc^2} = -1.9557 \times 10^{-4}T, \quad (5)$$

where

$k$  is Boltzmann's constant,  
 $m$  is the mass of the hydrogen atom, and  
 $c$  is the speed of light.

A change in the absolute temperature  $T$  of one degree K will give rise to a change in frequency of 1.38 parts in  $10^{13}$ . Since the bulb temperature is known to better than  $\pm 0.4^\circ\text{K}$ , the probable error due to this cause is of the order of 5 parts in  $10^{14}$ .

The value of the average of the square of the magnetic field  $H$  over the volume of the bulb affects the frequency of the oscillation in the amount  $\Delta f_M \approx 2750 \bar{H}^2$ . It is possible to measure the average value of the magnetic field  $\bar{H}$  in the bulb by inducing transitions among the  $F=1$ ,  $m_F=1, 0, -1$  states. The resonance is detected by observing a quenching or reduction in the maser output power. The transition resonant frequency (Zeeman frequency) is given by  $f_z \approx 1.4 \times 10^{-6} \bar{H}$  and is normally measured with an uncertainty of a few Hertz. The uncertainty  $\Delta H$  in  $\bar{H}$  gives an uncertainty in  $\Delta f_M$  through the relation

$$\Delta(\Delta f_M) \approx 5500 H \Delta H.$$

Normally the maser is operated at a field of 0.5 millioersted making the total offset

$$\frac{\Delta f_M}{f} = +4.8 \times 10^{-13}.$$

Assuming a 10 Hz uncertainty in  $f_z$ , corresponding to 7 micro-oersteds, the uncertainty in  $\Delta f/f$  due to magnetic effects is

$$\frac{\Delta(\Delta f_M)}{f} = \pm 1.3 \times 10^{-14}.$$

It will be noticed that there are two averaging methods involved in dealing with the magnetic field and that since  $\bar{H}^2$  is always greater than or equal to  $(\bar{H})^2$  there could be room for error in the determination of  $\Delta f_M$ . In this case, however, the possible error is not large since the effect of magnetic gradients would be detected by a large effect on the level of oscillation. An upper limit to the uncertainty due to these causes is estimated to be of the order of  $5 \times 10^{-14}$ .

The effect of cavity mistuning and the consequent "pulling" of the maser output frequency has already been discussed and the "tuned" condition has been described as the particular cavity resonance frequency chosen so that no output frequency shift results on changing the resonance linewidth. The sensitivity of this procedure depends in large part on the stability of the maser during the tuning process, the linewidth of the atomic resonance  $\Delta f_i$ , and the extent to which  $\Delta f_i$  can be broadened. There are several methods for broadening the line, such as by magnetic gradient quenching [17],

by coherently or incoherently inducing transitions among the Zeeman levels [18], or by spin exchange line broadening. The last method, which has been used for some time, offers the greatest possible differential broadening of the line. If one assumes that the stability of the maser and its reference can be expressed by a maximum excursion  $\pm \Delta f_b$  about the beat frequency, then the limits  $\pm \Delta f/f$  within which the maser can be tuned will be given by

$$\frac{\Delta f}{f} = 2 \frac{\Delta f_b}{f} \frac{1}{\left[1 - \frac{\Delta f_{i_2}}{\Delta f_{i_1}}\right]}. \quad (6)$$

Normally, the ratio  $\Delta f_{i_2}/\Delta f_{i_1}$  lies between 1.5 and 2.0; taking this value as 1.5 and putting  $\Delta f_b/f = 5 \times 10^{-14}$ , then  $\Delta f/f = 2 \times 10^{-13}$  is the maximum tuning error.

The error budget for the Varian hydrogen maser H-10 #4 is given in Table I.

TABLE I  
 CONTRIBUTIONS TO INACCURACY FOR VARIAN ATOMIC  
 HYDROGEN MASER H-10 #4

Source	$1\sigma$ estimate in parts in $10^{12}$
Uncertainty in wall shift correction	$\pm 0.42$
Uncertainty in temperature for 2nd order Doppler shift	$\pm 0.05$
Uncertainty in C-field, $\bar{H}_c$	$\pm 0.01$
Use of $\bar{H}_c^2$ for $\bar{H}_c^2$	$\pm 0.05$
Cavity mistuning	$\pm 0.2$
Random measurement error (1 hour)	$\pm 0.01$
Total $1\sigma$ estimate of accuracy capability for Varian H maser H-10 #4 (square root of sum of squares) $\pm 0.47 \times 10^{-12}$	

### B. Error Budget for NBS III Cesium Beam

A realistic error budget for the NBS III cesium beam standard was published [1] prior to the intercomparison described in this paper, giving  $\pm 5.6$  parts in  $10^{12}$  as a  $3\sigma$  estimate of accuracy uncertainty. However, the very stable signals available from the Varian hydrogen masers and recent theoretical work by Harrach [19] enabled a better determination to be made of some of the factors contributing to the uncertainty. Using one of the masers (H-10 #4) as an undisturbed reference frequency source, a set of evaluative experiments was performed on NBS III to study effects that result in systematic frequency offsets. These have been discussed in detail by Harrach and will only be summarized here.

The experiments consisted of measuring the radiation field dependence of the  $(F=4, m_F=0) \leftrightarrow (F=3, m_F=0)$  cesium resonance frequency under varied conditions of beam direction through the apparatus, excitation spectrum, and magnetic C-field polarity and magnitude. From the analysis and interpretation of the experimental results, it was inferred that fractional corrections of  $-2.2 \times 10^{-12}$  for dependence on radiation field intensity and of  $+0.4 \times 10^{-12}$  for second order Doppler shift should be applied to the measured resonance frequencies. A more complicated correction was necessary

for beam direction dependence of the resonance frequency. The beam direction was reversed four different times by interchanging the oven and detector components, with the intention of evaluating and eliminating the effect of a cavity phase difference. It was found that the process of beam reversal slightly altered the phase difference magnitude, so that the necessary corrections varied from about 2 to 5 parts in  $10^{12}$  for the different beam orientations, with an uncertainty of  $\pm 0.8$  part in  $10^{12}$  for each determination.

The uncertainties in the fractional frequency corrections plus contributions from other sources, itemized in Table II, produce a  $1\sigma$  estimate of accuracy capability of  $\pm 1.1 \times 10^{-12}$  for NBS III. The term "accuracy capability" refers to the accuracy attained when a set of evaluative experiments is performed, as distinct from the accuracy of the standard when it is used for an extended period of time in an undisturbed, routine fashion. The term accuracy is used as defined in reference [1].

TABLE II  
CONTRIBUTIONS TO INACCURACY FOR NBS III CESIUM  
BEAM FREQUENCY STANDARD

Source	$1\sigma$ estimate in parts in $10^{12}$
Uncertainty in average C-field magnitude, $\bar{H}_c$	$\pm 0.1$
Use of $\bar{H}_c^2$ for $\bar{H}_c^2$	$\pm 0.03$
Uncertainty in 1st and 2nd order Doppler shifts	$\pm 0.1$
Distortion from inequality of average C-field magnitudes in transition and drift regions, $\bar{H}_c(l) \neq \bar{H}_c(L)$	$\pm 0.03$
Uncertainty in C-field polarity-dependent shifts	$\pm 0.1$
Uncertainty in cavity phase difference (beam direction dependent shifts)	$\pm 0.8$
Cavity mistuning	$\pm 0.03$
Overlap of neighboring transitions	$\pm 0.1$
Uncertainty in power-dependent shifts	$\pm 0.16$
Random measurement error (1 hour)	$\pm 0.16$
2nd harmonic distortion of servo modulation	$\pm 0.16$
Miscellaneous servo system effects	$\pm 0.5$
Multiplier chain transient phase shifts	$\pm 0.33$
Total $1\sigma$ estimate of accuracy capability for NBS III (square root of sum of squares)	$\pm 1.1 \times 10^{-12}$

A comparison of the error budgets for the Varian maser (Table I) and the National Bureau of Standards cesium beam NBS III (Table II) shows that an apparatus-dependent effect is the major contributor to inaccuracy in each case. For the cesium beam, the frequency shift due to cavity phase difference is uncertain to  $\pm 0.8 \times 10^{-12}$ , or about 16 percent of its magnitude, while for the maser, the wall shift is considered known to  $\pm 0.42 \times 10^{-12}$ , or about two percent of its magnitude.

#### IV. HYDROGEN-CESIUM HYPERFINE FREQUENCIES RELATIONSHIP

An average value of the hyperfine separation of hydrogen in terms of that of cesium 133 can be obtained after suitable corrections are made for both the cesium device and the hydrogen maser.

A plot of  $\Delta f/f$  vs. measurement number and date of measurement is shown in Fig. 3. These data include

corrections made for magnetic field shift, power shift, phase shift, and second order Doppler shift of the cesium frequency. The data also include corrections for the effect of cavity mistuning in the hydrogen maser as determined by the procedure previously described. In order to relate the frequencies of the hyperfine separations of cesium and hydrogen to the synthesized frequencies, reference is made to the phase lock system in the upper half of Fig. 10. The varactor controlled 5 MHz quartz crystal oscillator is used to drive two frequency synthesizing systems. The first, which is simply a 280 times multiplier, provides about 2 milliwatts of power for the reference (local oscillator) input to the low noise (less than 10 dB noise figure) mixer where the maser signal is heterodyned to 20.405 MHz. This signal is amplified by about 110 dB and compared in phase with the output from the second synthesizing system driven by the 5 MHz oscillator. The latter system is a digital synthesizer which can be varied in increments of 0.01 Hz.

Analysis of the data of Fig. 3 gives a mean fractional frequency offset of  $0.1044 \times 10^{-10}$  with a root mean square deviation,  $\sigma$ , of  $7.4 \times 10^{-13}$  for 62 points. This represents a frequency offset of  $0.1044 \times 10^{-10} \times 1420.405 \times 10^6 = 0.0148$  Hz. Further corrections are made to bring the hydrogen frequency to the ideal conditions discussed in Section III.A. The fully corrected hydrogen maser frequency is obtained in the following way:

Synthesized frequency	1 420 405 751.6800 Hz
Offset frequency from Cs comparisons	+0.0148
Correction for wall shift	+0.0295
Correction for magnetic field	-0.0001
Correction for second order Doppler effect	+0.0622
	1 420 405 751.7864 Hz

The  $1\sigma$  accuracy estimate for the measurement is obtained by combining the error budgets of Tables I and II, giving an over-all accuracy of 1.2 parts in  $10^{12}$ . In terms of the frequency of the Cs<sup>133</sup> hyperfine transition ( $F=4, m_F=0$ )  $\leftrightarrow$  ( $F=3, m_F=0$ ), defined [20] as 9 192 631 770.0000 Hz, the frequency of the atomic hydrogen hyperfine transition ( $F=1, m_F=0$ )  $\leftrightarrow$  ( $F=0, m_F=0$ ) is

$$\nu_H = 1\,420\,405\,751.7864 \pm 0.0017 \text{ Hz.}$$

Measurements previously made for the hydrogen frequency in terms of cesium with the wall-shift corrections then used are given in Table III. The last number in the table is the preliminary value reported earlier after a partial analysis of the data. The reduction of the standard error from 0.0046 Hz to 0.0017 Hz was made possible through improved measurements by Varian Associates [13], [3] of the wall shift and its temperature dependence.

From the frequency comparison data in Table III, it is seen that the frequency of the NBS III cesium standard differs from the frequency of the Laboratoire Suisse de Recherches Horologiques, Neuchâtel, Switzerland (LSRH) cesium standard by  $-7 \times 10^{-13} \pm 1.2 \times 10^{-11}$ . This is the smallest discrepancy reported to date between these two national standards [25].

TABLE III  
MEASUREMENTS PREVIOUSLY MADE FOR THE HYDROGEN FREQUENCY IN TERMS OF CESIUM

Experiment	Bulb diameter, inches	Wall shift correction, Hz	Second order Doppler correction, Hz and temperature	Corrected frequencies 1 420 405 751. Hz plus values given below
Varian-Naval Observatory 1963 [21]	$6 \pm 0.1$	$0.039 \pm 0.008$	0.06 40°C	$0.827 \pm 0.02$
Harvard-Naval Observatory 1963 [15]	6.03	$0.0298 \pm 0.003$	0.0602 35°C	$0.800 \pm 0.028$
Varian-HP 1964 [22]	$5.5 \pm 0.1$	$0.040 \pm 0.0023$	0.0622 45°C	$0.778 \pm 0.016$
Varian-LSRH 1964 [23]	$5.5 \pm 0.1$	$0.040 \pm 0.0023$	0.0622 45°C	$0.785 \pm 0.016$
NASA-GSFC 1965 [24]	5.4	$0.043 \pm 0.0023$	0.062 45°C	$0.781 \pm 0.016$
NBS-Varian-HP 1966 [2]	6.87	$0.0320 \pm 0.0043$	0.0622 45°C	$0.7860 \pm 0.0046$

## V. DISCUSSION

The main purposes of this paper have been to report the data obtained from a set of measurements, to offer some measure of the state-of-the-art of time and frequency control, and to report an improved measurement of the frequency of the hydrogen hyperfine separation. Since the arts of frequency standard design and instrumentation for making measurements are still progressing rapidly, one can conclude that these measurements will be followed by further and better measurements.

Long term measurements, yielding average frequencies between standards of different origin, have shown excellent agreement between the Hewlett-Packard 5060A instrument and the NBS III cesium beam designated as the United States Frequency Standard. The relationship between the statistics of the two devices is reasonably well understood, and the discrepancy of the frequency averages of the two instruments is within their accuracy specification.

At present, the resolution of the measurements is limited by random noise of the apparatus. Thus far, no *unexplained* systematic effects are observed. Future designs of beam standards with stronger beam flux and perhaps narrower linewidths will result in the obtaining of better data on systematic frequency shifts of both the beam tube and the frequency lock electronics.

In the case of measurements between two hydrogen masers with phase lock systems, the instabilities introduced by the phase lock-synthesizer system and the measuring system are not as well understood. The levels of instability in the 1 to 100 second sampling time range, though lower than those observed for the cesium devices, cannot be accounted for by the presence of white additive noise of the first mixer in each receiving system. The phase fluctuations due to the frequency multipliers, the digital synthesizers, and the period counting system as well as those of the masers themselves must still be subjected to further study in the 1 to 100 second range. While the present data will pro-

vide a measure of the possible performance of such a system, there is no doubt that this preliminary effort can be improved by more careful engineering.

A comparison of various measurements having sampling times ranging from  $10^2$  to  $10^4$  seconds is given in Fig. 17. Here the data from the hp 5060A, NBS III, and H-10 #4 are given together. The value of  $\sigma$  for H-10 #4 is obtained by dividing the value of  $\sigma$  from H-10 #4 vs. H-10 #3 by the square root of 2. Both cesium instruments exhibit the  $\tau^{-1/2}$  behavior as discussed previously. The maser data show systematic variations at the  $10^{-14}$  level, which are probably due to cavity detuning effects that are largely thermal in origin. By incorporating automatic self-tuning methods, using the spin exchange line broadening method described earlier, it is possible that the maser stability can be kept to better than  $5 \times 10^{-14}$  for sampling times greater than  $10^4$  seconds.

The accuracy of the hydrogen-cesium ratio is limited by the characteristics of the devices used to extract the information from the two respective atoms. The maser, in confining the atoms in a bulb, subjects them to wall interactions. These must be measured as a function of reciprocal bulb diameter  $1/D$  and extrapolated to zero in  $1/D$ . In the case of the beam devices, the frequency resolution increases with increase of beam flux and inverse linewidth. Flux effects favor the construction of a short apparatus, and linewidth effects favor the construction of a long apparatus. It is important to control the intensity, spectral purity, and phase of the radio frequency excitation applied to the beam.

Further work in beam devices and masers will undoubtedly improve the accuracy of the hydrogen-cesium frequency ratio. The state-of-the-art has undergone considerable improvements, and the absolute accuracy of these devices is now at the  $10^{-12}$  level, being 1.1 parts in  $10^{12}$  for a cesium beam and 0.47 part in  $10^{12}$  for a hydrogen maser. Frequency and time measurements using atomic standards are now, more than ever, the most accurate and precise physical measurements which can be made.

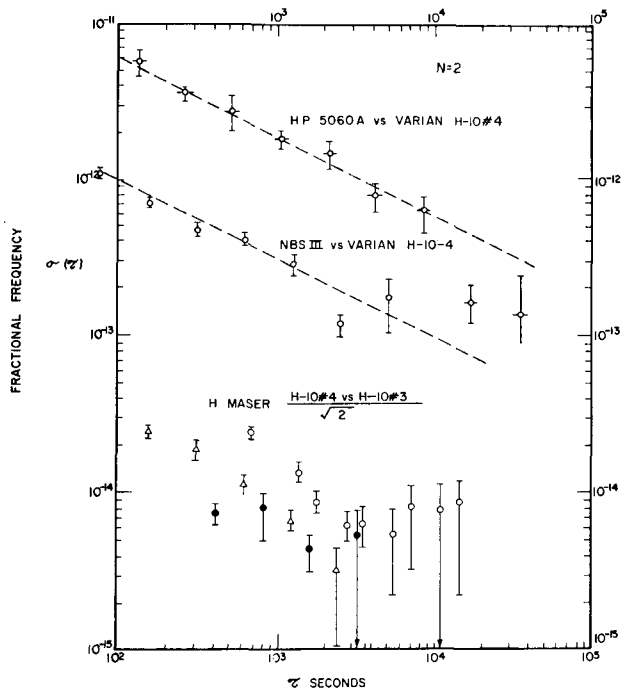


Fig. 17. Summary of frequency stability data as a function of sampling time  $\tau$ .

#### APPENDIX

The variance  $\sigma^2$  of the frequency fluctuations of a frequency standard is a useful parameter in determining quality factors for the unit. When finite data sampling is considered, this parameter is dependent on four independent variables of the analysis, i.e., the sample time  $\tau$ , the period of sampling  $T$ , the number of samples taken  $N$ , and the system bandwidth  $\omega_B$ .

For the experiments reported in this paper,  $T/\tau$  and  $\omega_B$  were constant for any given experiment, but they were not necessarily the same from one experiment to the next. For all  $\sigma$  versus  $\tau$  plots where  $T/\tau$  was a significant parameter, it was approximately equal to unity.  $N$  was always chosen equal to 2 and an ensemble average taken to determine a good statistical measure of the variance. The confidence limits are calculated on the assumption that each particular computed variance is uncorrelated with any other. A computer program based on the above method was used to analyze the data obtained in these measurements.

The variance is a function of the spectral density of the phase fluctuations of the system. By proper control of the above four analysis variables, it is possible to determine the frequency dependence of the spectral density of the phase fluctuations  $S_\phi(\omega)$ , as follows. If  $\langle \sigma^2(\tau, T, N, \omega_B) \rangle = k\tau^\mu$ , with  $k$  and  $\mu$  constant over a sufficiently large range of  $\tau$ , with  $-2 < \mu < +2$ , then

$$S_\phi(\omega) = h |\omega|^\alpha, \quad (7)$$

where  $\alpha = -\mu - 3$ , provided that  $N$ ,  $T/\tau$ , and  $\omega_B$  are kept constant and that  $\tau \gg 1/\omega_B$ . If  $\mu = -2$ , a degeneracy of  $\alpha$  occurs, and one may then use the dependence of the variance on the bandwidth,

$$\langle \sigma^2(\tau, T, N, \omega_B) \rangle = \frac{k}{\tau^2} |\omega_B|^{\alpha+1}, \quad \alpha > -1,$$

for determining the frequency dependence of the spectral density. For a detailed treatment of the above see Allan [4].

#### REFERENCES

- [1] R. E. Beehler, R. C. Mockler, and J. M. Richardson, "Cesium beam atomic time and frequency standards," *Metrologia*, vol. 1, pp. 114-131, July 1965.
- [2] R. Beehler et al., "An intercomparison of atomic standards," *Proc. IEEE (Letters)*, vol. 54, pp. 301-302, February 1966.
- [3] R. Vessot et al., "A direct frequency comparison of hydrogen masers in separate laboratories by concurrent monitoring of Loran C signals," *Proc. IEEE (Letters)*, vol. 54, pp. 303-304, February 1966.
- [4] D. W. Allan, "Statistics of atomic frequency standards," *Proc. IEEE*, vol. 54, pp. 221-230, February 1966.
- [5] L. S. Cutler and C. L. Searle, "Some aspects of the theory and measurement of frequency fluctuations in frequency standards," *Proc. IEEE*, vol. 54, pp. 136-154, February 1966.
- [6] R. E. Beehler et al., "A comparison of direct and servo methods for utilizing cesium beam resonators as frequency standards," *IRE Trans. on Instrumentation*, vol. I-11, pp. 231-238, December 1962.
- [7] R. Vessot, L. Mueller, and J. Vanier, "The specification of oscillator characteristics from measurements made in the frequency domain," *Proc. IEEE*, vol. 54, pp. 199-207, February 1966.
- [8] J. A. Barnes, "Atomic timekeeping and the statistics of precision signal generators," *Proc. IEEE*, vol. 54, pp. 207-220, February 1966.
- [9] D. Kleppner et al., "Hydrogen-maser principles and techniques," *Phys. Rev.*, vol. 138, pp. A972-A983, May 17, 1965.
- [10] J. Vanier, H. E. Peters, and R. F. C. Vessot, "Exchange collisions, wall interactions, and resettability of the hydrogen maser," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-13, pp. 185-188, December 1964.
- [11] J. Vanier and R. F. C. Vessot, "Cavity tuning and pressure dependence of frequency in the hydrogen maser," *Appl. Phys. Lett.*, vol. 4, pp. 122-123, April 1, 1964.
- [12] D. Kleppner, H. M. Goldenberg, and N. F. Ramsey, "Theory of the hydrogen maser," *Phys. Rev.*, vol. 126, pp. 603-615, April 15, 1962.
- [13] J. Vanier (to be published).
- [14] P. L. Bender, "Effect of hydrogen-hydrogen exchange collisions," *Phys. Rev.*, vol. 132, pp. 2154-2158, December 1, 1963.
- [15] S. B. Crampton, D. Kleppner, and N. F. Ramsey, "Hyperfine separation of ground-state atomic hydrogen," *Phys. Rev. Lett.*, vol. 11, pp. 338-340, October 1, 1963.
- [16] S. B. Crampton, "Hyperfine and spin exchange experiments with the atomic hydrogen maser," Ph.D. dissertation, Harvard University, Cambridge, Mass., 1964, unpublished.
- [17] J. Vanier and R. Vessot, "Relaxation in the level  $F=1$  of the ground state of hydrogen: application to the hydrogen maser," *IEEE J. of Quantum Electronics*, vol. QE-2, pp. 391-398, September 1966.
- [18] H. G. Andresen and E. Pannaci, "Servo-controlled hydrogen maser cavity tuning," *Proc. 20th Annual Symp. on Frequency Control*, 1966, to be published.
- [19] R. J. Harrach, "Some accuracy limiting effects in an atomic beam frequency standard," *Proc. 20th Annual Symp. on Frequency Control*, 1966, to be published, and "Radiation field dependent frequency shifts of atomic beam resonances," to be published in *J. Appl. Phys.*
- [20] *Comptes Rendus des Séances de la Douzième Conférence Générale des Poids*, Paris, France, October 6-13, 1964.
- [21] W. Markowitz et al., "Report on the frequency of hydrogen," *Frequency*, vol. 1, p. 46, July-August 1963.
- [22] H. E. Peter, et al., "Hydrogen maser and cesium beam tube frequency standards comparison," *Appl. Phys. Lett.*, vol. 6, pp. 34-35, January 15, 1965.
- [23] H. E. Peter and P. Kartaschoff, "Hydrogen maser frequency comparison with Swiss cesium atomic beam standard," *Appl. Phys. Lett.*, vol. 6, pp. 35-36, January 15, 1965.
- [24] E. H. Johnson and T. E. McGunigal, "Hydrogen maser frequency comparison with a cesium standard," NASA Technical Note TN D-3292, April 1966. The title and publication date of this technical note were not given correctly in reference [7] of Beehler et al. [2].
- [25] J. Bonanomi et al., "A comparison of the TA<sub>1</sub> and the NBS-A atomic time scales," *Proc IEEE (Correspondence)*, vol. 52, p. 439, April 1964.