An HF Array Antenna Electronically Scanned in Elevation

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***Abstract*—An HF receiving array antenna, consisting of ten 12 to 25 MHz horizontally-polarized log-periodic-dipole element antennas, spaced vertically 16 meters apart on the side of a 152-meter (500-foot) tower, has been erected near Boulder, Colo. It is capable of electronically scanning a sector in elevation ranging from 3.5° to 51° at 12 MHz, to 1.7° to 22° at 25 MHz. The sector was scanned with a single major lobe which had a vertical plane half-power beamwidth that varied from 7° at 12 MHz to 3° at 25 MHz. This array was a part of a system designed for studying the angle-of-arrival of downcoming radio waves from the ionosphere.**

A sinusoidal current distribution formed a single major lobe which was scanned in elevation by varying the period of the sinusoid. Scanning was accomplished electronically at an intermediate frequency; therefore, the increase in bandwidth of the received signal, caused by the rapid scanning, took place after conversion and did not necessitate an increase in bandwidth of the RF input circuitry. The scanning rate was 524 Hz with the sector scanned by an upgoing lobe and a downgoing lobe in each scanning cycle.

Photographs of the oscilloscope display are presented which show the vertical angle-of-arrival of 10 and 20 MHz signals from WWV. Comparison of sinusoidal distribution with conventional shows that the sinusoidal distribution results in half the beamwidth of the latter, 3 dB higher gain, and a lower minimum angle above horizon.

I. INTRODUCTION

BECAUSE of the importance of elevation angles-of-arrival in HF propagation studies, a ground-based receiving array antenna with a single major lobe capable of being scanned in elevation would be desirable. In the past, several attempts have been made to construct such an array. The most noteworthy of these efforts utilized endfire phasing to obtain the desired directivity in the vertical plane. Examples of such arrays are the MUSA,[1] the ISCAN,[2] and the MEDUSA.[3] The MUSA and ISCAN are linear endfire arrays, while the MEDUSA is a planar array whose major lobe can be steered in azimuth as well as in elevation.

These arrays have several disadvantages when used for HF elevation angle-of-arrival studies. They have an undesirably high degree of directivity in azimuth, and they must be many wavelengths long to achieve narrow vertical-plane beamwidths at low elevation angles. It is the low angles-of-arrival which are of primary interest in HF propagation studies.

In July, 1961, work commenced on a project at the National Bureau of Standards, Boulder Laboratories, Boulder, Colo., for the study of azimuthal and elevation angles-of-arrival of ionospherically propagated radio waves, both direct and backscattered, and their variations with time. A requirement of the project was to design and construct two HF receiving array antennas with major lobes capable of being electronically scanned at a rapid rate, one in azimuth, the other in elevation. These arrays were to be used to observe simultaneously, though independently, the azimuthal direction and elevation angle-of-arrival of downcoming HF radio waves. The two arrays, which were completed late in 1962, are shown in Fig. 1. This paper discusses only the vertical array.[1]

II. SYNTHESIS OF AN ILLUMINATION FUNCTION

The synthesis problem described here was directed toward obtaining a solution which was practical rather than mathematically exact or "optimum." The current distribution settled upon, besides producing a single lobe in elevation, does this without introducing super-gain, unequal element spacings, or exotic amplitude variations. All currents are in phase (although they may vary in sign) and element spacings are uniform. A Chebyshev current distribution is, however, introduced to limit the sidelobes. For purposes of computation, a -30 dB sidelobe level has been used. For practical purposes, a -20 dB sidelobe level can be readily obtained. Such a practical solution permits the use of electronic scanning which is a modified approach of that described by Cottony and Wilson.[4]

The illumination or excitation of a continuous vertical aperture, which is required to generate a single major lobe in elevation, can be determined using an elementary form of synthesis. The following derivation has been presented briefly by Cottony,[5] but because this paper may not be widely available, the derivation is given in detail here. Standard antenna array theory is used to determine the illumination $I(h)$ for a vertical aperture formed by a continuous sheet of horizontally polarized current elements. For the problem at hand, the pattern factor of these current elements will be assumed equal to unity since all interest during the development of the illumination function will be directed

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[1] A more detailed treatment of this vertical array is given in FitzGerrell *et al.*[9]

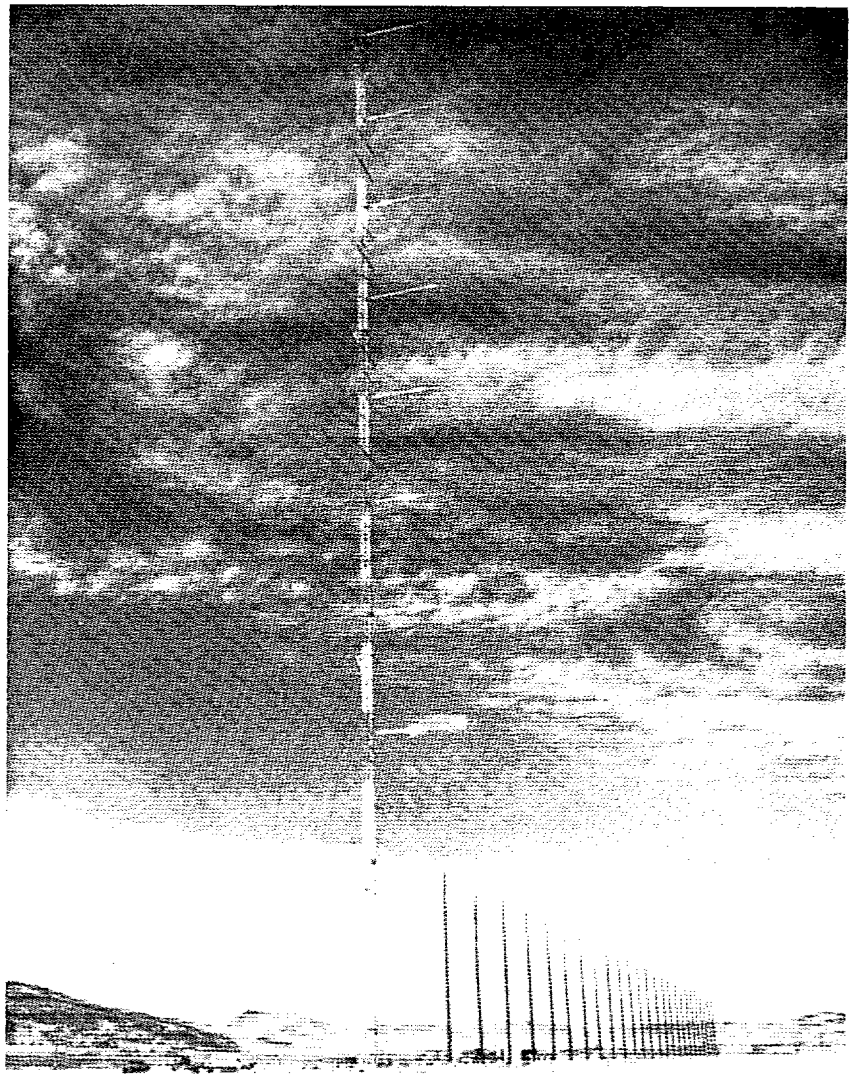

Fig. 1. The elevation-scan array and the adjacent azimuth-scan array.

to the vertical plane passing through the center line of the aperture and normal to the aperture. Fig. 2 presents this geometry, and shows the location of a single current element of length dl in the aperture, as well as its image in perfect ground.

The contribution to the radiated field $dE(\Delta)$ by a horizontally polarized current element $I(h)dh$ at a height h above perfectly reflecting ground is given by:

$$dE(\Delta) = -j\left[I(h)e^{j\beta h \sin \Delta} - I(h)e^{-j\beta h \sin \Delta}\right]dh, \qquad (1)$$

where

h = the height of the current element above ground,
$\beta = 2\pi/\lambda$,
λ = the signal frequency wavelength measured in the same units as h, and
Δ = elevation angle in degrees.

In this equation, $E(\Delta)$ is normalized with respect to the free-space field intensity due to a horizontal current element located at the origin.

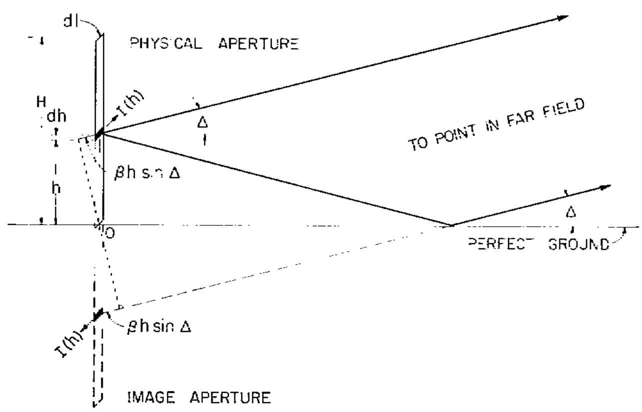

Fig. 2. Aperture geometry used to determine an illumination to produce a single major lobe in elevation.

From (1),

$$dE(\Delta) = 2I(h) \sin(\beta h \sin \Delta)dh, \qquad (2)$$

$$E(\Delta) = 2\int_0^H I(h) \sin(\beta h \sin \Delta)dh, \qquad (3)$$

where H = the total aperture height. A solution for $I(h)$ can be obtained by inspecting (3). A functional form is required for $I(h)$ which will maximize $E(\Delta)$ for any one specified elevation angle Δ_0, in the range $0 \leq \Delta \leq (\pi/2)$. That is, when $\Delta = \Delta_0$, $E(\Delta)$ should be maximum; when $\Delta \neq \Delta_0$, $E(\Delta)$ should be as small as possible. This idea, coupled with experience gained from previous work,[6],[7] suggests the use of a sine function for $I(h)$. Therefore, to maximize the radiation at some elevation angle Δ_0, let

$$I(h) = I_0 \sin (\beta h \sin \Delta_0), \tag{4}$$

where I_0 is the maximum value of $|I(h)|$. Now, using (3) and (4),

$$E(\Delta) = 2I_0 \int_0^H \sin (\beta h \sin \Delta_0) \sin (\beta h \sin \Delta) dh, \tag{5}$$

$$= I_0 H \left\{ \frac{\sin [\beta H (\sin \Delta_0 - \sin \Delta)]}{\beta H (\sin \Delta_0 - \sin \Delta)} - \frac{\sin [\beta H (\sin \Delta_0 + \sin \Delta]}{\beta H (\sin \Delta_0 + \sin \Delta)} \right\}, \tag{6}$$

$$= E_1(\Delta) + E_2(\Delta). \tag{7}$$

The field intensity $E(\Delta)$ can be considered to consist of two components, $E_1(\Delta)$ and $E_2(\Delta)$, each of which has a $\sin x/x$ form. Note that the beamwidth of these two components is determined by an effective aperture of width $2H$. The image aperture is seen to double the effective length of the physical aperture, and to reduce the vertical beamwidth by a factor of two. By using (4) instead of a uniform current distribution with a linear phase progression in the downward direction, an approximate 3 dB increase in directive gain is realized. The radiation pattern $|E_1(\Delta)|$ has a $|\sin x/x|$ form with the main lobe oriented at $\Delta = \Delta_0$. The radiation pattern $|E_2(\Delta)|$ is the mirror image of $|E_1(\Delta)|$ and has its major lobe oriented at $\Delta = -\Delta_0$. This lobe at $-\Delta_0$ has no physical significance because it appears below ground level, but its sidelobes do have a physical significance because some of them may appear in the region $0 \leq \Delta \leq (\pi/2)$. It is the reinforcement and cancellation of the sidelobes of $E_1(\Delta)$ and $E_2(\Delta)$ which cause the numerous closed contours, or spots, in the contour form radiation patterns which are described in Section IV.

The current distribution given by (4) can now be rewritten as

$$I(h) = \frac{-jI_0}{2} \left[e^{j\beta h \sin \Delta_0} - e^{-j\beta h \sin \Delta_0} \right]. \tag{8}$$

The illumination $I(h)$ can be considered as two components of equal amplitude which are opposite in sign and phase. For a specified elevation angle Δ_0, $\sin \Delta_0$ is a constant and the phase of the two components of $I(h)$ is then a linear function of h. This means that the sine function aperture distribution may be resolved, at any specified elevation angle, into two linear distributions, each having an amplitude $I_0/2$. This resolved aperture distribution produces the two main lobes as described previously—an imaginary one in the region below ground at an angle $-\Delta_0$, due to $E_2(\Delta)$, and one at $+\Delta_0$, due to $E_1(\Delta)$.

III. Application to an Array of Discrete Radiators

Now a transition will be made from the consideration of a continuous aperture to the consideration of an array of discrete, horizontally polarized element antennas with a uniform spacing equal to twice the height of the bottom antenna above the ground.

The elevation angle of the major lobe Δ_0 can be changed from some minimum angle to 90° by changing the period of (4). This minimum angle, for our purposes, occurs when the upper first sidelobe of $E_2(\Delta)$ coincides with the major lobe of $E_1(\Delta)$; at this point

$$\Delta_0 \approx \sin^{-1} \frac{3\lambda}{8H}.$$

For the discrete array, the maximum angle of interest is not necessarily 90°. A more important angle is determined when the electrical spacing of the element antennas is 180°. This angle, Δ_0 max, is the upper limit to Δ_0 under the restriction that only a single major lobe can occur in the sector $\Delta_0 \min \leq \Delta \leq \Delta_0 \max$. This angle is equal to the angular spacing between the major lobe and its grating lobes. For the practical use considered here, $\Delta_0 \max \leq 90°$. The minimum aimed angle is given approximately by

$$\Delta_0 \min \approx \left. \frac{3\lambda}{8(2r-1)h_1} \right|_{r=N}$$

where h_1 is the height of the bottom antenna above the ground, r is the antenna number counting from the bottom, and N is the total number of antennas in the array.

The equation for the current distribution as a function of antenna number (height) can be written in a form similar to (4) as

$$I(r) = I_r \sin [\beta(2r-1)h_1 \sin \Delta_0], \tag{9}$$

where $I_r = |I(r)|$ maximum. Initially, I_r is considered to be constant. Later it is shown that a Chebyshev distribution may be superimposed on I_r to minimize the sidelobe level. Now, from (9), Δ_0 max can be found by letting

$$\beta(2r-1)h_1 \sin \Delta_0 = (2r-1)\frac{\pi}{2},$$

giving

$$\Delta_0 \max = \sin^{-1} \frac{\lambda}{4h_1}. \tag{10}$$

The radiation pattern for this vertical array, over perfect ground, with a current distribution given by (9), can now be written by following the general form of (3).

$$|E'(\Delta)| = \left| \sum_{r=1}^{N} D(\Delta) I_r \sin \left[\pi(2r-1)\frac{t}{T} \right] \cdot \sin [\beta h_1 (2r-1) \sin \Delta] \right|, \tag{11}$$

where $D(\Delta)$ = directivity of the element antenna in elevation, and

$$\frac{t}{T} = \frac{2h_1 \sin \Delta_0}{\lambda}.$$

IV. Scanning the Sector in Elevation

To sweep the vertical sector Δ_0 min $\leq \Delta \leq \Delta_0$ max continuously, $\sin \Delta_0$ is made to increase with time as

$$\sin \Delta_0 = \frac{t}{T}\left(\frac{\lambda}{2h_1}\right), \tag{12}$$

where

T = the scanning period in seconds, and
t = time in seconds.

Equation (9) can be rewritten using (12) to get

$$I(r) = -j\,\frac{Ir}{2}\left[e^{j\pi(2r-1)t/T} - e^{-j\pi(2r-1)t/T}\right]. \tag{13}$$

It can be seen from (13) that as time progresses, the sinusoidal current distribution at any antenna position is the sum of two equal amplitude currents, one positive and one negative, one with its phase advancing with time and one with its phase retarding with time.

The contour-form radiation patterns given in Figs. 3(a) and (b) were computed using (11), and show $|E'(\Delta)|$ as a function of Δ and t/T for the ten-element array of log-periodic dipole antennas situated over perfect ground. For Fig. 3(a), $\lambda = 25$ meters and $h_1 = 8$ meters; for Fig. 3(b), $\lambda = 12$ meters and $h_1 = 8$ meters. Also from (13), it can be observed that I_r is not a function of height or time and can, therefore, be tapered to improve the major lobe versus sidelobe level—a Chebyshev taper, for instance.[8] This taper, or distribution, is applied with the center amplitude at the bottom antenna, because the image of the array forms one half of the effective array for which the taper must be calculated. Note that it is the reinforcement and cancellation of the equal-amplitude sidelobes of $|E_1(\Delta)|$ and $|E_2(\Delta)|$ for the discrete array which give the uniformly spotted appearance to Figs. 3(a) and (b).

Some understanding of the scanning array can be gained by examining Figs. 3(a) and (b) and (12). As t increases from near 0 to $T/2$, the major lobe moves from Δ_0 min to Δ_0 max. As t increases further from $T/2$ to near T, the first (original) major lobe moves up and into the imaginary region, $\sin \Delta_0 > 1$, as a new lobe, which was a grating lobe moving down until it met and reinforced the first lobe at $t = T/2$, sweeps the sector from Δ_0 max to Δ_0 min. The sector is swept twice in each scanning period. The grating lobes are relied upon for an infinite supply of major lobes as each old one passes out of the sector. If Δ_0 max $> \pi/2$ there will be a portion of the scanning period when no major lobe is present in the sector. If Δ_0 max $< \pi/2$ there will be multiple lobes in the sector for some portion of the scanning period. This latter situation occurs for the array described herein, as seen in Figs. 3(a) and (b).

In the offset-frequency scanning technique,[4] used to scan the major lobe of the array described here, $\sin \Delta_0$ is made to increase linearly with time in the electronic circuitry of the scanning system. The resulting phase variation of $I(r)$ can be introduced in converters of the receiving system which are connected to each individual antenna of the array. The function of the converter is to modulate the signal received by its associated antenna with two current distributions of opposite polarity with equal, but opposite, phase displacements. For a receiving system, this can be accomplished by modulating the received signal voltage $e_0 \sin \omega_0 t$ with two heterodyne voltages having this prescribed relationship.

For the rth converter these voltages will be

$$e_{r1} = e_0 \sin\left[\omega_0 t + (2r-1)\phi\right], \tag{14}$$

$$e_{r2} = -\,e_0 \sin\left[\omega_0 t - (2r-1)\phi\right]. \tag{15}$$

For a scanning antenna, ϕ is a function of time and will be denoted as $t\omega_s$ where ω_s, the angular frequency of scan, is 2π times the scan rate. This gives

$$e_{sr1} = e_0 \sin\left[\omega_0 + (2r-1)\,\omega_s\right] t, \tag{16}$$

$$e_{sr2} = -\,e_0 \sin\left[\omega_0 - (2r-1)\omega_s\right] t, \text{ and} \tag{17}$$

$$\tfrac{1}{2}(e_{sr1} + e_{sr2}) = e_0 \cos \omega_0 t\left[1 + \sin(2r-1)\omega_s t\right] - e_0 \cos \omega_0 t. \tag{18}$$

The sum of the two desired heterodyne voltages is equal to the basic, or center-frequency heterodyne voltage modulated by a voltage with a frequency equal to $(2r-1)$ times the angular frequency of scan, with the fundamental suppressed. This, in effect, describes the manner in which the heterodyne voltages were generated. A set of coherent phase-locked voltages, with frequencies equal to odd multiples of ω_s, were generated. These modulated the fundamental sideband-generator voltage, and each pair of sidebands was delivered to a converter while the fundamental was suppressed.

The receiver bandwidth requirement of the scanned array will be greater than the minimum bandwidth of the same array with a fixed lobe. The bandwidth of the vertical-scan system was determined only by the RF input circuitry of the converters, since scanning was accomplished at an intermediate frequency. This placed a 20 kHz bandwidth requirement on the second mixer and all of the circuitry that followed, not on the converter RF input.

V. Array Construction

The construction of the vertical array was started in May, 1962, and completed in December, 1962. Most of the details concerning this array can be found in FitzGerrell *et al.*[9] Fig. 1 shows this array and the adjacent azimuth-scan array as they are located at the Institute for Telecommunication Sciences and Aeronomy's Table Mountain field site north of Boulder, Colo. The vertical array consists of 10 log-periodic-dipole element antennas mounted on a 152 meter tower. These element antennas are uniformly spaced 16 meters apart with the bottom antenna 8 meters above the ground. The antennas used for this array are identical to each other and to those used in the azimuth-scan array. In the frequency range of 12 to 25 MHz, the VSWR of each element antenna is less than 2:1, the gain is about 7 dB above isotropic, the minimum front-to-back ratio is about 20 dB, and the E- and H-plane half-power beamwidths are approximately 75° and 115°, respectively.

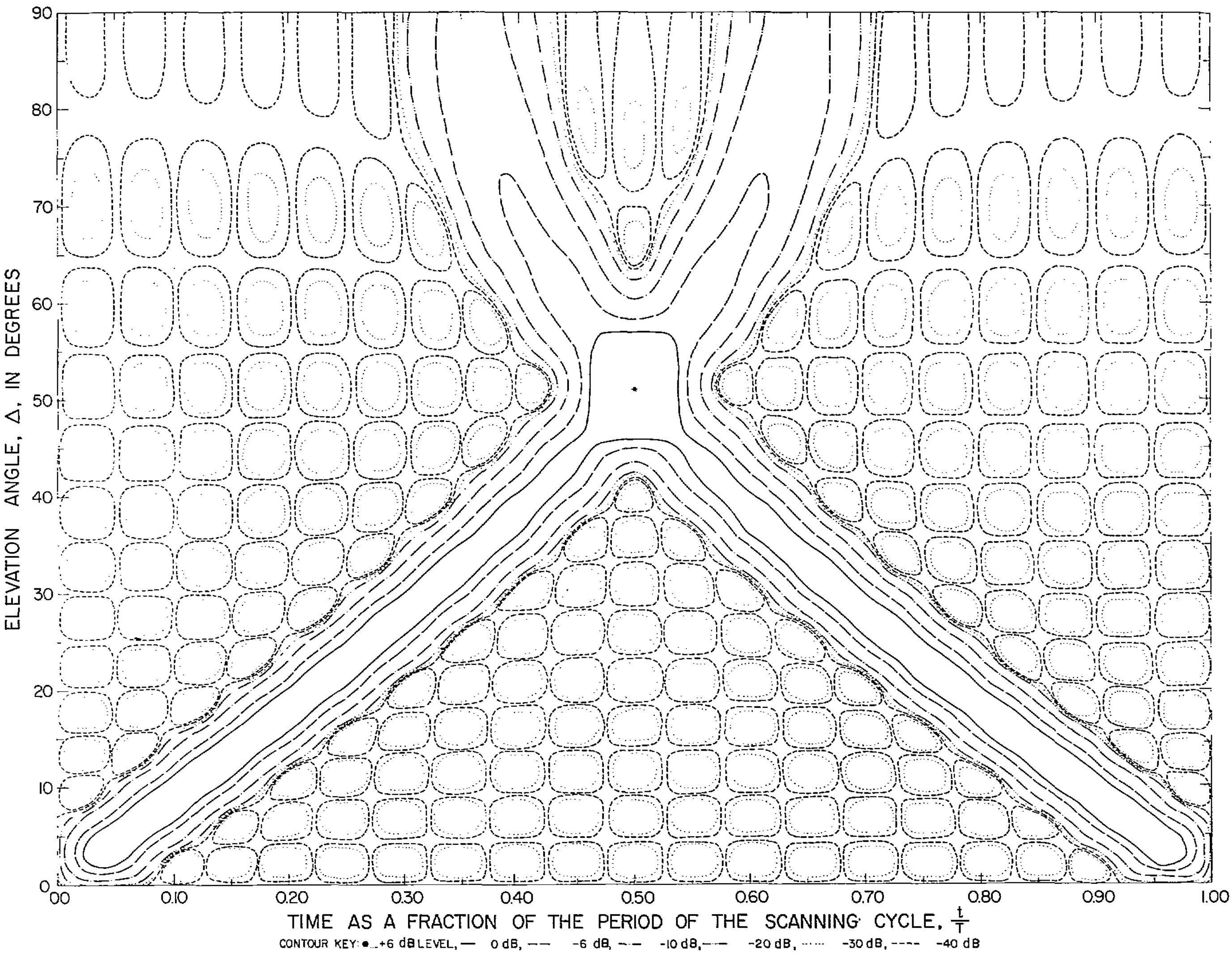

Fig. 3(a). Radiation pattern of the array as a function of elevation angle and time within a scanning cycle at 12 MHz.

VI. Results

The output signal of the scanning system was cyclic at a 524-Hz repetition rate, and was viewed on an oscilloscope using an *A*-scan type of display. The oscilloscope's horizontal sweep was triggered by a 524-Hz tuning fork frequency standard and the detected output was connected to the oscilloscope's vertical amplifier input. This arrangement displayed the signal amplitude as the ordinate, versus the sine of the elevation angle-of-arrival as the abscissa. Since the abscissa was actually calibrated in degrees of elevation angle, this scale was not linear.

Figs. 4(a) and (b) show the display of signals received at Boulder from WWV, Greenbelt, Md.—a 2450 km path. The photographs were rotated 90° clockwise to aid in their interpretation. The left-hand trace shows the received signal elevation angle versus amplitude. The right-hand trace is a simulated array antenna pattern obtained by applying equal amplitude calibration signals, alternating 180° in phase, to the converters. The converter of the bottom antenna received a positive signal, the next antenna received a negative signal, etc. This calibration simulated a signal arriving at the point of reinforcement of the two major lobes which sweep the sector. Thus, the simulated pattern permitted calibration of the elevation angle-of-arrival scale on this photograph. These figures give elevation angles-of-arrival for 10- and 20-MHz signals received from WWV. These signals appear at about 7° and 32° in Fig. 4(a) and at about 0° and 11° in Fig. 4(b). The response at about 69° in Fig. 4(a) can be disregarded, because it occurs at the point of reinforcement of the two major lobes where the array response is about 6 dB higher than it is in the remainder of the sector shown in this figure.

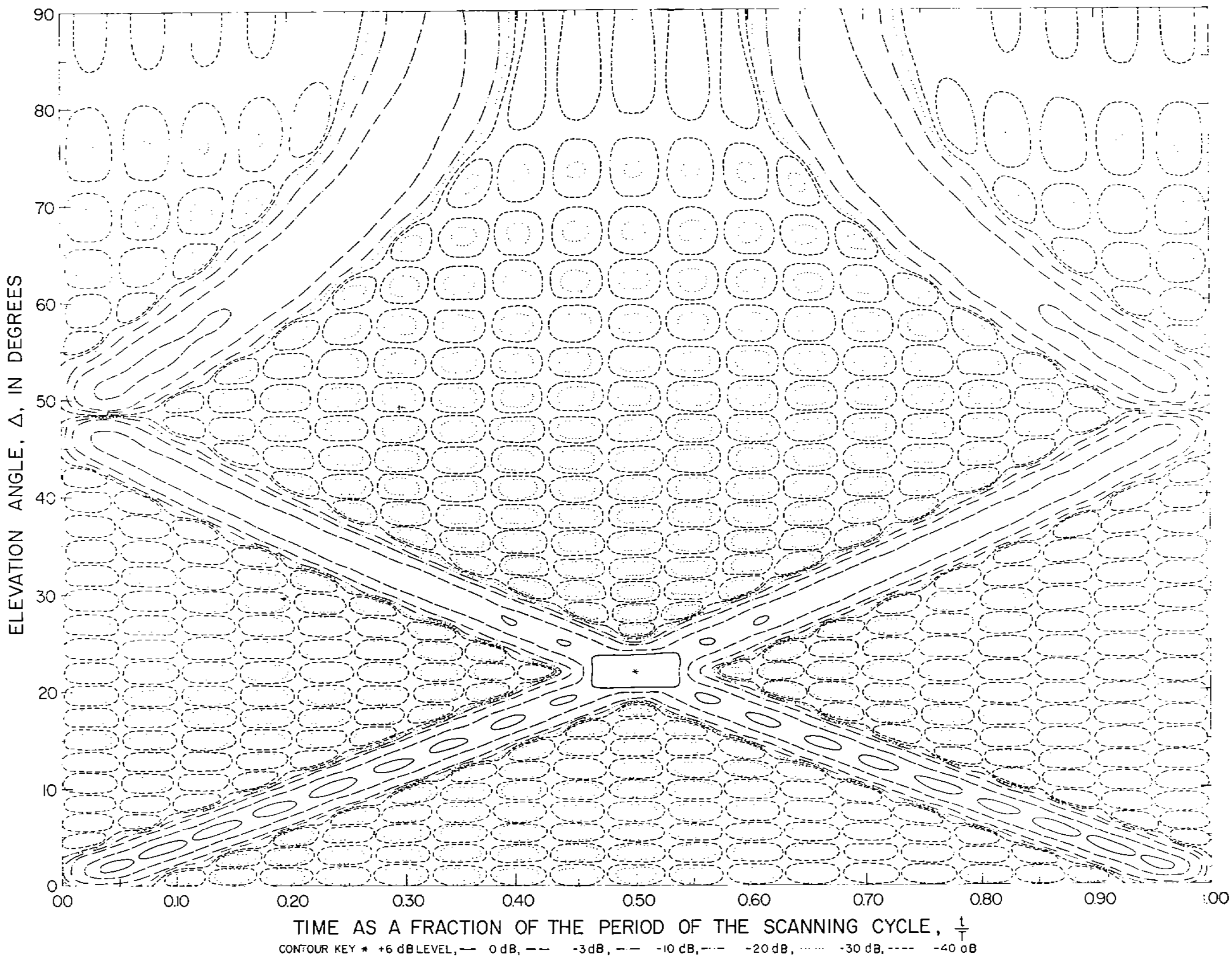

Fig. 3(b). Radiation pattern of the array as a function of elevation angle and time within a scanning cycle at 25 MHz.

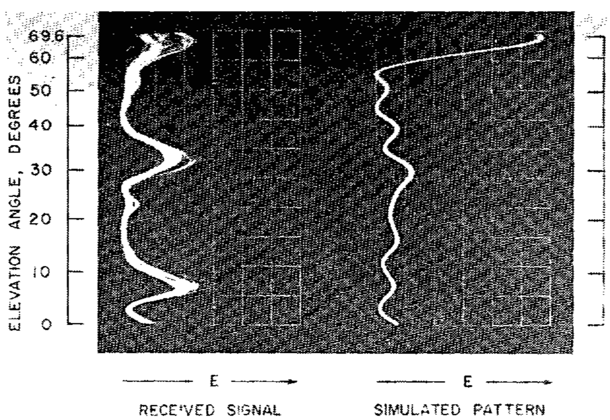

Fig. 4(a). Presentation of reception of WWV at 10 MHz showing received signals at elevation angles of 7° and 32°. Recorded at 1200 hours, December 6, 1962.

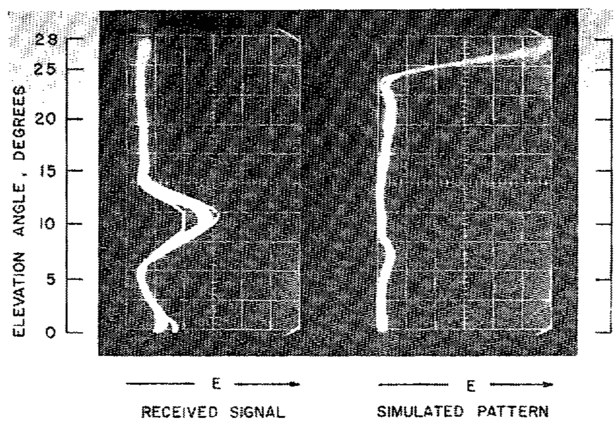

Fig. 4(b). Presentation of the reception of WWV at 20 MHz showing a received signal at an elevation angle of 0° and 11°. Recorded at 1000 hours, January 8, 1963.

VII. Discussion

The principal contribution of this work has been the introduction of the sinusoidal current distribution to a vertical aperture above a reflecting ground. Such distribution results in a single major lobe in elevation which can be scanned or steered by changing the period of the sinusoidal distribution.

The secondary contribution has been the development of electronic circuitry by which the sinusoidal distribution is generated and by which the major lobe is scanned in elevation.

The advantages of a vertical aperture over an endfire array are readily understood. At a frequency of 19.72 MHz, where the 152 meter height of the antenna is equal to 10 wavelengths, and at the "aimed" angle Δ_0 of 10°, the half-power beamwidth of the array described here, using sinusoidal illumination and Chebyshev taper for -20 dB sidelobe level, is 2.5°. An endfire array required to give the same width of the major lobe, at the same elevation angle, would have to be 1520 meters long. Further, the endfire array would almost have to be vertically polarized. In such a case, to get very low angles, it would need to be sited over highly conducting ground such as sea water. For high angles, over 45° above the horizon, the situation would be reversed; but high angles are not of interest to the application at hand, the study of long-distance ionospheric propagation.

The relative merits of sinusoidal illumination over other possible illuminations, such as conventional illumination with phase progression, are less readily appreciated. Through experience in presenting explanations of these merits, many misunderstandings have been encountered. To resolve these, computations of radiation patterns in elevation over the range $\Delta=0°$ to $\Delta=30°$ were carried out for the same array with "conventional" and sinusoidal illumination. The configuration of the array and the excitation currents are shown in Fig. 5. Fig. 5(a) shows the disposition of the elements in the existing array, and Fig. 5(b) shows the current amplitudes for the "conventional" illumination with Chebyshev taper, curve A, for a -20 dB sidelobe level. The scanning or steering is achieved by phasing the elements so as to direct the major lobe upward at the angle Δ_0. The element current amplitudes remain the same for all "aimed" angles. Fig. 5(c) shows the current amplitudes and polarities for the sinusoidal illumination. Besides the sinusoid, the amplitudes are also determined by a Chebyshev taper, curve B, corresponding to a -20 dB sidelobe level. For this illumination, the taper is that for one half of a 20 element array. The currents are all in phase, although polarities may be reversed. The actual distribution, curve C, is the product of the Chebyshev taper and the sinusoid. The amplitudes of the currents do change with Δ_0. The amplitudes shown here correspond to $\Delta_0=11.95°$.

Fig. 6 presents the computed radiation patterns of the array when illuminated by a sinusoidal distribution (solid line curves), and by a conventional illumination (dashed line curves). The figure has seven parts, one for each of the seven aiming angles Δ_0. The first three parts, 6(a) to (c), are intended to compare the patterns when aimed well above the horizon, and the fourth, 6(d), when the major lobe of the sinusoidal distribution is still clear of the horizon, while that for the conventional distribution is contiguous to the horizon. The last three parts, 6(e) to (g), illustrate the behavior of the array using the two illuminations when the major lobes are being reinforced or interfered with by the ground reflections. In all cases horizontal polarization and perfect ground reflection characteristics are assumed.

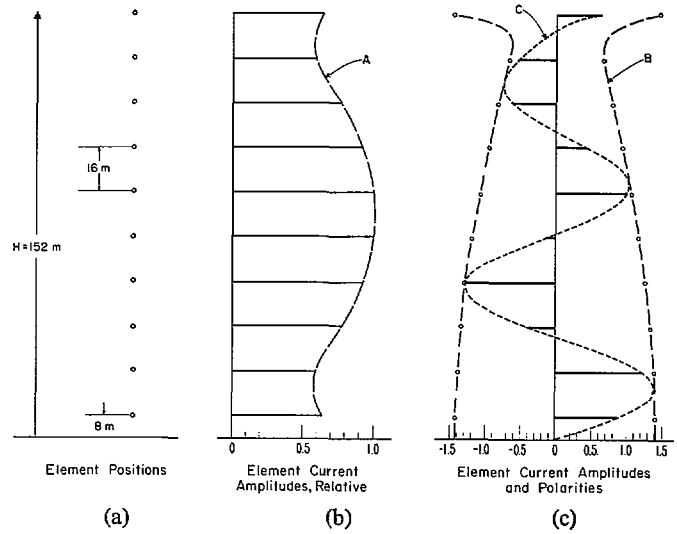

Fig. 5. Configuration of the 10-element array and the two aperture excitations used in comparison. (a) Positions of the elements of array above ground. (b) Element current amplitudes for "conventional" aperture illumination, using phase progression for scanning, with Chebyshev taper for a -20 dB sidelobe level. A is the envelope of element current amplitudes corresponding to Chebyshev taper for a 10-element array for a -20 dB sidelobe level. (c) Element current amplitudes and polarities, for sinusoidal aperture illumination for $\Delta_0=11.95°$, with superimposed Chebyshev taper for a -20 dB sidelobe level. B is the envelope of Chebyshev coefficients for one-half of a 20 element array for a -20 dB sidelobe level. Curve C is the envelope of element current amplitudes and is the product of the Chebyshev taper B and the sinusoidal distribution.

Four points of performance are to be compared: the relative gains of the array using the two illuminations, the half-power beamwidths of the major lobes, the shapes of the major lobes, and the behavior of the sidelobes. The relative gains of the array and the half-power beamwidths are tabulated in Table I. While the gain figures are presented to two decimal figures, and beamwidths to a tenth of a degree, in actual situation, even in the near ideal conditions of actual siting, the gain may be expected to vary by, possibly, one half of a decibel, and the beamwidths by a few tenths of a degree, due to errors in current excitations, irregularities in the ground, etc.

The element current amplitudes for the conventional array remain the same for all aimed angles. Therefore, the approximate power input to that array, neglecting mutual impedances, given by

$$\sum_{r=1}^{10} I_r^2$$

is constant. The response of the array, using either illumination, is expressed in decibels above that of the array with

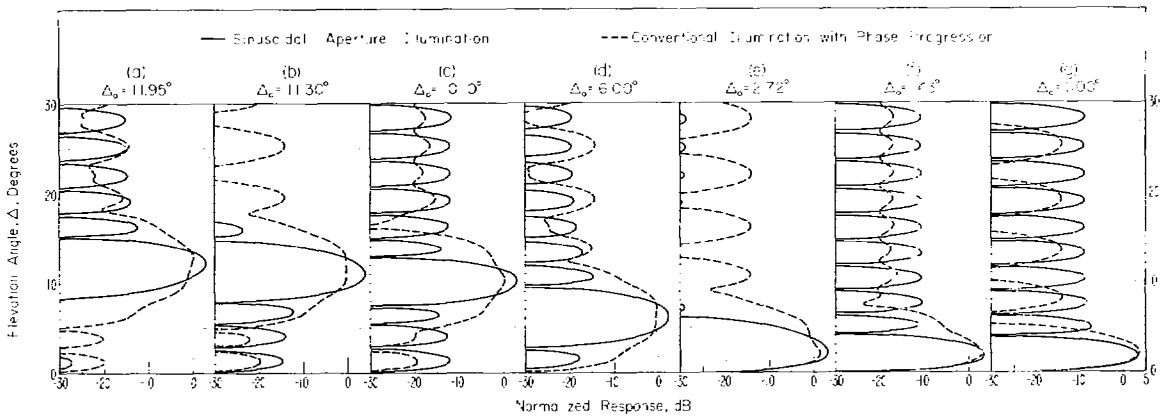

Fig. 6. Comparison of radiation patterns in elevation for the array of ten horizontal dipoles using sinusoidal and conventional aperture illuminations with superimposed -20 dB Chebyshev taper. All patterns normalized with respect to free space maximum for conventional illumination.

TABLE I

COMPARISON OF COMPUTED RELATIVE GAINS AND HALF-POWER WIDTHS OF MAJOR LOBE FOR TEN ELEMENT ARRAY USING SINUSOIDAL AND CONVENTIONAL ILLUMINATIONS

Aimed Angle Δ_0	Conventional Illumination		Sinusoidal Illumination	
Degrees	Relative Gain dB	Half-Power Beam-width Degrees	Relative Gain dB	Half-Power Beam-width Degrees
11.95	+0.19	5.4	+2.87	2.7
11.30	−0.31	5.7	+3.80	2.8
10.10	+0.38	4.7	+3.80	2.4
6.00	−0.16	5.4	+2.57	2.9
2.72	+1.48	4.1	+3.06	2.6
1.43	+3.02	2.4	+3.58	2.1
0.00	+3.60	2.3	+3.33	2.0

conventional illumination with no ground reflection (free space).

In Fig. 6(a) to (c) the aimed angle is sufficient for the major lobe of either array to be completely above horizon. The major lobe maximum for conventional array should then be 0.0 dB. However, the sidelobe level is illuminating the ground. This radiation is reflected to reinforce or reduce the energy in the major lobe. With a -20 dB sidelobe level, the maximum enhancement of the field can be 10 percent or $+0.83$ dB; the corresponding maximum reduction is also 10 percent or -0.92 dB. The gains of the array with either illumination are seen to fluctuate within these limits. The beamwidths of the array are also observed to be close to those in free space (2.5° for the sinusoidal and 5.2° for the conventional illumination).

Fig. 6(d) shows a transitional point where the major lobe of the array with conventional illumination begins to illuminate the reflecting ground. The degree of illumination is not, however, sufficient to affect either the gain or the beamwidth. In the last three parts of Fig. 6, the major lobes of the array, with either current distribution, illuminate the foreground. The 2.72° value of Δ_0 in 6(e) corresponds to an angle at which the height factor for conventional illumination is equal to 6 dB. The illumination of the ground, however, is relatively low and the reinforcement of the major lobe of the conventional array is only 1.48 dB. At $\Delta_0 = 1.43°$, the illumination of foreground by the conventional array is more intense, but the direct illumination is below maximum. A maximum reinforcement of 3.02 dB is realized, but its direction is 2.2° above horizon, not 1.43°. At $\Delta_0 = 0.00°$, the direct and reflected illumination are equal and a 3.60 dB major lobe reinforcement in the 2.2° direction is realized. The behavior of the sinusoidal illumination for $\Delta_0 = 2.72°$ and $\Delta_0 = 1.43°$ is normal, except that for the latter case, the direction of the maximum is 1.7° rather than 1.4°. The case of $\Delta_0 = 0.00°$ for the sinusoidal illumination requires explanation. Normally, the period of the sinusoid would become infinitely long and the illumination over a finite aperture would be zero. However, an equivalent result was obtained by computing the pattern for $\Delta_0 = 0.01°$ and by correcting for equal power input. This results in a triangular illumination modified by a Chebyshev taper. In an actual array, this result could be obtained by modifying the electronic circuitry. However, the gain and the beamwidth show no improvement over that for $\Delta_0 = 1.43°$. The direction of the major lobe maximum is likewise 1.7°.

The effect of secondary lobes reflected by the ground is much more important on the resulting sidelobes of the system. For the sinusoidal illumination, the direct radiation and reflected fields are either in-phase or out-of-phase. If in-phase and near equal in intensity, the sidelobe level will increase by 6 dB. If of opposite phase and near equal in intensity, almost complete cancellation should be observed. Since the sidelobes are very nearly equally spaced and of alternating polarity, the reinforcement or cancellation should be observed on all of the sidelobes to one side of the major lobe. Fig. 6(a) to (c) illustrates this. $\Delta_0 = 11.95°$ was selected to give reinforcement of sidelobes above the main lobe. The value $\Delta_0 = 11.30°$ in 6(b) was selected to show a 6 dB reinforcement on sidelobes below the main lobe. Fig. 6(c), with $\Delta_0 = 10.10°$, shows equal reinforcement of approximately 4 dB for sidelobes on either side of the major lobe.

The sidelobe behavior of the conventional array is made more complicated because the direct and reflected fields are completely in-phase (or out-of-phase) only in certain directions. These directions are given by angles,

$$\Delta_m = \sin^{-1} \frac{m}{4 \cdot \frac{10}{19} \cdot \frac{H}{\lambda}}, \qquad m = 1, 2, 3, \text{etc.}$$

For the frequency of 19.72 MHz and $\lambda = 15.2$ meters, Δ_m has values of 2.72°, 5.45°, 8.19°, etc. In these directions, provided the amplitudes of direct and reflected sidelobes are equal, a 6 dB reinforcement or a complete cancellation will occur. If the amplitudes are not equal, only partial reinforcement and minima, rather than nulls, will be observed. Fig. 6(f) is a good illustration of the latter. For the same reason, the varying relationship, the shape of the major lobe shows irregular form as the phase relationship between the direct major lobe and the reflected sidelobe changes with Δ.

The computed performance of the array with the two current distributions demonstrates rather conclusively that the sinusoidal current distribution is distinctly superior to the "conventional" distribution from the standpoint of gains (3 dB advantage), width of major lobe (one half that of the conventional), and the lower minimum angle of elevation of the major lobe.

From the standpoint of sidelobe level, the advantage is not as clear. While the total power content of the sidelobes is the same, there may be some advantage to having the sidelobes smoothed out. On the other hand, the -20 dB Chebyshev taper is rather pessimistic and was selected for illustrating the behavior of sidelobes. With reasonable care a -25 dB level should be attainable. In such case, with either array, the resulting sidelobe level should not exceed -19 dB.

The antenna system as described here was erected in 1962. Since then, the electronic circuitry has been modified. The sinusoidal illumination is still in use; however, the rapid scan was replaced by electromechanical steering which allows for a variable rate of change in direction and permits the major lobe to remain in a fixed direction as desired.

VIII. Conclusions

The use of a vertical-aperture array antenna rather than an endfire array results in an antenna with the major lobe narrow in elevation, but broad in azimuth. The elevation angle of the maximum is the same over all azimuthal directions. The use of horizontal polarization permits low minimum angles of elevation.

The sinusoidal illumination incorporated in this antenna effectively doubles the aperture of the antenna, as compared to that it would have had if a conventional illumination were used.

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Editor's Comments

The foregoing paper has aroused considerable interest among the technical reviewers. This is probably due to the fact that at first reading it violates two cherished assumptions, namely, 1) that for maximum gain we wish a uniform phase and amplitude distribution, and 2) that a 6 dB gain (Lloyd's mirror) is realized when an antenna is over a perfectly reflecting earth. The first of these is valid only if equal voltages are induced in the elements, and the second only if the antenna size is small compared to the mean height.

It is also of interest to note that the gain formula (in the presence of ground)

$$G \sim \frac{\left| \int_0^H f(x) \sin u_0 x dx \right|^2}{\int_0^H f^2(x) dx} \tag{1}$$

where

$$u_0 = \frac{2\pi}{\lambda} \sin \Delta_0$$

may be interpreted so that "$\sin u_0 x$" is the voltage induced in the elements and "$f(x)$" is a receiver sampling function. The numerator is then the received (summed) signal power and the denominator the incoherent addition of the sampled noise. Maximization of (1) is then equivalent to the optimum signal to noise ratio.

Further, by applying the calculus of variation to (1) it can be shown that the sinusoid is the optimum sampling function. If a square wave were used the gain would be 0.92 dB lower.

On transmission, the array sinusoidally fed would also realize the increased 3 dB gain. However, as half of our available transmitter power is standing idly by, the system gain is identical to a uniformly excited but progressively phased aperture. In the transmitting case, the square wave excitation (in and out of phase) has a 2.08 dB advantage, as all the transmitter power is then utilized.—John Ruze