

PHASE NOISE ISSUES IN FEMTOSECOND LASERS*

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ABSTRACT

This paper explores the affect of phase modulation (PM), amplitude modulation (AM), and thermal noise on the rf spectrum, phase jitter, timing jitter, and frequency stability of femtosecond lasers and other precision sources. Using these concepts we can suggest how some noise aspects of femtosecond pulsed lasers should scale.

Keywords: PM noise, FM noise, AM noise, frequency stability, timing jitter, phase jitter, frequency multiplication

1. INTRODUCTION

In this paper we review the basic definitions generally used to describe phase modulation (PM) noise, amplitude modulation (AM) noise, fractional frequency stability, timing jitter and phase jitter in a precision sources such as the femtosecond (fs) lasers. From these basic definitions we can then compute the affect of frequency multiplication or division on these measures of performance. We find that under ideal frequency multiplication or division by a factor N , the PM noise and phase jitter of a source is intrinsically changed by a factor of N^2 . The fractional frequency stability and timing jitter are, however, unchanged as long as we can determine the average zero crossings. After a sufficiently large multiplication factor N , the carrier power density is less than the PM noise power. This is often referred to as carrier collapse. Ideal frequency translation results in the addition of the PM noise of the two sources. The effect of AM noise on the multiplied or translated signals can be increased or decreased depending on the component non-linearity. Thermal noise added to a precision signal results in equal amounts of PM and AM noise. Each component affects the spectrum as described above. The upper and lower PM (or AM) sidebands are exactly equal and 100% correlated, independent of whether the PM (or AM) originates from random or coherent processes [1]. Thermal noise added to a precision signal results in equal amounts of PM and AM noise. Each component affects the spectrum as described above.

2. BASIC DEFINITIONS

2.1 DESCRIPTIONS OF VOLTAGE WAVE FORM

The output of a precision source can be written as

$$V(t) = [V_o + \varepsilon(t)][\cos(2\pi\nu_o t + \phi(t))], \quad (1)$$

where ν_o is the average frequency, and V_o is the average amplitude. Phase/frequency variations are included in the term $\phi(t)$ and the amplitude variations are included in $\varepsilon(t)$ [2]. The instantaneous frequency is given by

$$\nu = \nu_o + \frac{1}{2\pi} \frac{d}{dt} \phi(t). \quad (2)$$

The instantaneous fractional frequency deviation is given by

$$y(t) = \frac{1}{2\pi\nu_o} \frac{d}{dt} \phi(t) \quad (3)$$

The power spectral density (PSD) of phase fluctuations $S_\phi(f)$ is the mean squared phase fluctuation $\delta\phi(f)$ at Fourier frequency f from the carrier in a measurement bandwidth of 1 Hz. This includes the contributions at both the upper and lower sidebands. These sidebands are exactly equal in amplitude and are 100% correlated. Thus experimentally

$$S_\phi(f) = \frac{[\delta\phi(f)]^2}{BW} \quad \text{radians}^2/\text{Hz}, \quad (3)$$

$$BW \ll f, 0 < f < \infty,$$

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where BW is the measurement bandwidth in Hz. Since the BW is small compared to f , $S_\phi(f)$ appears locally to be white noise and therefore obeys Gaussian statistics. The fractional 1-sigma confidence interval for power spectral density measures of white noise is $1 \pm 1/N^{1/2}$, where N is the number of averages [3].

Often the PM noise is specified as single side band noise $L(f)$, which is defined as $1/2$ of $S_\phi(f)$. The units are generally given in dBc/Hz, which is short hand for dB below the carrier in a 1 Hz bandwidth.

$$L(f) = 10 \log [1/2 S_\phi(f)] \quad \text{dBc/Hz.} \quad (4)$$

Frequency modulation noise is often specified as $S_y(f)$ which is the PSD of fractional frequency fluctuations. $S_y(f)$ is related to $S_\phi(f)$ by

$$S_y(f) = [f^2/v^2] S_\phi(f), \quad 1/\text{Hz.} \quad (5)$$

In the laser literature one often sees the frequency noise expressed as the PSD of frequency modulation $S_\phi^*(f)$, which is related to $S_y(f)$ as.

$$S_\phi^*(f) = v^2 S_y(f) = f^2 S_\phi(f) \quad \text{Hz}^2/\text{Hz.} \quad (6)$$

Typically $S_y(f)$ is comprised of regions where the noise follows a power law dependence on f . The most common are given below in Eq. (7). The coefficients are identified as: h_2 is random walk FM, h_1 is flicker FM, h_0 is white FM, h_{-1} is flicker PM, and h_{-2} is white PM. Most precision sources have at least 3 of these noise types plus aging or drift.

$$S_y(f) = h_{-2} f^{-2} + h_{-1} f^{-1} + h_0 + h_1 f + h_2 f^2 \quad (7)$$

The amplitude modulation (AM) noise $S_a(f)$ is the mean squared fractional amplitude fluctuations at Fourier frequency f from the carrier in a measurement bandwidth of 1 Hz. Thus experimentally

$$S_a(f) = \left(\frac{\delta E(f)}{V_o} \right)^2 \frac{1}{\text{BW}} \quad 1/\text{Hz}, \quad (8)$$

$\text{BW} \ll f, 0 < f < \infty,$

where BW is the measurement bandwidth in Hz.

The rf power spectrum for small PM and AM noise is approximately given by

$$V^2(f) \cong V_o^2 [e^{-\phi_c^2} + S_\phi(f) + S_a(f)]. \quad (9)$$

Where $e^{-\phi_c^2}$ is the approximate power in the carrier at Fourier frequencies from 0 to f_c . ϕ_c^2 is the mean squared phase fluctuation due to the PM noise at frequencies larger than f_c [4]. ϕ_c^2 is calculated from.

$$\phi_c^2 = \int_{f_c}^{\infty} S_\phi(f) df. \quad (10)$$

The half-power bandwidth of the signal, $2 f_c$ can be found by setting $\phi_c^2 = 0.7$. The difference between the half-power and the 3-dB bandwidth depends on the shape of $S_\phi(f)$ [4].

2.2 FREQUENCY STABILITY IN THE TIME DOMAIN

The frequency of even a precision source is not stationary in time, so traditional statistical methods to characterize it diverge with increasing number of samples [2]. Special statistics have been developed to handle this problem. The most common is the two-sample or Allan variance (AVAR), which is based on analyzing the fluctuations of adjacent samples of fractional frequency averaged over a period τ . The square root of the Allan variance $\sigma_y(\tau)$, often called the ADEV, is defined as

$$\sigma_y(\tau) = \left\langle \frac{1}{2} [\bar{y}(t+\tau) - \bar{y}(t)]^2 \right\rangle^{1/2} \quad (11)$$

$\sigma_y(\tau)$ can be estimated from a finite set of frequency averages of length τ from

$$\sigma_y(\tau) = \left[\frac{1}{2(M-1)} \sum_{i=1}^{M-1} (y_{i-1} - y_i)^2 \right]^{1/2} \quad (12)$$

This assumes that there is no dead time between samples [2]. If there is dead time, the results are biased depending on type of PM noise. See [2] for details. $\sigma_y(\tau)$ can also be calculated from the $S_\phi(f)$ using

$$\sigma_y(\tau) = \left(\frac{\sqrt{2}}{\pi v_o \tau} \int_0^\infty H_\phi(f) [S_\phi(f)] \sin^4(\pi f \tau) df \right)^{1/2}, \quad (13)$$

where $H_\phi(f)$ is the transfer function of the system used for measuring $\sigma_y(\tau)$ or δt below. See Table 1 [2]. If $H_\phi(f)$ has a low pass characteristic with a very sharp roll off at a maximum frequency f_h , it can be replaced by 1 and the integration terminated at f_h . Practical examples usually require the exact shape of $H_\phi(f)$.

2.3 PHASE JITTER

The phase jitter $\delta\phi$ is computed from the PM noise spectrum using

$$\delta\phi = \int_0^\infty [S_\phi(f)] H(f) df \quad (14)$$

Generally $H(f)$ must have the shape of the high pass filter or a minimum cutoff frequency f_{min} used to exclude low frequency changes for the integration, or $\delta\phi$ will diverge due to random walk FM, flicker FM, or white FM noise processes. $H(f)$ usually has a low pass characteristic at high frequencies to limit the effects of flicker PM and white PM. See Table 1.

2.4. TIMING JITTER

Recall that $\sigma_y(\tau)$ is the fractional frequency stability of adjacent samples each of length τ . The time jitter δt over a period τ is the timing error that accumulates after a period τ . δt is related to $\sigma_y(\tau)$ by

$$\frac{\delta t}{\tau} = \frac{\delta v}{v} = \sigma_y(\tau) \quad \delta t = \tau \sigma_y(\tau) \quad (15)$$

Table 1 shows the asymptotic forms of $\sigma_y(\tau)$, δt , and $\delta\phi$ as a function of τ , f_{min} , and f_h for the 5 common noise types at frequency v_o and Nv_o . It is interesting to note that for white phase noise, all three measures are dominated by f_h . For random walk frequency modulation (FM) and flicker FM, $\sigma_y(\tau)$ is independent of f_h and instead is dominated by $S_\phi(1/\tau)$ or $S_\phi(f_{min})$. Also, the timing jitter is independent of N as long as we can still identify zero crossings, while the phase jitter, which is proportional to frequency, is multiplied by a factor N . Typical sources usually contain at least 3 of these noise types.

3 EFFECTS OF FREQUENCY MULTIPLICATION, DIVISION, AND TRANSLATION

3.1 FREQUENCY MULTIPLICATION AND DIVISION

Frequency multiplication by a factor N is the same as phase amplification by a factor N . For example 2π radians is amplified to $2\pi N$ radians. Since PM noise is the mean squared phase fluctuation, the PM noise must increase by N^2 . Thus

$$S_\phi(Nv_o, f) = N^2 S_\phi(v_o, f) + \text{Multiplication PM}, \quad (16)$$

where Multiplication PM is the noise added by the multiplication process.

Table 1. $\sigma_v(\tau)$, δt , and $\delta\phi$ as a function of τ , f_{\min} , and f_h at carrier frequency ν_o and $N\nu_o$.

Noise type	$S_\phi(f)$	$\sigma_v(\tau)$	δt at ν_o or $N\nu_o$	$\delta\phi$ at ν_o	$\delta\phi$ at $N\nu_o$
Random Walk FM	$[\nu^2/f^4]h_2$	$\pi[(2/3)h_2\tau]^{1/2}$	$\tau\pi[(2/3)h_2\tau]^{1/2}$	$\nu[(1/(3f_{\min}^3)h_2)^{1/2}]$	$N\nu[(1/(3f_{\min}^3)h_2)^{1/2}]$
Flicker FM	$[\nu^2/f^3]h_1$	$[2\ln(2)h_1]^{1/2}$	$\tau[2\ln(2)h_1]^{1/2}$	$\nu[(1/(2f_{\min}^2)h_1)^{1/2}]$	$N\nu[(1/(2f_{\min}^2)h_1)^{1/2}]$
White FM	$[\nu^2/f^2]h_0$	$[\{1/(2\tau)\}h_0]^{1/2}$	$[(\tau/2)h_0]^{1/2}$	$\nu\{[(1/f_{\min}) - (1/f_h)]h_0\}^{1/2}$	$N\nu\{[(1/f_{\min}) - (1/f_h)]h_0\}^{1/2}$
Flicker PM	$[\nu^2/f^1]h_1$	$[1/(2\pi\tau)] [1.038 + 3\ln(2\pi f_h \tau) h_1]^{1/2}$	$[1/(2\pi)] [1.038 + 3\ln(2\pi f_h \tau) h_1]^{1/2}$	$\nu[\ln(f_h/f_{\min})h_1]^{1/2}$	$N\nu[\ln(f_h/f_{\min})h_1]^{1/2}$
White PM	$[\nu^2/f^2]h_2$	$[1/(2\pi\tau)][3f_h h_2]^{1/2}$	$[1/(2\pi)][3f_h h_2]^{1/2}$	$\nu[f_h h_2]^{1/2}$	$N\nu[f_h h_2]^{1/2}$

We see from Eqs. (9), (10) and (16) that the power in the carrier decreases exponentially as e^{-N^2} . After a sufficiently large multiplication factor N , the carrier power density is less than the PM noise power. This is often referred to as carrier collapse [4]. Ideal frequency translation results in the addition of the PM noise of the two sources [2]. The half-power bandwidth of the signal also changes with frequency multiplication.

Frequency division can be considered as frequency multiplication by a factor $1/N$. The effect is to reduce the PM noise by a factor $1/N^2$. The only difference is that there can be aliasing of the broadband PM noise at the input to significantly increase the output PM above that calculated for a perfect divider [5]. This effect can be avoided by using narrow band filter at the input or intermediate stages.

3.2 EFFECT OF FREQUENCY TRANSLATION

Frequency translation has the effect of adding the PM noise of the input signal ν_1 and the reference signal ν_o to that of the PM noise in the nonlinear device providing the translation.

$$S_\phi(\nu_2, f) = S_\phi(\nu_o, f) + S_\phi(\nu_1, f) + \text{Translation PM.} \quad (17)$$

Thus dividing a high frequency signal, rather than mixing two high frequency signals generally produces a low frequency reference signal with less residual noise.

3.3 EFFECT OF ADDITIVE NOISE

The addition of a broadband noise signal $V_n(t)$ to the signal $V_o(t)$ yields a total signal

$$V(t) = V_o(t) + V_n(t). \quad (18)$$

Since the noise term $V_n(t)$ is uncorrelated with $V_o(t)$, $1/2$ the power contributes to AM noise and $1/2$ the power contributes to PM noise.

$$\text{AM } V_n(t)/\sqrt{2} \quad \text{PM } V_n(t)/\sqrt{2}. \quad (19)$$

$$L(f) = \frac{s_\phi(f)}{2} = \frac{s_a(f)}{2} = \frac{V_n^2(f)}{4V_o^2} \frac{1}{\text{BW}}, \quad (20)$$

where BW is the bandwidth in Hz.

These results can be applied to amplifier and detection circuits as follows. The input noise power to the amplifier is given by $kTBW$. The gain of the amplifier from a matched source into a match load is G_o . The noise power to the load is just $kTBW G_o F$, where F is the noise figure. The output power to the load is P_o . Using Eq. (19) we obtain

$$L(f) = \frac{s_\phi(f)}{2} = \frac{s_a(f)}{2} = \frac{V_n^2(f)}{4V_o^2} \frac{1}{\text{BW}} = \frac{2kTBWFG_o}{4P_o \text{BW}} = \frac{kTFG_o}{2P_o} = -177 \text{ dBc/Hz}, \quad (21)$$

for $T=300\text{K}$, $F=1$, $P_o/G_o = P_{in} = 0 \text{ dBm}$.

4.0 APPLICATIONS TO FEMTOSECOND COMB DIVIDERS

There are several aspects of the design and construction of femtosecond pulsed laser systems that depend critically on general noise considerations [6], [7]. These are the selection of the pulse repetition rate (PRR), obtaining a good low frequency readout signal, and phase locking to a stable low frequency reference. The focus here will be on general scaling rules rather than the specification of performance, which will change rapidly with technology development.

4.1 SELECTING THE PRR

Femtosecond comb dividers are in essence very high order frequency multipliers. The basic oscillation comes from the PRR of the oscillator. This is typically 100 MHz to a few GHz. A mode-locked fs pulse laser provides ultra-short pulses (~30 fs) at the pulse repetition rate PRR. This yields a comb of frequencies given by

$$\nu_o = n\text{PRR} + \nu_{\text{offset}}, \quad (22)$$

where n is the harmonic number and ν_{offset} is the offset of the comb from zero. The shortness of the pulse requires phase coherence between all the comb lines. The PM noise is thought to come from both the fluctuation of PRR and ν_{offset} . At this point it is not entirely clear which noise type dominates, and in fact that may change for different configurations and PRR. The advantage of the higher PRRs is that the frequency multiplication number is lower and the spectrum simpler because the power is distributed over a fewer number of harmonics. The disadvantage is that the PM noise of the PRR contribution to the comb lines is expected to be higher than obtained with a lower PRR.

The PRR is to first order the group velocity divided by the round trip path length L . Therefore, higher PRRs mean lower round trip path lengths. The fluctuations in equivalent fractional length changes $\delta L/L$ lead to fractional changes in frequency $\delta\nu/\nu$. The equivalent statement about the PM noise is that the PM noise at a given output frequency scales as $1/L^2$ as long as δL is independent of L . The 100 MHz laser could have up to 20 dB lower PRR contribution to the PM noise at the optical but perhaps not enough power in the comb lines and a much more dense spectrum. Therefore, it appears that one should choose the PRR as low as possible and still obtain the necessary power in the output comb lines to perform the desired frequency comparisons.

4.2. OBTAINING A GOOD READOUT SIGNAL

The low frequency output signal derived from the fs laser to be used for frequency comparisons and for constructing a stable time scale should be of order 100 MHz or higher [8]. The errors in making precision measurements at a lower frequency like 5 MHz preclude obtaining the full potential stability of femtosecond laser systems. The detector noise adds PM noise and frequency fluctuations that are independent of the comb separation (until the separation approaches the bandwidth of the detector).

The diamonds (Series 1) of figure 1 show the approximate PM noise spectrum of the 9th harmonic of a fs laser operating with a PRR of 99 MHz. [9]. The close-in noise exhibits a f^{-4} or flicker FM behavior, while the broadband noise is white PM. The f^{-4} part of the spectrum is thought to be intrinsic to the fs laser system while the white PM part is due to the readout system. See below. The true broadband PM noise performance is hidden by the detection system.

At an output frequency of 564 THz, the multiplication factor is approximately 633,000. This corresponds to increasing the PM noise spectrum by 116 dB. Typically an optical linewidth of 5 to 10 MHz is observed. From this measurement we can estimate the intrinsic PM noise of the PRR using Eqs. (9) and (10), to be no larger than -188 dBc/Hz at an offset of 5 MHz. The squares (Series 2) represent estimated PM noise based on the optical linewidth of approximately 10 MHz [8] and broadband white FM noise, while the triangles assume broadband flicker PM noise.

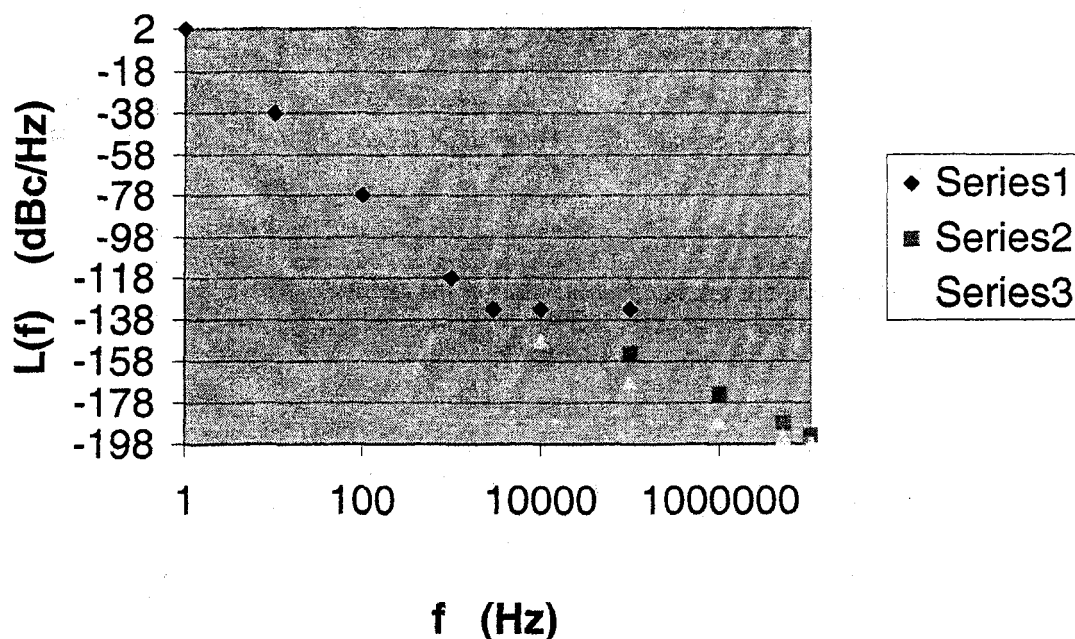


Figure 1. Approximate PM noise of the PRR separation measured on of a fs laser with a PRR of approximately 99 MHz. The diamonds (Series 1) are measured data. The squares (Series 2) are estimated PM noise assuming that the broadband noise is white FM, while the triangles (Series 3) assume flicker PM noise.

Using the arguments above I expect that the flicker FM behavior to scale as $1/L^2$, while the white PM should be constant, when observed at 890 MHz. I also expect that the optical comb linewidths will decrease somewhat with increasing L.

We can predict the white PM level of Figure 1 from the detected signal level of 0.3 mA rms into a 50-ohm detector. The bandwidth of the detector is approximately 6 GHz. Since this signal is spread over approximately $6 \text{ GHz}/100 \text{ MHz} = 60$ lines, this corresponds to a signal power in each of the detected comb lines of

$$\text{Power/line} = (0.0003)^2 50/60 \text{ W or } -41 \text{ dBm.} \quad (22)$$

Using Eq. (21) and assuming a noise figure of roughly 4 dB, we find an expected white PM level of

$$\text{White PM} = -177 + 41 + 4 = -132 \text{ dBc/Hz.} \quad (23)$$

This agrees very well with the data of Figure 1 and should be roughly the same whether detecting 99 MHz, 890 MHz, or 6 GHz. Using Table 1 we can calculate the fractional frequency stability $\sigma_y(\tau)$ for each noise type. The random walk FM (f^4 part of the spectrum) leads to

$$\sigma_y(\tau) = \pi \left(\frac{3\tau}{(890\text{MHz})^2} \frac{2}{3} \right)^{0.5} = 5 \times 10^{-9} \tau^{0.5}. \quad (24)$$

Note that this contribution to $\sigma_y(\tau)$ grows as $\tau^{0.5}$, is independent of the measurement bandwidth, and probably would not change at different detection (readout) frequencies as it is due to the fs laser noise and not the detection system. The white PM contribution, on the other hand, is due to the detection process and depends on the readout frequency and the bandwidth of the detection process. If we assume some sort of counting system at 890 MHz with a bandwidth of 10 MHz we obtain

$$\sigma_y(\tau) = \frac{1}{2\pi\tau} \left(\frac{3(10\text{MHz})1.2 \times 10^{-13}}{(890\text{MHz})^2} \right)^{0.5} = 3.5 \times 10^{-13} \tau^{-1}. \quad (25)$$

$\sigma_y(\tau)$ would be approximately 9 times higher for detection at 100 MHz and 6.7 times smaller for detection at 6 GHz. $\sigma_y(\tau)$ can be substantially reduced by decreasing the detection bandwidth. For example reducing it from 10 MHz to 10 kHz would reduce $\sigma_y(\tau)$ by approximately a factor of 32.

The detected comb separation or readout signal can be used for phase locking the PRR to a more stable intermediate oscillator to improve the short-term PM noise and frequency stability. Since the resolution and stability for detecting frequency improves as $1/(\text{comb separation})$ until one reaches the bandwidth of the detector, the best performance would be obtained at a readout of roughly in the 6 to 40 GHz range. This leads to the problem of dividing from the GHz range to roughly 100 MHz which is a standard reference frequency typically used for comparison with other high stability clocks and to generate time scales.

NIST has developed microwave frequency dividers that provide excellent PM noise and phase stability [10]. A 40 GHz to 20 GHz or 20 GHz to 10 GHz divider would support a fractional frequency stability of better than $1 \times 10^{-16} \tau^{-1}$. We have also built a synthesizer that outputs both 100 MHz and a frequency near 6 GHz, 9.192 GHz, or 10.007 GHz with a range of a few MHz and a fractional resolution of 1×10^{-17} . The internal fractional frequency stability is approximately $1 \times 10^{-14} \tau^{-1}$ improving to $1 \times 10^{-18} \tau^{-1}$ at 1 day [11]. Such a unit could be phase locked to a comb separation near 10 GHz (or 20 or 40 GHz divided down to 10 GHz) and output a precision 100 MHz signal that can be adjusted in frequency with a fractional resolution of 1×10^{-17} .

5. DISCUSSION

I have explored the affect of phase modulation (PM), amplitude modulation (AM), and thermal noise on the rf spectrum, phase jitter, timing jitter, and frequency stability of fs lasers and other precision sources. Under ideal frequency multiplication or division by a factor of N, the PM noise and phase jitter of a source is changed by a factor of N^2 . After a sufficiently large N, the carrier power density is less than the PM noise power, often referred to as carrier collapse. Thermal noise added to a precision signal results in equal amounts of PM and AM noise. Each component affects the spectrum as described above. The upper and lower PM (or AM) sidebands are exactly equal and 100% correlated, independent of whether the PM (or AM) originates from random or coherent processes. Using these concepts I have suggested how some noise aspects of femtosecond pulsed lasers should scale.

For example, in selecting the best PRR frequency it would be very helpful to have additional measurements on the random walk FM noise and the linewidth of the optical comb lines as a function of the PRR. I expect the PM noise due to fluctuations in the PRR scaling to scale as PRR^2 . If the PM noise in the comb offset is higher than that of the high multiplication of the PRR, it will be necessary to control this effect first.

The low frequency readout should be 100 MHz or higher to be able to make frequency measurements with uncertainties of order 1×10^{-17} or better [8]. The detector thermal noise creates white PM noise that will limit the short-term frequency resolution. This effect can be minimized by detecting a frequency of $N(\text{PRR})$ that is near the 3-dB bandwidth of the detector. NIST has developed microwave dividers that could be used to divide signals as high as 40 GHz down to 10 GHz with very low noise. A new NIST synthesizer could further divide the 10 GHz to 100 MHz and still maintain an internal long-term frequency accuracy of 1×10^{-17} for 20 minutes averaging or 1×10^{-18} for 1 day averaging [11].

6. REFERENCES

- [1] F.L. Walls, "Correlation Between Upper and Lower Sidebands," IEEE Trans. Ultrason., Ferroelectrics, and Freq. Cont., 47, 407-410, 2000.
- [2] D.B. Sullivan, D.W. Allan, D.A. Howe, and F.L. Walls, "Characterization of Clocks and Oscillators", NIST Tech. Note 1337, 1-342, 1990.

- [3] F.L. Walls, D.B. Percival, and W.R. Ireland, "Biases and Variances of Several FFT Spectral Estimators as a Function of Noise Type and Number of Samples," Proc. 43rd Ann. Symp. Freq. Control, Denver, CO, May 31-June 2, 336-341, 1989. Also found in [1].
- [4] F.L. Walls and A. DeMarchi, "RF Spectrum of a Signal After Frequency Multiplication; Measurement and Comparison with a Simple Calculation," IEEE Trans. Instrum. Meas., **24**, 210-217, 1975.
- [5] A. SenGupta and F.L. Walls, "Effect of Aliasing on Spurs and PM Noise in Frequency Dividers," Proc. Intl. IEEE Freq. Cont. Symp., Kansas City, MO, June 6-9, 2000.
- [6] Thomas Udem, Jorg Reichert, Ronald Holzwarth, Scott Diddams, David Jones, Jun Ye, Steven Cundiff, Theodor Hansch, and John Hall, "A new type of frequency chain and its application to fundamental frequency metrology," to be published in Hydrogen II, Ed. S. Karshenboim, Springer 2001
- [7] Yun Ye, John Hall, and Scott A. Diddams, "Precision phase control of an ultrawide-bandwidth femtosecond laser: a network of ultrastable frequency marks across the visible spectrum," Optics Letters, **25**, pp. 1675-1677, 2000.
- [8] Lisa M. Nelson and Fred L. Walls, "Environmental effects in mixers and frequency distribution systems" Proc. 1992 IEEE Freq. Cont. Symp., Hershey PA, 183-187, 1992
- [9] Unpublished data from Eugene Ivanov.
- [10] E. S. Ferre-Pikal and F. L. Walls, "Microwave regenerative dividers with low PM noise, IEEE Trans. UFFC., **46**, 216-219, 1999
- [11] A. Sen Gupta, D. Popovic, and F. L. Walls, "Cesium frequency synthesis: A new approach," Special Issue UFFC Trans., 615-619, 1999

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