The Effect of Harmonic Distortion on Phase errors in Frequency Distribution and Synthesis

F.L. Walls* and F. G. Ascarrunz+
*Time and Frequency Division, NIST, +SpectraDynamics Inc. Boulder CO, USA

Abstract

This paper explores the effect of harmonic distortion on phase errors in frequency distribution and synthesis. Harmonic distortion gives rise to large timing errors in phase or zero crossing detectors. These large timing errors may cause unacceptable sensitivity to environmental conditions or signal fluctuations. Two types of phase detectors are considered in this paper. The timing error in an ideal zero crossing detector is calculated as a function of harmonic number and the phase of the harmonic relative to the fundamental. The result is

$$\delta t_{max} = \sum_{n=2}^{\infty} v_n \Big/ v_1\omega_1,$$

where v_1 is the amplitude of the carrier frequency ω_1, and v_n is the amplitude of the nth harmonic. The timing errors of double balanced mixers are in principle insensitive to even harmonics of the RF signal, and their sensitivity to higher odd harmonics fall roughly as 1/n. Experimental data from several mixers at 5 MHz and different drive levels show that this model is a first-order approximation to real devices. The low sensitivity to the second harmonic is an important advantage over true zero-crossing detectors because it is difficult to significantly reduce the second harmonic without introducing large phase shifts due to filtering networks. Balanced mixers offers approximately 20 dB improvement over the true zero-crossing detector when used in the linear range of operation. Harmonic distortion is an important parameter in the design of systems with timing stability less than about 100 ps.

Introduction

With the potential of new frequency standards with short-term frequency stability less than $1\times10^{-14}\ \tau^{-1/2}$, it has been necessary to re-examine the origin of timing errors in measurement systems as well as frequency distribution and synthesis building blocks. It is desirable to have these subsystems maintain timing errors of less than approximately 1 ps. General criteria for frequency metrology to support these standards are discussed in [1], where the requirements on voltage-standing-wave-ratio (VSWR), the temperature coefficients of the cable delay, and errors in phase detectors for the design of ultra stable synthesis and distribution systems, are discussed. References [2,3], demonstrate that distribution amplifiers which satisfy the requirements set forth in [1] can be made at 5 to 10 MHz and at 100 MHz.

This paper explores the one remaining problem that has not been considered: the effect of harmonic distortion on phase errors in frequency synthesis and distribution systems. Harmonic distortion can cause large timing errors, which may give rise to unacceptable sensitivity to environmental conditions or signal fluctuations. The harmonic distortion may be present in the source or arise within the distribution amplifiers. Harmonic distortion is converted to timing errors by the phase detector. It is not the timing error itself which causes problems but the changes in this error due to signal amplitude fluctuations or phase fluctuations. These amplitude and phase fluctuations are generally due to environmental effects but could have other systematic origins.

Two types of phase detectors are considered in this paper. The first is a true zero-crossing detector and the second is a balanced mixer. To simplify the discussion, the local oscillator signal is assumed to be an ideal sine wave and all the harmonic distortion is in the RF input signal. Clearly if the local oscillator signal also has harmonic content there will be additional terms in the timing error which can be treated as discussed below.

True zero-crossing detector

In many timing and frequency synthesis systems, a digital zero-crossing based phase detector is used. These devices generally trigger some type of counter which essentially counts the time between zero crossings. Harmonic distortion causes the zero crossing to be a function of the phase and amplitude of the harmonics as well as the fundamental. Fluctuations in the time of the zero crossing therefore compromise the accuracy of the system. The analysis to determine the timing error for an ideal zero crossing detector is straightforward. The input signal V can be represented as a summation of the carrier with frequency ω_1 and amplitude v_1 with harmonics $n\omega$ of amplitude v_n and phase ϕ_n relative to the fundamental:

$$V = v_1 \cos(\omega_1 t) + \sum_{n=2}^{\infty} v_n \cos(n\omega_1 t + \phi_n). \quad (1)$$

Since we are interested in what happens at the zero crossings when the phase of the fundamental is a multiple of $\pi/2$, we linearize about that point with a Taylor series expansion:

$$V = v_1 \cos(\omega_1 t_0) + \sum_{n=2}^{\infty} v_n \cos(n\omega_1 t_0 + \phi_n) - \left[v_1\omega_1 \sin(\omega_1 t_0) + \sum_{n=2}^{\infty} v_n n\omega_1 \sin(n\omega_1 t_0 + \phi_n) \right](t - t_0) \quad (2)$$

At the zero crossing $V = 0$ and $\omega_1 t_0 = \pi/2$. After simplification the time error Δt, due to the harmonic content of the signal can be expressed as

$$\Delta t = (t - t_0) = \frac{\sum_{n=2}^{\infty} v_n \cos\left(\phi_n + n\frac{\pi}{2}\right)}{v_1\omega_1 + \sum_{n=2}^{\infty} v_n n\omega_1 \sin\left(\phi_n + n\frac{\pi}{2}\right)} \quad (3)$$

As Eq. (3) shows, the timing error depends on the amplitude and phase of the harmonics. The worst case timing error is

$$\Delta t_{max} = \frac{\sum_{n=2}^{\infty} v_n}{v_1 \omega_1}, \quad (4)$$

which occurs when the phase of the nth harmonic is an integral multiple of $\pi/2$ relative to the phase of the fundamental. In frequency distribution applications, the amplitude and phase of the harmonics may exhibit fluctuations due to environmental effects, changes or mismatch in the load impedance or dispersion along a transmission line. These timing errors are also important in frequency synthesis or phase-locking applications since they are converted to frequency errors by the phase-locked loops. For a signal with a carrier frequency of 5 MHz and a second harmonic of -25 dBc the timing error could be as much as 1.8 ns depending on the phase of the harmonic relative to the fundamental. A 10% change in the amplitude of the harmonic; that is from -25 dBc to -25.5 dBc would cause a 180 ps error. These errors could be greater if other harmonics are present as well. At a carrier frequency of 100 MHz the timing error due to a second harmonic of -25 dBc is 9 ps.

Double Balanced Mixers

Balanced mixers used as phase detectors are not so easily analyzed so we have experimentally measured the phase errors due to harmonic distortion up to the ninth harmonic. A 5 MHz source is split the first branch is filtered to suppress the harmonic content to well below -60 dBc.

This signal is applied to the LO port of the mixer. The harmonic frequency to be tested is generated at -30 dBc and offset by 3 kHz from $n\omega_1$. This signal is then summed to the second branch as shown in Figure 1. Variable attenuators are used to adjust RF and LO power at the mixer ports for the various phase error measurements. The low harmonic power assures linear conversions within the phase detector. The 3 kHz IF signal is low pass filtered and analyzed on an FFT analyzer. This approach assures that measurements yield the worst case timing error.

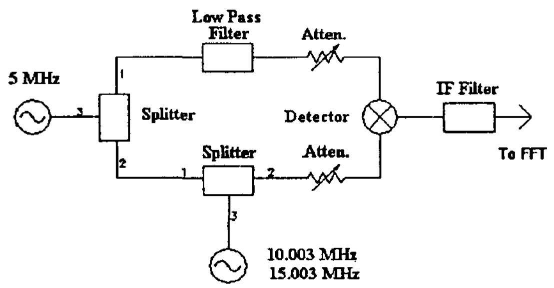

Figure 1. The setup for measuring the timing error due to distortion in double balanced mixer phase detectors.

The timing errors due to harmonic distortion in balanced mixers are different than for the zero crossing detector. In principle balanced mixers are insensitive to even harmonics of the reference oscillator and the sensitivity to the odd harmonics fall roughly as 1/n. This trend is apparent in Figs. 2, 3, 4 and 5, where there is suppression of the timing error due to the even harmonics. However leakage from port to port causes even and odd harmonics to mix down to DC causing at the IF port, additional DC components which are interpreted as phase shifts or timing errors. These timing errors are about 15-25 dB smaller than for the zero-crossing detector.

Four different low frequency (.1-500 MHz) mixers commonly used in our synthesizers and measurement equipment, were tested at LO power ranging from 4 to 13 dBm and RF power of 1 to 10 dBm. The timing errors normalized to 0 dBc are displayed in Figs. 2-5 for four different combinations of LO and RF power at the mixer ports. For a specific harmonic power the timing error is obtained by multiplying the time error read from the chart by v_n/v_1 where v_n is the amplitude of the harmonic and v_1 is the amplitude of the carrier. The data for LO ranges of 4 to 13 dBm and RF ranges of 1 to 10 dBm are displayed in tabular form in Tables 1-4.

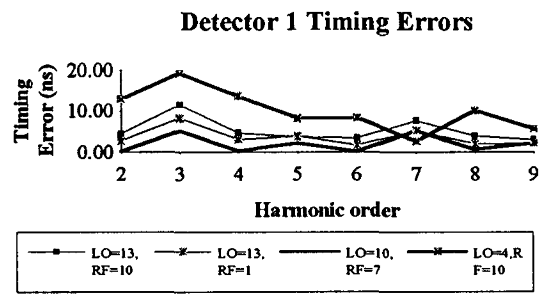

Figure 2. The timing error for a single balanced mixer phase detector w/ nominal LO of +10 dBm normalized to a harmonic level of 0 dBc for a carrier frequency of 5 MHz. For a specific harmonic level the timing error is obtained by multiplying the time error read from the chart by v_n/v_1.

Figure 2 displays the timing errors for Detector #1, a single balanced mixer optimized for use as a phase detector (it has very low conversion loss) for a carrier frequency of 5 MHz. The timing error for the even harmonics is generally much lower than for the odd harmonics. Even in the worst case, the timing error at 5 MHz for this phase detector is lower than 31.8 ns. Recall that for a true zero-crossing detector the normalized error for each harmonic is 31.8 ns for a carrier frequency of 5 MHz. For a 100 MHz carrier the timing error of a true zero-crossing detector sensitivity is 1.6 ns. The lowest sensitivity to harmonic distortion occurs when the power at the LO port is +10 dBm and the power at the RF port is +7 dBm. Detector #2 is a general purpose double balanced mixer.

Detector 2 Timing Errors

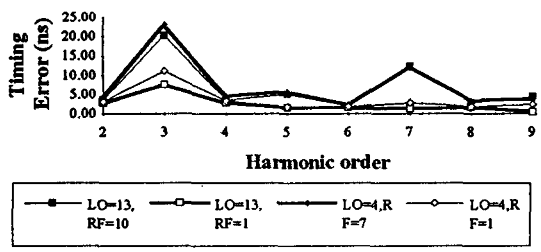

Figure 3. The timing error for a double balanced mixer w/ nominal LO power of +7 dBm normalized to a harmonic level of 0 dBc for a carrier frequency of 5 MHz. For a specific harmonic level the timing error is obtained by multiplying the time error read from the chart by v_n/v_1.

Figure 3 shows that the best performance is attained when the mixer is driven at +13 dBm on the LO port and +1 dBm on the RF port. This power level configuration corresponds to the mixer being driven in its linear operating range; the RF power level is well below the LO power level. High power levels at both the RF and LO ports is the configuration commonly used when a double balanced mixer is used as a phase detector. In this configuration the mixer is operating in saturated mode. Operating in saturated mode may provide the lowest conversion loss but the sensitivity to harmonic distortion increases by an order of magnitude. For detector 3 the sensitivity to the third harmonic can be reduced by having a high LO power and a low RF power at the expense of a slightly higher sensitivity to the other harmonics. In general detector 3 and detector 4 have trends similar to the previous two mixers.

Detector 3 Timing Errors

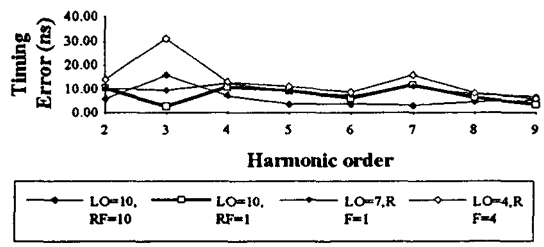

Figure 4. The timing error for a double balanced mixer w/ nominal LO power of +15 dBm normalized to a harmonic level of 0 dBc for a carrier frequency of 5 MHz. For a specific harmonic level the timing error is obtained by multiplying the time error read from the chart by v_n/v_1.

Detector 4 Timing Errors

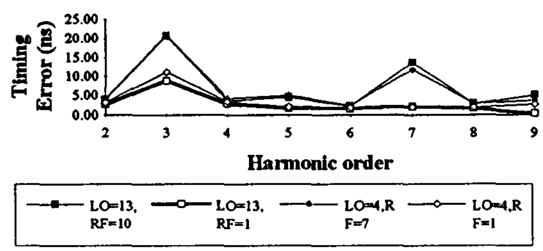

Figure 5. The timing error for a double balanced mixer w/ nominal LO of +7dBm, normalized to a harmonic level of 0 dBc for a carrier frequency of 5 MHz. For a specific harmonic level the timing error is obtained by multiplying the time error read from the chart by v_n/v_1.

Discussion

Harmonic distortion changes the zero crossing of a signal by an amount which depends on the amplitude and phase of each harmonic present in the signal. The timing error which results becomes problematic with signal and phase fluctuations due to environmental effects such as temperature and humidity, aging in crystal oscillators, aging in the bias components of active devices or changes in load impedance and transmission line characteristics. A balanced mixer phase detector offers a significant improvement over a true zero-crossing detector. The timing error due to the even harmonics present in the signal are suppressed and if the mixer is operated linearly, the sensitivity to the 3rd harmonic decreases 10 to 20 dB. Increasing the carrier frequency from 5 to 100 MHz should help to reduce the effects by about a factor of 20.

If a timing stability of 1 ps is required and one assumes that the harmonic content is stable to approximately 3%, then the harmonic content of the signal should not exceed -60 dBc when a true zero-crossing detector is used to determine time or phase. If a balanced mixer is used in place of the true zero-crossing detector, these requirements can be reduced to a harmonic content of approximately -45 dBc at 5 MHz or -20 dBc at 100 MHz. The distribution amplifiers described in [3,4] easily satisfy these requirements.

References

[1] F.L. Walls, L.M. Nelson, and G.R. Valdez, "Designing for frequency metrology at the 10-18 level", Proc. 6th EFTF, pp 477-481, 1992.

[2] A. DeMarchi, F. Mussion, and M. Siccardi, "A high isolation low noise amplifier with near unity gain up to 100 MHz," Proc. 1993 IEEE Symp. on Freq. Contr., 1993, pp 216-219.

[3] C.W. Nelson, F.L. Walls, M. Siccardi, and A. DeMarchi, "A new 5 and 10 MHz high isolation distribution amplifier," Proc. 1994 IEEE Symp. on Freq. Contr., 1994, pp. 567-571.

[4] M. Siccardi, S. Romisch, F.L. Walls, and A. DeMarchi, "A low noise 100 MHz distribution amplifier for precision metrology," to be published in Proc. 9th Eur. Freq. and Time Forum, 1995.

Table 1.

5 MHz Carrier	Timing Error in (ns) of the nth harmonic							
Power at Detector #1	2nd	3rd	4th	5th	6th	7th	8th	9th
LO=13dBm RF=10dBm	4.55	11.56	4.71	3.78	3.45	7.64	3.92	3.01
LO=13dBm RF=7dBm	4.01	11.56	4.65	2.31	2.62	4.82	2.94	1.31
LO=13dBm RF=4dBm	1.08	13.27	1.18	1.22	0.49	2.90	0.23	2.39
LO=13dBm RF=1dBm	2.77	8.18	3.11	3.87	1.83	5.34	2.08	2.28
LO=10dBm RF=10dBm	0.70	8.28	0.67	2.15	0.91	6.42	0.89	1.36
LO=10dBm RF=7dBm	0.28	5.16	0.34	2.25	0.30	5.16	0.74	2.28
LO=10dBm RF=4dBm	3.11	10.07	3.37	3.49	1.92	4.44	2.36	4.20
LO=10dBm RF=1dBm	7.90	6.50	8.57	7.64	4.93	8.28	5.34	4.20
LO=7dBm RF=10dBm	5.73	10.54	5.60	2.74	3.08	2.56	4.05	0.81
LO=7dBm RF=7dBm	4.01	2.80	4.01	3.65	2.10	5.93	1.99	3.83
LO=7dBm RF=4dBm	6.21	9.84	6.65	5.16	3.83	6.42	4.29	5.34
LO=7dBm RF=1dBm	10.18	8.57	10.91	8.87	6.42	8.47	6.88	4.99
LO=4dBm RF=10dBm	12.97	19.18	13.74	8.18	8.47	2.74	10.07	5.66
LO=4dBm RF=7dBm	15.59	13.89	15.41	9.84	8.67	8.18	9.18	4.44
LO=4dBm RF=4dBm	14.38	13.74	15.59	10.79	8.87	12.67	9.50	8.37
LO=4dBm RF=1dBm	15.77	14.89	17.09	12.53	9.84	11.42	10.54	7.04

Table 2.

5 MHz Carrier	Timing Error in (ns) of the nth harmonic							
Power at Detector #2	2nd	3rd	4th	5th	6th	7th	8th	9th
LO=13dBm RF=10dBm	3.57	20.32	3.49	5.04	1.99	12.38	2.90	4.44
LO=13dBm RF=7dBm	3.26	20.55	3.37	4.20	1.81	6.00	2.36	2.47
LO=13dBm RF=4dBm	2.87	15.59	3.01	2.28	1.67	2.25	1.75	4.55
LO=13dBm RF=1dBm	2.74	7.55	2.77	1.52	1.63	1.34	1.63	0.49
LO=10dBm RF=10dBm	3.70	21.03	3.65	5.34	2.10	14.22	3.11	5.79
LO=10dBm RF=7dBm	3.22	21.52	3.33	4.44	1.79	7.90	2.36	2.84
LO=10dBm RF=4dBm	2.90	17.70	3.08	2.59	1.71	2.47	1.77	5.47
LO=10dBm RF=1dBm	2.84	9.39	2.90	1.54	1.73	2.06	1.75	0.30
LO=7dBm RF=10dBm	4.34	22.80	4.29	6.07	2.47	16.90	3.65	7.29
LO=7dBm RF=7dBm	3.29	21.27	3.45	4.65	1.85	9.39	2.50	3.04
LO=7dBm RF=4dBm	2.97	18.11	3.18	2.90	1.77	2.39	1.85	5.66
LO=7dBm RF=1dBm	2.84	10.42	2.94	1.56	1.75	2.65	1.71	1.28
LO=4dBm RF=10dBm	4.87	21.03	4.76	6.28	2.84	16.90	4.01	7.55
LO=4dBm RF=7dBm	4.29	23.06	4.50	5.73	2.44	12.10	3.29	3.92
LO=4dBm RF=4dBm	3.70	19.18	3.96	3.78	2.20	3.53	2.31	5.93
LO=4dBm RF=1dBm	3.15	11.16	3.29	1.83	1.94	2.90	1.81	2.36

Table 3.

5 MHz Carrier	Timing Error in (ns) of the nth harmonic						
Power at Detector #3	2nd	3rd	4th	5th	6th	7th	8th	9th
LO=13dBm RF=10dBm	4.60	14.38	5.28	1.85	2.62	2.59	3.22	2.62
LO=13dBm RF=7dBm	1.31	21.03	1.73	3.57	0.96	8.28	0.89	6.88
LO=13dBm RF=4dBm	6.88	8.87	7.13	4.20	3.92	7.46	4.29	3.92
LO=13dBm RF=1dBm	22.53	17.29	23.87	14.89	13.42	7.21	14.72	6.07
LO=10dBm RF=10dBm	6.00	15.95	7.38	3.74	3.87	3.18	4.65	5.47
LO=10dBm RF=7dBm	10.30	10.42	11.16	8.97	6.07	11.83	6.21	5.47
LO=10dBm RF=4dBm	17.29	15.95	17.90	12.53	10.07	7.38	11.04	5.60
LO=10dBm RF=1dBm	10.42	2.74	10.91	9.29	6.21	11.83	6.57	3.45
LO=7dBm RF=10dBm	1.47	24.99	2.36	2.20	0.95	5.66	1.03	3.70
LO=7dBm RF=7dBm	5.73	18.74	5.73	6.28	1.49	9.72	3.45	6.81
LO=7dBm RF=4dBm	10.18	19.18	11.42	9.50	6.81	7.04	7.64	8.37
LO=7dBm RF=1dBm	10.30	9.61	12.53	9.39	7.64	11.16	8.09	6.96
LO=4dBm RF=10dBm	8.87	23.06	5.04	2.71	3.01	5.93	3.01	2.18
LO=4dBm RF=7dBm	10.07	35.72	10.79	7.38	6.14	5.79	7.38	12.97
LO=4dBm RF=4dBm	14.22	31.11	13.12	11.16	8.87	15.95	8.47	6.00
LO=4dBm RF=1dBm	19.63	15.59	11.56	10.54	7.72	13.27	8.09	5.93

Table 4.

5 MHz Carrier	Timing Error in (ns) of the nth harmonic							
Power at Detector #4	2nd	3rd	4th	5th	6th	7th	8th	9th
LO=13dBm RF=10dBm	3.96	20.79	3.61	4.65	2.28	13.74	2.94	5.16
LO=13dBm RF=7dBm	2.87	19.63	3.01	3.78	1.69	6.42	2.03	1.56
LO=13dBm RF=4dBm	2.77	16.32	2.97	2.36	1.67	1.75	1.81	4.55
LO=13dBm RF=1dBm	2.68	8.87	2.87	1.63	1.60	2.15	1.63	0.40
LO=10dBm RF=10dBm	3.65	20.55	3.57	4.99	2.15	14.55	2.80	6.21
LO=10dBm RF=7dBm	3.01	20.32	3.18	4.10	1.79	8.09	2.23	2.23
LO=10dBm RF=4dBm	3.01	18.32	3.29	2.84	1.85	1.77	2.10	5.16
LO=10dBm RF=1dBm	2.87	9.95	3.08	1.75	1.71	2.08	1.71	1.42
LO=7dBm RF=10dBm	4.24	21.77	4.15	5.60	2.50	16.51	3.22	7.21
LO=7dBm RF=7dBm	3.33	21.03	3.53	4.60	1.99	9.95	2.53	2.90
LO=7dBm RF=4dBm	2.97	17.49	3.29	3.04	1.83	2.33	1.99	4.93
LO=7dBm RF=1dBm	2.71	10.07	2.94	1.73	1.61	1.94	1.63	2.03
LO=4dBm RF=10dBm	5.04	21.52	4.87	6.07	2.94	17.49	3.78	8.28
LO=4dBm RF=7dBm	4.05	21.03	4.29	5.16	2.41	11.83	3.04	3.83
LO=4dBm RF=4dBm	3.78	18.74	4.10	3.92	2.31	4.05	2.53	4.93
LO=4dBm RF=1dBm	3.26	11.16	3.49	2.13	1.92	2.01	1.99	2.80