Trapped ions, entanglement, and quantum computing*

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ABSTRACT

A miniature, elliptical ring rf (Paul) ion trap has been used in recent experiments toward realizing a quantum computer in a trapped ion system. With the combination of small spatial dimensions and high rf drive potentials, around 500 V amplitude, we have achieved secular oscillation frequencies in the range of 5-20 MHz. The equilibrium positions of pairs of ions that are crystallized in this trap lie along the long axis of the ellipse. By adding a static potential to the trap, the micromotion of two crystallized ions may be reduced relative to the case of pure rf confinement. The presence of micromotion reduces the strength of internal transitions in the ion, an effect that is characterized by a Debye-Waller factor, in analogy with the reduction of Bragg scattering at finite temperature in a crystal lattice. We have demonstrated the dependence of the rates of internal transitions on the amplitude of micromotion, and we propose a scheme to use this effect to differentially address the ions.

Keywords: quantum computing, quantum logic, ion traps, laser cooling and trapping

1. INTRODUCTION

Since the development of useful algorithms for quantum computation[1,2], there has been an explosion of work toward realizing a practical quantum computer. One of the more attractive systems for implementation is a string of trapped ions[3,4], and already a quantum logic gate has been demonstrated with a single trapped ion[5]. In this paper, we report on further progress toward achieving quantum logic in a trapped ion system. We describe a miniature, elliptical ring, rf (Paul) trap used in our current experiments. The addition of a static potential can be used to reduce the micromotion of several ions confined in this type of trap. We then describe progress toward individual addressing of ions using the Debye-Waller factors due to rf micromotion. Other recent results, including cooling two ions to their collective motional ground state as well as observations of heating, will be described in a separate publication[6].

2. ELLIPTICAL RF PAUL TRAP

The ion trap used in our current experiments, shown schematically in Figure 1, is based closely on the trap used in previous experiments[7]. The ring and endcap electrodes are made from 125 μm thick beryllium foil. The ring electrode was formed by punching a hole in the foil with a pinpoint and then widening the hole into an elliptical shape using a thin tungsten wire as a file. The edges of the hole were smoothed by "flossing" the hole with the tungsten wire. The aspect ratio of the ring is approximately 3:2, and the elongated axis is approximately 525 μm long. The endcaps are made by cutting a 250 μm slot in a similar piece of foil.

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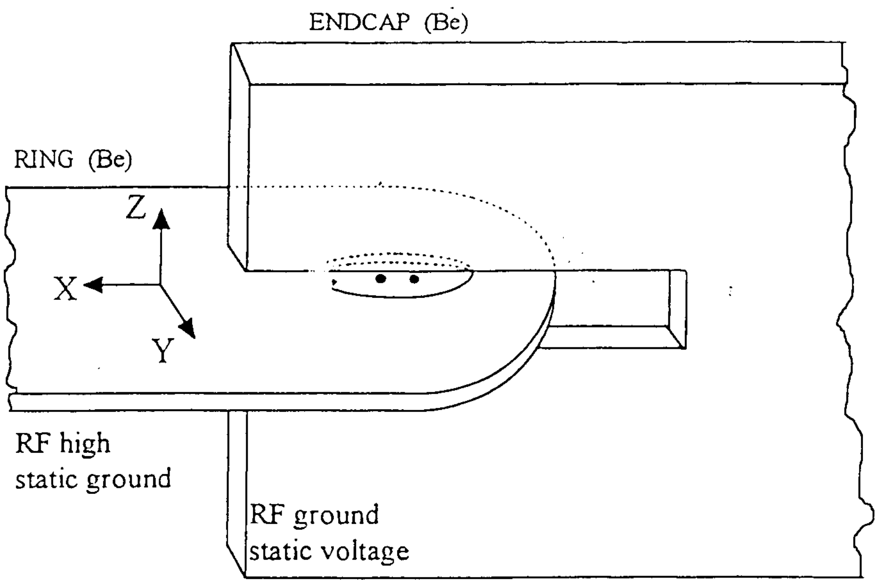


Figure 1. A schematic of the elliptical ion trap used in recent studies. Ions align along the elgongated, weak direction of the elliptical ring electrode. The endcaps may be biased with a static electric potential to adjust the spring constants of the trap.

The rf drive is generated by feeding a copper quarter-wave coaxial line, resonant at about 240 MHz, that is inside the vacuum system[7]. The rf is coupled in at the base of the resonator with a single loop of wire, the shape of which is empirically adjusted to impedance-match the 50 Ω rf source. The ring is clamped to the end of the center conductor, and the typical voltage amplitude on the ring is 550 V for 1 W input rf power. The endcaps may be biased with a static potential to adjust the trap frequencies. There are, in addition, four shim electrodes (not shown) that allow us to null out stray electric fields.

The classical motion of an ion in a rf quadrupole trap is described by the Mathieu equation.[8,9] In the pseudopotential approximation, the solution for the motion may be broken up into a fast, small amplitude motion at the rf drive frequency, termed the micromotion, and a slower, larger amplitude motion that describes the position of the ion averaged over a period of the rf drive, which is called the secular motion. The secular oscillation frequencies are observed through resonant detection[7,10]. A plot of the secular frequencies as a function of the electrical potential applied to the endcaps is shown in Figure 2. The measured frequencies have been fit with the functions

$$\omega_x = \frac{\Omega}{2}\sqrt{\alpha a + (\alpha q)^2/2}\,, \tag{1}$$

$$\omega_y = \frac{\Omega}{2}\sqrt{(1-\alpha)a + (1-\alpha)^2 q^2/2}\,, \text{ and} \tag{2}$$

$$\omega_z = \frac{\Omega}{2}\sqrt{-a + q^2/2}\,, \tag{3}$$

where $\Omega = 2\pi \times 238.3$ MHz is the rf drive frequency, α is the geometric ellipticity parameter, and a

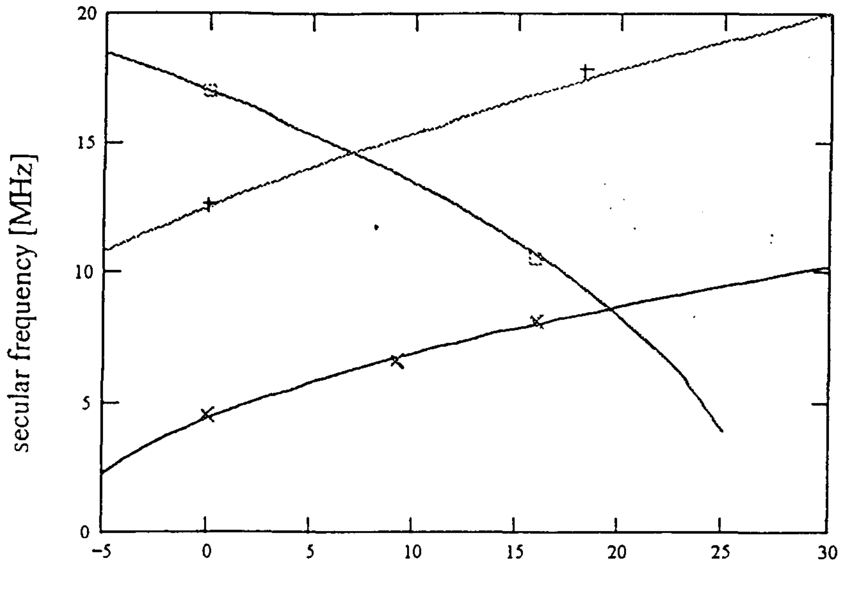


Figure 2. The secular motional frequencies as a function of the voltage applied to the endcaps. Note that the voltage is actually negative with respect to the ring electrode. The x symbols are for the x frequencies, the open squares are for the z frequencies, and the crosses are for the y frequencies. The lines are fits to Equations 1 through 3.

and q are static and rf parameters in the Mathieu equation for the classical amplitude of motion[9]. The parameter α is defined so that the electrical potential has the form $\Phi \propto \left[\alpha x^2 + (1-\alpha)y^2 - z^2\right]$. Thus, for a spherical quadrupole trap, $\alpha = 0.5$. Due to the asymmetry of the endcaps, the ellipticity parameter is $\alpha = 0.36$ even when the ring is circular[7]. The elliptical ring used in the current trap enhances the ellipticity of the fields, and we extract $\alpha = 0.26$ from a fit to the data.

In the type of trap described here, the electric fields vanish at a single point. In the absence of stray fields, a single ion in the trap is confined in the vicinity of this zero-field point. Thus, in the pseudopotential approximation ($q \ll 1$), there is almost negligible micromotion when a single ion is cooled to the zero-point state of secular motion. In the case of two ions crystallized in the trap, Coulomb repulsion pushes the ions away from this zero-field region. Thus there is significant micromotion even when two ions are cooled to the zero point of secular motion. The kinetic energy T_μ of micromotion is equal to the rf pseudopotential energy U_{rf} at the ion equilibrium positions[11]. However, the addition of a static potential U_0 can reduce the micromotion of two ions relative to the case of a pure rf trap. The ion separation is reduced by using a static potential to strengthen the confinement along the axis on which the ions lie, which in turn decreases the amplitude of the rf fields at the ion equilibrium positions. The potential energy at the "squeezed" equilibrium is a sum of the rf pseudopotential and static potential contributions, $U = U_{\mathrm{rf}} + U_0$, and the ratio of micromotion kinetic energy to total potential energy, $T_\mu / U = U_{\mathrm{rf}} / \left(U_{\mathrm{rf}} + U_0\right)$, is less than 1.

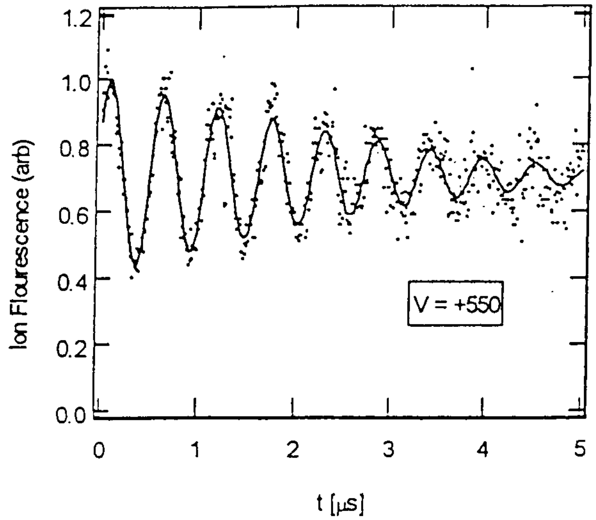
In the elliptical rf trap, the equilibrium positions of two cold crystallized ions lie along the direction of the weakest spring constant. In the absence of a static potential, the weakest confinement is on the long axis of the elliptical ring, which we label the $\hat{x}$ axis (see Figure 1). As Figure 2 shows, the three trap frequencies may be altered by applying a voltage to the endcaps. In particular, a negative potential on the endcaps decreases the $\hat{z}$ confinement while strengthening the confinement in the plane of the ring. As the ions are squeezed together, they maintain their equilibria along the $\hat{x}$ axis so long as $\omega_x < \omega_z$. To determine the maximum reduction in micromotion, consider the limiting case $\omega_x = \omega_z$. In this case, the ratio of micromotion kinetic energy to total potential energy is $T_\mu / U = \alpha$. Thus the physical geometry of the trap determines the maximum suppression of micromotion, which can be substantial for small values of α.

3. MICROMOTION AND DEBYE-WALLER FACTORS

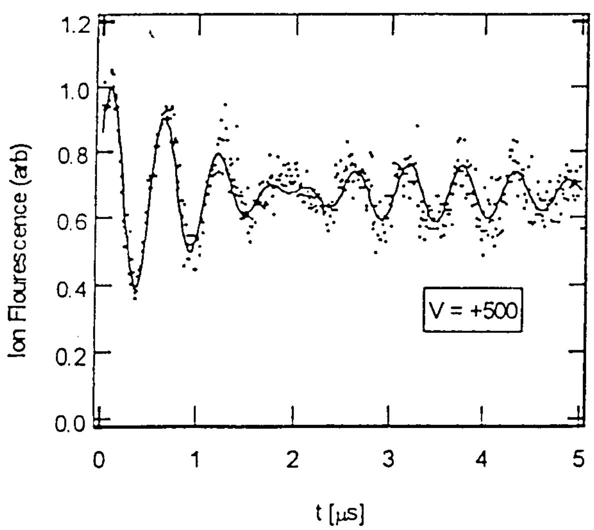
Quantum logic operations on a string of ions require the ability to differentially address the ions. This can be accomplished by focusing a laser so that only one ion is illuminated and the others are in the dark. However, the scheme outlined in the previous section to reduce the micromotion, whereby the ions are squeezed together using a static potential, excacerbates the problems of illuminating only one ion. In this section we present an alternative method for differential addressing that does not require tight focussing of a laser beam. In this scheme, the ions are uniformly illuminated, and the internal transition strengths are differentially modified by the motion of the ions. This sort of effect is familiar in crystallography, where the amplitude of the Bragg diffraction peak is a sensitive function of the random motion of the atoms in the crystal. The reduction of Bragg scattering at elevated temperatures is governed by the Debye-Waller factor.

In the case of two ions, the amplitude for driving stimulated Raman transitions between two internal states is sensitive to the motion of the ion, and we call the motional terms that modify the Raman transition rate the Debye-Waller factors[4]. For our system of trapped ions, both the thermal motion of the ion and the micromotion contribute to a reduction in the rate of stimulated Raman transitions. In the frame of the oscillating ion, the applied laser light is frequency modulated at the ion motional frequencies due to the Doppler shift. We are here concerned with the micromotion, so the applied laser light appears to have sidebands at the rf frequency, and the power in the central frequency is depleted. If the modulation index due to the motion is m, then matrix elements of transitions driven by the central, or carrier, frequency are proportional[12] to the Bessel function $J_0(m)$. This reduction of the transition strength can be manipulated by applying external fields. In particular, an applied electric field along the $\hat{x}$ direction displaces the ions so that the micromotion of one ion is reduced while that of the other is increased. This will increase the Rabi frequency of one ion at the expense of reducing the Rabi frequency of the other ion. If the ratio of the Rabi frequencies can be made equal to 2, then a pulse of radiation can be a π pulse for one ion and a 2π pulse for the other ion. Alternatively, the micromotion amplitude of one ion may be so large that $J_0(m) = 0$, allowing individual addressing of the other ion. The applied electric field may then be reversed to address the second ion.

In the experiment, an electric field along the $\hat{x}$ direction is generated by applying a potential to the four shim electrodes. For a variety of applied fields, the Rabi frequencies of the two ions on the carrier transition was observed[6,10], as shown in Figure 3. The data are fitted to a simple function with two independent frequencies. These pairs of frequencies are plotted in Figure 4 as a function of applied voltage on the shim electrodes. This clearly demonstrates the ability to adjust the Rabi frequencies. The implementation of quantum logic with this scheme is now under experimental investigation.



(a)



(b)

Figure 3. Rabi flopping on the carrier Raman transition for two ions. The detected flourescence oscillates in time as the ions flop between a "bright" state and a "dark" state.[6,10] In the upper trace, the ions have equal amplitudes of micromotion, and hence their Rabi frequencies are identical. In the lower curve, the ions have different amplitudes of micromotion, and this is reflected in two different Rabi frequencies. The fit is to two decaying sinusoids. The electric potential applied to the shim electrodes is indicated.

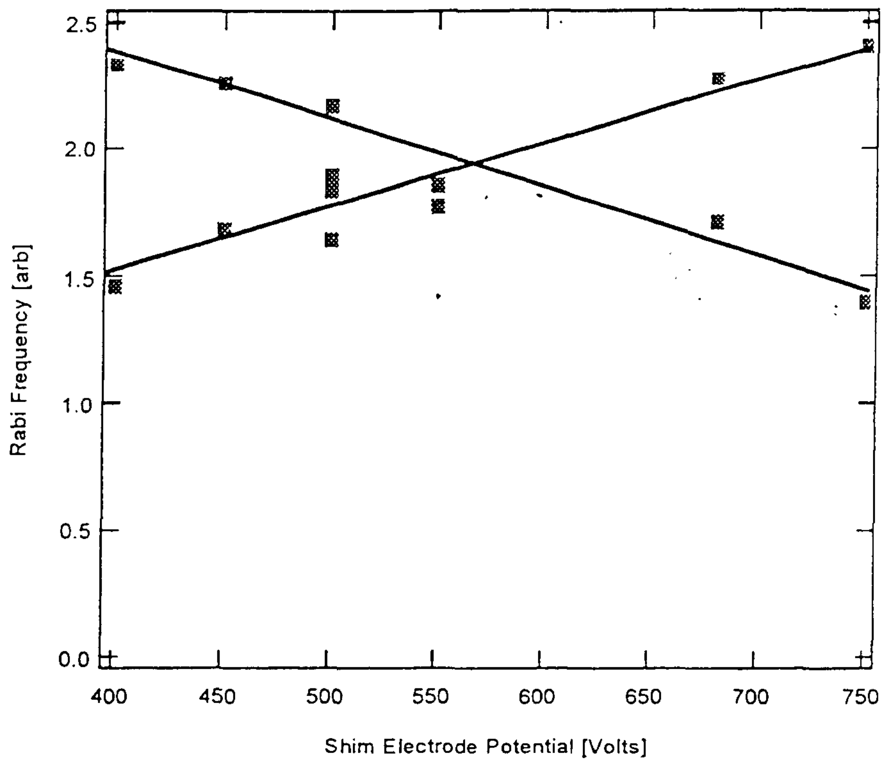


Figure 4. As the ions are displaced by an applied voltage on the shim electrodes, the Rabi frequencies are different for the two ions. The lines are only to guide the eye.

4. OUTLOOK

Achieving nontrivial quantum computation in a trapped-ion system requires a number of difficult steps. In this paper, we have discussed one possibility for individual addressing. Combined with our recent achievement of cooling two ions to the ground state of their collective motion[6], this puts simple quantum logic operations on two ions within reach. Future work will revolve around implementing some scheme for individual addressing in order to entangle the states of the ions. In the longer term, linear ion traps will be pursued in order to achieve quantum logic with more than two or three ions.

5. ACKNOWLEDGEMENTS

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