Superconducting Resonators: High Stability Oscillators and Applications to Fundamental Physics and Metrology

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ABSTRACT

Superconducting oscillators have achieved better frequency stability than any other device for averaging times of 10 l.c.s. to 1000 l.c.s. This high stability results from the use of solid niobium resonators having Q factors greater than 10^{10}. Such oscillators have direct applications as clocks and spectrally pure sources. They may also be used for accurate measurements of many physical quantities and to perform a variety of experiments on fundamental constants, relativity, and gravity waves.

INTRODUCTION

The best frequency stability ever reported is 3×10^{-16} which was achieved by a superconducting cavity stabilized oscillator. Although superconductivity is not fundamental to such performance, it results in two very important advantages beyond the usual qualities which characterize a high stability resonator: The very high Q which is obtainable (up to 10^{11}) implies both extremely narrow linewidth and low intrinsic noise; the small effective nonlinearity of the superconductor makes it possible to use relatively high power levels. The combination of these properties and others makes it possible to optimize the stability of oscillators from extremely short times to many days.

A variety of different types of superconducting resonators are in use: lumped circuit, stripline, cavity and Fabry-Perot. They span a frequency range from DC to near 100 GHz. Plated superconductors can be employed, but bulk niobium fired in a high vacuum furnace has produced the highest Q. Many different types of oscillators have also been developed or proposed for use with superconducting resonators. These include use of the superconducting resonator in the feedback path of an amplifier to produce an active oscillator and use of the superconducting resonator in some type of phase bridge to produce a control signal for stabilization of an independent oscillator. The factors which currently limit the performance of superconducting oscillators are not fundamental. The long-term stability is probably determined by ambient influences such as temperature and mechanical stability of critical components. The spectral purity is primarily limited by vibrations and stored energy in the resonator.

Because of their exceptional frequency stability, superconducting oscillators outperform other frequency standards in a number of applications. For example, they have the best stability of any device over the range of averaging times needed for long baseline interferometry. In addition, they should provide unsurpassed spectral purity for use in the synthesis of far-infrared frequencies. Superconducting oscillators are also interesting for a variety of fundamental physics experiments. It is possible to use them to test for time variations in the fundamental constants, to search for gravity waves, and to test the metricity of gravity. Since many physical quantities can be transduced to a frequency, high stability oscillators are excellent candidates for state-of-the-art metrology. Superconducting oscillators are particularly well-suited for some temperature, length and electrical parameter measurements.

SUPERCONDUCTING RESONATORS

A variety of different types of superconducting resonators may be used for high stability oscillators and for applications to fundamental physics and metrology. They include lumped circuit, stripline, cavity and Fabry-Perot resonators. The choice of a resonator type for a particular application involves the consideration of many factors. In general, however, a resonator with a high quality factor (Q) and a very stable resonant frequency is desirable. For applications in which a physical quantity is transduced to a frequency shift, the resonator frequency must be sensitive to the physical quantity, yet insensitive to other conditions. The factors which affect the Q and the frequency of a resonator are discussed below.

Resonator Q. Superconducting resonators with high Q have been studied most extensively for application to particle accelerators which use resonators in the range of 100 MHz to 10 GHz. In this frequency range, resonators made of niobium, Nb_3Sn and lead have all achieved unloaded Q's (Q_o) greater than 10^9. Niobium resonators have been most extensively studied. They have achieved high Q_o's in the range of 10^{10} to 10^{11} for TM_{010}-mode cylindrical cavities at 9 GHz[1] and 1.3 GHz[2], for TE_{011}-mode cavities at 3 GHz[3] and 9 GHz[4], for a helical cavity at 90 MHz[5], and for a re-entrant cavity at 400 MHz[6]. Much less work has been performed on Nb_3Sn resonators. Nonetheless a TE_{011}-mode cavity at 9 GHz has achieved a Q_o of 6×10^9[7]. Although there has been extensive work on lead resonators, Q_o's larger than 10^9 have been limited to TE_{011}-mode cavities which have nearly zero electric field on their surface. A Q_o of 4×10^{10} has been reported for a lead TE_{013}-mode cavity at 12 GHz[8].

An understanding of the relationship of the Q_o to the resonator mode and the operating temperature and frequency comes from the study of the various types of losses in the resonator as well as the mode characteristics.

First, a superconducting resonator whose dielectric is vacuum is considered. In this case, the losses are determined by the magnitude of the magnetic field, H, at the surface of the superconductor. The dissipated power density, p, in the superconductor is

$$p = \frac{1}{2} RH^2, \qquad (1)$$

where R is the total surface resistance of the superconductor. For a particular resonator and mode the unloaded Q is related to the surface resistance by the geometrical factor:

$$Q_o = \Gamma/R. \qquad (2)$$

The geometrical factor is a measure of the ratio of the resonator-stored energy to the integral of the magnetic field squared over the surface of the resonator.

The geometrical factor varies over a wide range depending on the resonator type. A right-circular-cylindrical cavity with its diameter equal to its length has $\Gamma = 750\Omega$ for the TE_{011}-mode and

$\Gamma = 300\Omega$ for the TM_{010}-mode. A Fabry-Perot resonator can have a high Γ of $1 \times 10^4\Omega$[5], whereas a helical cavity can have a low Γ of 5Ω[5]. Thus, the choice of a resonator type can strongly influence Q_o.

The surface resistance of superconductors[10] is approximated by the following relation for temperature less than one-half the superconducting transition temperature (T_c) and for frequency (ω) well below the superconducting energy gap:

$$R = R_o\ \omega^{1.7} \exp\left(-\frac{\Delta_o T_c}{T}\right) + R_{res}, \tag{3}$$

where R_o is a constant and $\Delta_o = \Delta(0)/kT_c$ is the reduced energy gap. The first term of Eq. 3 is the surface resistance (R_s) of the superconducting state, and the second term is the residual surface resistance (R_{res}) which represents other types of losses at the surface. R_{res} as low as $5 \times 10^{-10}\Omega$ at 90 MHz[5] and $3 \times 10^{-9}\Omega$ at 9 GHz[1] has been reported. Although very low R_{res} can be achieved, it is important to note that at higher temperatures and frequencies R_s, the first term in Eq. 3, may dominate the total surface resistance because of its strong exponential temperature dependence. For example, $R_s = 2.6 \times 10^{-5}\Omega$ at 4.2 K and $R_s = 2.5 \times 10^{-9}\Omega$ at 1.3 K for niobium at 10 GHz. Thus at 10 GHz, a superconducting niobium TM_{010}-mode cavity must be operated below 1.3 K to reach a Q_o of 10^{11}. The parameters for the first term in Eq. 3 are given in Table I for niobium, Nb_3Sn and lead. These parameters should be adequate for the frequency range 100 MHz to 100 GHz.

Table I

Superconductor	Δ_o	T_c (K)	$R_o(\Omega)$
Nb[11]	1.88	9.25	7.13×10^{-22}
Nb_3Sn[7, 16]	2.1	18.	5.05×10^{-22}
Pb	2.05	7.2	5.15×10^{-22}

If the volume of a superconducting resonator is filled with a dielectric, the loss tangent, δ of the dielectric must also be considered. The loss tangent of dielectrics at low temperature has not been extensively investigated. Bagadasarov, et al. have recently reported a very low dielectric loss tangent of 2.4×10^{-8} for sapphire at 2 K[13]. This measurement was made on a solid sapphire 3 GHz TM_{010}-mode cavity coated with lead. This work is continuing.

Resonator Frequency. The frequency of a superconducting resonator is a result of the resonator geometry and mode, the dielectric within the space of the resonator, and the RF properties of the superconductor. Although the characteristics which determine the resonant frequency are relatively well-fixed, they are nonetheless sensitive to the operating conditions of the resonator and also to external conditions. These sensitivities are discussed below.

(a) Resonator Stored Energy. The frequency of a superconducting resonator is shifted from its value at zero stored energy by electromagnetic radiation pressure[1] and by the nonlinear superconducting surface reactance[14]. The frequency shifts for both of these effects are to lower frequencies and are proportional to the stored energy. The electromagnetic-radiation-pressure frequency shift is dependent on the resonator geometry, the mode and the elasticity and size of the structure forming the resonator. If a resonator and its structure are scaled by a factor α, while the stored energy is scaled by α^3 (same energy density), then both the resonator frequency and electromagnetic-radiation-pressure frequency shift scale as $1/\alpha$. Thus, the radiation-pressure fractional frequency shift remains constant. On the other hand, the nonlinear-surface-reactance frequency shift depends only on the resonator geometry and mode. If a resonator is scaled by a factor α, while the stored energy is scaled by a factor α^3, then the resonator frequency scales as $1/\alpha$ and the nonlinear-surface-reactance frequency shift scales as $(1/\alpha)^2$. Thus, the fractional frequency shift due to the nonlinear surface resistance scales as $1/\alpha$.

The frequency shift due to the electromagnetic radiation pressure is smallest in a cavity with a massive structure. For example[15] a circularly cylindrical TM_{010}-mode 8.6 GHz massive niobium cavity (the length of the cavity and the thickness of the walls are about equal to the cavity radius) has a stored-energy coefficient for the fractional frequency shift of $-1.5 \times 10^{-6}\ J^{-1}$. This coefficient is probably dominated by the nonlinear surface reactance contribution. On the other hand, a weakly supported resonator has a large frequency shift due to the electromagnetic radiation pressure. For example, an unsupported helical niobium resonator at 130 MHz[16] has a very large stored-energy coefficient for the fractional frequency shift of $-5.5 \times 10^{-3}\ J^{-1}$. This coefficient is dominated by the electromagnetic radiation pressure.

The frequency shift coming from the stored energy has two effects on resonator frequency stability. First, power fluctuations in the source result in fluctuations of stored energy and thus the resonant frequency. Second, the nonlinear coupling of the electromagnetic resonator mode to a mechanical mode of the resonator structure by the electromagnetic radiation pressure produces ponderomotive oscillations above some threshold stored energy[16]. These oscillations involve the transfer of energy between the electromagnetic and mechanical resonators, and they result in very large frequency and amplitude modulation of the electromagnetic resonator. For most applications, the resonator is operated well below the threshold for ponderomotive oscillations.

(b) Resonator Temperature. The frequency of a superconducting resonator is shifted from its absolute zero temperature value by both thermal expansion and the temperature dependence of the superconducting surface reactance. The frequency shifts for both of these effects are downward with increasing temperature. The frequency shift due to the thermal expansion is dependent only on the structure forming the resonator. If the structure of a resonator is scaled by a factor α, the fractional frequency shift due to thermal expansion would remain constant. On the other hand, the fractional frequency shift due to the temperature dependence of the superconducting surface reactance would scale as $1/\alpha$. The temperature coefficient of the frequency shift places some requirement on temperature stability in order to achieve a particular level of frequency stability.

An example of the temperature coefficient for the fractional frequency shift is given in Fig. 1 for a circularly cylindrical TM_{010}-mode solid niobium cavity at 8.6 GHz[17]. There are three contributions to the temperature coefficient: the temperature dependence of the surface reactance (skin depth), the lattice thermal expansion and the electronic thermal expansion. The skin depth contribution has a very strong temperature dependence which is exponential ($\exp(-\Delta_0 T_c/T)$). The electronic thermal expansion contribution has about the same functional form as the skin depth contribution, but its magnitude is much less (2%). The lattice thermal expansion contribution has a much weaker temperature dependence which is T^3. At 8.6 GHz, for the TM_{010}-mode niobium cavity, the skin depth and thermal expansion contributions cross at 1.2 K. The use of a dielectric structure for the resonator, rather than a structure formed with a superconductor, can yield a lower thermal expansion contribution to the temperature coefficient for the fractional frequency shift[18].

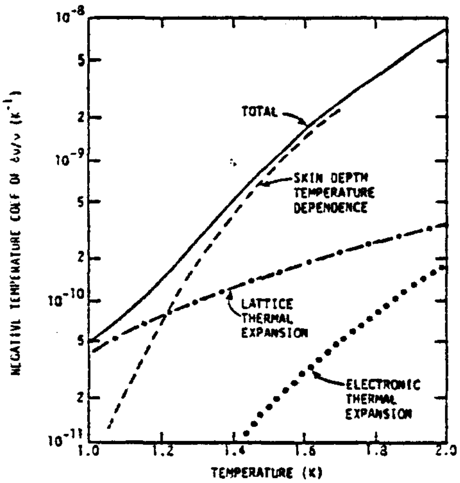

Fig. 1. Temperature coefficient of $\delta\nu/\nu$ for a niobium cavity.

(c) Gravity and Acceleration. On Earth, the largest frequency shift of this type is the result of the resonator structure self-weight, which places the resonator structure itself under strain. Again a massive structure is desirable to reduce the frequency shift on the surface of the Earth. An estimate of the fractional frequency shift of a massive TM_{010}-mode niobium cavity at 8.6 GHz supported at one end with its axis vertical in the gravitational field on the surface of the Earth[19] gives a fractional frequency shift of from 10^{-8} to 10^{-9}, which corresponds to an acceleration coefficient for the fractional coefficient for the fractional frequency shift of from 10^{-9} to $10^{-10}s^2m^{-1}$. This frequency shift is relatively large, and it makes the resonant frequency quite sensitive to tilt with respect to the gravitational field. A substantial reduction in the acceleration coefficient could be made by supporting the TM_{010}-mode cavity at its mid-plane so that the upper half is under compression and the lower half under tension. Strain in the resonant structure is also produced by linear acceleration and rotation of the resonator and by the gradient of the gravitational force. Although these effects are small for an Earth-fixed resonator, they can become quite important for resonators in free-falling satellites and space probes.

(d) Radiation. Penetrating radiation, such as cosmic radiation, affects the dielectric characteristics within a resonator, whether it is a vacuum or a solid dielectric. Thus, the radiation causes a frequency shift. The magnitude of the frequency shift due to radiation appears to be small. It has been measured by bringing a 5 mCi Co^{60} source close to a solid niobium cavity with a vacuum dielectric[19]. When the Co^{60} source was brought from a large distance to within 12 cm of the niobium cavity, the fractional frequency shift was less than 1×10^{-12}.

(e) External Pressure. Substantial frequency shifts can result from pressure applied externally to a resonator. This pressure may be the result of the hydrostatic pressure of a liquid or gas or the magnetic pressure due to the ambient magnetic field. The hydrostatic pressure is usually reduced by operating the resonator in a vacuum, and the magnetic pressure is reduced by shielding the resonator from the external magnetic field, and under these conditions the fractional frequency shift is very small.

(f) Vibrations. Mechanical vibrations of a resonator modulate its resonant frequency, which of course directly affects the spectral density of phase fluctuations. The nature of the mechanical vibrations and their affect on the resonant frequency depends on the geometry and mode of the resonator and on the elasticity, geometry, and density of the resonator structure. Although probably small, a frequency shift proportional to the vibrational energy can come about through the anharmonicity of the lattice potential of the resonator structure.

(g) Structural Changes. To this point, it has been assumed that the resonator structure is unchanged except for its single-valued dependence on parameters such as pressure, temperature, etc. It is possible, however, for the atomic arrangement of the resonator structure to change. The energy to change the atomic arrangement can either come from residual stress within the resonator structure created when it is fabricated, or it can come from external sources such as temperature, pressure, gravity or radiation. It is probably not possible to analyze the affect of these energy sources on structural changes and the consequent frequency jumps or drift. The best measure of the affect of these energy sources on structural changes may come from direct observation of the fractional frequency drift in superconducting resonators, which has been observed to be as low as 8×10^{-15}/day[20].

OSCILLATOR DESIGN

Because of the tremendous stability potential of superconducting resonators, a variety of techniques have been used to construct superconducting oscillators. The goals of this research have varied. Some oscillators have been constructed to illustrate feasibility, some to accomplish modest stability goals for further research on superconducting resonators, and others to achieve the ultimate frequency stability over some range of Fourier frequencies or averaging times. As a result, the achieved frequency stability for each technique is probably a poor indication of its capabilities. Instead of making such a comparison, this paper will outline some of the advantages or disadvantages of each method from the point of view of achieving the best possible frequency stability. The techniques discussed here use the superconducting resonator in three different ways--as the sole resonator of an oscillator circuit, as an auxiliary resonator to stabilize a free-running (noisy) oscillator, or as a filter which provides no feedback to the source.

Figure 2b illustrates how an oscillator may be realized using a superconducting resonator and a unilateral amplifier. Oscillation can occur when the amplifier gain exceeds the losses and the total phase shift around the loop is a multiple of 2π rad. Automatic gain control or limiting is necessary in order to produce oscillations at the desired power level. The resonator may be used in either transmission or reflection but the transmission mode is preferable because the insertion loss of the resonator suppresses spurious modes of oscillation which do not lie in its pass bands. This technique has received considerable attention because of its simplicity[21, 22]. The only element which needs to be located in the dewar is the superconducting cavity which can be connected to the room temperature amplifier by long lengths of transmission lines. However, this virtue is its major detraction when state-of-the-art frequency stability is desired. Changes in the phase length of the transmission lines produce proportional frequency shifts. If $\Delta\phi$ is the phase change from any source, the fractional frequency shift is

$$\Delta\nu/\nu = \Delta\phi/2Q_L \,. \tag{4}$$

The phase changes due to such factors as thermal expansion and vibrations are sufficiently large in a cryogenic system that they totally dominate the short term stability and drift of such an oscillator.

One solution to this problem is to use an amplifier which functions in the same low temperature environment as the resonator and is connected to it by short rigid transmission line. It has been proposed to use a cryogenic travelling wave maser in a unilateral amplifier design[23]. Alternatives are tunnel diode amplifiers and varactor diode parametric amplifiers. Both of these devices function by generating a negative conductance at the resonator frequency. Since they are bilateral, they can simply be connected to the superconducting cavity through an impedance-transforming network as shown in Fig. 2a. When the negative conductance of the amplifier exceeds the positive load conductance of the resonator, oscillation results. Jiminez and Septier have demonstrated the feasibility of the tunnel diode superconducting oscillator[24] and further work is being done by Braginski et al.[25, 26]. One of the authors (Stein) is developing a superconducting non-degenerate parametric oscillator[24]. The major advantage of the tunnel diode oscillator is that it requires only dc bias power for operation. On the other hand, there are several disadvantages. Shot noise in the tunnel junction limits currently available tunnel diode amplifiers to an effective noise temperature of 450K at 9GHz[28]. In addition, the very low operating voltage limits the theoretical output power to 1mW at 10 GHz from commercially available devices (having peak current less than 20 mA). If other problems were solved

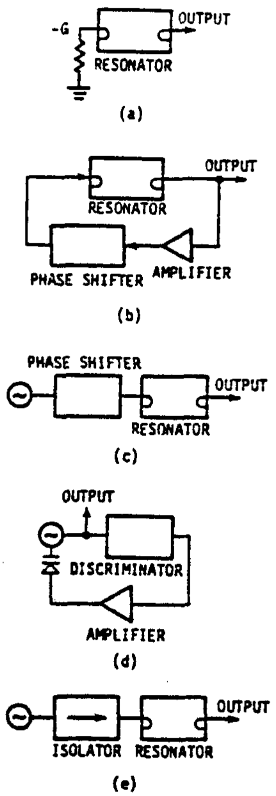

Fig. 2. Block diagram illustrating several superconducting frequency sources: (a) Negative resistance oscillator. (b) Loop oscillator, (c) stabilized voltage-controlled oscillator, and (e) passive filter.

these two difficulties could limit the frequency stability of the tunnel diode superconducting oscillator. In contrast, cooled parametric amplifiers have demonstrated 20 K noise temperatures and room temperature nondegenerate parametric oscillators have produced more than 100 mW at 9 GHz[28, 29].

The most widely studied technique for realizing a superconducting oscillator has been the stabilization of a free-running oscillator with a superconducting resonator[24]. One technique for accomplishing this, called cavity stabilization, is shown in Fig. 2c. The oscillator is injection-locked by the power which is reflected from the superconducting resonator. The stabilization factor, which is the ratio of the free-running oscillator frequency fluctuations to the cavity-stabilized oscillator frequency fluctuations, is given by the ratio of the Q of the superconducting cavity to the Q of the free-running oscillator[30]. Since the frequency fluctuations of an oscillator are inversely proportional to its resonator Q,[31] the best possible performance of a cavity stabilization system reduces to that of an oscillator built with the superconducting cavity as its only resonator. There are two major disadvantages to this technique. First, room temperature oscillators such as klystrons and Gunn-effect devices have extremely high noise temperatures; second, the frequency offset from the center of the resonance is proportional to the line length between the oscillator and the cavity just as in the loop oscillator.

The most successful superconducting oscillator technique to date is the use of active feedback to stabilize a voltage controlled oscillator[32]. The superconducting resonator is the frequency-sensitive element of a discriminator which generates an output voltage proportional to the frequency difference between the oscillator and the center of the superconducting cavity resonance[33]. Although this system also has a long path length between the room temperature oscillator and the superconducting cavity, it is possible to design the discriminator so that the dependence of the oscillator frequency on this path length is greatly reduced. This is accomplished by using phase modulation sidebands on the carrier frequency to provide the reference for locating the plane of the detuned short of the superconducting cavity. Despite the fact that this technique also uses a noisy room temperature oscillator, its performance is not limited by this fact. This is true because in such a system it is possible to greatly multiply the phase vs. frequency slope of the resonator by using external amplifiers. In this way the frequency fluctuations of the free-running oscillator may be reduced until the performance level is determined by the microwave detectors.

Figure 2e shows a superconducting resonator being used to filter the output of an oscillator. This application is particularly important when the oscillator is to be used as a source for frequency multiplication. For example, present state-of-the-art quartz crystal oscillator may be multiplied to 0.5 THz before the carrier is lost in the phase noise pedestal. However, if the same oscillator is filtered by a passive superconducting cavity with loaded Q equal to 2×10^9 it could in principle be multiplied to 100 THz[34].

The conclusion of the above discussion is that two types of superconducting oscillators appear most promising for improved stability: stabilization of a VCO and the all-cryogenic oscillator. The fundamental limitations of the two devices are similar, so the most important differences at this time are the practical problems of implementation: The VCO stabilization system has all the critical elements outside the dewar where they are readily available for adjustment and experimentation, but they are necessarily sensitive to problems of temperature fluctuations and vibration. On the other hand, the active oscillator is compact and totally contained in the highly controlled cryogenic environment. Such an oscillator will, however, present some new technical difficulties such as heat dissipation and device parameter fluctuations.

THEORETICAL OSCILLATOR NOISE PERFORMANCE

For the purpose of this discussion, the frequency fluctuations of any oscillator based on a superconducting resonator can be separated into two categories: Statistical fluctuations around the center of the resonance and the perturbations of the resonant frequency itself. The second category determines the ultimate performance level for any oscillator which is relatively independent of its design. It includes temperature, power level, and mechanically-induced frequency shifts which were discussed in a previous section.

If the center of the resonance is sufficiently constant, then other sources of noise will determine the ultimate stability of the superconducting oscillator system. The frequency fluctuations about the center of the resonance are highly dependent on the design of the particular oscillator; however, a lower limit corresponding to the case where all noise sources are filtered by the resonator can be determined. If the perturbing noise is white, then the phase of the oscillator does a random walk[35, 36, 37]. The one-sided spectral density of the phase fluctuations, in a form appropriate for microwave resonators, is given by

$$S_\phi(f) = \left(\frac{\nu_o}{f}\right)^2 \frac{kT}{2P_a\, Q_E\, Q_L} \qquad (5)$$

where ν_o is the operating frequency, k is the Boltzman constant, T is the absolute temperature, P_a is the power dissipated in the load, and Q_E and Q_L are the external and loaded Qs respectively. In the case of the superconducting cavity with $Q_E = 10^{10}$, $Q_L = 5 \times 10^9$, $P_a = 10^{-3}$W, $\nu_o = 10^{10}$ Hz, and T = 1K,

$$S_\phi(f) = 10^{-20}\ \mathrm{Hz}/f^2.$$

The active element in a practical oscillator will dominate the thermal noise. In this case T must be interpreted as the effective noise temperature of the device. Such noise temperatures vary from approximately 20 K for varactor parametric amplifiers to more than 10^4 K for a transferred-electron device.

Another important limitation on the stability of a superconducting oscillator is additive noise which results from a white noise voltage generator at the output of the oscillator. In an ideal oscillator the additive noise is due to output buffer amplifiers or a user device. The spectral density of the phase fluctuations is

$$S_\phi(f) = kT'/2P_a, \qquad (6)$$

where T' is the effective noise temperature of the circuitry which sees the output of the oscillator. If the effective noise temperature is 300 K and the available power is 10^{-3} W, then

$$S_\phi(f) = 2 \times 10^{-18}/\mathrm{Hz}.$$

In this case the additive noise dominates the oscillator spectrum for Fourier frequencies greater than .07 Hz. Under the same conditions, the rms fractional frequency fluctuations are given by

$$\sigma_y(\tau) = \left[\left(\frac{8.3 \times 10^{-21}}{\tau^{\frac{1}{2}}}\right)^2 + \left(\frac{4.3 \times 10^{-20} f_h}{\tau}\right)^2\right]^{\frac{1}{2}}, \qquad (7)$$

where f_h is the noise bandwidth of the measurement system.

Equations (5) and (6) show that both the random walk of phase and the additive noise can be reduced by increasing the available power. However, this technique is limited by several factors. The nonlinearity of the resonator couples amplitude and phase modulation and may ultimately limit the stability. If this is not a problem, then at some field level the resonator breaks down. Finally, high power levels may exceed the dynamic range of the user device such as a mixer in a super-heterodyne receiver.

SUPERCONDUCTING OSCILLATOR FREQUENCY STABILITY

The superconducting cavity stabilized oscillator (SCSO), which has been extensively investigated by the authors [33, 38], exemplifies the potential of superconducting cavities to achieve very high frequency stability. An ensemble of three nearly independent SCSOs was used to evaluate their performance. Each SCSO employed a massive niobium TM_{010}-mode 8.6 GHz cavity operating at about 1.2 K with a temperature stability of 10 μK. The unloaded Qs of these superconducting cavities ranged from a few times 10^9 to 7×10^{10}. The frequency stability of these SCSOs is discussed below. The potential frequency stability of SCSOs has not yet been fully realized; for short term stability, circuits utilizing a superconducting cavity as a transmission filter would be more appropriate.

Frequency Domain Measurements. The spectral density of phase fluctuations $S_\phi(f)$ has been measured as a function of Fourier frequency for the SCSO. The results of these measurements[39] for the 8.6 GHz SCSOs are summarized in Fig. 3. There are several aspects of S_ϕ which are important to discuss. First, for Fourier frequencies between 100 Hz and 50 kHz, the baseline for S_ϕ is white and it is 10^{-12} rad^2 Hz^{-1}. This level is the result of the noise characteristic of the 1 MHz amplitude detector used in the SCSO and it does not represent the limit of what may be achieved with a superconducting resonator. For frequencies below 100 Hz, the baseline of S_ϕ makes a transition to flicker frequency noise ($S_\phi \propto f^{-3}$). Superimposed on the baseline for Fourier frequencies below 500 Hz are what appear to be bright lines. These bright lines have been attributed to mechanical vibrations transmitted to the cavity from the floor.[1] The peak in S_ϕ at a Fourier frequency of 250 kHz comes from the specific character of the SCSO feedback frequency response which has an open loop gain of one at about 300 kHz. Thus, there is not enough feedback gain in this region to improve the phase fluctuations of the Gunn oscillator which is stabilized by the superconducting cavity. For the highest Fourier frequencies, S_ϕ is dropping because of the characteristics of the Gunn oscillator itself.

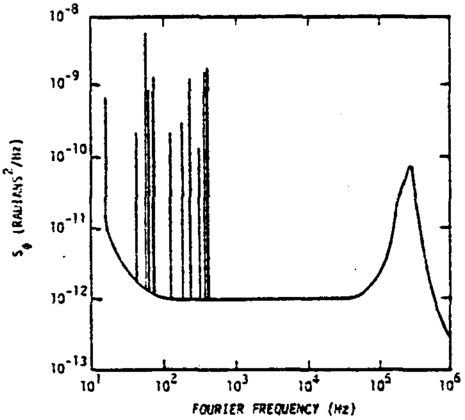

Fig. 3. Spectral density of phase fluctuations S_ϕ as a function of Fourier frequency for an 8.6 GHz superconducting-cavity stabilized oscillator.

Time Domain Measurements. The two-sample Allan variance for the 8.6 GHz SCSOs has been measured[39] as a function of sampling time with a noise bandwidth of 10 Hz, and is shown in Fig. 4 by the open circles. The figure has three regions of interest. First, for sampling times (τ) less than 30 s, the Allan variance (σ^2_y) has the form $\sigma_y = 10^{-14}/\tau$. For the noise bandwidth of 10 Hz, σ_y is dominated by the low frequency bright lines in S_ϕ rather than its white component. Second, σ_y reaches its noise floor of 3×10^{-16} for sampling times

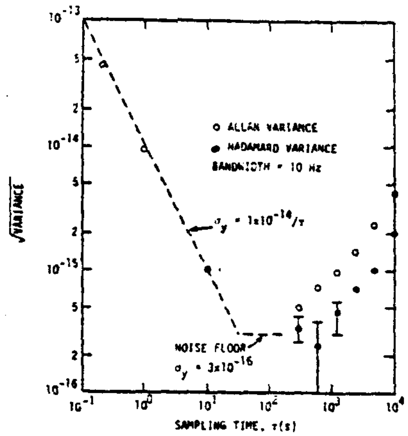

Fig. 4. The Allan and Hadamard variances as a function of sampling time, τ. The data are for a superconducting-cavity stabilized oscillator operating at 8.6 GHz.

between 30 and 100 s. Third, for $\tau > 100$ s, σ_y is dominated by drift in the SCSO, and therefore increases. The presence of drift is demonstrated by computing the second difference variance using three samples. This variance, represented by dots in Fig. 4, removes some of the drift and extends the noise floor to 1000 s.

<u>Frequency Drift</u>. The frequency drifts of the SCSOs have been measured both among the three SCSOs and with respect to an ensemble of cesium atomic frequency standards for periods of about twelve days. The linear drifts of the SCSO's with respect to the cesium frequency standards are typically about 10^{-14} day^{-1} and are both positive and negative.[20] At this time there is no evidence that the drift is the result of any structural changes in the cavities. The drift is probably accounted for by external influences on the cavity and the electronic circuits. In addition to linear drift, the frequency of the SCSOs has a daily variation with an amplitude of a few times 10^{-14}. These periodic variations are correlated with variations of tilt in the floor in which the SCSO dewar is mounted.

APPLICATIONS

Although all of the uses of superconducting cavities result from their low electromagnetic losses, substantially different advantages can be gained from this one property. There are applications where the superconductivity serves primarily to reduce the large amounts of power which would be dissipated in conventional devices; these applications are not discussed here. A second group uses the very high Q to realize exceptional filter characteristics such as narrow bandwidth and high ratio impedance transformation. Low loss also means that practically any object placed in a superconducting cavity will dominate its performance; thus many materials properties may be accurately transduced to a frequency with high resolution and low noise. As a result of their state-of-the-art stability, superconducting oscillators are beginning to find a wide variety of applications either directly as clocks, as components of oscillator systems, or in a variety of physics experiments involving clocks and time. Finally, there are some special applications which are rather unique and are discussed separately from the others.

RF Superconductivity. One of the first applications of superconducting resonators was to provide an RF power source of adequate stability to measure many of the properties of the superconducting resonator itself. In this application, a voltage-tunable microwave oscillator excites the superconducting resonator. In combination with a dc phase bridge, the superconducting resonator produces a frequency discriminator voltage which locks the voltage-tunable oscillator to the frequency of the superconducting resonator. This stabilization technique, which is also applied to superconducting accelerators and superconducting particle separators, avoids the need to provide rather expensive and sometimes cumbersome frequency synthesis of the RF power source from a high-quality quartz crystal oscillator.

Filters. Perhaps the most obvious application of high Q superconducting cavities is as narrow band filters. Below approximately 100 MHz, lumped element resonators are preferred, but at higher frequencies various types of cavities are used: helical structures below 500 MHz; quarter-wave reentrant structures between 500 MHz and a few GHz; low-order mode structures from a few GHz to 10 GHz or more; and Fabry-Perot resonators at higher frequencies. In addition to the obvious advantage of very narrow bandwidth, for example, 0.1 Hz at 10 GHz, cavities have very pure resonance modes. The geometry can often be chosen so that spurious modes are separated by an octave or more. In contrast, quartz crystal filters have spurious modes usually spaced only 0.1 percent from the desired mode.

Superconducting cavities can be mechanically tuned over a wide range, but the instability introduced by the tuning mechanism counteracts the most important property of the resonators. Very fine, stable tuning can be achieved by controlling the temperature of the resonator--the total fractional tunability is on the order of 10^{-10}. A second method which yields a tuning range of 10^{-4} is optical tuning by the photodielectric effect. This has been realized by placing a high resistivity semiconductor wafer in the gap of a quarter-wave reentrant cavity where the RF electric field is very high[40]. When a light beam is directed on the semiconductor its dielectric constant changes, thereby producing a large shift in the cavity resonant frequency. Step changes in the frequency can be made in less than 10 ms but the presence of the semiconductor degrades the band-width of the filter: a Q of 10^6 at 1 GHz has been achieved for such a device.

The narrowband, tunable filters which can be realized with superconducting cavity filters may be applicable to some communications or radar receivers. However, it is more likely that they will be used in highly specialized devices. For example, a superconducting resonator may be used to obtain strong coupling between an evaporated Josephson junction and an electromagnetic field. This is particularly valuable when the junction is used as an oscillator. If such a junction is not incorporated into a resonant structure, the large shunt capacitance shorts out high-frequency voltages. The theoretical maximum output power from a Josephson junction is $0.58\, I_o V$, where I_o is the critical current and V is the bias voltage.[41] Theoretically, a typical junction can output about 10^{-7} W at X band, but only about one percent of this is observed from waveguide-coupled evaporated junctions. In addition to performing the required impedence matching to the junction, the superconducting resonator also provides narrow band filtering of the output signal. Unfortunately, this property detracts from the major advantage of the Josephson junction oscillator--it has a tuning sensitivity 10^5 times greater than conventional oscillators.[42] In order to preserve this property, it will be necessary to use superconducting cavities with extremely agile frequency tuning.

Superconducting resonators may also be useful as high-ratio impedance transformers for use with superconducting antennas. This application also has the disadvantage of narrow bandwidth. Superconducting antennas are not considered advantageous today over conventional devices.[42]

Transducers. Superconducting cavities make attractive transducers for a variety of quantities because they introduce negligible perturbations and their output is usually in the form of a frequency which is easily and accurately measured. Since the frequency of a cavity resonator is approximately inversely proportional to one of its dimensions, it is very natural to use such a resonator to measure changes in that length. The resolution is limited by the frequency instability of the resonator; the best achieved stability of superconducting cavity oscillators corresponds to a random noise level of 3×10^{-16} cm. This performance can be improved in principle by designing the resonator so the controlling dimension is very small. It has been predicted that a quarter-wave reentrant cavity would permit the resolution of 10^{-17} cm for a one-second integration time.[43] However, it has not been demonstrated that the necessary Q can be obtained with such a resonator design. There is an interest today in superconducting cavity length transducers for the detection of gravity waves. In such an experiment, the cavity itself could be used as the antenna, but more likely, two cavities would be coupled to a traditional bar antenna in a way which would cancel a substantial fraction of the frequency noise in the exciting oscillator. By appropriate design, the resonator dimensions and therefore frequency can be made sensitive to extremely small changes in Earth's gravity. A gravimeter with a sensitivity of 10^{-10} g and a drift of 5×10^{-8} g/day has been reported.[44]

Substantial use has been made of superconducting cavities to measure a variety of properties of materials at low temperature. The technique is to place a sample within the resonator and to measure either the frequency or Q as a function of the parameters of interest. The advantage of this technique is that it does not require ohmic contacts and can be used with randomly shaped, powdered or liquid samples. Dielectric constants very near unity and loss tangents as small as 10^{-9} can be measured because of the high stability and very low losses of the superconducting cavity itself. Many semiconductor properties have been measured including relaxation time, lifetime, Fermi level, trap ionization energy, trap density, capture cross section, free carrier density and trap population.[45]

Some properties of liquid helium have also been studied this way. The thermal expansion has been measured with a sensitivity, in terms of fractional density change, of 4×10^{-9}.[46] This sensitivity is sufficient to yield quantitative data concerning the dispersion relation for thermal phonons in liquid helium. The damping of oscillations of the liquid He through a small orifice can be studied by using the frequency of a superconducting cavity to sense the level of liquid helium within it.[47] This data has been used to study the quantization of vorticity in superfluid helium.

Several interesting and useful devices can be implemented using superconducting cavities. A thermometer can be made for the temperature range from .25 to .6 K by filling a cavity with He^3 vapor in equilibrium with the bulk liquid.[46] Changes in density, which are reflected in frequency changes, are interpreted in terms of the temperature of the gas. The accuracy of such a thermometer is estimated to be 0.2 percent. A nuclear radiation detector can be made by placing a properly doped semiconductor crystal on the stub of a quarter-wave reentrant cavity.[48] Below 70 K, the charge carriers created by the absorption of radiation are trapped for very long times at sites in the forbidden band. As a result, the frequency of the cavity shifts proportionally to the total absorbed dose. A detector for low levels of light can be implemented in a similar way only in that case the frequency shift results from the photodielectric effect.

Oscillators and clocks. The excellent spectral purity and medium term stability (up to about one day) of superconducting oscillators have numerous applications. These are divided into three categories in the following discussion: oscillators used as components of instruments; oscillators used as clocks to provide timing functions for complex instrument systems; and oscillators used to perform experiments based directly on clock performance.

Oscillators with very good spectral purity (short-term stability) are important elements of many instruments and measurement techniques. One example is the use as the source oscillator for frequency multiplication from microwave to infrared or higher frequencies. The process of multiplication by an integer n increases the phase noise power by n^2. This creates a severe practical problem because once the integrated white phase noise becomes comparable to 1 rad, it is no longer possible to identify the coherent signal component.[34] Traditional multiplication schemes require the use of several intermediate steps with independent oscillators at each step to reduce the phase noise to an acceptable level. Superconducting oscillators can be used in principle to accomplish high order multiplication in a single step. They have two important advantages in this application: they can operate at frequencies at least as high as 10 GHz and theoretically can produce a signal whose spectral purity is limited by the characteristics of the multiplier. For example, it has been predicted that a state-of-the-art commercial 5 MHz quartz crystal oscillator may be multiplied to .5 THz before the carrier is lost, but the same signal when filtered by a 10 GHz superconducting cavity with $Q = 10^{10}$ can be multiplied directly to 100 THz.

One of the authors (Stein) is testing a superconducting-cavity parametric oscillator at 9.2 GHz for use at the National Bureau of Standards.[27] Also Viet Nguyen at the University of Paris IX (Orsay) is constructing an SCSO for the purpose of multiplication. Both of these devices employ a TM_{010}-mode niobium resonator with two coupling ports. One of the two ports is used by the oscillator or stabilization circuit, and the second port is a transmission port for the useful output power. Both of these devices should have substantially reduced $S_{\phi}(f)$ for higher Fourier frequencies than that shown for the SCSO in Fig. 1. The improvement comes from the filtering character of the cavity at the transmission port.

Certain types of radar also depend critically on the spectral purity of their local oscillator. Return signals from nearby stationary clutter mix with the phase-noise sidebands and limit the signal-to-noise ratio of the true Doppler signal. For example, if a 1 GHz radar has target velocity detection down to 40 m/s and Doppler bandwidth of 10 KHz, then in order to achieve 80 dB sub-clutter visibility it is necessary to use a local oscillator whose phase noise is more than 120 dB below the carrier for Fourier frequencies greater than 200 Hz.[49]

A third application for short-term stable oscillators is as flywheel oscillators in atomic frequency standards. Since the time domain frequency stability, $\sigma_y(\tau)$, cannot improve faster than $1/\tau$, the medium term performance of the standard is limited by the flywheel oscillator if its stability is worse than that of the atomic frequency discriminator at the attack time of the feedback loop. Quartz crystal oscillators do not degrade the performance of current atomic standards, but if expected improvements in these standards are made, then improved flywheel oscillators will be needed and superconducting oscillators are a possible candidate. Flywheel oscillators are also used for autotuning hydrogen masers, a process whereby cavity-pulling and spin exchange frequency shifts are simultaneously reduced.[50] Present autotuning systems utilize a pair of masers, one of which could be replaced by a superconducting oscillator, thereby realizing a significant cost reduction.[51]

Superconducting oscillators can also be used to provide time for complex instrumentation and measurement systems such as radio astronomy and radar ranging. The desirability of superconducting oscillators for these applications range from cost savings to the potential for significant improvements in performance.

Very long baseline interferometry (VLBI) using independent clocks may have the following clock performance requirements for certain types of experiments: initial 1 μs synchronization of the start of recording; total time error of less than 1 ns over a five-hour observation period to insure that all the data have the same initial offset error; and sufficient coherence to guarantee that the recorded signals can be cross-correlated.[32] For a 10 GHz system and an observation time of one hour, the coherence requirement is met by an oscillator with a noise floor $\sigma_y = 5 \times 10^{-15}$. The required noise floor decreases inversely with both the operating frequency and the observation time. The requirements are approximately met by both hydrogen frequency standards (masers and passive devices) and superconducting oscillators. The masers are very expensive (~\$250 K) whereas the other two devices appear to cost only about one third as much and may have the possibility of extending the frequency for some types of VLBI to the region of 100 GHz. One of the authors (Turneaure) has analyzed the phase coherence of two standard SCSOs at 8.6 GHz for application to VLBI experiments with 30-and 60-minute cycling times.[39] For 30 min., the rms phase error is 31 m rad; and for 60 min, it is 87 m rad. Turneaure at Stanford is currently constructing an SCSO for VLBI at Owens Valley Radio Observatory.

Various types of navigation systems need state-of-the-art clocks. Since both the navigation requirements and the techniques are somewhat flexible, it is difficult to place fixed requirements on clock performance. Typical performance goals are discussed here for two navigation systems.

The NASA Deep Space Net (DSN) utilizes a network of radar stations to track spacecraft which have left earth orbit. The current capability of the system is approximately 5 m range resolution and 1 μrad angular resolution. Planners foresee the need for approximately an order of magnitude improvement in resolution for some missions which will be flown in the early 1980s such as Jupiter Orbiter (JOP).[53] The clock stability requirements depend on the method of range measurement. One approach which has been suggested is to replace most of the coherent (two way) Doppler ranging with non-coherent (one way) Doppler measurements. The latter technique, also called wideband VLBI, has the advantage that it substantially reduces the tracking time for accuracy comparable to current coherent tracking methods. However, it places the most stringent requirements on clock performance. When daily calibrations are used, the frequency must be constant to 1.5×10^{-14} over one day. Weekly calibrations are preferable to reduce cost and operating time but the frequency stability requirement becomes 3.2×10^{-16} over one day.[53]

An experiment has been proposed to use the DSN to detect gravitational waves. The passage of a gravitational wave pulse past the earth and spacecraft produces an identifiable signature in the range information. This experiment places the strictest stability requirements on the frequency standards. To do a feasibility experiment the frequency stability must be 3×10^{-16} for the duration of the experiment, 40 s to 4000 s, which can be achieved with a state-of-the-art superconducting oscillator system. A desirable experiment would require frequency stability of 3×10^{-18}.[54] It is probable that a superconducting oscillator using a cavity with $Q = 10^{11}$ would eventually be capable of reaching this performance level.

The Global Positioning System (GPS) is a multisatellite system intended to provide earthbound navigation with a precision of about 10 m. Each satellite carries high stability clocks which need to keep time to 10 ns over 10 days. Several hydrogen frequency standards are being developed for this purpose. Depending on future system requirements, superconducting oscillators could become desirable for this application.

There are several fundamental physics experiments which are based directly on superconducting oscillator performance. Two of these, a red shift experiment and a fundamental constants experiment, are performed by comparing the frequency of a superconducting oscillator to the frequency of an atomic standard based on a hyperfine transition such as cesium or hydrogen. In the red shift experiment one looks for a term in the frequency ratio that has a period of one solar day. Because of the Earth's rotation in the Sun's gravitational field, various theories of gravity predict

$$\frac{\nu_{hyperfine}}{\nu_{superconducting}} = A[1 + 10^{-12}(\Gamma_o - \tfrac{1}{2}T_1) \cos (2\pi\tau/1 \text{ solar day})] \quad (8)$$

where Γ_o and T_1 are zero for any metric theory of gravity such as the general theory of relativity.[54] With available standards it would in principle be possible to set an upper limit of 10^{-3} on $\Gamma_o-\frac{1}{2}T_1$. The Eotvos experiment already places a limit of 10^{-8} on Γ_o and may place a limit on T_1.

If instead of analyzing the data for diurnal effects, they are fit to a linear drift model, then the same experiment may be analyzed to yield an upper limit on the time rate of change of the fine structure constant. The ratio of the frequencies of a hyperfine standard to a superconducting oscillator is

$$\frac{\nu_{hyperfine}}{\nu_{superconducting}} = Bg(\frac{m}{M})\alpha^3 \quad (9)$$

where m is the mass of the electron, M is the mass of the nucleus, and g is the gyromagnetic ratio of the nucleus; B is a constant. By comparing a superconducting oscillator to a cesium standard for 12 days it has been determined that $(1/\alpha)(d\alpha/dt)$ is less than 4×10^{-12}/ year with 68 percent probability.[20] The quality of this experiment was determined by the data link connecting the laboratories where the standards were located. Significant improvements may result from a comparison of superconducting oscillators and hydrogen standards in the same laboratory. Although astronomical and geophysical measurements have set a tighter upper limit, they have the disadvantage of averaging possible changes over periods on the order of 10 percent of the age of the universe.

A laboratory experiment has been proposed to use a superconducting resonator excited by a superconducting oscillator to measure the Lense-Thirring effect--the dragging of inertial frames by a rotating mass. A toroidal superconducting waveguide is centered on the rotation axis of an axisymmetric object. The wave travelling in the same direction as the rotating body takes less time to complete one round trip than the counterrotating wave. As a result, the interference pattern rotates around the waveguide. For a 5000 Kg mass with 50 cm radius rotating at an angular velocity of 2×10^3 rad/s, the angular velocity of drag has been estimated to be 2×10^{-20} rad/s.[56] The required waveguide Q is 5×10^{16} and the stability of the exciting oscillator must be 1×10^{-17}. This stability is probably achievable, but it is impossible to say if such high Qs can be achieved. There are many other problems at least as difficult as these, making this experiment extremely problematic.

<u>Special Devices</u>. Occasionally superconducting cavities can be used to solve some unusual problems. One example of this is the

reduction of the "cavity phase shift" problem in cesium beam frequency standards. At the present time, the most significant limitation in the accuracy of cesium beam frequency standards is the extent to which the phase difference of the RF fields in the two interaction regions can either be nulled or measured. If superconducting cavities were used, the variations of the microwave phase across the apertures of the cavities would become very small and it would be possible to measure the intercavity phase shift with high precision. This approach is not being tried at the present time because of the difficulty of using cryogenic cavities in an atomic beam device and because there is another promising technique with fewer difficulties.

Another novel suggestion is a superconducting cavity gyroscope. One possible configuration is the toroidal microwave cavity which was described earlier in the discussion of the Lense-Thirring effect. The position of the nodes in such a cavity experiences a phase shift proportional to the rotation speed of the cavity and the frequency of the exciting radiation. This is just the Sagnac effect which is used today in laser gyroscopes. Because of the difficulty in optical detection of fringes, laser gyroscopes are constructed today in the form of ring oscillators. These devices must be biased in order to overcome a dead zone at low rotation rates; systematic errors introduced by the bias degrade the performance of the gyroscope. The superconducting version of such a gyroscope is less sensitive by a factor of 10^4 to 10^5 because of the wavelength difference. However, this is offset by the fact that it is possible to detect a much smaller fraction of a fringe at 10 GHz than at visible frequencies. Consequently, it may be possible to construct a Sagnac-type gyroscope (using superconducting cavities) which operates in the interferometric mode with no dead zone and is competitive in sensitivity with the best mechanical gyroscopes.

CONCLUSIONS

There are desirable applications for superconducting cavities with Qs up to 10^{17} and for superconducting oscillators with stabilities as good as 3×10^{-18}. The best Q achieved to date is 5×10^{11} and the best stability is 3×10^{-16}. Since it is generally possible to find the center of a resonance line to one part per million, it is reasonable to expect a stability of 2×10^{-18} will be achieved using present technology within ten years. On the other hand, Q improvement to the 10^{17} level will require major advances in superconducting technology. The likelihood of it happening within the next ten years, if ever, cannot be reasonably assessed.

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