Bragg Diffraction from Crystallized Ion Plasmas

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Plasmas, the ionized states of matter, are usually hot and gaseous. However, a sufficiently cold or dense plasma can be liquid or solid. A one-component plasma (OCP) consists of a single charged species embedded in a uniform, neutralizing background charge (1). Aside from its intrinsic interest as a simple model of matter, the OCP may be a good model for some dense astrophysical plasmas (2), such as the crusts of neutron stars or the interiors of white dwarfs, where the nuclei are embedded in a degenerate electron gas. According to calculations, a classical, infinite OCP freezes into a bcc lattice when the Coulomb coupling parameter

\[ \Gamma = \frac{e^2}{4\pi\epsilon_0 a W S k_B T} \]  

is approximately equal to 170 (3). Here, \( \epsilon_0 \) is the permittivity of the vacuum, \( e \) is the charge of an ion, \( k_B \) is Boltzmann's con-

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stant, $T$ is the temperature, and $a_{\text{W-S}}$ is the Wigner-Seitz radius, defined by $4\pi a_{\text{W-S}}^3/3 = 1/n_0$, where $n_0$ is the particle density; $\Gamma$ is the ratio of the Coulomb potential energy of neighboring ions to the kinetic energy per ion.

Ion plasmas can be confined and brought to thermal equilibrium in Penning traps. Such systems have static thermal equilibrium properties equivalent to those of an OCP, where the magnetic field takes the place of the background charge (4–6). Calculations (7) and experiments (8) for approximately spherical plasmas having $N \approx 10^3$ to $10^4$ ions show concentric shell structures, dominated by surface effects. Calculations by Dubin and O’Neil (9, 10) suggest that a bcc lattice might begin to form in the center when the number of concentric shells is greater than about 30, which corresponds, for a spherical plasma, to $N \approx 10^3$. Ordered structures of tens of thousands of ions have been observed in a radio-frequency (rf) quadrupole storage ring (11) and in a linear rf trap (12) but, because of the elongated shapes of these structures, surface effects dominated and bulk structure was not observed.

Tan et al. have reported Bragg diffraction patterns from laser-cooled ions in a Penning trap (13). For approximately spherical plasmas with 200,000 ions or more, the patterns were consistent with bcc ordering but not with face-centered cubic (fcc) ordering. However, the Bragg patterns were smeared into circles by the rotation of the plasma about the magnetic field axis, so it was not possible to distinguish between scattering by a single crystal and scattering by several crystals or to determine the orientation of the crystals. Here we report the observation of time-resolved (stroboscopic) Bragg diffraction patterns, from which the effect of the plasma rotation is removed (14).

In our experiment (Fig. 1), the $^9\text{Be}^+$ ions were confined in a cylindrical Penning trap, consisting of an electrostatic quadrupolar potential and a uniform magnetic field $B = 4.465$ T, parallel to the $z$ axis. The radial electric field leads to a rotation, at frequency $\omega_0$, of the plasma about the $z$ axis. For a given $N$, an equilibrium state of the plasma can be parameterized by $T$ and $\omega_0$ (4–6). In the limit of low $T$, approached in our experiments, the plasmas are uniform-density spheroids. For $N = 10^6$, a spherical plasma at a typical density of $4 \times 10^8$ cm$^{-3}$ has a diameter of 1.7 mm.

The ions are cooled by a laser beam propagating along the $z$ axis and tuned slightly lower in frequency than a hyperfine-Zeeman component of the $2S - 2P$ to $2p - 2P$ resonance at 313 nm. The laser power is approximately 50 $\mu$W and is focused at the ion plasma to a diameter of about 0.5 mm. We estimate that $T \approx 10$ mK (15, 16). For a typical value of $n_0 = 4 \times 10^8$ cm$^{-3}$, this results in $\Gamma \approx 200$. A series of lenses forms an image of the diffraction pattern on an imaging photodetector.

We used two methods to derive a timing signal for stroboscopic detection of the Bragg diffraction patterns. The first (passive method) is based on detecting a photon from a diffracted beam after it has passed through an aperture (Fig. 1). The second (active method) is based on phase-locking the rotation of the plasma to an applied rotating electric field (17, 18).

Two types of imaging detectors were used. One (the MCP-RA detector) is an imaging photomultiplier tube (PMT) based on a microchannel-plate (MCP) electron multiplier and a multielectrode resistive anode (RA) for position sensing. For each photon, the position coordinates are derived from the current pulses collected from the different parts of the RA. The other is a charge-coupled device (CCD) camera coupled to an electronically gateable image intensifier.

Time-integrated diffraction patterns were obtained with both the MCP-RA detector and the CCD camera. Before attempting to observe crystal diffraction patterns, we tuned the frequency of the laser beam from several gigahertz to $\sim 10$ MHz below resonance, causing $T$ to vary from above to below the liquid-solid transition temperature. The duration of the frequency sweep was about 10 to 30 s. About 30% of the time, we observed a pattern consisting of several sharp rings, indicating that a crystal had been formed (13, 14). Figure 2, which is consistent with a bcc lattice rotating about a 100 (fourfold symmetry) axis (19), is an example of such a pattern.

In order to compare quantitatively the observed Bragg diffraction pattern to a calculated one, it is necessary to know $n_0$, which can be determined from $\omega_0$ [equation 10 of Bollinger et al. (6)]. In (13), $\omega_0$ and $n_0$ were determined from the aspect ratio $\alpha = z_0/T_0$, where $z_0$ and $Z_0$ are, respectively, the radial and axial diameters of the plasma [equation 16 of Bollinger et al. (6)]. The uncertainty in $\omega_0$ determined by fitting the side-view images is $\pm 5\%$. If there are discrete Bragg diffraction peaks, $\omega_0$ can be determined accurately (to about 0.1%) from time correlations between scattered photons (Fig. 1). A typical correlation spectrum is shown in figure 4(a) of Tan et al. (14). As reported in (13), 14 time-integrated

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Fig. 1. Experimental setup. Laser light is directed through the ion plasma in the Penning trap. A diffraction pattern is created at a plane beyond lens 2, where rays that are parallel leaving the plasma are focused to a point. A mirror, placed near that plane, deflects the light to an imaging photodetector. An aperture placed inside a hole in the mirror allows diffracted light to be detected by a photomultiplier tube (PMT). The aperture is placed off the axis of the optical system, so the PMT generates a timing signal as the diffraction pattern rotates.

Fig. 2. Time-integrated Bragg diffraction pattern obtained with the CCD camera. Rotation of the plasma causes the diffraction spots to be smeared into circles. The long rectangular shadow is due to the laser beam deflector. The small circular shadow is due to the hole in the mirror. The four linear shadows forming a large square are due to a wire mesh. Here, $\omega_0 = 2\pi \times 128$ kHz, $n_0 = 3.90 \times 10^8$ cm$^{-3}$, $N = 5 \times 10^5$, $\alpha = 1.00$, and $2r_0 = 1.35$ mm.
Bragg diffraction patterns were analyzed for an approximately spherical plasma having 270,000 ions. Patterns for a larger data set, in which \( \omega_i \) was determined by photon correlation, are shown in Fig. 3. The positions of the peaks agree with those calculated for a bcc lattice, to within the 2.5% uncertainty of the angular calibration. They disagree by about 10% with the values calculated for an fcc lattice. The ratio of the peak positions of the first five peaks agree to within about 1% with the calculated ratios for a bcc lattice. The scatter of the data is much reduced relative to that of figure 3 of Tan et al. (13), reflecting the more accurate \( \omega_i \) determination.

In principle, Fig. 3 provides information on the orientations of the crystals. If the crystals formed with random orientations, we would expect Fig. 3 to show a greater number of diffraction peaks at \( \{211\} \) Bragg reflections) than at \( \{220\} \) Bragg reflections), whereas it actually shows the reverse. This data set showed a preference for alignment of the crystals with a \((100)\) axis along the magnetic field direction. Preliminary observations indicate that the degree to which the magnetic field direction coincides with the symmetry axis of the trap electrodes influences the crystal orientations.

Tan et al. have noted (13) that not all of the diffraction rings allowed for various orientations of a bcc lattice were seen at any given time. This indicated that the portion of the plasma having bcc ordering included at most a few crystals rather than many randomly oriented crystallites. Figure 4 is an example of a time-resolved diffraction pattern obtained with the passive timing method and the CCD camera. In this case, the diffraction spots all line up on a square grid, consistent with a single bcc crystal oriented so that the incident laser beam is along a \((100)\) axis. For these data, an angular calibration was made with an uncertainty of less than 1% with a mask. The agreement between the observed and calculated grid spacing was \( \sim 1\% \).

In order for a diffracted beam to form, \( \mathbf{k}_i \) and \( \mathbf{k}_s \) must differ by a reciprocal lattice vector (Laue condition) (20). In a typical x-ray crystal diffraction case, satisfying the Laue condition for many spots requires that the incident radiation have a continuous range of wavelengths. Here, the Laue condition is relaxed because of the small size of the crystal, so a pattern is obtained even with monochromatic radiation. If the diameter of the region of the plasma having crystalline order is \( L \), the mismatch in reciprocal space can be about \( 2\pi/L \). The diameter of this plasma was \( \sim 1.36 \) mm. In Tan et al. (13), approximate lower limits for \( L \) of 150 \( \mu \)m and 240 \( \mu \)m were determined from the widths and intensities of the Bragg peaks, respectively. For this plasma, \( a_{wgs} = 8.5 \) \( \mu \)m, and the cubic lattice spacing is 17 \( \mu \)m. A cube 240 \( \mu \)m wide would be about 14 lattice spacings in diameter and would contain about 6000 ions.

We also observed patterns that were consistent with single bcc crystals nearly aligned along other directions, including \( \{111\} \), \( \{115\} \), \( \{012\} \), \( \{113\} \), \( \{110\} \), and \( \{013\} \). A pattern consistent with a single bcc crystal oriented along a \((115)\) direction is shown in Fig. 5. Some time-resolved patterns were observed that were not consistent with a single crystal but were consistent with two or more crystals having a fixed relative orientation.

With approximately spherical plasmas (\( \alpha \) between 0.6 and 1.4), different diffrac-
tion patterns were observed on different cooling cycles. With more oblate plasmas, the same pattern was observed each time. A very oblate plasma resembles the planar geometry considered by Dubin and O'Neil (9, 10), in which a stack of bcc (110) planes was predicted to have the lowest energy when there are about 60 or more planes. For some cases with fewer planes, a stack of fcc (111) planes has lower energy. In a time-resolved diffraction pattern from a plasma having \( \alpha = 0.38 \) (Fig. 6), the most intense diffraction spots form a rectangular array, consistent with a bcc lattice oriented along a (110) direction, that is, a stack of (110) planes. Weaker diffraction spots, forming a hexagon, are also seen. These appear at the lowest temperatures. The expected positions of the spots for the [220] Bragg reflections of an fcc lattice oriented along a (111) direction, that is, a stack of (111) planes, are at the vertices of the hexagon overlay. An ideal hexagonal close-packed lattice, oriented along the [001] direction, would generate the same hexagonal spot pattern. However, it would also generate another hexagonal spot pattern at a smaller radius, which is not observed.

Simulations of ion plasmas show hexagonal patterns resembling fcc (111) planes on the layers nearest the surface (7). The hexagonal diffraction pattern in Fig. 6 could be the result of scattering from surface layers, and the rectangular pattern could result from scattering from the central region. Some spots in Fig. 6 do not match either the rectangular grid or a hexagonal lattice. They may be due to scattering from a transition region that is neither bcc nor fcc. Further examination of oblate plasmas with different thicknesses may enable the transition from surface-dominated structure to bulk behavior in a finite, strongly coupled OCP to be studied.

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Changes in the West Antarctic Ice Sheet Since 1963 from Declassified Satellite Photography

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Comparison of declassified satellite photography taken in 1963 with more recent satellite imagery reveals that large changes have occurred in the region where an active ice stream enters the Ross Ice Shelf. Ice stream B has widened by 4 kilometers, at a rate much faster than suggested by models, and has decreased in speed by 50 percent. The ice ridge between ice streams B and C has eroded 14 kilometers. These changes, along with changes in the crevassing around Cravy Ice Rise, imply that this region’s velocity field shifted during this century.

One of the major uncertainties in the Intergovernmental Panel on Climate Change’s projection of future sea level is the uncertain behavior of the West Antarctic Ice Sheet (1). It was much larger during the last glacial maximum 20,000 years ago, and its retreat rate then has been rapid at times (2). Current behavior does not indicate that it is now retreating rapidly, but areas of rapid change have been discovered and the potential for unstable behavior remains under study.

The thick West Antarctic Ice Sheet is grounded on a submarine bed contained in an extensive rift basin coated with thick marine sediments and is subject to high geothermal heat flow (3). Discharge of West Antarctic ice is dominated by rapidly moving ice streams. These ice streams feed floating ice shelves; the transition from grounded to floating ice occurs at the “ grounding line.” Occasionally ice shelves ground, forming ice rises that the ice shelf must flow around. Between ice streams, the ice accumulates to form higher elevations that slowly flow laterally into the ice streams across heavily crevassed shear margins.

Declassified intelligence satellite photography (DISP) recently made available affords a direct view of the ice sheet’s configuration in the early 1960s, greatly extending the limited surface observations made during the International Geophysical Year in 1958 to 1959. Here, we analyzed changes in the mouth of ice stream B from the downstream tip of ridge B/C (between streams B and C) to the area just downstream of Cravy Ice Rise (Fig. 1) (4).

The DISP data were collected on 29 and 31 October 1963 (5). The DISP frames we used were 4 inch by 5 inch (10.16 cm by 12.7 cm) contact negatives, which we scanned at 600 dots per inch to convert them to digital form. They were collected by the cartographic camera onboard a Corona mission satellite and have a ground spatial resolution of about 150 m. Our second data set is a mosaic of two images from the advanced very high resolution radiometer (AVHRR) collected on 12 November 1980 and 8 December 1992. These images were obtained from the U.S. Geological Survey (USGS) World Wide Web site as part of an Antarctic mosaic and have a

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