

A COORDINATE FREQUENCY AND TIME SYSTEM

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SUMMARY

A coordinate frequency and time system, suitable for extension to worldwide coverage, is described in relation to the form evolving in the United States. It consists of a network of component primary and associated stations, at fixed locations and altitudes, and a reference coordinating component at a reference location and altitude.

Each primary station is a source of coordinate frequency and time signal emissions steered directly in rate and epoch by a steering element; for coordination purposes this element is offset slightly in rate from an associated independently running "proper" local atomic standard. The local independent atomic standards are each evaluated for internal accuracy of realization of the standard Cesium frequency value assigned in the International System (SI) of Units, and for reliability and stability of operation.

A unified local atomic standard for the system serves as the coordinating component. It is defined in terms of a weighted average of the independent local atomic standard frequencies whose weights are chosen on the basis of reliability, stability, internal accuracy, and independence. It is a "proper paper standard" regarded as normally located at the system origin, at a standard reference altitude. In its definition, the frequency of the unified atomic standard is taken equal to the value $f(\text{Cs})$.

When completely coordinated, the carrier frequencies and signal epochs received at the origin from a participating station have average values equal to the assigned coordinate values when measured by the unified atomic standard. But if the measurement were made by the unified standard at the emission site, this would not be the case, in general, because of the small Pound-Rebka carrier frequency shift. Moreover, referred to emissions from a coordinated station A, there is no fractional difference in average rate and frequency of signals received at A, if they are emitted from another coordinated station, B.

An analysis of this network leads to the necessary conditions on the average frequencies emitted from the participating stations when coordinated, and to the frequency values of the independent local atomic standards referred to the unified one.

The system could be extended internationally, by regarding the national unified standards as components of an international one whose assigned frequency would equal $f(\text{Cs})$. Again, a physical link must be established between the components in real time. This leads to the recognition of small individual frequency offsets of emissions and of national unified standards needed to achieve a well-defined international coordinate frequency and time system.

I. INTRODUCTION

A. General Consideration

There are three aspects of a time and frequency information dissemination system which require three quite distinct types of activity by associated personnel.

1) The system must be administered. This means that its effects on the users served by it must be continually evaluated to see if their needs are met. Policy decisions must be taken with an eye toward future developments, as well as toward safeguarding of commonly accepted practice, and technical feasibility.

2) It must be operated and maintained consistently and efficiently. By its nature all component portions of the system must be coordinated continually to insure internal consistency. Internal records of the regular operations, adjustments, changes in system, and data collected by monitoring activity must be kept in a uniform way and be available to system administrative, operating, and engineering staff.

3) The technical characteristics of the system, including its limitations, must be investigated and continually reexamined in the light of technological advances and the requirements of the system users and potential users. There must be a clear understanding of the existing system physically and from an information theory point of view. This understanding must be both on a conceptual and operational level and be both deep enough and broad enough to yield a proper technical perspective concerning the relation of the system to others, existing or proposed.

These three requirements account for the sextuple authorship of this short paper. The authors can be identified roughly, but not exclusively, in pairs with these aspects. They are paired because the system proposed herein stems from experience with and observation of the joint effort made between two major institutions of the many government agencies interested in establishing a single coordinate frequency and time system for the United States. In making this coordination effort we have observed that it may be feasible to extend its use with small modifications to wider coverage, an aim explicitly stated in the "terms of reference" of Study Group VII of the CCIR. In passing, it should be noted that the terms "coordinated" and "coordinate" -- since they refer to an agreed upon method for referring time to a common reference, are almost synonymous

and may be used in the present text interchangeably (except for syntax).

There are, of course, many other individual contributions, including industrial concerns and government agencies, to this systematic effort in addition to the six authors and their respective two agencies. It perhaps suffices to cite in this respect the members of U. S. Study Group VII.

B. The USNO-NBS Coordination

As a guide in discussing the present coordination effort we list seven prerequisites descriptive of the kind of coordinate time system being set up and maintained (although not yet completely formalized!) in the United States for general use.¹

1) The system contains component member stations and laboratories.

2) Each member has similar identifiable items of equipment or portions of equipment systems, which we shall call elements of the local system component.

3) The component members of the system are evaluated and given statistical weights on the basis of certain criteria agreed upon by the members.

4) Adjustment procedures are formulated in the sense of designating at what portions of the system coordinating adjustments are to be made, and when they are to be made, and tolerance limits of various kinds are specified.

5) Maintenance procedures are formulated and followed in the sense of collection, recording, and reporting of data to be used in order to maintain the smooth operation of the system.

6) Provisions are made for the participation of associated member stations, whose atomic clocks are used at the station for control and stability, but are not maintained sufficiently independently to be included in the component weighting procedure of the system. Such stations are monitored and emit signals within the prescribed tolerance limits -- i. e. ,

they disseminate the coordinate time and frequency information in the way agreed upon, but do not have a commercial or government standards laboratory directly working with them as part of the system.

7) A reference location and initial epoch must be designated; a reference time scale (a "paper" clock) must be defined as an average (using the weights chosen in item 3) of the component independent atomic frequency and time standards. It should be noted that the system origin so specified should be in terms of a physical object--e. g. , a laboratory building, and a well-defined physical event. Moreover, the record of differences between the reference scale and other well-defined scales such as UT2 and ET must be kept continuously as a necessary part of the coordinate time system.²

Since the independent local atomic standards and clocks are usually averages of several physically distinct but similar atomically controlled or calibrated clocks, the name independent local mean standard will be used for them. (Sometimes we shall use the term: "local atomic standards" as an alternative one.) Similarly, the average time scale at the system spatial origin will be designated the national mean scale --or sometimes the "unified atomic standard."

We can retrace these points in terms of the present USNO-NBS coordinated effort for the United States.

1) Obviously, the two present components of the system are:

(1) the U. S. Naval Observatory in Washington, D. C. , in close association with the laboratories of NRL and commercial standards laboratories which furnish the USNO with information and assistance in evaluating their atomic Cesium standard oscillators and clocks.

(2) the National Bureau of Standards Laboratory at Boulder, Colorado.

2) The essential basic elements for each component have been identified and are illustrated schematically in Figure 1. They are:

(a) local atomic Cesium frequency standards and independent mean time scales. At the USNO Time Service Division, in Washington there is maintained a set of from ten to sixteen independent atomic clocks whose readings are recorded and averaged almost every day by a statistical weighting procedure. The average reading is the independent mean paper time scale IM (USNO). This scale is also known as A. 1. The rate of running of each clock in the set is controlled by the radiation frequency characteristic of the well-known energy transition of atomic Cesium. The statistical procedure chosen insures a high degree of internal stability, reliability, and independence of IM (USNO) from other physical systems. Although not a direct realization in the sense of standards laboratories of the base units of time interval and frequency adopted in the SI, this "proper" time scale yields a close approximation. It is an example of a local atomic standard for the coordinate time system described herein.

At the NBS Time and Frequency Division, in Boulder there is maintained a local Cesium atomic frequency standard known as NBS-III, used to calibrate the rates of running of a set of crystal-oscillator and atomic-oscillator controlled clocks. A statistically weighted average of those readings is used to compute the "paper" atomic time scale known as AT (NBS). The internal accuracy of the reference frequency standard, NBS-III, is continually evaluated to determine the confidence with which the standard realizes the SI base unit of time interval, the second (or its inverse, the hertz). The statistical procedure is chosen to ensure that the AT (NBS) scale realizes the internal accuracy of the local atomic frequency standard, at this moment, 5×10^{-12} (3σ).

Redundancy and reliability are furnished by the crystal and atomic clock system, while long-term stability, as well as accuracy, is computed from the calibrations relative to the NBS-frequency standard. This portion of the NBS system is an example of an independent local proper atomic standard for the coordinate time system, and may be designated IM (NBS). Its scale is identical with what has been known as NBS-A, but is now known as AT (NBS).

(b) coordinating elements whereby necessary corrections and adjustments for coordination are introduced into the local control equipment or perhaps are kept only as a record and guide for coordination. Some detailed methods for the calculation of these corrections are discussed in a later section. No standard name has yet been given to these internal system records; but each component certainly keeps such records. It is probable that they should be continuously available to both components.

(c) Steering elements can be identified at both component stations. They are the respective clocks and oscillators used to control or steer the carrier frequencies, signal pulse rates, and time epochs. One may designate them as coordinate clocks and frequency standards. They should be set physically to maintain the agreed coordination. At the USNO, the steering element is called the master clock, but might be designated for system purposes TC(USNO). Similarly, the NBS time scale or clock keeping coordinate time may be called TC(NBS). When these clocks and scales become coordinated internationally, they could be designated UTC(USNO) and UTC(NBS) respectively; the U denotes "universal", and the TC denotes "coordinated time".

(d) There are, of course, radio emissions of time signals and carrier frequencies closely associated in spatial location with each component. For NBS, this is WWV and WWVL, and for the USNO, this is NSS. There are other emissions closely associated with them, but, because they are not from nearby radio stations, we prefer to distinguish them as "associated stations".

(e) Finally, at each locale there are monitoring and maintenance facilities whereby data required for maintaining continuous coordination are collected and recorded.

3) The initial coordination between the USNO and NBS took place on 1 October 1968. It was necessary at that time to correct relatively large divergences in rate of the respective emissions amounting to about 8 parts in 10^{13} . Hence, it was decided, as an interim measure, without following the format discussed here, to shift the steering rates sufficiently over a period of time, to eliminate the major portion of this discrepancy.¹

This has now been accomplished. Accordingly, more refined adjustments for coordination, which will take into account small effects such as the Pound-Rebka gravitational red-shift,³ and relative random walks of the two local atomic standards, will be made in the future. In order to do this, it has been decided to attach equal weight to the two standards. In our notation, this means $\alpha_1 = \alpha_2 = 0.5$. Briefly, this decision resulted from a consideration of several incommensurate requirements at the two components. It is very important that the coordinate scale have nearly one "second" as the base unit of time at the coordinate time origin. This is the "second", as defined in the International System (SI) of units. It is part of the mission of NBS to attempt to realize, via the frequency standard NBS-III, this unit. At the same time, it is also important to keep the coordinate time scale as stable as possible, and insure its reliability, of prime consideration for the USNO.

4) Every few months, as a result of continuous monitoring via radio observations and portable clock intercomparisons, a new adjustment of the steering elements and the radio emissions must be made, to insure that the frequencies and rates of all system emissions, as observed at the USNO (i. e., near sea level), have the nominal values assigned them, measured relative to the unified mean standard of the system. Because of differences produced by propagation, and differences in rate between the local mean standards and the system's unified one, the rates as emitted from the stations and measured by the respective local standards will not have the nominal values but will be slightly offset. This is discussed more fully in the next section.

5) The USNO maintains a large portable clock service, and is charged by the DOD with assuring uniform standards of practice in frequency and time throughout the DOD. Coupled with the radio monitoring facilities both at NBS and the USNO, data are collected which result in a value for the fractional rate difference, S_{12} , between the USNO and NBS independent mean standards.

Based in part on this measurement, and on the weight value chosen, other quantities of importance for maintaining coordination are inferred, as described in the third section.

6) We mention briefly the stations and emissions which can be considered as associated ones. This means that NBS-station WWVH, in Hawaii, being at a different altitude, should emit signals at a rate very slightly different than the rate of WWV -- even when both rates were measured by ideally identical local standards at the two sites. The Loran-C chain and the forthcoming Omega system, as well as other U. S. Navy standard frequency and time signal stations may be considered as associated stations; ⁴ monitoring data yields differences between their emitted signal rates and those necessary to be correctly coordinated with this system. Broadcast stations, too, if considered as associated coordinate time and frequency stations need similar information.

7) The USNO continually make observations of star transits, and this information is used by the International Bureau of Time (BIH) in constructing the astronomical time scale known as UT2 (roughly the same as Greenwich Mean Time). The knowledge of UT2 and the coordinate time system emissions is a necessary prerequisite for maintaining the coordinate time scale and UT2 as different aspects of the same coordinate time system. Necessary redundancy and convenience in using it for navigational purposes is thereby built into the system.

We have designated the USNO as the spatial origin location of this national coordinate time system. The initial epoch chosen is, provisionally, set at 1 January 1958 at 0000 UT.

II. THE MEANING OF COORDINATE TIME AND METRIC TIME

Coordinate time systems as spatially extended physical objects have long been envisioned in physics, and employed in astronomy. The importance of giving an explicit operational definition of such a system has recently been stressed by several authors -- notably in a presidential address to the Royal Astronomical Society by D. H. Sadler. ^{5,6,7.}

It is our purpose here to explain the distinction between coordinate time and metric time. Very briefly, and roughly, it is analogous to the distinction between master-slave radio stations, and independently running oscillators.

More exactly, let us imagine many clocks, chosen for their stability and reliability, and because each, when calibrated by an atomic frequency standard at the same location, is determined to run freely at a rate very closely approximating that specified in the International System. The clocks can then be distributed over a large area and at different fixed altitudes, at fixed locations on the earth. Each then continues to run with its proper (Fr: propre \equiv self) rate. It is a local atomic standard, and can be used to measure rates and time intervals in terms of the SI base units for frequency and time. It is therefore called a "metric" instrument. This agrees with the strict mathematical sense of metric since we do not envision the possibility of changes in location or "small" accelerations as affecting its rate. It is a good piece of laboratory equipment for measuring time intervals.

Now, two such real clocks when in juxtaposition will normally diverge very slowly, but randomly, in reading and rate, because of tiny random effects which are not eliminated. A suitable average can produce an average reading and rate, which is presumably more stable --

and the stability properties of such a mean clock may be very desirable. However, one cannot necessarily say that the average clock is a better realization of the SI unit of time than either one of the two. This is certainly a matter for further investigation in each case. More important, however, is the fact that two such clocks at a distance would run quite independently, and at their own (proper) rates.

However, to obtain a time epoch and a frequency which is uniform over a large spatial volume, a uniform physical link with some standard clock must be established. It is a remarkable fact that radio signal pulses or carriers, used to establish such a link, and when measured by two standard metric clocks A and B, at a distance from each other, show very little difference in frequency or rate relative to these. This is illustrated in Figure 2. However, a small systematic effect has been observed in the frequency of electromagnetic radiation when it is propagated through a difference, gH , in gravitational potential, over an altitude difference H —even though the gravitational acceleration field, \vec{g} , is a constant. This difference is sometimes ascribed to a difference in rate of the metric clocks. But the argument from physical grounds shown in Figure 3, indicates that the radio waves, or photons, themselves change energy, and therefore frequency, by having work done on them as they move in a direction with a component parallel to the field vector \vec{g} . The computed value of this effect on radio carrier frequencies between Boulder, Colorado, and Washington, D. C. is 1.8 parts in 10^{13} or an accumulative effect over a year of $6 \mu s$. (phase difference in time units).

This can be described geometrically by attributing different radii of curvature to different portions of the spacetime map on which are plotted events happening to two clocks, A and C, at a distance and at different altitudes. In this way the relative difference between the received pulse rate at C and the metric clock rate at C can be portrayed.

This is shown in Figure 4.

So it is easy to see, as also shown on Figure 5, that one coordinates time at different locales and altitudes, by sending radio pulses from an origin (say at sea level) so that the coordinate clocks at various locales are "slaved" to the agreed time scale at the system origin. Naturally other propagation effects and random effects must be taken into account, by a suitable collection of data. But the main point made here is that coordinate times (and frequencies) must be carefully determined by establishing physical links with a suitable origin time scale. Moreover, it is by no means obvious that all such physical links will lead to the same coordination. Hence, it is essential to define the method used to establish this link very carefully, and to follow well-prescribed procedures in measuring (by metric clocks) and recording the "behavior" of the extended system.⁸

In this system, coordinate clocks slaved together by radio means are needed for synchronization and epoch interpolation. Independent, proper, metric clocks are needed for time interval and rate measurement and calibration. Radio dissemination is needed in order to define the time coordinate spacing operationally. A unified system mean clock at the origin forms, together with adequate records, a useful reference and transfer standard, if its rate is nearly that prescribed in the International System.

III. PRINCIPLES AND ANALYSIS OF THE SYSTEM

We have enunciated, at least by implication, four main principles necessary for the quantitative definition of the kind of coordinate time system emerging from the USNO-NBS coordination.

- 1) A national mean standard or unified atomic clock and frequency standard is defined by the weighting process at the system origin. Its fractional rate difference from the average of the set of local mean atomic standard rates is zero.
- 2) Natural astronomical events are related to the national mean clock readings via UT2.
- 3) No radio waves or pulses used to synchronize the system should "disappear" -- ideally there is conservation of phase so that in every closed circuit (including equipment) the net phase change is zero.

4) Disseminated coordinated frequencies and rates measured as received at the origin in terms of the national mean standard must have their assigned (i. e., nominal) coordinate values.

In view of these requirements we may write down certain relationships between measured, inferred, or assigned quantities and quantities needed for making adjustments in emissions in order to attain coordination. First, let us define a few symbols. Let

N = number of component stations and laboratories in the system.

$\alpha_1, \alpha_2, \dots, \alpha_N$: weights assigned to the respective components.

$S_{12}, S_{13}, \dots, S_{1N}$: measured or inferred fractional frequency deviations of the rate of the first local mean atomic standard (its metric element) from the other component local mean time and frequency standards, as if in juxtaposition.

F_1, F_2, \dots, F_N : fractional frequency deviations of the rate of the respective local mean standards from the national one.

E_1, E_2, \dots, E_N : fractional frequency deviations of steering elements from local mean standards--recorded in the coordinating elements and introduced into the steering elements to compensate for random and systematic differences of the component local standards and emissions from the national mean at the system origin.

G_1, G_2, \dots, G_N : fractional frequency deviations introduced by the gravitational Pound-Rebka shift into the radio time and frequency signals used for synchronization and dissemination as they travel to the system origin.

D_1, D_2, \dots, D_N : ideal fractional frequency deviations of the respective emitted signals from their respective local mean atomic standards in order to follow the presently defined UTC system.

It should be understood that the frequencies as actually emitted and received may not at any one time agree with the ideal values. The system records should show the ideal values, and the departures from them which would be regarded as errors in the coordinate time system.

Figure 6 is a schematic diagram of a national system with N components. On it, one may trace for each component, the part of the component circuit in which each of the quantities defined above appear.

The national mean standard is denoted by the letters UAS (unified atomic standard).

On the figure, certain quantitative relations are shown which we now derive.

$$\sum_{i=1}^N \alpha_i = 1 \quad (1)$$

By the first principle, we may also write

$$\sum_{i=1}^N \alpha_i F_i = 0 \quad (2)$$

The definition of the pairwise measured (or inferred) fractional frequency deviation of the first local standard from the others yields

$$\left. \begin{aligned} S_{12} &= F_1 - F_2 \\ S_{13} &= F_1 - F_3 \\ &\vdots \\ S_{1N} &= F_1 - F_N \end{aligned} \right\} \quad (3)$$

It is useful to note that $S_{ij} = F_i - F_j$ and therefore that $S_{1j} = S_{1i} + S_{ij}$, for i and j having values $1, 2, \dots, N$.

One immediately infers that

$$F_1 = \sum_{j=2}^N S_{1j} \alpha_j, \quad (4)$$

a relation which may be used in place of Equation (2).

The gravitational radiation shifts are easily evaluated from the differences in altitude of the component emission stations from the system reference altitude. Let these altitude differences be H_1, H_2, \dots, H_N , in kilometers.

Then

$$G_j = \frac{gH_j}{c^2} \quad (j=1, \dots, N) \quad (5)$$

where $g = 9.8 \text{ m/s}^2$

$$c = 3 \times 10^8 \text{ m/s.}$$

This effect must be included to help satisfy principle 4).

The present UTC-system necessitates the insertion of a constant frequency offset for all UTC-coordinated emissions from the nominal assigned values in order to partially synchronize the time signals with UT2. Thus we have, at present

$$D_j = E_j - 300 \times 10^{-10}. \quad (6)$$

There are discussions at present aimed at eliminating this large frequency offset, and attaining the very rational requirements of the navigators by a different, simpler, and improved means. When, and if, this is done, the distinction between D and E will vanish.

Until then, all UTC-carrier signals used for calibration purposes ought to be corrected on reception by this amount.

In any event, this frequency offset only furnishes part of the information required for a more accurate immediate knowledge of UT2, important to many users. Such information is, or can be, supplied by voice or simple code on the same emissions.

Principles 3) and 4) lead to the equations

$$E_j + F_j + G_j = 0, \quad (j=1, 2, \dots, N) \quad (7)$$

That is, for $j=1$, for example, the ideal fractional frequency deviation, E_1 , between the first local mean atomic standard and the emitted signal rates, added to the gravitational shift, G_1 , in the emitted radiation, ought to exactly compensate for the inferred random difference, F_1 , in rates of the first local mean standard and the national unified mean standard. The difference between E_1 and D_1 can be ignored, since it is both added and subtracted in making the first, or any, circuit in the system.

Now, if one is given all the α_j -values, and measures, or infers from observations, all the S_{1j} -values, then all the F_j -values can be determined from Equations 1, 3, and 4. The E_j -values are determined from (7), and the D_j -values from (6). One represents most of these relations easily on a frequency level diagram as in Figure 7.

Associated with each component station there are two levels: the level determined by the fractional frequency difference, S_{1j} , of the first station's local mean standard from the j^{th} local mean standard, and the level determined by the gravitational shift, G_j , determined from the altitude difference of the station from the system origin reference level.

Positive quantities are represented by vectors pointing up, negative ones by vectors pointing down. Hence, a station at a high altitude will have a gravitational level below the reference level, indicated by UAS in Figure 7, and conversely. Figure 8 also depicts the situation for the case $N=2$ as it existed for the USNO-NBS coordination. As explained, however, the adjustments made did not quite follow this procedure because of the large value of S_{12} . (The values of G_1 and G_2 were taken to be zero for this initial adjustment.)

IV. CONCLUSIONS

Let us summarize the foregoing discussions, briefly. In order to set up this coordinate time system, there seem to be nine essential procedural steps.

(1) The components must be identified-- both the associated and the member stations and laboratories.

(2) The essential system elements belonging to each component member must be identified: the local mean atomic standard, the coordinating element, the steering element, and the local coordinated radio station emitting specified time and frequency signals.

(3) The reference origin in space and time must be specified.

(4) Intercomparison measurements of the component local mean atomic time and frequency standard rates and epochs must be made, leading to the values of S_{1j} , ($j=2, \dots, N$).

(5) Statistical weights (α) must be chosen for each component.

(6) The national mean standard is then defined by determining the deviations (F) of all the local independent mean atomic standards from it.

(7) Astronomical events must be measured in terms of the unified national mean standard, utilizing UT2.

(8) The desired ideal emission rate offsets (E) from their respective local mean standards must be determined and the present UTC value D ($D = E - 300 \times 10^{-10}$) determined for UTC coordinated emissions.

(9) Continuous monitoring procedures must be set up to continually redetermine and publish -- for the system at least -- the weights and

- (a) F - values
- (b) E , D - values
- (c) Differences between actual and ideal emission rates and epochs.
- (d) Differences between actual and ideal reception rates and epochs.

If these procedures are permanently implemented, and the present USNO-NBS coordination appears to assure that this will be the case, the system can become a model for more extensive worldwide application, and for other national coordinate time systems. It already is a specific well-defined portion of what is often called the "National Measurement System".⁹

Some modifications may be needed for worldwide use. For example, the weighting process becomes a truly knotty problem, with political overtones. To insure proper statistical procedures, it is essential that all component members keep truly independent standards, and evaluate them exhaustively for reliability, stability, and accuracy. A certain amount of knowledge of how each of these national standards and their components (observatories, laboratories, and radio stations) are linked by physical means and coordination procedures must be available to the designated international coordinating agency. We are fortunate that there is already in existence a traditional agency, the International Bureau of Time (BIH), which receives all the essential astronomical information to construct the UT2 scale, and acts as coordinator for the present UTC system. In fact,

the BIH maintains an average atomic scale of time which satisfies some of the prerequisites to serve as an international mean time scale and standard (the scale known as A. 3). Questions concerning the weighting procedure, i. e., how to use the data furnished the BIH -- (both national mean scale data and individual component data) should be answered.^{10, 11, 12.}

Again, the CCIR has established an International Working Party charged for the moment with considering the improvement of the UTC system to meet modern requirements. So it would be natural for this group, IWP VII/1, to continue to act as an advisory body to the FAGS (steering committee for the BIH) and to the BIPM (the international bureau engaged in metric standardization activities). These are clearly interdisciplinary matters which are the concern of the sciences of astronomy, geophysics, radio engineering, physics, and politics. This is a matter for future consideration in the CCIR.¹³

ESSENTIAL ELEMENTS: COMPONENT MEMBER STATION

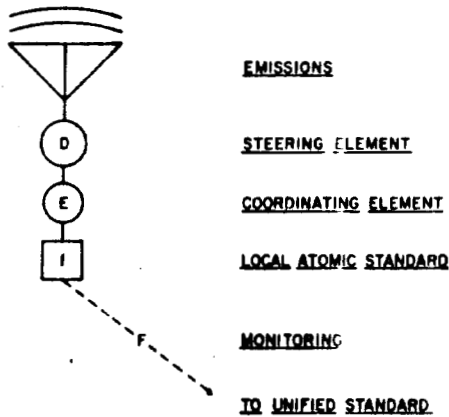
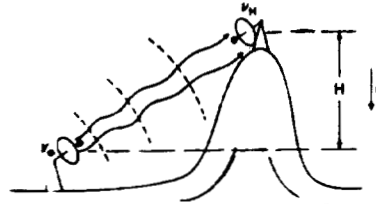


Figure 1: The symbols D, E, F refer to fractional frequency offsets explained in the text. The symbol "f" stands for the rate or frequency of the local mean clock or frequency standard--close to the prescribed SI-value.

POUND-REBKA EFFECT

PRINCIPLE: FOR PHOTONS AND RADIO WAVES, SELF-ENERGY IS:
 $E = h\nu + mc^2$



$$\frac{g}{c^2} = \frac{9.8 \text{ m/s}^2}{9 \times 10^{16} \text{ m}^2/\text{s}^2} = 1.09 \times 10^{-13} / \text{km (ALTITUDE)}$$

$$\approx 1.8 \times 10^{-13} \text{ mae (ALTITUDE)}; 6 \mu\text{s/yr/mile}$$

— SEND SIGNAL UP
 MEASURE (SI) FREQUENCIES

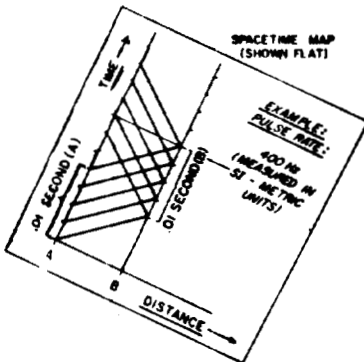
- ① AT SEA-LEVEL: $mc^2 = h\nu_0$
- AT ALTITUDE H: ENERGY = $h\nu_H$
- ② ALMOST CONSTANT FORCE DOWNWARD, ACTING OVER DISTANCE H, REMOVES ENERGY: $-mgH$
- ③: $h\nu_H - h\nu_0 = -mgH = -h\nu_0 \frac{g}{c^2} H$
- ④ P-R EFFECT:

$$\frac{\nu_H - \nu_0}{\nu_0} = -\frac{g}{c^2} H = -G_H$$

Figure 3: The Pound-Rebka, or gravitational frequency shift has been experimentally verified to one percent for gamma rays using the Mössbauer effect. It is a consequence of the general relativistic theory of gravitation.

DISTANT COMPARISONS OF RATES, FREQUENCIES, AND ACCUMULATED TIME INTERVALS

DISTANT COMPARISONS OF RATES, FREQUENCIES, AND ACCUMULATED TIME INTERVALS



METRIC VALUE OBTAINED FROM LOCAL ATOMIC STANDARDS

COORDINATE SPACING OBTAINED BY DISSEMINATION FROM AVERAGE (UNIFIED) STANDARD AT ORIGIN

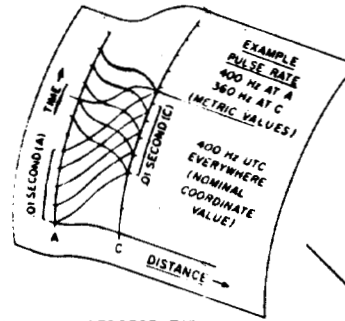
Figure 2: The schematic spacetime map shows radio pulses transmitted to B and returned to A to synchronize their clocks. A and B are separated by about 2.25×10^3 km (~1400 miles).

CASE 1: "A" AND "B" AT SAME ALTITUDE AND:

- (a) NO SYSTEMATIC EFFECTS (E.g. POUND-REBKA, CLOCK RATE DIFFERENCES)
- (b) NO RANDOM EFFECTS (E.g. PROPAGATION, CLOCK DISPERSION)
- ∴ COORDINATE SPACING = METRIC UNIT

NO DISTINCTION REQUIRED BETWEEN COORDINATE SPACING AND METRIC INTERVALS

SPACETIME MAP (SHOWN AS SURFACE OF REVOLUTION)



AGAIN CASE 2: "C" IS AT GREATER ALTITUDE THAN "A" AND:

- (a) ONLY SYSTEMATIC EFFECT SHOWN IS POUND-REBKA - EXAGGERATED !!
- (b) NO RANDOM EFFECTS SHOWN

METRIC INTERVAL ≠ COORDINATE SPACING EXCEPT AT "A", IF ORIGIN IS CHOSEN AT "A"

DISTINCTION REQUIRED BETWEEN NOMINAL COORDINATE VALUES AND MEASURED METRIC VALUES (SI)

A PHYSICAL REPRESENTATION - SHOWS WAVES AFFECTED BY WORK OF GRAVITY SHOWS CLOCK RATES UNAFFECTED BY GRAVITY POTENTIAL ∴ METRIC IS SHOWN UNDISTORTED; COORDINATE SPACING IS SHOWN DISTORTED.

Figure 4: A and C are again separated by about 2.25×10^3 km. Their coordinate clocks are synchronized by radio pulses, and the rates of these are measured by the metric clocks running at their own (proper) SI rates. The curvature of the schematic map is introduced to show that the intervals of the two metric clocks are equal at the two respective locations differing in altitude. Only altitude changes produce the P-R effect, although for pictorial purposes, horizontal distances as well are combined with differences in altitude.

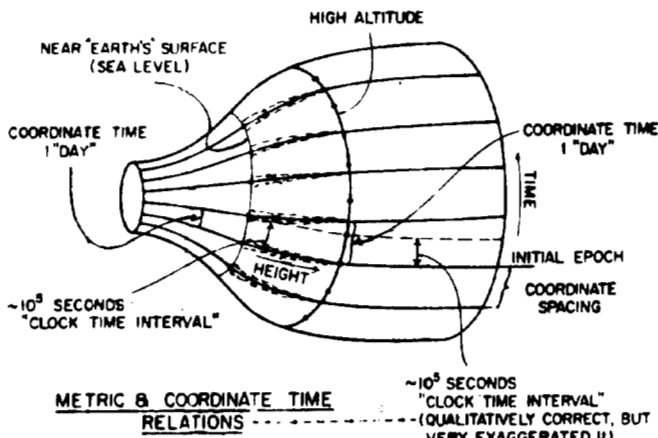
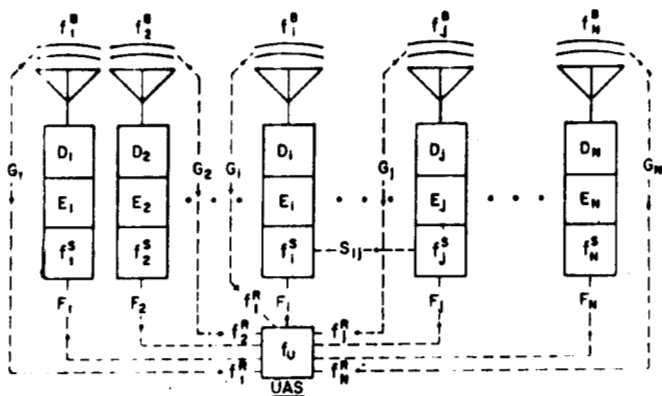


Figure 5: This is an even more exaggerated portrayal of the spacetime curvature needed to show the difference between coordinate intervals and metric intervals. The center of the "earth" is at the smallest cross section of the "bottleneck". The generators of the surface follow roughly the variation in gravitational potential with altitude. Unlike Figure 4, no horizontal distances are involved.

COORDINATE FREQUENCY AND TIME NETWORK AND MEASUREMENT SYSTEM



$$F_j = F_i - S_{ij} \quad \alpha_j = 1 \quad \alpha_j F_j = 0 \quad E_j + F_j + G_j = 0 \quad G_j = \frac{g}{c^2} H_j$$

Figure 6: The coordinate time system resembles an electrical network. Ideal values of emitted frequencies f_j^B , are indicated, as are ideal values f_j^R , of received frequencies, and the rates f_j^S of the standards, and f_u of the system unified mean standard. Not shown are the observed emitted frequencies f_j^E , and actually received frequencies f_j^A . The measured or inferred rate intercomparison of the i^{th} and j^{th} local standards is indicated by the fractional frequency difference S_{ij} .

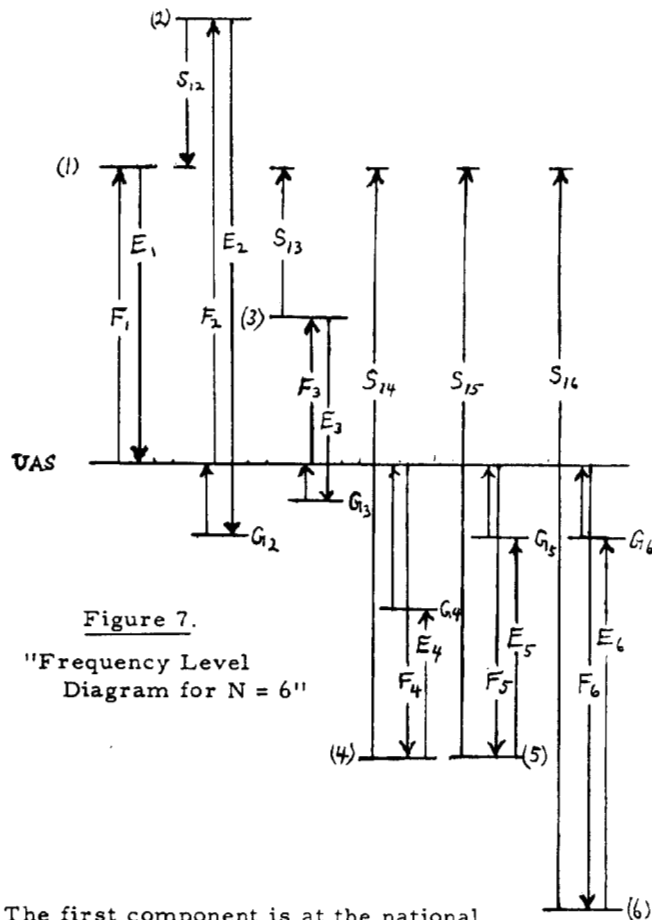
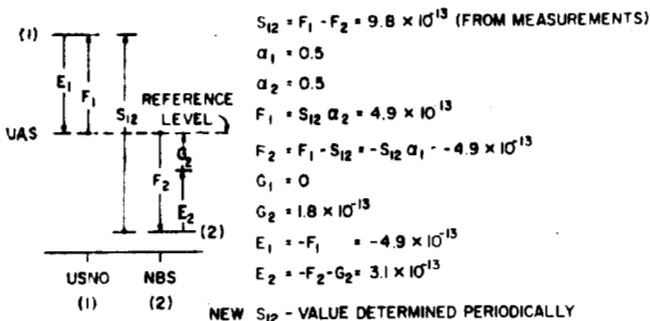


Figure 7.

"Frequency Level Diagram for N = 6"

The first component is at the national mean reference level, (UAS), so $G_1 = 0$. Positive quantities are associated with upward arrows. Weights are not shown.

FOR USNO-NBS COORDINATION AND SYSTEM (1 OCT., 1968) [N = 2]



$$\text{CHOOSE } \begin{cases} \alpha_1 = \alpha_2 \\ G_1 = 0 \text{ (~SEA-LEVEL)} \end{cases}$$

Figure 8: The values shown are of the correct orders of magnitude, but are for illustrative purposes only. Presently, the difference in rates of the local mean standards is considerably less, so that the importance of the gravitational shift is relatively greater.

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