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Lasing without Inversion: the Road to New Short-Wavelength Lasers

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Abstract—Lasing Without Inversion (LWI) is a novel technique in laser physics which holds the promise of a solution to one of the most fundamental problems in short-wavelength laser research. Making use of quantum coherence and interference effects, this technique allows lasing even if only a small fraction of population is in the excited state, i.e., even if the population of the active medium is not inverted. The intense theoretical work on the subject has been verified by several experiments which confirmed that LWI is indeed possible. The efforts of researchers are presently directed to demonstrate LWI in the UV and X-ray spectral regions.

1. INTRODUCTION

As a general rule, generation of coherent radiation becomes increasingly difficult as the wavelength of the radiation becomes shorter. A principal source of this difficulty is the problem one faces in creating a population inversion. Since the invention of the laser [1], the requirement of a population inversion has been considered to be fundamental to laser operation. Indeed, the emissive and the absorptive profiles of an optically active medium (e.g. atomic medium) are usually reciprocal: if the stimulated emission rate is given by

$$\text{rate of stimulated emission} = Bn\rho_{aa}, \quad (1)$$

then the stimulated absorption rate goes as

$$\text{rate of stimulated absorption} = Bn\rho_{bb}. \quad (2)$$

Here, B is the Einstein B-coefficient; n is the number of photons per unit volume in the lasing mode; and ρ_{aa} and ρ_{bb} are the populations of the upper and the lower lasing levels, respectively. It follows, therefore, that in order to obtain a net amplification, i.e., to have a stimulated emission rate faster than stimulated absorption, more atoms should be in the upper state a than in the lower state b.

The problem arises as the wavelength (frequency) of the $a \longrightarrow b$ transition becomes shorter (larger). In this case the spontaneous emission rate, which is given by the Einstein coefficient A grows as the cube of the transition frequency ω:

$$\text{probability of spontaneous emission} = A = B\frac{\hbar\omega^3}{\pi^2 c^3}. \quad (3)$$

Under these conditions, excited atoms undergo a rapid transition back to the ground state leaving the medium uninverted. Thus, fast spontaneous emission increases the difficulty of creating a population inversion and hence of realizing short-wavelength lasers.

This consideration was one of the main motivations in the development of a completely new approach to the problem, which is now widely referred to as Lasing Without Inversion (LWI). Theoretical work carried out at the end of 80s [2] pointed out that under certain conditions it is possible to achieve amplification and lasing even if the population of the active medium is not inverted. The key mechanism, which is common to all of the proposed schemes, is quantum coherence and interference in multilevel systems. In particular, it was shown that, if coherence is established between certain atomic states, different absorption processes may interfere destructively, leading to the reduction or even cancellation of stimulated absorption. At the same time, the stimulated emission probability may remain intact, leading to the possibility of gain even if only a tiny portion of the population is in the excited state.

Since that time, significant theoretical and experimental efforts have addressed the problem. Various schemes were proposed and detailed theoretical analyses were conducted. This work paved the way for several experiments which have shown inversionless amplification [3]. The logical culmination of these efforts was the demonstration of cw inversionless laser oscillation [4, 5].

Inversionless lasers have been shown to have some unique properties such as nonclassical photon statistics and subnatural narrow spectral features, as well as many interesting applications such as long-wave intrasubband semiconductor lasing [6]. However, the generation of short-wavelength radiation remains the most important potential application of LWI. One should note at this point that working at shorter wavelength represents a new regime for inversionless lasers. In particular, several effects which become extremely important for LWI with large frequency up-conversion have not been considered in the previous theoretical studies of LWI. Likewise, the proof-of-principle experimental demonstrations that have taken place so far with visible light will find considerable complications on the way to the UV and X-ray regime. The aim of this paper is to address some aspects of LWI in the short-wavelength region, analyze the complications that arise in this regime and point out possible solutions.

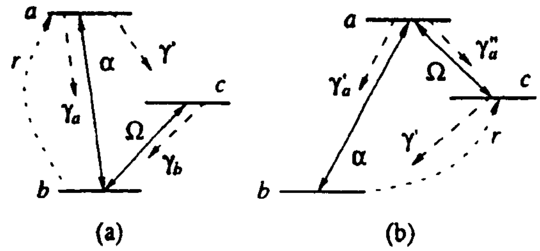

Fig. 1. Two frequently discussed schemes of LWI: (a) V-scheme; (b) Λ-scheme. α represents the lasing field and Ω represents the driving field. The possibility of incoherent pumping is indicated by *r*.

2. LASING WITHOUT INVERSION: THEORY AND EXPERIMENT

2.1. Theoretical Foundations

There exists a number of distinct approaches to LWI, differing in the way quantum coherence and interference are established. We focus here on the schemes where coherences are established by optical fields. Such schemes have been the most practical in the lab and are the only schemes in which LWI has been observed so far. Figure 1 shows two typical examples of such schemes: the so-called V- and Λ-schemes. In both of these schemes, atoms are prepared by a coherent driving field resonant with optically allowed transitions ($b \longrightarrow c$ for the V-scheme and $a \longrightarrow c$ for the Λ-scheme). The absorption of the probe (lasing) field resonant with the $a \longrightarrow b$ transition (both cases) is then strongly suppressed. This suppression allows amplification and lasing even if the atomic population is not inverted.

We describe such systems theoretically by an atomic density matrix interacting with classical fields. First we consider the V-scheme. The evolution of the off-diagonal density matrix elements (ρ_{ij}) is described by the following set of equations [8]:

$$\dot{\rho}_{ab} = -\Gamma_{ab}\rho_{ab} - i\alpha(\rho_{aa} - \rho_{bb}) - i\Omega\rho_{ac}, \tag{4}$$

$$\dot{\rho}_{cb} = -\Gamma_{cb}\rho_{cb} - i\Omega(\rho_{cc} - \rho_{bb}) - i\alpha\rho_{ca}, \tag{5}$$

$$\dot{\rho}_{ac} = -\Gamma_{ac}\rho_{ac} + i\alpha\rho_{bc} - i\Omega^*\rho_{ab}, \tag{6}$$

where α and Ω are the Rabi frequencies of the probe and driving fields respectively, and Γ_{ij} are the complex relaxation rates which include detunings, finite linewidths etc. We note that the above equations do not completely describe the dynamics of the system under consideration. To obtain a complete description we also need evolution equations for the populations of the levels. The form of these equations will depend on the particular choice of incoherent pumping mechanism. In order to obtain LWI, some form of such pumping is needed to populate the upper level, although a total inversion is not required. Nonetheless, we can make some general conclusions without considering the equations for the diagonal density matrix elements.

We solve for ρ_{ab} and find the gain coefficient as a function of the populations. In a weak probe field limit, and under conditions of exact resonance (when the detunings of the driving and probe fields from their corresponding resonances are zero), we obtain the gain per unit length:

$$\begin{aligned} \mathrm{V gain} &= \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\mathrm{Im}\frac{\rho_{ab}}{\alpha} \\ &= \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\frac{(\rho^0_{aa} - \rho^0_{bb}) + |\Omega|^2/(\gamma_{ac}\gamma_{bc})(\rho^0_{bb} - \rho^0_{cc})}{\gamma_{ab} + |\Omega|^2/\gamma_{ac}}, \end{aligned} \tag{7}$$

where N is the density of atoms, λ_l is the wavelength of the lasing field, $\gamma_{i\to j}$ is the decay rate on the $i \longrightarrow j$ transition, and ρ^0_{ii} is the population of level i calculated to the zeroth order in the probe field.

A similar analysis can be done for the Λ-scheme. In this case, the equations of motion are

$$\dot{\rho}_{ab} = -\Gamma_{ab}\rho_{ab} - i\alpha(\rho_{aa} - \rho_{bb}) + i\Omega\rho_{cb}, \tag{8}$$

$$\dot{\rho}_{ac} = -\Gamma_{ac}\rho_{ac} + i\Omega(\rho_{cc} - \rho_{aa}) + i\alpha\rho_{bc}, \tag{9}$$

$$\dot{\rho}_{cb} = -\Gamma_{cb}\rho_{cb} - i\alpha\rho_{ca} + i\Omega^*\rho_{ab}, \tag{10}$$

resulting in a linear gain coefficient:

$$\begin{aligned} \Lambda\mathrm{gain} &= \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\mathrm{Im}\frac{\rho_{ab}}{\alpha} \\ &= \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\frac{(\rho^0_{aa} - \rho^0_{bb}) + |\Omega|^2/(\gamma_{ac}\gamma_{bc})(\rho^0_{cc} - \rho^0_{aa})}{\gamma_{ab} + |\Omega|^2/\gamma_{bc}}, \end{aligned} \tag{11}$$

where ρ^0_{ii} are again calculated to the zeroth order in the probe field.

At this point we may discuss the physical origin of LWI. Let us first consider a V-scheme. From (7), we see two contributions of the upper level coherence that modify the gain or absorption. The first one is due to the dynamic Stark effect [9]. It is represented by the term proportional to $|\Omega|^2$ in the denominator of (7) and leads to reduced absorption or gain depending on the sign of

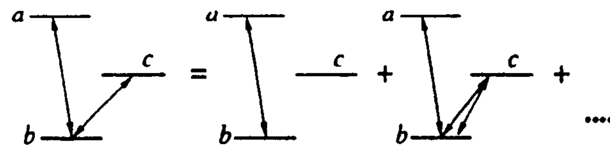

Fig. 2. Quantum interference of different absorption processes is the explanation for lasing without inversion in a V-type LWI.

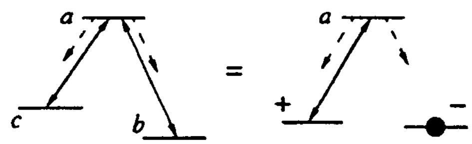

Fig. 3. Dressed state basis for Λ-type LWI. Without incoherent pumping the population is optically pumped into state $|-\rangle$, which is uncoupled from the optical fields. However, LWI cannot always be explained as lasing with inversion between states $|a\rangle$ and $|+\rangle$.

the gain coefficient. The physical origin of this contribution is the splitting of dressed states, which occurs due to the presence of a strong field coupling bare states b and c. A probe field which is resonant with the bare states is thus detuned from the dressed sublevels, leading to a reduction of both resonant absorption and gain. There exists, however, another important contribution that also modifies the absorption properties of the system. This contribution is represented by the term proportional to $|\Omega|^2$ in the numerator of (7). Its physical origin can be interpreted as a quantum interference of various absorption processes. In the presence of a strong driving field, an atom has several ways to absorb a probe photon (Fig. 2). In addition to direct single-photon absorption, it can, for example, first absorb and emit a pair of photons of the driving field, and then absorb a probe photon. Different absorption processes can interfere destructively leading to a reduction of absorption and enhancement of gain. Alternatively, reduction of absorption and enhanced gain can be viewed as a result of the Fano-type interference between the states dressed by the strong field Ω [10].

Although the analytical expression for gain looks similar in the case of the Λ-scheme, the physics of the Λ-scheme is somewhat different. Suppose, first, that there is no incoherent pumping. In this case all population of the three-level Λ system interacting with the two coherent fields is optically pumped into a superposition of the states b and c (Fig. 3):

$$|-\rangle = \frac{\alpha^*|c\rangle - \Omega^*|b\rangle}{\sqrt{|\alpha|^2 + |\Omega|^2}}, \tag{12}$$

which is decoupled from the optical fields. The population can leave this superposition only due to dephasing of the lower level coherence at a rate proportional to γ_{bc}. Since this dephasing rate can be made very slow (compared with an optical decay rate), the medium becomes transparent. This phenomenon is called the coherent population trapping and has been intensively studied in the past [11].

One might expect that LWI in a Λ-scheme occurs due to a "hidden" inversion in the dressed state basis, that is, due to an inversion between the upper state a and the $|+\rangle$ combination of the levels c and b, which is coupled to the optical fields (see Fig. 3):

$$|+\rangle = \frac{\alpha|b\rangle + \Omega|c\rangle}{\sqrt{|\alpha|^2 + |\Omega|^2}}. \tag{13}$$

This is, however, not true for most cases. The reason is that incoherent pumping out of the lower states is always accompanied by dephasing, which leads to population transfer into the coupled state. As a result, it turns out that incoherent pumping produces more population in the state $|+\rangle$ than in the upper lasing level. The system, however, displays LWI even under these conditions. This is possible due to the presence of Fano-type interferences between the states dressed by the strong field Ω [12], similar to those discussed above for the V-scheme.

Although (7) and (11) can give some useful insight into the origin of the gain in LWI, they are of little use in practice. The reason for this is that the populations ρ^0_{ii} depend on the driving field strength as well as upon the particular choice of the incoherent pumping scheme. Therefore a careful analysis of each particular pumping scheme is necessary. In general, this will give different conditions for gain without inversion for different schemes and methods of pumping. The most important particular cases are

- V-scheme without incoherent pumping into the auxiliary state c. In this case, the population difference $\rho^0_{bb} - \rho^0_{cc}$ can be expressed in terms of the population of the state ρ_{cc}, which leads to

$$\mathrm{V gain} = \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi} \tag{14}$$
$$\times \gamma_{ac}\frac{(\rho^0_{aa} - \rho^0_{bb}) + \gamma_c/(\gamma_a + \gamma_c + 2\gamma_{\text{phase}})(\rho^0_{bb})}{|\Omega|^2},$$

where γ_c and γ_a are the total decay rates out of levels c and a respectively; γ_{phase} is the dephasing rate on the dipole-forbidden transition $a \longrightarrow c$, and we have assumed the limit of a strong driving field $|\Omega|^2 \gg \gamma_{ac}\gamma_{cb}$. From (14), one can see that the contribution from quantum interference plays a significant role only if $\gamma_c/(\gamma_a + \gamma_c + 2\gamma_{\text{phase}})$ is comparable to unity. In particular, this implies

$$\gamma_a \le \gamma_c + 2\gamma_{\text{phase}} \le \gamma_c. \tag{15}$$

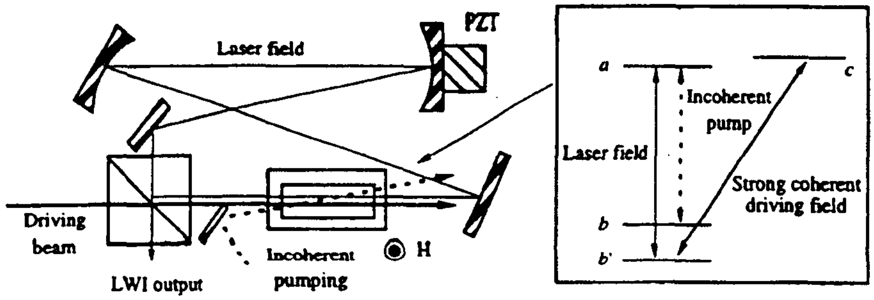

Fig. 4. Schematic of the experimental setup and simplified level scheme used for the first LWI oscillator in Rb.

Therefore, the total decay rate out of level *a* should be at least of the same order or less than the decay out of level *c*.

- Λ-scheme with direct pumping from the lower lasing state into the upper state. In this case, the condition for gain without inversion is [12]

$$\gamma'_a \le \gamma''_a, \tag{16}$$

where we used the notation of Fig. 1b, i.e., $\gamma''_a \equiv \gamma_{a\to c}$ and $\gamma'_a \equiv \gamma_{a\to b}$.

- Λ-scheme with indirect pumping into the upper state [13]. In this case there is no constraint on the decay rates involved, and the expression for the gain is

$$\Lambda\text{gain} = \frac{3\lambda_l^2 N\gamma'_a}{4\pi} \times \frac{|\Omega|^2 r\gamma''_a}{\left(\gamma'_a|\Omega|^2 + r(\gamma'_a+\gamma''_a)^2/4\right)\left(|\Omega|^2 + r(\gamma'_a+\gamma''_a)/4\right)}, \tag{17}$$

where r is the incoherent pumping rate and we assumed that $r \ll \gamma'_a, \gamma''_a$. Note that in the limit of strong driving field ($\Omega \gg \sqrt{r(\gamma'_a+\gamma''_a)}$) the gain scales as $r\gamma''_a/|\Omega|^2$, i.e., it is proportional to the decay rate on the transition $a \longrightarrow c$.

2.2. *Experimental Confirmation*

There are now several experimental demonstrations of inversionless amplification of light and cw LWI oscillators.

While early experiments [3] have demonstrated amplification without population inversion in the pulsed transient regime, it was not until recently that experiments demonstrating cw amplification and inversionless laser oscillation have been carried out. We focus on the latter.

The first demonstration of LWI oscillation [4] was based on the V-scheme (Fig. 4). The experiment was carried out in a Rb atomic vapor cell using diode lasers. The coherence was created by a narrow-band 780-nm extended cavity diode laser tuned to the D_2 ($5S_{1/2} \longrightarrow 5P_{3/2}$) absorption line of ^{87}Rb. Incoherent pumping was provided by a broad-band laser diode tuned to the D_1 ($5S_{1/2} \longrightarrow 5P_{1/2}$) absorption line. In the first set of experiments, a weak coherent probe field at 795 nm (D_1) was used to test the absorptive properties of the system. The frequency of the driving laser was tuned within the Doppler profile of the D_2 absorption line, while the frequency of the probe laser was swept through the D_1 line.

The results of the probe transmission measurements are shown in Fig. 5a. Curve *1* shows the result without the incoherent pump laser. Within the broad Doppler absorption profile of the transmission spectrum of the probe beam, there are two transmission peaks, which appear in the presence of the driving laser. They are due to the optical pumping of specific velocity groups by the drive laser and correspond to the pumping of the population from the level *b* to the level *b'* via different hyperfine sublevels of the state $c(5P_{3/2})$. When the incoherent pump is added (curve *2* in Fig. 5a), an increase in the transmission of the probe at the first transmission peak is observed; it corresponds to amplification of the weak probe field. The amplification occurs in spite of the fact that the results of optical pumping by the drive field are diminished by the incoherent laser (it optically pumps atoms back to the state *b'*). Consequently, the absorption of the probe increases in all other parts of the spectrum.

To verify the absence of a population inversion, experiments with different linewidths for the probe laser were carried out. As the linewidth of the probe laser was increased, the amplification disappeared. Since the weak probe does not affect the populations of the levels and its linewidth only enters into the dephasing rate of the upper-level coherence (in fact, it leads to an increase of this rate) this proves that the observed amplification is not due to population inversion, and that it is LWI due to quantum interference effects.

When the probe field is blocked and the cell is placed into a ring resonator, self-sustained laser oscillations

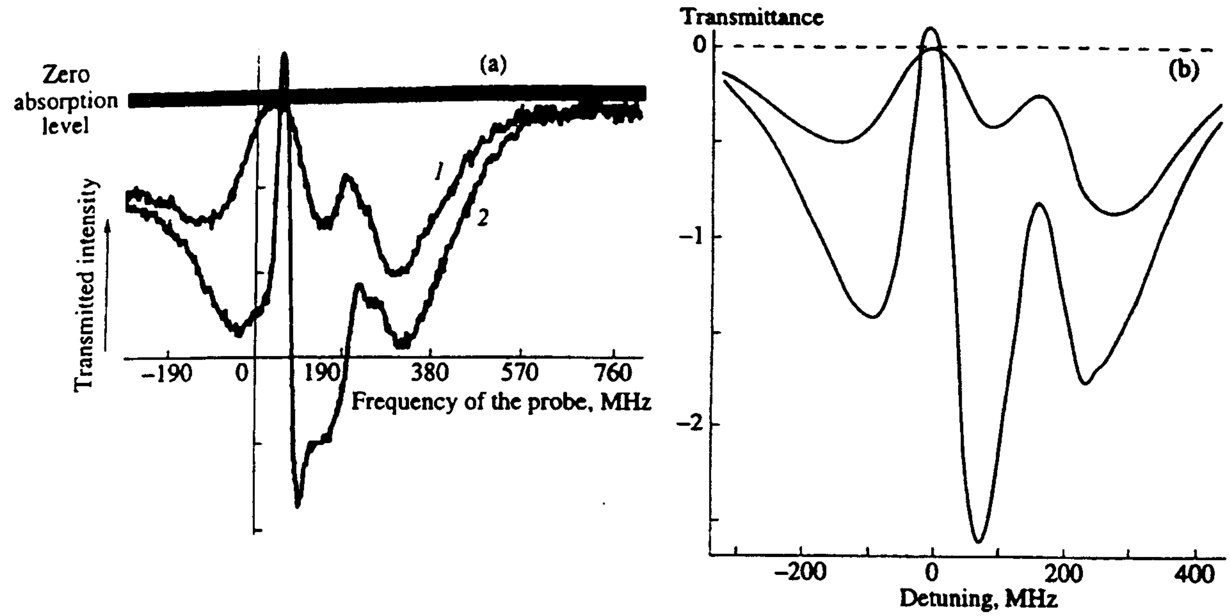

Fig. 5. (a) Experimentally measured transmission of the probe laser as a function of its frequency. Curve *1* was taken without the incoherent laser (pump); curve *2* was taken with the pump. (b) Calculated absorption (gain) coefficient for a weak probe field as a function of its frequency in the vicinity of the D_1 absorption line of Rb in the presence of the driving field tuned to the D_2 absorption line. Detuning is from the center of the absorption line.

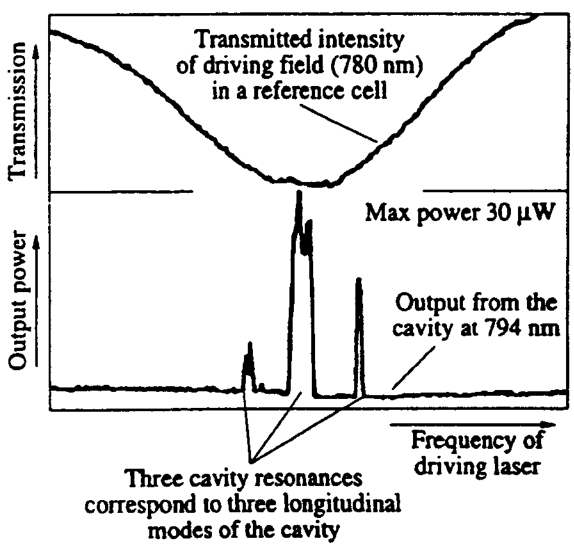

Fig. 6. Results of the LWI oscillator experiment: dependence of the driving field absorption (curve *1*) in the reference cell and corresponding dependence of the cavity output power at 794 nm (curve *2*) as a function of the frequency of the driving field. Three peaks of curve *1* correspond to three longitudinal modes of the cavity (with frequency separation ~170 MHz).

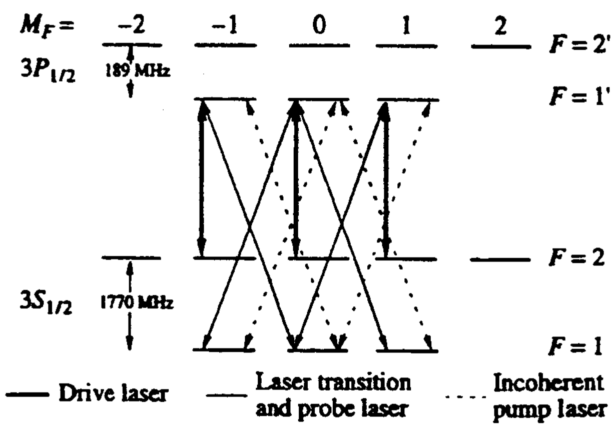

Fig. 7. Level scheme and corresponding optical fields used for the LWI oscillator in Na.

were observed in the cavity at the frequency corresponding to the probe transition of the D_1 line (Fig. 6).

In our other experiment, a Λ-type inversionless laser was realized in a Na atomic beam (Fig. 7). The coherent preparation was accomplished by a cw dye laser which coupled the $F = 2$ ground state ($3S_{1/2}$) hyperfine sublevel to the $F = 1$ sublevel of the $3P_{1/2}$ state (D_1 line). Amplification and cw LWI oscillation took place on the transition from this upper state to the $F = 1$ hyperfine sublevel of the ground state.

Similarly to the Rb case, the absorptive properties were tested using a separate probe laser whose transmission through the atomic beam was measured as a function of frequency (Fig. 8). Curve *1* shows the absorption of the probe as it is tuned across the atomic resonance. When the drive field was present, a narrow transmission peak was observed in the center of absorption profile (curve *2*). This occurs in spite of the fact that the drive field optically pumps most of the population into the $F = 1$ hyperfine sublevel of the ground state that is being coupled by the probe laser. It corresponds to electromagnetically induced transparency and is closely related to coherent population trapping, as discussed in Section 2.1. When incoherent pumping was added (curve *3* in Fig. 8b), a gain peak appeared in the center of the absorption line. After surrounding the Na beam with a ring laser cavity, and blocking the probe laser, cw laser oscillation was observed at the probe frequency.

We would like to emphasize several important aspects of the above experiments. First, these demonstrations have proven that LWI can work in real atoms and in the presence of many complicating factors, which can easily destroy coherences. Such factors include Doppler broadening, Zeeman sublevels, hyperfine structure and transient effects. Second, the real atomic structure and all of the effects mentioned above are not necessarily negligible and were taken into account by our theoretical treatment. For example, in both of the experiments described, the gain appeared only for certain combination of field polarizations and only when these fields were tuned to certain hyperfine sublevels. Moreover, the experiments in Rb required the presence of a static magnetic field which shifts different Zeeman sublevels by different amounts. Such effects can not be described within the framework of the simplified three- or four-level models and require special consideration which take into account the real level structure. For the Rb experiment such a treatment was done by solving the density matrix equations for a 32-level model,

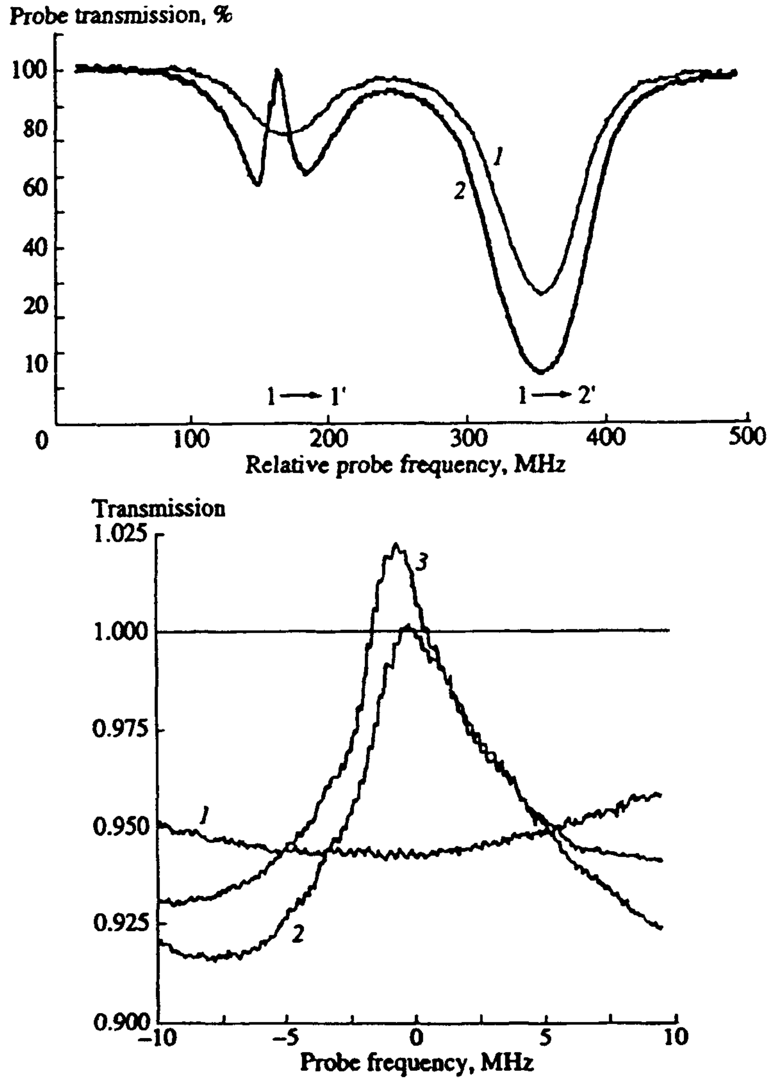

Fig. 8. Probe laser transmission as a function of probe laser frequency for the Λ-scheme in a sodium atomic beam. Curve *1* is the transmission of the probe in the absence of other fields. Curve *2* is the transmission of the probe in the presence of a driving field, showing electromagnetically induced transparency. Curve *3* is the probe transmission, in the presence of drive and pump fields, showing LWI gain.

which included all hyperfine and Zeeman sublevels of the D_1 and D_2 absorption lines, and then by averaging over a Maxwellian velocity distribution. Such a model gives a reasonable agreement with experiment, as can be seen in Fig. 5b.

Thus, the present experiments clearly confirm the basic principles of LWI. These principles do not depend upon the frequencies of the fields involved. The particular requirements and conditions for LWI, however, do change as one moves in the direction of shorter wavelengths.

3. SHORT-WAVELENGTH REALITY

As was discussed in the Introduction, generation of short-wavelength radiation is an obvious application of LWI. When the coherence is prepared by external optical fields, e.g., in the Λ- and V-schemes, one prefers that the generated radiation have a frequency larger than the frequency of the driving field. In other words, LWI can be used in this case for frequency up-conversion of radiation.

We have already noted above that the basic principles of LWI do not depend upon the frequencies of the fields under consideration. This follows from the fact that the form of the equations of motion for atomic coherences and populations do not change with the transition frequencies. These frequencies, however, do affect the equations of motion since some parameters of the systems, e.g., atomic decay rates or detunings, depend upon them. LWI gain is, in turn, very sensitive to changes of these parameters. In this case, several effects, which were not critical for the visible-frequency experiments, become important.

In particular, the role of Doppler broadening changes significantly. Although effects due to Doppler broadening were noted in the early experiments [14], their influence becomes especially destructive if the frequencies of the driving and probe fields are significantly different. The physical origin of this problem can be understood as follows. When coherence is prepared by the driving optical field, the gain (or absorption) does not change significantly as long as the two-photon resonance condition is fulfilled, i.e. as long as the detunings of the probe and driving fields from their corresponding atomic transitions are the same. The linear Doppler effect results in additional detunings. Specifically, for an atom moving with velocity V, the effective detunings of the lasing and driving fields, $\bar{\Delta}_l$ and $\bar{\Delta}_{dr}$, can be represented as

$$\bar{\Delta}_l = \Delta_l + \frac{\omega_l}{c}V_z, \tag{18}$$

$$\bar{\Delta}_{dr} = \Delta_{dr} + \frac{\omega_{dr}}{c}V_{z'}, \tag{19}$$

where Δ_l and Δ_{dr} are the detunings in the absence of Doppler broadening, ω_l and ω_{dr} are the frequencies of the probe and driving fields, and V_z and $V_{z'}$ are the projections of atomic velocity on the directions of their propagation.

Suppose that both fields have frequencies corresponding to resonance with zero-velocity atoms and that the driving and probe fields are propagating in the same direction. If the frequencies ω_l and ω_{dr} are similar, then the driving and probe fields are detuned from the resonance frequencies of moving atoms by approximately equal amounts. In this case, the two-photon contribution to the gain (or absorption) does not change much with Doppler detuning. This is illustrated in Fig. 9a, where the contribution to the gain and absorption of the different velocity groups is shown for the Λ-scheme (Fig. 1b). The parameters are chosen such that zero-velocity atoms amplify the probe field even though the population of the upper state a is roughly a tenth of the population of the lower lasing level b. In the case when the lasing and driving frequencies are identical, the contribution to the gain changes slowly with Doppler detuning. In this case the gain peak is practically insensitive to the Doppler broadening (as is apparent from Fig. 9b). This was precisely the case in the proof-of-principle LWI demonstrations described above.

The situation changes completely when the frequencies ω_{pr} and ω_{dr} become different. Although for small Doppler detunings the atoms still contribute positively to the gain, for larger Doppler detunings their contribution is negative (Fig. 9a), i.e., they reduce the overall gain (Fig. 9b). The magnitude and the center of the absorptive contribution is directly related to the frequency difference between the lasing and the driving fields: as the latter increases, the magnitude of the absorption increases and its maximum moves towards small detunings, i.e. towards slower atoms.

In light of this problem, one can choose between two approaches to preserve LWI gain. One way is to reduce the magnitude of the Doppler broadening. This can be done by using, for example, a well-collimated beam of atoms. However, it is difficult to achieve a high density of atoms in such a beam. Another approach to the problem makes use of the fact that the width of the gain peak increases with the Rabi frequency of the drive field due to power broadening. By increasing the power of the driving field, one can therefore move the absorption peak to larger Doppler detunings (Fig. 10a) and partially compensate the destructive influence of Doppler broadening (Fig. 10b). In this approach, the Rabi frequency of the drive field should be on the order of the Doppler width. There exists an extra price one has to pay for such a Doppler-free operation. As the Rabi frequency of the drive field (Fig. 10b) increases, the maximum possible gain decreases. To keep the value of the gain sufficiently high, one therefore has to increase the atomic density. Since available atomic density and laser power at the driving frequency are limited, a trade-off must be made between reasonable Doppler width on one hand and driving power and number of atoms on the other.

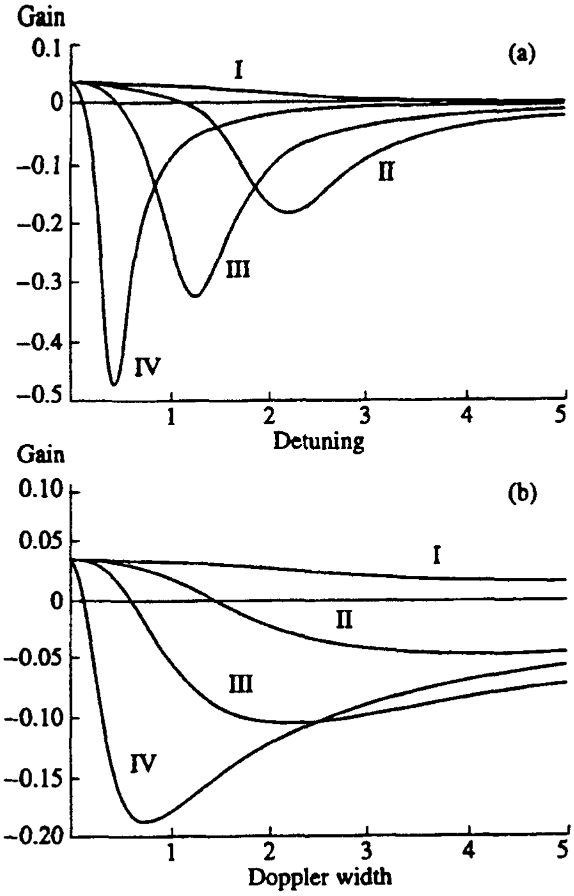

Fig. 9. Influence of the Doppler broadening on the LWI gain. (a) Contribution to the gain by different velocity groups as a function of Doppler detuning, (b) total gain averaged over the velocity distribution as a function of the Doppler width: (*1*) $\omega_l = \omega_{dr}$, (*2*) $\omega_l = 1.2\omega_{dr}$, (*3*) $\omega_l = 1.5\omega_{dr}$, (*4*) $\omega_l = 3\omega_{dr}$. Parameters are $\Omega = \gamma_a$, $R = \gamma = 0.1\gamma_a$, γ_a being the total decay out of the upper state *a*.

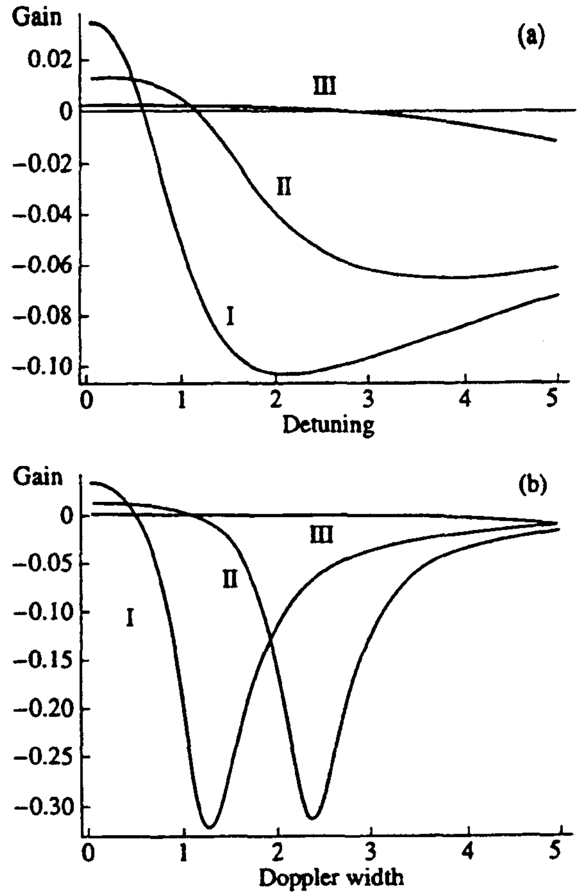

Fig. 10. Influence of the Doppler broadening on the LWI gain for different Rabi frequencies of the driving field. (a) Contribution to the gain by different velocity groups as a function of Doppler detuning, (b) total gain averaged over the velocity distribution as a function of the Doppler width: (*1*) $\Omega_{dr} = \gamma_a$, (*2*) $\Omega_{dr} = 2\gamma_a$, (*3*) $\Omega_{dr} = 4\gamma_a$. Parameters are $\omega_l = 3\omega_{dr}$, $R = \gamma = 0.1\gamma_a$.

In Section 2.1 we pointed out that gain without inversion implies certain conditions on the atomic decay rates. Usually these conditions require the decay on the driving transition to be faster than the decay on the lasing transition. Unfortunately, this is usually not the case when the frequency of the lasing field significantly exceeds that of the driving field. Indeed, as (3) suggests, the decay rate, (proportional to the Einstein B coefficient) increases with the cube of the frequency of the corresponding transition. Thus, to have gain without inversion, one should choose a driving transition that will have the B coefficient (B_{dr}) larger than that of the lasing transition (B_{pr}). Such a combination of transitions is not impossible in real atoms; however, this requirement leads to some interesting consequences which we consider below.

The B coefficient is responsible for the rate of stimulated processes. Therefore, the condition $B_{dr} > B_{pr}$ implies that the probability of a stimulated process on the driving transition should be larger than the probability of a stimulated process on the lasing transition. As a result, absorption of the driving field sets an upper limit on how much gain can be achieved.

Let us consider, for example, the Λ-type inversionless amplifier (Fig. 1b), in which the driving and probe fields are propagating in the same direction, which we take as the z-axis. Neglecting transverse variations of the fields amplitudes, the propagation equations for the slowly varying amplitudes of the fields can be represented as

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\alpha = \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\mathrm{Im}\frac{\rho_{ab}}{\alpha}\alpha, \tag{20}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\Omega = \frac{3\lambda_d^2 N\gamma_{a\to c}}{4\pi}\mathrm{Im}\frac{\rho_{ac}}{\Omega}\Omega, \tag{21}$$

where, as before, α and Ω are the Rabi frequencies of the probe and driving fields, respectively, and λ_p and λ_d are their wavelengths. To find amplification of the probe field one has to solve these two equations simultaneously.

Some conclusions, however, can be made without actually solving them. Let us consider the case when both the probe and driving fields are in resonance with their corresponding transitions. In the limit when the probe field is sufficiently weak, the absorption coefficient for the driving field can be represented as [cf. (8)–(10)]

$$\begin{aligned}\text{absorption of drive} &= \frac{3\lambda_d^2 N\gamma_{a\to c}}{4\pi}\frac{\mathrm{Im}\rho_{ac}}{\Omega},\\ &= \frac{3\lambda_d^2 N\gamma_{a\to c}}{4\pi}\frac{\rho_{cc}^0-\rho_{aa}^0}{\gamma_{ac}}.\end{aligned} \tag{22}$$

Using (11) for the linear laser gain and assuming that $\rho_{bb}^0 > \rho_{aa}^0$ (LWI condition), we obtain

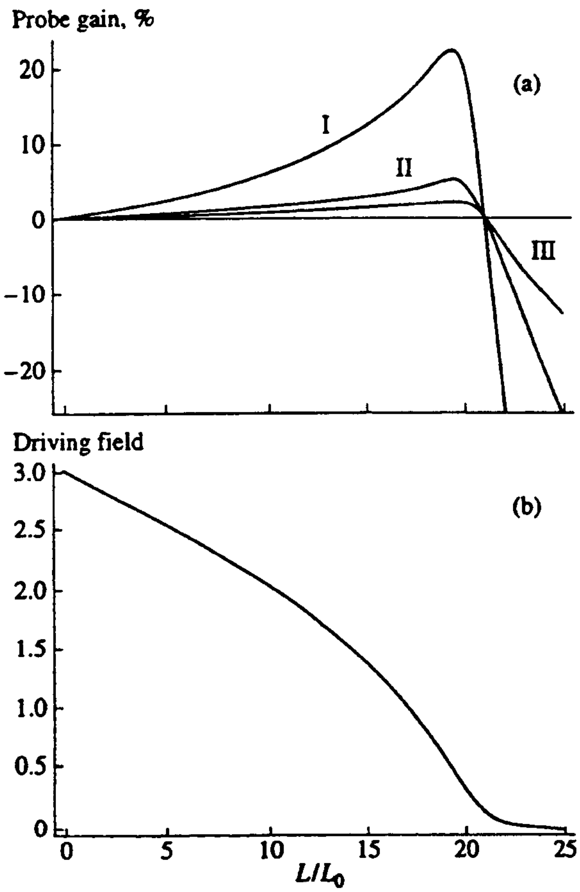

Fig. 11. Absorption of the driving field and LWI gain as a function of the interaction length for copropagating probe and driving fields. (a) LWI gain as a function of the interaction length, (b) amplitude of the driving field as a function of the interaction length: (*1*) $\omega_l = \omega_{dr}$, (*2*) $2\omega_{dr}$, (*3*) $3\omega_{dr}$.

$$\begin{aligned}\text{probe gain} &= \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\mathrm{Im}\frac{\rho_{ab}}{\alpha}\\ &= \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\\ &\quad\times\frac{(\rho_{aa}^0-\rho_{bb}^0)+|\Omega|^2/(\gamma_{ac}\gamma_{bc})(\rho_{cc}^0-\rho_{aa}^0)}{\gamma_{ab}+|\Omega|^2/\gamma_{bc}}\\ &\le \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\frac{|\Omega|^2/(\gamma_{ac}\gamma_{bc})(\rho_{cc}^0-\rho_{aa}^0)}{\gamma_{ab}+|\Omega|^2/\gamma_{bc}}\\ &\le \frac{\lambda_l^2\gamma_{a\to b}}{\lambda_d^2\gamma_{a\to c}}\frac{|\Omega|^2}{\gamma_{bc}\gamma_{ab}+|\Omega|^2}\times\text{absorption of drive}\\ &\le \frac{\lambda_l^2\gamma_{a\to b}}{\lambda_d^2\gamma_{a\to c}}\times\text{absorption of drive}.\end{aligned} \tag{23}$$

So we conclude

$$\frac{\text{probe gain per unit length}}{\text{absorption of drive per unit length}} \le \frac{\lambda_p^2\gamma_{a\to b}}{\lambda_d^2\gamma_{a\to c}}. \tag{24}$$

The meaning of this result can be understood as follows: if the wavelength of the probe (lasing) field is shorter than that of the driving field and $\gamma_{a\to b} < \gamma_{a\to c}$ the ratio above is clearly less than unity. On the other hand, it is unacceptable to have a driving intensity lower than a certain value, since the medium then becomes highly absorbing for the probe field. Thus, one is forced to choose the parameters such that the absorption of the driving field is not too large. In this case (24) sets an upper limit for the probe gain and the latter does not grow with atomic density and interaction length as (7) and (11) might suggest.

These ideas are illustrated in Fig. 11, where the results of numerical integration of (21) are shown. As the interaction length is increased, the probe field gain (Fig. 11a) reaches its maximum value at the point where the intensity of the driving field becomes small due to absorption losses (Fig. 11b). The maximum gain decreases as the wavelength of the probe field increases and becomes extremely small for large λ_l/λ_d. This dependence follows directly from the $(\lambda_l/\lambda_d)^{-2}$ scaling of (24).

To summarize, the dissipative absorption of the driving field causes serious problems in the short-wavelength operation of inversionless lasers, since it limits the maximum possible LWI gain. The relatively straightforward solution to this problem would be to use a setup with perpendicular driving and lasing fields. One can, in principle, arrange an atomic beam with a rectangular (or oval) shape, such that the interaction distance for the driving field is much shorter than that of the probe field (Fig. 12). In such a setup one can

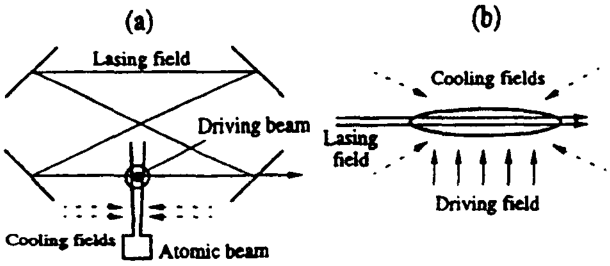

Fig. 12. Schematic of the experimental setup in which lasing and driving fields are propagating in perpendicular directions. Laser cooling can be used to reduce the Doppler broadening.

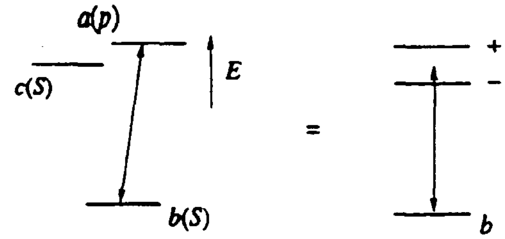

Fig. 13. A dc electric field can be used to create a coherence between two closely spaced atomic levels of different parity, e.g., S and P states.

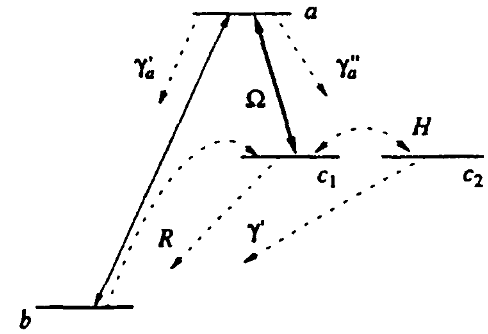

Fig. 14. A simple four-level model in which a dc magnetic field can be used for LWI gain enhancement. c_1 and c_2 model two Zeeman sublevels of the atom. The direction of the dc magnetic field should be perpendicular to the axis of quantization.

make the interaction length in the direction of propagation of the probe field sufficiently long to ensure high gain while at the same time keeping the absorption of the driving field (which is propagating in the perpendicular direction) small. The disadvantage of such a configuration would be its increased sensitivity to Doppler broadening. Indeed, such a geometry does not allow any cancellation of the two-photon Doppler broadening. In order to ensure very low transverse Doppler broadening, such an oval-shaped beam could be laser-cooled as shown in Fig. 12b.

4. LWI GAIN ENHANCEMENT VIA STATIC ELECTRIC AND MAGNETIC FIELDS

The problems discussed in the previous section apply only to those schemes in which coherences are prepared by optical fields. However, this is not the only way to establish atomic coherence. In particular, static electric or magnetic fields can be used to establish coherence between different atomic energy levels. Since such a static field is not absorbed by the medium, the problems discussed in the previous section do not play a major role even in the short-wavelength spectral region.

A classic example of this scheme is where coherence is established between two closely spaced atomic energy levels by a dc electric field [15] (Fig. 13). Such a scheme was successfully applied to observe enhanced second harmonic generation with reduced absorption in atomic hydrogen. However, this scheme was not considered for short-wavelength LWI [16], possibly because of the need to have two closely spaced sublevels with different parity and large values of the dc electric field.

We choose here a different approach and show an example of how static fields can be used in combination with strong optical fields to enhance inversionless gain. In particular, we consider a four-level atomic system in the presence of a weak magnetic field (Fig. 14). Two states, (c_1 and c_2) are taken to be degenerate in the absence of the magnetic field. They can represent, for example, two Zeeman sublevels. An optical driving field with the correct polarization will couple one of these states (say c_1) with the upper lasing level a. The weak magnetic field applied in the direction perpendicular to the axis of quantization (see Fig. 14), causes the Larmor precession, which also induces coherence between sublevels c_1 and c_2. To see this, we consider the equations of motion for the evolution of the off-diagonal density matrix elements:

$$\dot{\rho}_{ab} = -(\gamma_{ab}+i\Delta)\rho_{ab} - i\alpha(\rho_{aa}-\rho_{bb}) - i\Omega\rho_{c1b}, \quad (25)$$

$$\begin{aligned}\dot{\rho}_{c1b} = &-(\gamma_{cb}+i\Delta)\rho_{c1b} + i\Omega^*\rho_{ab}\\ &- i\Omega_m\rho_{c2b} - i\alpha\rho_{c1a},\end{aligned} \quad (26)$$

$$\dot{\rho}_{c2b} = -(\gamma_{cb}+i\Delta)\rho_{c2b} - i\Omega_m\rho_{c1b} - i\alpha\rho_{c2a}, \quad (27)$$

$$\begin{aligned}\dot{\rho}_{ac1} = &-\gamma_{ac}\rho_{ac1} + i\Omega_m\rho_{ac2}\\ &- i\Omega(\rho_{aa}-\rho_{c1c1}) + i\alpha\rho_{bc1},\end{aligned} \quad (28)$$

$$\dot{\rho}_{ac2} = -\gamma_{ac}\rho_{ac2} + i\Omega_m\rho_{ac1} + i\alpha\rho_{bc2} + i\Omega\rho_{c1c2}, \quad (29)$$

$$\begin{aligned}\dot{\rho}_{c1c2} = &-\gamma_{c1c2}\rho_{c1c2}\\ &+ i\Omega_m(\rho_{c1c1}-\rho_{c2c2}) + i\Omega^*\rho_{ac2}.\end{aligned} \quad (30)$$

Here, Δ is the detuning of the probe field from resonance with the $a \longrightarrow b$ transition. $\Omega_m = g\mu_B B$ is the Larmor precession frequency, and we have assumed that the driving field of the Rabi frequency Ω is resonant with the $a \longrightarrow c_1$ transition.

From (25)–(30), we see that the magnetic field here plays a similar role to that of the driving field. In fact, it is equivalent to a driving field with a Rabi frequency equal to the Larmor precession frequency. As a result, in the presence of such an effective field the optical properties of this system are completely modified.

Solving (25)–(30) to the first order in the probe field, we obtain the following expression for the linear gain G on the lasing transition:

$$G = \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}\mathrm{Im}\frac{\rho_{ab}}{\alpha} = \frac{3\lambda_l^2 N\gamma_{a\to b}}{4\pi}$$

$$\times\frac{1}{\gamma_{bc}|\Omega|^2+\gamma_{ab}(\Omega_m^2+\gamma_{bc}^2)}[(\rho_{aa}^0-\rho_{bb}^0)(\gamma_{bc}^2+\Omega_m^2) \tag{31}$$

$$+|\Omega|^2\gamma_{bc}/\gamma_{ac}(\rho_{c1c1}^0-\rho_{aa}^0)$$

$$+\Omega_m^2(1+\gamma_{bc}/\gamma_{ab})(\rho_{c1c1}^0-\rho_{c2c2}^0)],$$

where we have assumed that $|\Omega| \gg \gamma_{bc}, \gamma_{ac}, \Omega_m$. One can see that the Larmor precession leads to the presence of the term proportional to the population difference $\rho_{c1c1}^0 - \rho_{c2c2}^0$, which plays an important role. Solving equations for the diagonal elements of the density matrix and substituting the zeroth-order populations into (31), we find that

$$\text{probe gain} = \frac{3\lambda_l^2 N\gamma_a'}{4\pi}$$

$$\times\frac{\Omega_m^2[(R-\gamma')\gamma_a''-2\gamma_m\gamma_a']}{(\gamma_{bc}|\Omega|^2+\gamma_{ab}\Omega_m^2)(\gamma_a''R+2\gamma_m\gamma_a'+\gamma'\gamma_a''+2\gamma_m R)}, \tag{32}$$

were R is the incoherent pumping rate and γ' is the decay rate of the optically forbidden transition as indicated in Fig. 14, $\gamma_{bc} = R + \gamma'$,

$$\gamma_m = \frac{\gamma'|\Omega|^2\gamma_{ac}+2\Omega_m^2(\gamma_{ac}+\Omega_m^2)}{\gamma_{ac}|\Omega|^2},$$

and we have assumed additionally that $\gamma_{bc} \ll \Omega_m \ll \gamma_{ab}$. Under these conditions $\gamma_m \ll \gamma_a', \gamma_a''$. The inversion can be represented as

$$\rho_{aa}-\rho_{bb} = \frac{2\gamma_m R-2\gamma_m\gamma_a'-\gamma'\gamma_a''}{\gamma_a''R+2\gamma_m\gamma_a'+\gamma'\gamma_a''+2\gamma_m R}. \tag{33}$$

We see that the system displays gain without inversion if $\gamma_a' + \gamma'\gamma_a''/(2\gamma_m) > R > \gamma' + 2\gamma_m\gamma_a'/\gamma_a''$. It is also interesting to note that LWI in this system does not imply any constrains on the decay rates. Let us turn now the question of the absorption of the driving field. From (25)–(30) we have

$$\text{drive absorption} = \frac{3\lambda_d^2 N\gamma_{a\to c}}{4\pi}\mathrm{Im}\frac{\rho_{ac1}}{\Omega}$$

$$= \frac{3\lambda_d^2 N}{4\pi}\frac{\gamma_a''\Omega_m^2}{\gamma_{ac}|\Omega|^2}\frac{\gamma_a''R-2\gamma_m R}{\gamma_a''R+2\gamma_m\gamma_a'+\gamma'\gamma_a''+2\gamma_m R}, \tag{34}$$

where we have used the same assumptions as in deriving (32). Combining (32) and (34), we find that, if $\gamma_a \gg R \gg \gamma'$ and $|\Omega| \gg \sqrt{\gamma_{ab}/\gamma_{bc}}\,\Omega_m$, then

$$\frac{\text{probe gain per unit length}}{\text{absorption of drive per unit length}} \sim \frac{\lambda_l^2\gamma_{a\to b}\gamma_{ac}}{\lambda_d^2\gamma_{a\to c}\gamma_{bc}}. \tag{35}$$

This expression is identical to the condition (24) except for the last factor, which is given by the ratio of the decay rate of the optical coherence to the decay rate of the coherence of the dipole-forbidden transition. The latter factor can actually be so large, that the ratio in (35) can easily be made greater than unity. In such a scheme, therefore, considerable amplification of the probe field occurs *without increased absorption of the driving field*. In other words, the presence of magnetic field enhances the LWI gain. Since the absorption of the driving field becomes especially destructive for short-wavelength inversionless lasers, such a scheme is of particular interest for LWI with large frequency up-conversion.

We finally note that in certain cases it is possible to use a microwave field on the dipole-forbidden transition (or optical field on the transition with very slow decay rate) for the coherent preparation of the system as was pointed out, e.g., in [17]. In such schemes LWI implies somewhat different conditions and may no longer be restricted by the absorption of the driving field. In this case, however, the aspects of dynamical stability of lasing become of critical importance as will be discussed elsewhere [18].

5. CONCLUSION

In this paper we have reviewed the basic principles of Lasing Without Inversion (LWI) and described the first realizations of LWI oscillators. We considered several aspects of inversionless lasers operating in the short-wavelength region. Although the basic principles of LWI are the same regardless of the wavelengths of the fields involved, the practical conditions for realization of LWI are quite different for the short-wavelength spectral region. We pointed out the problems specific for this region and outlined possible approaches to solve these problems.

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