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EFFECT OF DRIFT ON TOTALDEV

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Abstract

Estimates of the frequency stability of clocks and oscillators are given by statistics of frequency fluctuations. At long-term averaging times, TOTALDEV is a recommended statistic over ADEV, the Allan deviation. Simulation of common noise types with drift are used to compare TOTALDEV and ADEV. Comparisons show that the response of TOTALDEV is consistent at long term with drift taken out whereas the response of ADEV is not.

results in a  $\tau^{-1}$  behavior, usually a rise of the last point in ADEV, small deviations in the value of the drift coefficient upon removal cause dramatic variations in ADEV. This is a problem in reliably interpreting long-term stability of an oscillator or frequency standard with a given model of drift taken out. A proposed solution is not to use  $\hat{\sigma}_y(\tau)$  at long-term (large  $\tau$ ) but rather attempt to extrapolate shorter-term measurements with justification [4].

INTRODUCTION

The currently recommended statistic for characterizing frequency stability is the Allan deviation (ADEV) which is the square root of the average two-sample standard variance of frequency fluctuations denoted  $\hat{\sigma}_y(\tau)$  [1].

Determining and removing a polynomial (particularly a drift coefficient) over a measurement interval before computing ADEV has meaning only if the procedure is physically correct. Drift removal is model-dependent [2]. A common method involves quantifying drift as an overall second difference of  $\{x(t)\}$  which, when removed, results in the usual ADEV being precisely 0 at the longest averaging time [3]. Since drift

A statistic called TOTALDEV which retains the intuitive simplicity and range of applicability of ADEV has been developed [5]. TOTALDEV does not respond with 0 when using any method of drift removal. After drift removal, TOTALDEV's value of frequency stability at the longest averaging time is consistent with the true power-law behavior characteristic of the noise process being measured and is less dependent on the method of drift removal. Additionally for a common set of noise processes, TOTALDEV (1) has the same mean as ADEV and (2) has lower variability [6].

BACKGROUND

In this paper, we discuss effects of linear frequency drift on TOTALDEV. TOTALDEV is recommended instead of traditional ADEV in cases where improved confidence at long-term averaging times is desirable [5]. Ordinarily drift is removed

from a time-series of phase- or frequency-difference data before applying either TOTALDEV or ADEV, since these statistics are designed to quantify statistical trends in the non-deterministic variations of data.

Occasionally, in the long term it is hard to distinguish drift which has a  $\tau^{+1}$  trend in ADEV relative to a divergent noise type like Random Walk FM (RWFM), which has a  $\tau^{+1/2}$  trend in ADEV because the number of samples is small and therefore confidence is low. For this reason, in the long term it is advantageous to use the assumption and principles in TOTALDEV since it is a better estimate of frequency stability than ADEV. By modifying the original procedure involved in TOTALDEV, we have constructed a suitable statistic which in simulation is stable (although slightly biased at the longest term) in the presence of drift term and has the confidence advantage of TOTALDEV. The procedure is given by the following:

For  $x(1), \dots, x(N)$ , remove a slope and constant (endmatching procedure) to produce  $x_o(1), \dots, x_o(N)$ , where  $x_o(1) = x_o(N) = 0$ . Adjoin  $x_o(N+1), \dots, x_o(2N-1)$ , where  $x_o(N+j) = -x_o(N-j)$ ,  $j=1$  to  $N-1$ . For  $\tau=m\tau_o$ , TOTALVAR (with square-root as TOTALDEV) is given by

$$\hat{\sigma}_{TOTAL}^2(\tau) = \frac{1}{2\tau^2} \frac{1}{N-m-1} \sum_{n=1}^{N-m-1} [x_o(n) - 2x_o(n+m) + x_o(n+2m)]^2. \quad (1)$$

Notice that  $n+m$  stops just short of  $N$ ; that avoids an identically zero second difference. Practically speaking, it does not matter very much whether this zero-valued second difference is included or not. If  $n+m$  is greater than  $N$ , then the absolute value of an earlier second difference is unnecessarily

duplicated since TOTALDEV uses a periodic assumption for  $\{x_o(n)\}$ .

This  $\hat{\sigma}_{TOTAL}^2(\tau)$  calculation is the same as computing the usual fully overlapped AVAR estimate (with square root as ADEV) for  $\tau=m\tau_o$  on the time-deviation vector  $x_o(1), \dots, x_o(N+m-1)$ , so that if we have a routine that does fully overlap AVAR, we can apply it to the endmatched and periodically extended subvector of  $\{x_o(n)\}$ .

### METHODS OF DRIFT REMOVAL

Typically drift is consistent (reproducible) enough to be considered predictable (deterministic) and is modeled as proportional to the square of measurement time as given by

$$x(t) = x_o + x_{RATE}(t) + x_{DRIFT}(t) + x_{NOISE}(t) \quad (2)$$

$$\text{or } x(t) = x_o + Rt + \frac{D^2}{2}t^2 + \epsilon(t), \quad (3)$$

where  $x_o$  = initial time difference at  $t_{START}$ .

$Rt = x_{RATE}(t)$ , where  $R$  is the rate,  $\frac{D^2}{2}t^2 = x_{DRIFT}(t)$ , where

$D$  is the drift, and  $\epsilon(t) = x_{NOISE}(t)$ .

We assume that parts of this function other than  $\epsilon(t)$  (the residual noise) are measurable, systematic quantities (deterministic). The noise is considered random and not deterministic in any known way. Because frequency difference function  $y(t)$  is the first difference (sampled first derivative) of time difference function  $x(t)$ , variances of the difference of these frequency differences (like ADEV and TOTALDEV) depend only on  $D$  and  $\epsilon(t)$ . That is,  $x_o$  and  $R$  have no frequency

deviates as a function of averaging time, hence no variance (in terms of differences in  $y(t)$ ).

We assume that all drift is the linear least squares fit to  $\bar{y}(t)$  or its equivalent at  $\tau = T/2$ , the overall second-difference to  $x(t)$ . Since for  $T$  data length the drift function is

$$\frac{D^2}{2}t^2, \text{ subtracting twice the overall}$$

second difference to  $x(t)$  data should remove this function in all cases, it seems reasonable that it should be used in all cases regardless of the noise type. Removing a linear least-squares fit to frequency is, however, not identical to removing an overall second difference since  $\bar{y}(t)$  is dependent on  $\tau$ . The slope at  $x(t)$  as given by

$$\bar{y}(t) = \frac{x(t) - x(t-\tau)}{\tau} \approx \dot{x}(t). \quad (4)$$

When we remove drift from the data, the residuals are often not white Gaussian. The drift has been properly specified, but possibly nondeterministic divergent noise components still remain in the data; hence we deem that our drift estimate is not optimum. In fact, depending on the kind of noise prevalent over the length  $T$  of the observation window, we modify our model (eq. (3)) and apply a specific kind of regression analysis which yields closer Gaussian residuals for period  $T$  to determine the drift  $D$  [2]. The kind of noise at the longest averaging time ( $\tau = T/2$ ) and its corresponding optimum drift estimator is in Table 1 ( $\tau_0$  is the minimum averaging time).

**TABLE 1. Noise type and the corresponding optimum drift estimate**

NOISE	REMOVE DRIFT AS
White PM	Linear Least-Squares Fit to Time $x(t)$ at $\tau = \tau_0$
White FM	Linear Least-Squares Fit to Frequency $y(t)$ at $\tau = \tau_0$
Flicker FM or Random Walk FM	Twice the Overall Second Difference To Time $x(t)$ at $\tau = T/2$

We assume that these noise processes are in the presence of drift to begin with. These methodologies have been shown to be optimum in closed (analytic) solutions [7].

### PROBLEM

From a practical standpoint, it is often desirable to measure data for long enough that Flicker FM and/or Random Walk FM are observed. This means that we remove drift as twice the overall second-difference to time  $x(t)$ . Now for an illustration of the problem at hand, we simulated a noise process consisting of short-term white FM (WHFM) at  $ADEV = 2 \times 10^{-12} \tau^{1/2}$ , medium-term flicker FM (FLFM) at  $ADEV = 1 \times 10^{-12}$ , and long-term constant linear frequency drift of  $4 \times 10^{-11}$  over a full observation of 1025 points. The resulting ADEV plot of 10 such simulations is shown in figure 1. Short term  $\tau^{1/2}$  behavior and long term  $\tau^1$  (drift) behavior are clearly visible with flicker ( $\tau^0$ ) not as visible but nevertheless present. We then created 100 independent identically distributed simulations, removed the drift estimate using the overall mean second

difference of each series  $\{x(n)\}$ , and calculated ADEV. Figure 2 shows all results overlaid and shows that the effect of this estimated drift removal on ADEV is dramatic. It indicates that ADEV does not properly report the expected FLFM value in long term when the usual estimate of drift is removed from the original data.

This problem is not to be confused with the fact that the most probable value for ADEV at  $\tau=T/2$  is 0 since there is only one degree of freedom. This is not an issue of probability. Figure 2 indicates that  $\sigma_y(\tau=T/2)$  equals 0. Indeed it is common for the "last" (longest  $\tau$ ) or next to last value of a  $\sigma_y$  vs.  $\tau$  plot to be totally unrelated to the rest of the plot. The last values are usually optimistic, that is, the points (especially the last one) approach  $\sigma_y(\tau) = 0$ , a physically desirable result but with drift removed it is usually a wrong result.<sup>1</sup> A proper estimation of overall second difference as drift and its subsequent removal requires that the Allan variance be exactly 0 at  $\tau$  equals half the length of time  $T/2$  of any observation window  $T$ . Then why is it not always 0? One reason is that for short data sets the remaining data residuals after drift removal may contain a turn-on and turn-off transient

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<sup>1</sup>The estimate of linear frequency drift can be determined from a change of slope in  $y(t)$  equal to

$$\frac{\text{Last } \bar{y} - \text{First } \bar{y}}{\tau=T/2}$$

If this is "removed" from any data, we find  $\sigma_y^2(\tau=T/2)$  is precisely 0, a pesky result to say the least. One expects the long- $\tau$  values to go down but no one has addressed the problem of the last  $\tau$  value being 0 when obviously the variance is not really 0.

due to the rectangular window function which leaks power into ADEV in the long term [8]. This can affect the values of the variance reported at and near  $\tau = T/2$ . In other words, when drift is properly removed, the "last" points of a typical ADEV plot are not the variance of the residuals at all but rather are an artifact of the statistic of small samples. Another reason may relate to accumulated round-off error. Computations lose an ability to resolve zero when numbers become smaller.

In cases where the last point is obviously false as judged by the rest of  $\sigma_y(\tau)$  plot, it is thrown out. Worse, however, if it seems plausible and is judged somehow as "okay," it is retained and can be used to conform to some expected or suitable long-term behavior. From our present understanding of ADEV, the inclusion of any "error bars" representing a defined confidence interval for the last deviate (at  $\tau = T/2$ ) seems to be defiant of common sense when drift is removed.

### SOLUTION

We define frequency stability as the degree to which an oscillating signal produces the same value of frequency for any interval  $\Delta t$  throughout a specified period [9]. As a consequence, ADEV properly assumes that a time series of phase differences  $\{x(n)\}$  cannot be treated as a sample population of random variables like numbers picked out of a hat. However it does not matter if the series has all signs reversed, is run backwards, includes a slope or offset, starts at a different time origin, or is treated as circular. This is shown pictorially in figure 3. TOTALDEV (square root of eq (1)) is a statistic which is constructed based on these facts. In particular, TOTALDEV uses an assumption

that the series can be circularly represented with different averageable time origins. In contrast to figure 2, figure 4 shows 100 plots of calculations of TOTALDEV using the same original data as figure 2, that is, with estimated drift removed. Here the long-term frequency stability reported by TOTALDEV is more consistent with the expected power-law trend.

### CONCLUSION

Using test suites of data, TOTALDEV has been shown to be a considerable improvement over ADEV, especially at longer averaging times where fewer samples are available [10]. This is because TOTALDEV averages a greater number of possible frequency variations contained in finite-length data as compared to ADEV. TOTALDEV's appeal is that it reports frequency instability in the same manner as ADEV. The authors have shown that TOTALDEV long-term results can still be properly interpreted with drift removed.

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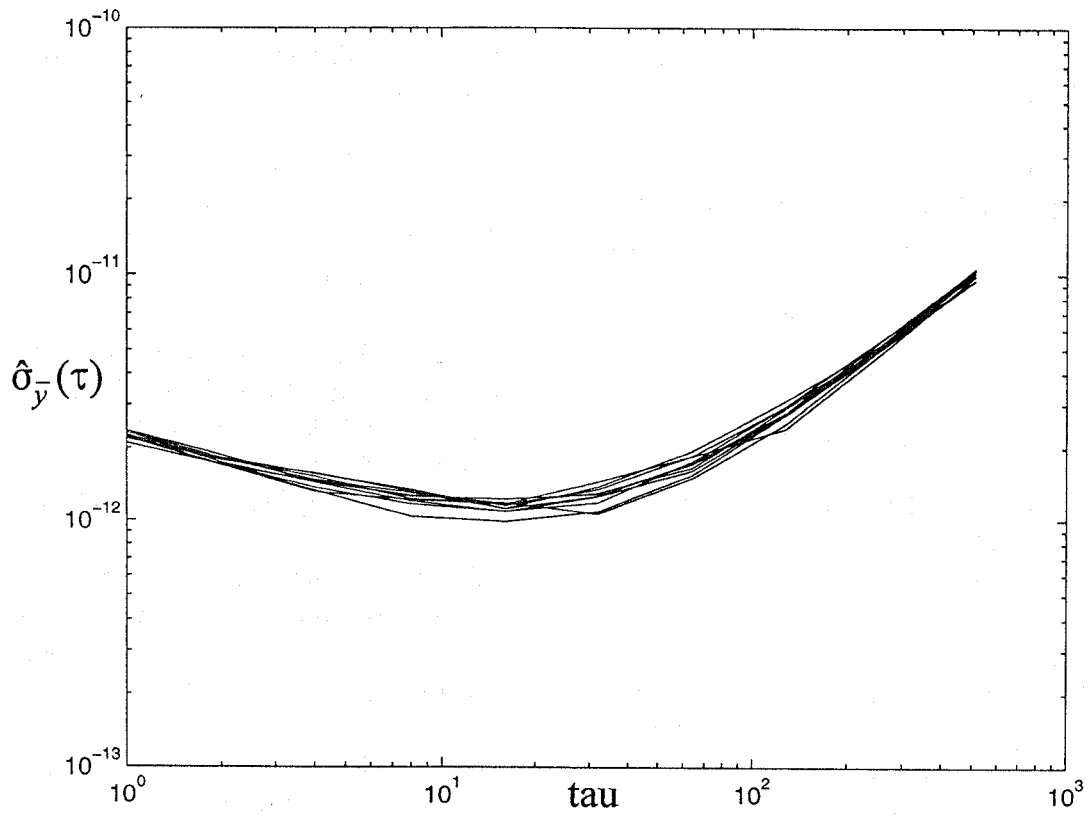


Fig. 1. ADEV of ten simulations of White FM (WHFM), Flicker FM (FLFM) and drift.

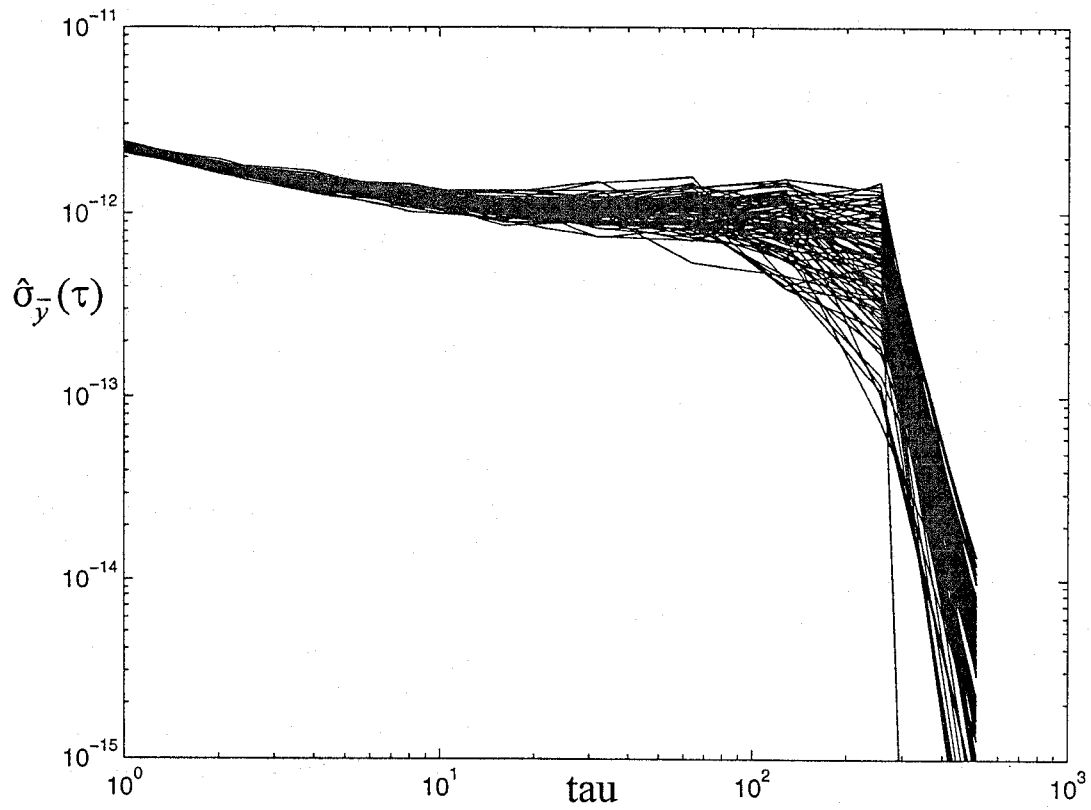


Fig. 2. ADEV of 100 simulations of White FM (WHFM), Flicker FM (FLFM) and drift as in figure 1 with estimated drift removed. Note the downshoots in the last points of ADEV.

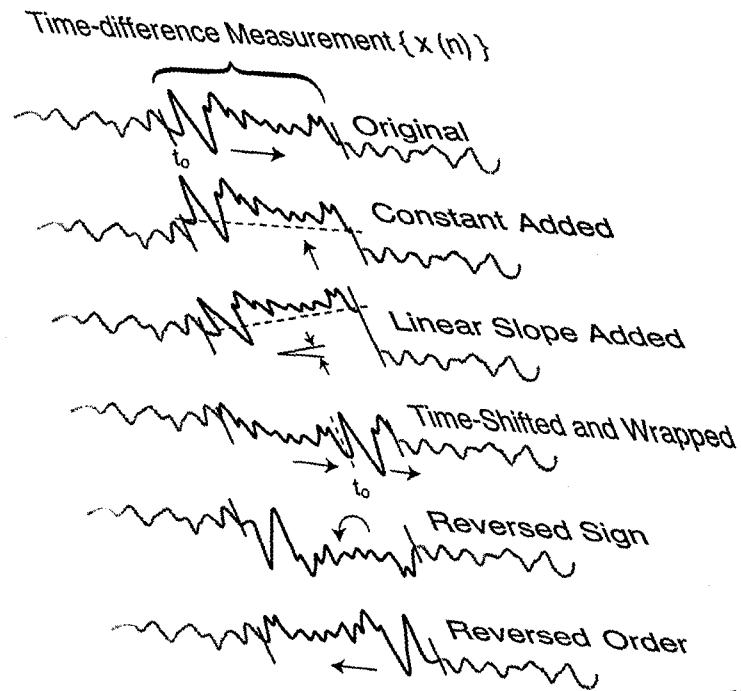


Fig. 3. Procedures that do not affect characterizations of frequency stability.

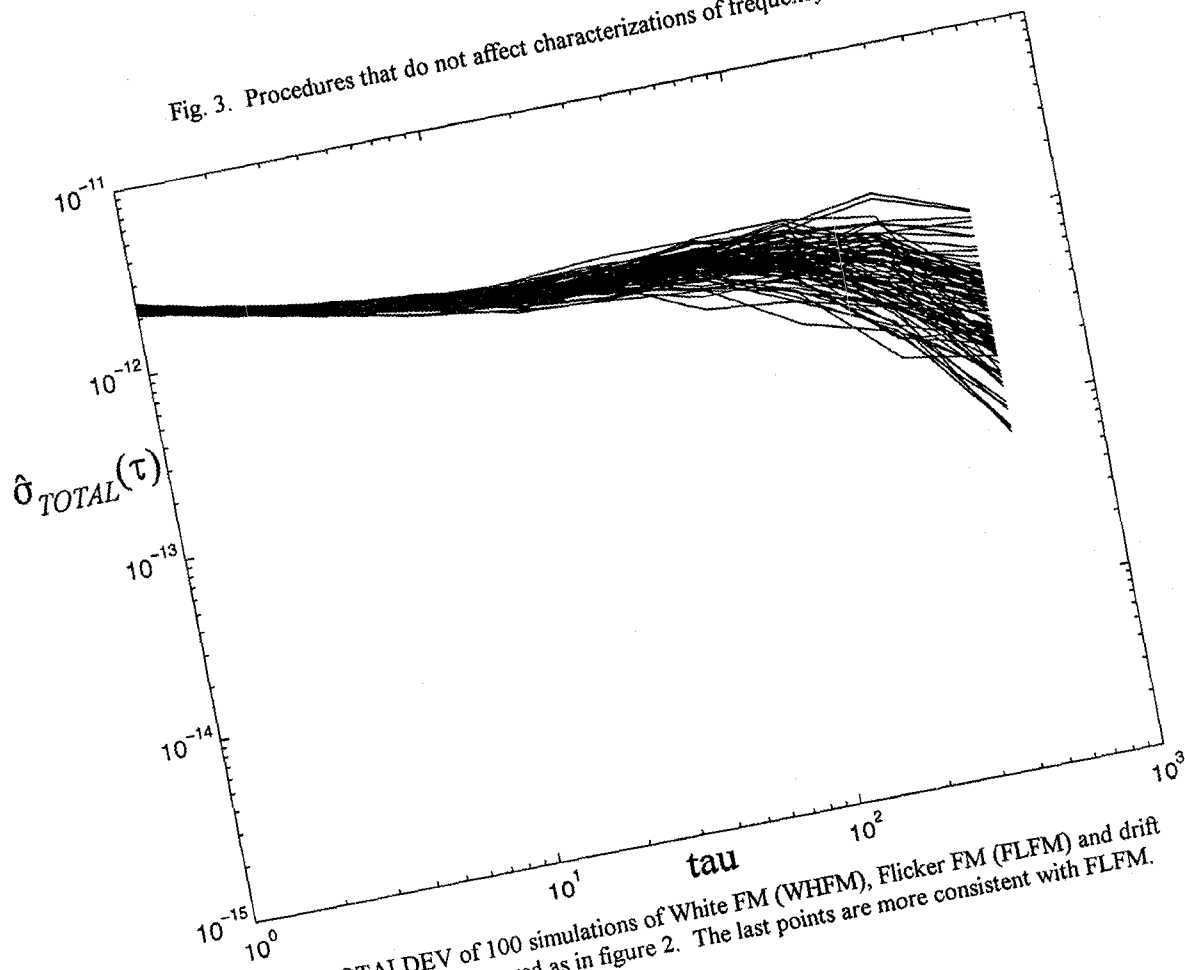


Fig. 4. TOTALDEV of 100 simulations of White FM (WHFM), Flicker FM (FLFM) and drift with estimated drift removed as in figure 2. The last points are more consistent with FLFM.