Cooling methods in ion traps[1]

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I. Introduction

Trapping of ions is used to make measurements that are difficult or impossible to perform by other techniques, because the ions can be held for long periods in a well-controlled environment. Cooling of the ions has several beneficial effects for such experiments. First, it increases the time that the ions remain in the trap. Second, it can lead to increased precision for measurements of masses, magnetic moments, and optical or microwave spectra. Third, some phenomena, such as the formation of spatially ordered structures of ions, can be observed only at low temperatures.

We use the term "cooling" to mean a reduction in the velocities of the ions. Use of the term does not imply that the ions are in thermal equilibrium with each other nor that they have a well-defined temperature. The motional modes that are cooled may be random or coherent. The secular motion in a Paul (rf quadrupole) trap and the axial motion in a Penning trap are examples of random modes. The micromotion in a Paul trap and the magnetron rotation in a Penning trap are examples of coherent modes. Laser cooling is very effective and has been used to obtain temperatures much less than 1 K.[1],[2] However, since laser cooling depends on the energy level structure of the ions and on the availability of narrowband radiation sources matched to the level spacings, it has been applied only to a few kinds of ions. Other cooling methods are more generally applicable.[3],[4] Resistive cooling, active feedback cooling, or rf sideband cooling can be applied to all ions, since they depend only on the charge and mass of the ions. Collisional cooling is also widely applicable. Sympathetic cooling is the cooling of one ion species through Coulomb collisions with another, laser-cooled ion species. It has been demonstrated with ions in Penning traps[5],[6] and for (a few) ions in Paul traps.[7]

In spectroscopy the main reason for cooling ions is to reduce Doppler shifts. If the transition is broadened by the first-order Doppler shift, which is linear in velocity, cooling the random motion will reduce the observed linewidth. Methods other than cooling can eliminate the first-order Doppler shift. Dicke narrowing[8] and two-photon absorption[9] are two examples. Reduction of the second-order (time dilation) Doppler shift, however, requires cooling.

Dicke narrowing has been applied to many ion trap experiments. It occurs when the atoms or ions are confined to spatial dimensions less than about half the wavelength of the radiation that excites the transition. The lineshape contains a sharp component, which is free of the first-order Doppler effect. In addition, the lineshape contains a broad pedestal or a discrete series of sidebands, depending on whether the motion of the ions is random or periodic. The pedestal or sidebands are reduced in intensity as the spatial confinement is improved.

For radiofrequency or microwave transitions, this effect can be seen without any special cooling techniques. As long as the ions are confined in the trap, they are also confined to dimensions less than the wavelength. Hyperfine resonances a few hertzes wide (Q's of 10^9 to 10^{10}) were observed in early work with uncooled $^3He^+$ ions[10] and with $^{199}Hg^+$ ions.[11] For optical transitions, which have much shorter wavelengths, cooling to temperatures less than 1 K is required to obtain Dicke narrowing. A Dicke-narrowed optical spectrum has been observed in laser-cooled, trapped Ba^+ ions[12] and in Hg^+ ions.[13]

Precise measurements of masses and g-factors also benefit from cooling.[14]–[17] Reduction of the velocities lowers uncertainties due to the relativistic mass increase and due to anharmonicities of the electric potentials. Cooling the magnetron motion in a Penning trap confines ions to regions of smaller spatial extent. Over a small region, the magnetic field can be more uniform and the electric potential more nearly quadratic.

Cooling of trapped ions is also useful in studies of plasmas and ordered structures. With sufficient cooling so that the Coulomb potential energy between nearest neighbors is much greater than their average kinetic energy, the ions can form ordered structures. Ordered, crystal-like structures of highly-charged micrometer particles of aluminum in a Paul trap were observed when the particles were cooled with background gas.[18] Similar structures of a few laser-cooled atomic ions have recently been observed in Paul traps.[7],[19]–[21] A concentric shell structure of several thousand laser-cooled Be^+ ions has been observed in a Penning trap.[22]

II. Resistive cooling

Resistive cooling involves connecting the trap electrodes to an external circuit, so that the ions dissipate their energy by inducing electric currents in the circuit. If there are

no sources of heating besides the coupling to the external circuit, the ions come to equilibrium at the temperature of the external circuit. The method is, in principle, applicable to all ions.

The cooling time can be estimated from a simple model.[3] Consider a single ion of mass m and charge q. The ion is bound by a harmonic restoring force to the interior of a parallel plate capacitor. The separation between the plates is $2z_0$. The ion oscillates in the direction perpendicular to the plates (the z direction) at a frequency ω_z. The plates of the capacitor are connected by a resistor R. The capacitance C of the plates is assumed to be small, so that $R \ll 1/(\omega_z C)$. If the ion moves with a velocity v_z, electric charges are induced on the plates, and a current $i = qv_z/2z_0$ flows through R.[23] Let $W_z = m\langle v_z^2 \rangle$ be the total (kinetic plus potential) energy of the ion. The brackets denote a time average. The average rate at which the moving ion does work on the external circuit is

$$-\frac{dW_z}{dt} = \langle i^2 R \rangle = \frac{q^2 R W_z}{4mz_0^2}. \tag{1}$$

The energy is damped with a time constant equal to

$$t_0 = \frac{4mz_0^2}{q^2 R}. \tag{2}$$

This model can be applied to the axial (z) motion in either a Penning or a Paul trap. In those cases, the external circuit is connected between the two endcap electrodes. It could also be applied to the x or y mode in a Paul trap or the cyclotron mode in a Penning trap. In such a case, the ring electrode must be split, and the external circuit connected between the two halves. This method cannot be applied to the magnetron mode in a Penning trap, because the energy in the magnetron mode, which is mostly potential, *decreases* as the amplitude of the mode increases. Dissipation of energy leads to an increase in the diameter of the magnetron orbit and an increase in the magnetron velocity. As will be discussed in Section III, active feedback can be used to reverse this process.

In a real trap, as opposed to an infinite parallel plate capacitor, the expression for the induced current is $i = B_1 q v_z/(2z_0)$, where B_1 depends on the geometry of the electrodes. The cooling time t_0 is proportional to B_1^{-2}. Methods for calculating B_1 have appeared in the literature.[24],[25] For typical trap designs, $B_1 \approx 0.8$.

Equation (2) shows that resistive cooling is most effective for ions with high charge-to-mass ratios. For a given mass and charge, the cooling time can be reduced by decreasing z_0 or increasing R. In practice, the high value of R is usually obtained by using a parallel inductance L to cancel the reactance due to the capacitance C of the trap electrodes. The effective value of R is then $Q/(\omega_z C)$, where Q is the quality factor of the tuned circuit. Typically, $Q \approx 1000$. However, Cornell *et al.*[16] have used a superconducting tank circuit with $Q = 25\ 000$. This reduces the axial cooling time to 6 s for N_2^+ ions.

Consider N independent ions in a trap. Let $i = \sum_{j=1}^{N} i_j$ be the total current induced by the ions in the external circuit, where i_j is the current induced by the jth ion. Since the ions are assumed to be independent, $\langle i_j i_k \rangle = \delta_{jk} i_0^2$, where the brackets denote a time average and where i_0 is the rms current induced by a single ion. Thus, the total mean-squared induced current, and hence the rate of energy loss, is N times that of a single ion:

$$\langle i^2 \rangle = \sum_{j=1}^{N}\sum_{k=1}^{N}\langle i_j i_k \rangle = \sum_{j=1}^{N}\langle i_j^2 \rangle = N i_0^2. \tag{3}$$

The average energy of N ions is N times the average energy of a single ion. Thus, the time constant for damping the energy of N independent ions is the same as that for a single ion, provided that the relaxation of energy between the center-of-mass mode and the internal modes is fast. This is not necessarily the case, particularly if the trap potentials are very nearly harmonic.

Resistive cooling of the axial motion of a gas of electrons in a Penning trap was observed by Dehmelt and Walls.[26] The cooling time was about 0.1 s. Since there was very little heating from other sources, the temperature of the electrons could be brought down to the temperature of the external circuit. This system was studied in more detail by Wineland and Dehmelt.[27]

Resistive cooling of protons in a Paul trap was demonstrated by Church and Dehmelt.[28] The temperature without resistive cooling was about 12 000 K. Coupling to an external circuit reduced the temperature to 900 K and increased the storage time from 100 s to 3000 s. The cooling time was a few seconds. Heating caused by the rf trap fields ("rf heating") prevented cooling to the temperature of the external circuit.

Resistive destabilization (heating) of the magnetron motion of electrons in a Penning trap was observed experimentally by White, Malmberg, and Driscoll.[29] A resistance was connected between two sections of the cylindrical ring electrode. The magnetron mode is a displacement of the center of the electron plasma from the central axis of the trap. The magnetron mode is also called the $l = 1$, $k_z = 0$ diocotron mode, since it is an excitation that has an $e^{il\theta}$ azimuthal dependence, where $l = 1$, and no dependence on z. The displaced plasma rotates around the axis due to the drift caused by crossed electric and magnetic fields ($\vec{E} \times \vec{B}$ drift). This displacement grew exponentially with time. The observed variation of the growth rate with resistance was in agreement with theory.

III. Active-feedback cooling

Cooling trapped ions by negative electrical feedback has been discussed theoretically and demonstrated experimentally, though it has not yet been widely applied. Like resistive cooling, it can be applied to all ions.

Dehmelt *et al.*[30] have treated theoretically the case in which the feedback is applied continuously, to a single, harmonically bound ion. A displacement of the ion from its equilibrium position induces a signal on, for example, one of the endcap electrodes. Let this signal be amplified

by a factor g and fed back with reversed phase to the other endcap electrode. The fed-back signal forces the ion back toward the equilibrium position. The cooling time of the ion is $(1+g)$ times shorter than for ordinary resistive cooling. However, there is a trade-off between the cooling rate and the final temperature. The thermal noise voltage of the external circuit resistance is also amplified, so the final temperature is $(1+g)$ times higher than for resistive cooling. One potential problem with this method is that the correction signal can be picked up by the detector and interfere with the signal from the ion. This effect could be reduced by coupling part of the correction signal, with a phase reversal, to the detector. If there is more than one ion, the cooling is not as straightforward. To the extent that the electric field from the electrodes is uniform over the ion cloud, only the center of mass of the ions can be detected and corrected, not the deviations of the individual ions from the center of mass. Continuous feedback has been used to cool ions in Penning traps, but details have not been published. (See footnote 26 of Ref. [4].)

Resistive destabilization of the magnetron motion of an electron plasma in a Penning trap was described in Section II. The magnetron mode can be damped with active feedback similar to that described in the preceding paragraph. A signal is picked up on one section of the ring electrode and fed back on another section, with the proper gain and phase shift. Cooling of the magnetron mode by this method has been demonstrated experimentally.[31],[32] The magnetron mode is coherent; that is, the displacements of the individual ions have the same phase. Thus, the cooling of a plasma is similar to that for a single ion.

Stochastic cooling is a kind of negative feedback cooling which has successfully been applied to antiprotons in storage rings.[33] In stochastic cooling, a signal proportional to the average displacement of the particles is picked up on one section of the storage ring. It is amplified and fed back to a set of electrodes, called a kicker, at another section of the ring. The timing is adjusted so that the signal arrives at the electrodes at the same time as the particles. If there were only a single particle in the ring, its displacement from the desired orbit could be brought to zero by proper adjustment of the gain and phase of the amplifier. In practice, there are many particles, whose signals are not resolved. At best, only the center-of-mass deviation of a sample of particles can be brought to zero in a single application of feedback. Stochastic cooling depends on having a spread in the energies of the particles, so that, after a period of time, called the mixing time, the center-of-mass deviation again becomes nonzero and more energy can be extracted. This mixing time is analogous to the time required for coupling of energy between the center-of-mass and internal modes in an ion trap.

Stochastic cooling has been extended to traps by Beverini *et al.*[34]–[36] In a trap, the pick-up and kicker electrodes cannot be separated physically, as they can be in a storage ring. In order to avoid feed-through from the kicker to the pick-up, the detection and correction are separated in time from each other. The amplitude and phase of the center-of-mass motion is measured. Then a short electric pulse is applied, with the proper amplitude and phase to stop the center-of-mass motion. The process is then repeated.

To show that cooling can be obtained by this method, we consider a simplified model.[35] Consider N noninteracting ions, each oscillating at a fixed frequency ω_k $(k = 1, 2, \ldots, N)$. In the real case, there is a spread in frequencies due to space charge and the anharmonicity of the trap potential, but the frequency of a given ion is not fixed. The axial position of the kth ion is

$$Z_k = A_k \sin(\omega_k t + \phi_k). \tag{4}$$

The position of the center of mass is

$$Z_B = \frac{\sum_k A_k \sin(\omega_k t + \phi_k)}{N} \equiv A_B \sin(\omega_B t + \phi_B), \tag{5}$$

where A_B is the instantaneous center-of-mass amplitude.

We assume that the frequencies and amplitudes of the individual ions are distributed randomly. The time average of the square of the center-of-mass amplitude is then

$$\langle A_B^2 \rangle = \langle A^2 \rangle / N, \tag{6}$$

where $\langle A^2 \rangle$ is the ensemble average of the squared amplitude of a single ion. The instantaneous value of A_B^2 fluctuates around the mean value given by Eq. (6) as the phases of the ions change with respect to each other. Let σ_ω be the standard deviation of the ω_k's from their mean value ω_0. The time required for A_B^2 to change significantly (the mixing time) is on the order of $\tau_m = \sigma_\omega^{-1}$.

Suppose a short electric field pulse of amplitude E is applied to the ions at time t^*. The pulse duration τ is assumed to be much shorter than the oscillation period of the ions. Before the pulse, the position of the kth ion is

$$Z_k = A_k \sin(\omega_k t + \phi_k). \tag{7}$$

After the pulse, it is

$$Z_k' = A_k' \sin(\omega_k t + \phi_k'). \tag{8}$$

The change in the squared amplitudes is

$$A_k'^2 - A_k^2 = \left(\frac{qE\tau}{m\omega_k}\right)^2 + 2\frac{qE\tau}{m\omega_k} A_k \cos(\omega_k t^* + \phi_k), \tag{9}$$

where m and q are the mass and charge of an ion. The first term on the right side of Eq. (9) is positive and causes heating. The second term on the right side of Eq. (9) can be positive or negative, depending on the argument of the cosine.

Suppose that the amplitude of the applied field is

$$E = -g\frac{mA_B\omega_0}{q\tau}, \tag{10}$$

where g is the gain. The average change in the squared amplitudes of the particles is

$$\langle A'^2 \rangle - \langle A^2 \rangle = (g^2 - 2g)A_B^2. \tag{11}$$

The ions are cooled if $0 < g < 2$, which makes the right side of Eq. (11) negative. The optimum cooling is obtained for $g = 1$. For this case, we take a statistical average and use Eq. (11) to obtain

$$\langle A'^2 \rangle - \langle A^2 \rangle = -\langle A_B^2 \rangle = -\langle A^2 \rangle / N. \qquad (12)$$

The fractional change in $\langle A^2 \rangle$ is $1/N$ for one feedback pulse. Another pulse can be applied after a period of about τ_m. Thus, the optimum damping rate is approximately $1/(N\tau_m)$.

A more detailed calculation[35] shows that the energy per ion decreases to a limiting value $\epsilon_{\rm lim}$ which depends on the electrical noise and on the gain g. The cooling time $\tau_{\rm cool}$ also depends on g. There is no advantage to increasing g above about 0.5, since $\epsilon_{\rm lim}$ increases rapidly, while $\tau_{\rm cool}$ decreases slowly. For $g < 0.5$, there is a trade-off between $\epsilon_{\rm lim}$ and $\tau_{\rm cool}$, such that their product is approximately independent of g. If the electrical noise is due to thermal noise in the circuit,

$$\epsilon_{\rm lim}\tau_{\rm cool} \approx k_B T t_0, \qquad (13)$$

where k_B is Boltzmann's constant, T is the electrical noise temperature, and t_0 is the time constant for resistive cooling given by Eq. (2).

Beverini *et al.* have carried out an experimental demonstration of stochastic cooling of ions in a Penning trap.[36] The temperature was measured from the noise current induced on the trap electrodes by the ions. They measured a time constant for stochastic cooling of 55 ± 10 s, in good agreement with their calculation. The time constant for resistive cooling was about 160 s. The effectiveness of the cooling was verified by varying the phase of the feedback signal. For a 180° phase shift, heating occurred.

Stochastic cooling is most useful when the ions are initially at high energies. At low energies, the signal-to-noise ratio suffers, and stochastic cooling is less effective.

IV. Collisional cooling

Trapped ions can be cooled by collisions with charged or neutral particles which have lower temperatures. The collisions may lead to loss of the ions through charge-exchange or other reactions, or they may cause perturbations of the energy levels. However, in some cases the benefits of the cooling outweigh these disadvantages. One advantage of collisional cooling over resistive cooling is that it works with ions having a wide range of energies and oscillation frequencies. Because of the required high-Q tuned circuit, resistive cooling works for only a narrow range of oscillation frequencies. Two applications of collisional cooling will be given here: buffer gas cooling of ions in a Paul trap and electron cooling of antiprotons in a Penning trap. The latter is a form of sympathetic cooling (the cooling of one ion species by another). The case where one of the ion species is laser cooled is discussed in Section VI.

Consider collisions of ions in a Paul trap with neutral molecules which have lighter masses than the ions. Major and Dehmelt[37] showed that such collisions should damp the secular motion. The micromotion should not be much disrupted by a collision, but only slightly modified in amplitude and phase, so the trapping should not be affected. Other theoretical work on buffer gas cooling of ions in a Paul trap has been done by Blatt *et al.*[38] and Vedel *et al.*[39]

The final ion temperature results from a balance between heating and collisional cooling. Usually, rf heating[28] is the dominant source of heating. The mechanism of rf heating has recently been clarified. Computer simulations show that rf heating results from the existence of a chaotic regime for a system of many interacting ions in a Paul trap.[40] The chaos does not require stochastic forces. It results from the deterministic, classical dynamics of the coupled, nonlinear system. There are also crystalline or quasiperiodic states, in which the ions do not absorb energy from the rf trapping field. Chaos results in a continuous, rather than a discrete, power spectrum of the motion of the ions. This allows the ions to absorb energy from the rf field. If the density of the ions becomes low enough, their motions become nearly independent and are not chaotic. The rf heating then stops. This is why ions can be confined for long times even without cooling.

In an early experiment, Wuerker *et al.*[18] confined charged micrometer particles of aluminum in a Paul trap. The particle motions could be damped by background gas at a pressures up to 1.3 Pa (10^{-2} Torr). When the pressure was high enough, the particles crystallized into regular patterns. These patterns corresponded to the spatial configurations which minimized the effective potential energy of the system. The time required for the particles to crystallize, starting from a disordered state, increased as the pressure was reduced. An increase in the confinement time of heavy ions (Hg^+) by the introduction of a light buffer gas (Ne) was observed by Dawson and Whetton.[41] This increase in confinement time was interpreted as an effect of collisional cooling. However, the ion temperature was not measured directly.

Schaaf *et al.*[42] measured the temperature of Ba^+ ions confined in a Paul trap in the presence of He buffer gas. They measured the spatial density distribution of the ions by scanning a laser beam across the trap and observing the fluorescence. (Knight and Prior[43] had previously used a similar method to measure the density distribution of Li^+ ions in a Paul trap.) Schaaf *et al.* found the density distribution to be well described by a Gaussian in both the radial and axial directions. Since the strength of the confining potential was known, the ion temperature could be inferred from the width of the spatial distribution. The temperature of the ions decreased as the He pressure was increased. At a pressure of 5×10^{-4} Pa, the temperature was reduced by a factor of 3. At higher pressures, the ions were rapidly lost from the trap.

Cutler *et al.*[44],[45] have collisionally cooled $^{199}Hg^+$ ions with He for use in a microwave frequency standard. The frequency standard is based on measurements of the ground-state hyperfine separation, which is approximately

40.5 GHz. The ions are prepared in the ($F = 0$) ground-state hyperfine level by optical pumping with 194 nm radiation from a ^{202}Hg resonance lamp. This radiation is nearly resonant with the transition from the ($F = 1$) level of the ground state to the $6p\ ^2P_{1/2}$ state. If 40.5 GHz microwave radiation repopulates the $F = 1$ level, the trapped ions absorb and scatter radiation from the lamp, and the scattered radiation is detected by a photomultiplier tube.

The frequency of the hyperfine resonance increased rapidly, by about 5 parts in 10^{12}, as the He pressure was increased from 0 to about 1×10^{-4} Pa.[44] Cutler *et al.* interpreted this as a reduction of the second-order Doppler shift, due to collisional cooling of the secular motion. A slow, linear increase in frequency with pressure remained, which they interpreted as pressure shift of the hyperfine resonance. The fractional pressure shift was 4×10^{-11} Pa^{-1}. The maximum He pressure was approximately 10^{-3} Pa. The addition of He increased the ion storage time, increased the ion density, and stabilized the second-order Doppler shift. The storage time was 2.5×10^3 s.

The amplitude of the micromotion depends only on the intensity of the rf electric fields at the positions of the ions. The fields go to zero at the center of the trap and increase in all directions outward from the center. Hence, the second-order Doppler shift due to the micromotion depends on the spatial distribution of the ions in the trap. The spatial distribution depends on the number of ions, the temperature of the secular motion, and the trap parameters. The He collisions do not cool the micromotion directly, but by cooling the secular motion, they increase the ion density, so the ions are closer to the center of the trap. This reduces the micromotion second-order Doppler shift, since the rf electric fields sampled by the ions are reduced.

Cutler *et al.*[44] measured the second-order Doppler shift of the $^{199}Hg^+$ resonance. For 10^6 ions and typical trap parameters, it was -1.2×10^{-12} times the resonance frequency. An analysis of the sidebands of the Dicke-narrowed spectrum of the hyperfine resonance showed that the temperature of the secular motion was reduced to approximately 500 K by the He.[45] Without cooling the kinetic energy of the ions was about 2 eV, which corresponds to a temperature for the secular motion of 8000 K.

Neutral buffer gas cooling is not used in Penning traps, since such collisions would quickly drive the ions out of the trap. Unlike the Paul trap, which is stable in all directions, the Penning trap is unstable in the radial (x-y) direction. Although the particle orbits are stable in the absence of collisions, there is no restoring force in the radial direction. Rather, there is a balance between an electrostatic force directed away from the axis and a magnetic ($\vec{v} \times \vec{B}$) force directed toward the axis.

However, cooling of one ion species by another *charged* species is feasible in some cases in a Penning trap. The Penning trap can simultaneously hold ions of different charge-to-mass ratios. The stability of the trapping is not destroyed by collisions between the different kinds of ions. There must be some means of removing heat from the second species.

Electron cooling is used in storage rings to cool protons or other particles.[46] In this method, a beam of electrons is sent through a straight section of the storage ring, with a velocity that matches the average velocity of the circulating particles. In the frame moving with the average velocity of the ions, the ions lose energy to the electrons. Since the electrons pass through once and are dumped, they do not have to be cooled. Like stochastic cooling, electron cooling was first demonstrated in storage rings and has recently been adapted to ion traps.

Gabrielse *et al.*[47] have demonstrated cooling of trapped antiprotons by collisions with trapped electrons. Cold antiprotons can be applied to tests of CPT invariance (through measurements of their inertial mass) and to measurements of the gravitational force on antimatter.

In the experiment of Gabrielse *et al.*, pulses containing approximately 3×10^8 5.9 MeV antiprotons were extracted from a storage ring and directed toward the trap. After passing through a titanium window and an aluminum plate, some of the antiprotons were slowed down to 3 keV or less and were captured in a Penning trap. A maximum of about 60 000 antiprotons were captured. The Penning trap was made up of several cylinders, which can be held at different electric potentials. The magnetic field was 6 T.

The longitudinal energy distribution was measured by lowering the confining potential at one end of the trap, so that antiprotons that had an energy greater than the potential could escape. The antiprotons were annihilated when they struck the vacuum enclosure. The charged pions that were created were detected by scintillators. Soon after being trapped, the antiprotons had a broad energy distribution, about 2 keV wide. This distribution was almost unchanged after 2.7 d (in the absence of electron cooling).

Cooling of the antiprotons was observed when electrons were introduced to the trap. To improve the control and monitoring of the electron cooling, a small harmonic well was created inside the trap to hold the electrons. This was done by adjusting the potentials of five of the cylindrical electrodes to approximate a quadrupole potential over a short region. Approximately 10^7 electrons were loaded into the trap from a field-emission point. Since the electrons had low energies, they collected at the minimum of the harmonic well. The electrons cooled, by emitting synchrotron radiation, to the temperature of the surrounding apparatus (4.2 K) with a time constant of 0.1 s. The electrons induced a noise voltage across a parallel resonant circuit, which was connected between one of the electrodes and ground potential. This signal was used to determine the number of trapped electrons.[27]

If antiprotons were introduced after the electrons had been loaded and cooled, the voltage across the resonant circuit increased, due to heating of the electrons by the antiprotons. The time constant for cooling of the antiprotons was approximately 10 s. The energy distribution of the antiprotons was measured after cooling. Energy widths

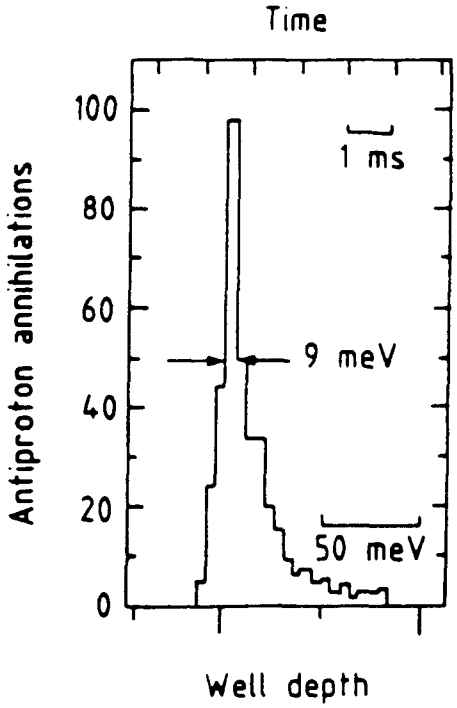

Figure 1: Graph of the number of antiprotons detected as a function of the well depth, which is reduced linearly in time. The width of this distribution reflects the energy distribution of the trapped antiprotons, which are cooled by electrons. (From Ref. [47].)

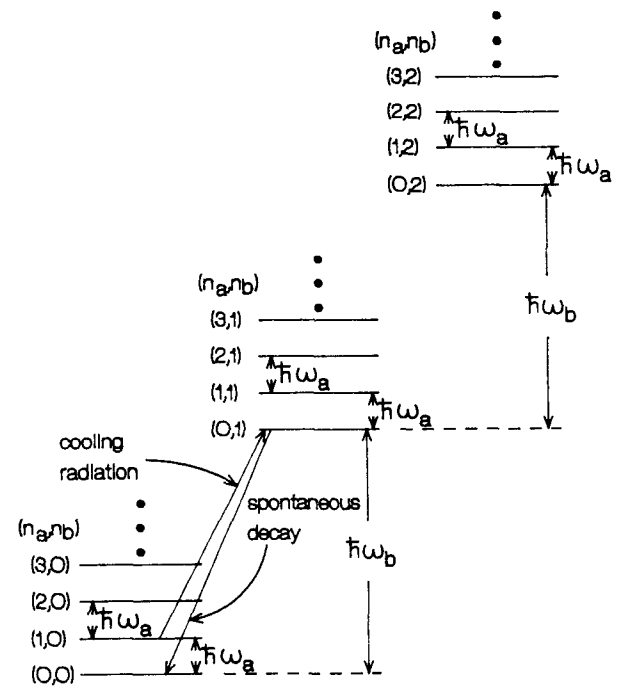

Figure 2: Energy level structure for a particle which is suitable for sideband cooling, having two modes, labeled a and b.

as small as 9 meV were observed (see Fig. 1). Part of this energy spread may have been due to conversion of the space charge potential energy to kinetic energy, as the antiprotons escaped from the trap. Thus, the actual energy spread of the kinetic energies of the antiprotons in the trap may have been even lower.

V. Sideband cooling

Rf sideband cooling and laser cooling (also known as optical sideband cooling) were developed for very different types of measurements. Rf sideband cooling was developed to improve measurements of elementary particle g-factors and of ion masses, by confining the particles to smaller spatial volumes.[48] Laser cooling was developed to reduce Doppler shifts in atomic spectroscopy.[49],[50] However, the two methods are fundamentally the same. Both can be described in terms of mode coupling by a parametric drive.

Figure 2 is a generalized diagram of the mode structure of the cooled system. The system contains two modes, represented by the subscripts a and b. We assume that $\omega_a \ll \omega_b$, so the energy levels separate into manifolds as shown. For simplicity, we assume the level separations in each manifold are the same, so that the energy is $\hbar(n_a\omega_a + n_b\omega_b)$, but this condition is not essential. The cooling relies on (1) parametric coupling of the two modes and (2) relaxation of the b mode by spontaneous decay at a rate γ_b. Spontaneous decay of the a mode is assumed to be negligible. Here, the a mode is a mode of oscillation of an ion, which is to be cooled. Figure 2 shows one part of the cooling process. The particle is initially in the state $(n_a = 1, n_b = 0)$. It absorbs a photon of energy $\hbar(\omega_b - \omega_a)$ from the externally applied field and is driven to the state $(n_a = 0, n_b = 1)$. The particle then spontaneously emits a photon of energy $\hbar\omega_b$ and makes a transition to the state $(n_a = 0, n_b = 0)$. In this process, system's energy is reduced by $\hbar\omega_a$. This is the basic idea of sideband cooling. Heating occurs if the coupling causes absorption of photons of energy $\hbar(\omega_b + \omega_a)$.

In general the process is more complicated. Other absorption processes of the form $(n_a, n_b) \rightarrow (n_a - 1, n_b + 1)$ can be driven by the same applied coupling. The entire distribution of n_a and n_b must be taken into account. Also, even when the coupling is tuned to resonance, other nonresonant transitions can occur, since the resonances are broadened by spontaneous emission.

For rf sideband cooling, ω_b and ω_a are rf frequencies. For example, the b mode might be the axial motion and the a mode might be the magnetron motion, in which case the a mode energy level structure is inverted. Cooling then occurs when the applied coupling is at frequency $(\omega_b + \omega_a)$.

For laser cooling, ω_b is an optical frequency. In this case, the b mode is just a two-level system ($n_b = 0$ or

1). The recoil of the cooled particle upon the spontaneous emission of a photon of frequency ω_b leads to heating. Let $\langle \Delta n_a \rangle$ be the average change in n_a per spontaneous emission. Then,

$$\hbar\omega_a \langle \Delta n_a \rangle = R \equiv (\hbar\omega_b)^2/2mc^2, \tag{14}$$

where R is the recoil energy (the kinetic energy of an initially stationary particle of mass m after emission of a photon of frequency ω_b). We assume that the photon is emitted into free space. That is, the emitted radiation is not influenced by cavity effects. Two cases of laser cooling will be treated separately: $\omega_a \ll \gamma_b$ (sidebands unresolved) and $\omega_a \gg \gamma_b$ (sidebands resolved).

Sideband cooling is closely related to methods of dynamic nuclear orientation.[51] The method of polarization by forbidden transitions is directly analogous to sideband cooling. In this method, an applied rf field drives transitions between states which differ in both the nuclear and electronic spin quantum numbers. This corresponds, for example, to the $(n_a = 1, n_b = 0) \longrightarrow (n_a = 0, n_b = 1)$ transition in Fig. 2. The electronic spin relaxes quickly $[(n_a = 0, n_b = 1) \longrightarrow (n_a = 0, n_b = 0)]$. The nuclear ($a$) mode reaches the same polarization as the electronic (b) mode.

Vyatchanin[52] has discussed, in general terms, the cooling of a quantum oscillator (or two-level system) by parametric coupling to another quantum oscillator (or two-level system) which is more strongly damped. This treatment is directly applicable to sideband cooling.

A. Rf sideband cooling

Rf sideband cooling has been used with great success, first on electrons stored in Penning traps[15] and more recently on atomic or molecular ions stored in Penning traps.[14],[16] It has been especially helpful in reducing the radii of the magnetron orbits of a few particles. Cooling of the magnetron motion is especially important, since most external perturbations, such as collisions with neutral gas, cause the magnetron radius to grow. Without cooling, the ions would eventually strike the ring electrode and be lost. With rf sideband cooling, the ions can be held in a Penning trap for as long as desired.

The case of rf sideband cooling which has received the most attention is when the a mode is the magnetron mode ($\omega_a = \omega_m$) and the b mode is the axial mode ($\omega_b = \omega_z$). This is similar to other cases. Since the a energy levels are inverted, the frequency of the applied drive is tuned to cause transitions of the form $(n_a, n_b) \longleftrightarrow (n_a + 1, n_b + 1)$. Spontaneous emission occurs at frequency ω_b, through coupling of the axial motion to a tuned circuit. We assume that damping of the magnetron motion, in the absence of sideband cooling, can be neglected. Finally, we assume that $\hbar\omega_z \ll k_B T_z$, where T_z is the temperature of the axial motion, and that the axial mode is nearly in thermal equilibrium with the external circuit. The average axial oscillation quantum number $\langle n_z \rangle$ is much greater than 1.

We wish to calculate the magnetron cooling rate. Let the ion be subjected to a parametric coupling in the form of a time-varying potential,[15],[17],[53]

$$V' = \frac{V_d}{d^2} xz\cos(\omega_d t), \tag{15}$$

where d has the dimensions of a length, and x and z are coordinates of the ion. Consider the case where $\omega_d = \omega_z + \omega_m$. The basic mechanism for cooling in this case is as follows: Suppose the x component of the magnetron motion is initially of the form $x = r_m \cos(\omega_m t)$. Substituting this expression for x into Eq. (15), we see that there is a z component of electric field ($-\partial V'/\partial z$) at frequency ω_z, which can excite the axial motion. The axial motion is then damped by the tuned circuit, at a rate γ_z. By this parametric process, the magnetron motion is transferred to the axial motion, which is then removed from the ion by the tuned circuit. We can show from Eq. (15) that the magnetron-induced axial motion gives rise to an electric field in the x direction, and that this field has frequency ω_m and the correct phase to damp the magnetron motion.[17],[53]

If saturation can be neglected, that is, if the axial damping is fast compared to the rate of energy transfer between the axial and magnetron modes, then the magnetron amplitude decays with a rate,[17],[53]

$$\gamma_m \approx \frac{q^2 V_d^2}{4m^2\omega_z\gamma_z(\omega_c - 2\omega_m)}. \tag{16}$$

Brown and Gabrielse[17] have considered a more general case, where saturation is allowed and where ω_d is not necessarily equal to $\omega_z + \omega_m$.

Figure 3 shows rf sideband heating and cooling of a single electron in a Penning trap. The magnetron radius r_m, plotted on the vertical axis, increases when the drive is applied at angular frequency ω_d equal to $\omega_z - \omega_m \equiv 2\pi(\nu_z - \nu_m)$. It shrinks when $\omega_d = \omega_z + \omega_m$.

If damping of both the axial and magnetron motions can be neglected, this type of parametric coupling leads to a sinusoidal exchange of the energies in the two modes. Parametric coupling of modes of ions in a Penning trap has been studied theoretically, for this case, by Cornell *et al.*[54]

The exchange of excitation between modes in a Penning trap, by coupling with a parametric drive, has been used to cool the cyclotron motion of ions in a recent experiment at MIT.[16] In this experiment, the ratio of the masses of CO^+ and N_2^+ was measured to an accuracy of 4×10^{-10}. This is the most accurate mass comparison to date. A short rf pulse of the appropriate amplitude and duration exchanges the phases and actions of the cyclotron and axial modes. The action of the rf pulse is mathematically like that of a π pulse applied to a two-level system. Since the axial motion is kept cold by resistive cooling, application of the π pulse cools the cyclotron motion. The axial motion is heated immediately after the pulse, but is cooled again resistively. Ordinary rf sideband cooling,

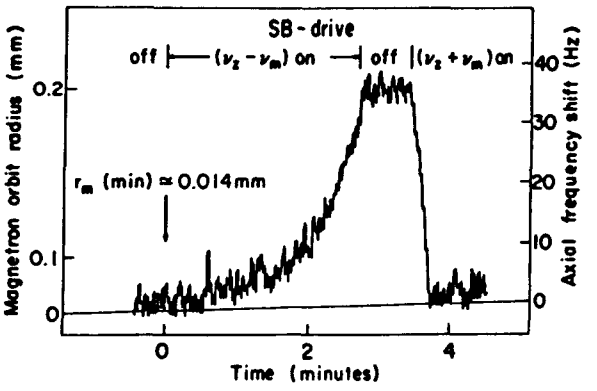

Figure 3: Rf sideband heating and cooling of the magnetron mode of a single electron in a Penning trap. The vertical scale is the axial oscillation frequency. This is related to the magnetron radius because of a deliberately applied magnetic inhomogeneity "bottle." (From Ref. [15].)

with cw excitation at frequency $\omega_z + \omega_m$ was used to cool the magnetron motion.

For both cw and pulsed parametric coupling, the minimum achievable temperature is determined by heating of the axial motion by thermal noise. When the axial motion is coupled to the tuned circuit, it has an amplitude due to random thermal noise. This causes a random excitation of the magnetron motion, given by Eq. (15), through the parametric coupling between the modes. The resulting noise excitation of the magnetron motion (or cyclotron motion in in the case of the MIT group) gives the limit for sideband cooling. This theoretical limit can be calculated from the equations of motion,[17],[53] by thermodynamic arguments,[55] or by detailed-balance arguments.[17] The result, for the case where $k_B T_b \gg \hbar\omega_b$, is

$$|T_a(\min)| = \frac{\omega_a}{\omega_b}|T_b|. \qquad (17)$$

Cooling is routinely observed in several experiments, but the temperatures achieved are somewhat higher than the limit given by Eq. (17).

B. Optical sideband cooling (resolved sidebands)

In optical sideband cooling, the b mode is an internal optical transition in a trapped particle. Generally, the transition is an electronic transition (at an optical frequency) which relaxes radiatively by spontaneous emission at the rate γ_b. The quantum number n_b takes only the values 0 and 1. Here we consider the resolved sideband case ($\gamma_b \ll \omega_a$), where ω_a is the oscillation frequency of the cooled mode of the bound particle. (The unresolved sideband case will be discussed in Section V. C.) Optical sideband cooling of trapped ions and neutral atoms has been an active field of research.[1],[2] New methods of laser cooling, which are distinct from optical sideband cooling, have recently been developed for neutral atoms.[56]–[59] These methods have not yet been applied to trapped ions.

The parametric coupling from the ground state vibrational manifold ($n_b = 0$) to the excited state vibrational manifold ($n_b = 1$) is done with narrowband radiation. The width $\Delta\omega_b$ of the radiation should be less than or on the order of γ_b for optimal cooling. It should be resonant with the first lower sideband at frequency $\omega_b - \omega_a$. If the recoil energy R [defined in Eq. (14)] is less than the vibrational energy $\hbar\omega_a$, then sideband cooling is possible. (Otherwise the cooling which results from the photon absorption is undone by the recoil from the photon emission.) An atom absorbs photons of energy $\hbar(\omega_b - \omega_a)$ and reemits photons of average energy $\hbar\omega_b$. Hence, on the average, each scattered photon reduces the atom's vibrational energy by $\hbar\omega_a$ and the atom's vibrational quantum number n_a by 1. In this way, it is possible to make the mean quantum number $\langle n_a \rangle$ much less than 1, provided that $R \ll \hbar\omega_a$. If $\langle n_a \rangle \ll 1$ for all motional degrees of freedom, then the atom resides in the ground state level of the confining potential most of the time.

Optical sideband cooling of the thermal degrees of freedom of many simultaneously trapped particles is possible in principle. However, the kinetic energy in coherent degrees of freedom usually limits the minimum attainable kinetic energy. The higher energies of the coherent motions in both the Penning trap and in the Paul trap are consequences of Coulomb repulsion.

In a Penning trap, any ion that does not lie on the symmetry axis of the trap rotates about the axis, due to crossed electric and magnetic fields ($\vec{E} \times \vec{B}$ drift). This rotation gives rise to a kinetic energy that depends quadratically on the radial distance of the ion from the trap axis. It might be possible for a few ions to lie along the symmetry axis. However, if there are many ions, the combined effects of the trapping fields and the mutual Coulomb repulsion of the ions will force some of the ions to lie at a finite distance from the axis.

In a Paul trap, the Coulomb repulsion between ions is balanced by a force from the trapping potential, directed toward the center of the trap. Only one ion can occupy the center of the trap, where the applied trap fields go to zero. Any offset of an ion from the trap center leads to micromotion of the particle at the frequency of the applied rf field. The kinetic energy in this nonthermal motion cannot be reduced by sideband cooling. So, although it is possible to reduce the kinetic energy of the secular motion for more than one ion, the kinetic energy in the micromotion remains undiminished and can be substantial. This limitation could be overcome in a Paul trap with linear geometry.[60]–[62] In such a trap, the magnitude of the rf field approaches zero on a line, rather than at a point.

Even for a single ion, optical sideband cooling in a Penning trap is difficult. The magnetron, cyclotron, and axial frequencies cannot be degenerate for stable trapping. This leads to a complicated sideband spectrum.[63] The spectrum contains sidebands not only at the fundamental motional frequencies (for example, $\omega_b \pm \omega_m$), but also all intermodulation products ($\omega_b + j\omega_c' + k\omega_z + \ell\omega_m$, where j, k, ℓ are any integers). At high temperatures this spec-

trum is nearly continuous. Therefore, precooling to near the Dicke limit ($x_{\rm rms} \ll \lambda/2$) is required before sideband cooling is done. Here, $x_{\rm rms}$ is the rms value of the x coordinate of the ion, and $\lambda = 2\pi c/\omega_b$)

Attaining the lowest vibrational energies (the minimum $\langle n_a \rangle$) for all degrees of freedom would require cooling with a laser tuned to an individual sideband for each of the three motional degrees of freedom. Finally, the condition that the recoil energy R be much less than any of the motional energies is more readily satisfied in the Paul trap than in the Penning trap.

For both Paul and Penning traps, the sideband cooling limit for a single ion is

$$\langle n_a \rangle = \frac{1}{4}\left(\alpha + \frac{1}{4}\right)(\gamma_b/\omega_a)^2, \tag{18}$$

where α depends on the angular distribution for photon emission and is of order 1. Equation (18) is valid when the intensity of the cooling radiation is below saturation and when $\gamma_b/\omega_a \ll 1$. The limit results from a balance between cooling and heating. The most important cooling process is on-resonance absorption of a photon on the lower sideband ($n_a \longrightarrow n_a - 1$), followed by emission on the carrier ($n_a \longrightarrow n_a$). (In the Dicke limit, almost all emission is on the carrier.) The most important heating processes are (1) off-resonance $n_a \longrightarrow n_a + 1$ transitions, followed by emission on the carrier, and (2) off-resonance $n_a \longrightarrow n_a$ transitions, followed by emission on the lower sideband ($n_a \longrightarrow n_a + 1$). This result was derived by Neuhauser *et al.*[64] and others.[55],[65] Wineland *et al.*[63] considered the effect of finite laser bandwidth and also explicitly treated resolved optical sideband cooling in a Penning trap.

Lindberg[66] and Javanainen *et al.*[67] have derived the steady-state energy of a single two-level atom, confined in a one-dimensional harmonic well of frequency ω_a, for arbitrary laser frequency, laser intensity, and γ_b/ω_a:

$$\langle E_a \rangle \equiv \hbar\omega_a\left(\langle n_a \rangle + \frac{1}{2}\right) = \tag{19}$$
$$\frac{\hbar}{-4\Delta}[\omega_a^2(\Delta^2 + \gamma_2^2 + 6\kappa^2) + 4\gamma_2^2(\Delta^2 + \gamma_2^2 + 2\kappa^2)]^{-1}$$
$$\times\{\alpha[\omega_a^2(\Delta^2 + 5\gamma_2^2 + 4\kappa^2 - \omega_a^2)^2$$
$$+4\gamma_2^2(\Delta^2 + \gamma_2^2 + 2\kappa^2 - 2\omega_a^2)^2]$$
$$+\omega_a^2(\Delta^2 + \gamma_2^2 + 2\kappa^2)(\Delta^2 + 5\gamma_2^2 + 8\kappa^2 + \omega_a^2)$$
$$+4\gamma_2^2[(\Delta^2 + \gamma_2^2 + 2\kappa^2)^2 + 2\kappa^2(\Delta^2 - 3\gamma_2^2 - 3\omega_a^2)]\}.$$

This result holds in the limit that $R/(\hbar\omega_a) \ll 1$. Here, κ is the Rabi frequency, which is proportional to the square root of the laser intensity, and $\gamma_2 \equiv \gamma_b/2$. The detuning Δ is defined as $\Delta \equiv \omega - \omega_b$, where ω is the laser frequency. It must be negative (laser frequency below resonance), or else there is no steady state. Equation (20) reduces to Eq. (18) in the limit $\kappa \to 0$, $\gamma_2/\omega_a \to 0$, and $\Delta \to -\omega_a$.

The value of $\langle n_a \rangle$ can be determined experimentally from the relative transition strengths for on-resonance absorption on the first lower and upper sidebands.[63],[68] When $\langle n_a \rangle \ll 1$, the strength of resonant absorption on the lower sideband ($\omega_b - \omega_a$), which is proportional to $\langle n_a \rangle$, approaches zero. When the ion is in the lowest kinetic energy state ($n_a = 0$), it is no longer possible to extract vibrational quanta from the ion. The strength of the upper sideband ($\omega_b + \omega_a$) is proportional to $\langle n_a \rangle + 1$. Thus, if the lower and upper sidebands are probed with saturating power, the ratio of their absorption strengths becomes independent of power and directly gives $\langle n_a \rangle$.

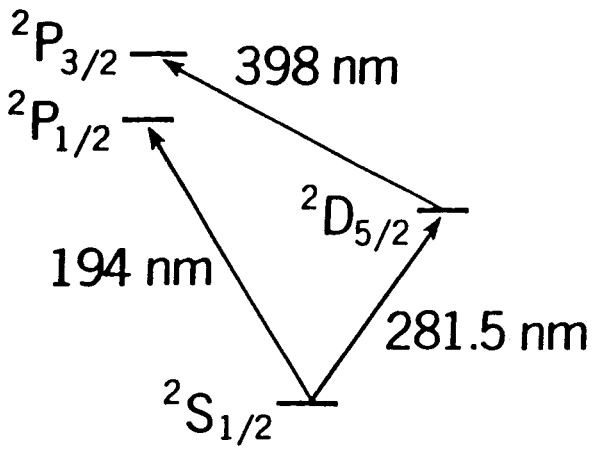

Figure 4: A simplified energy level diagram of Hg^+, showing the transitions used for optical sideband cooling. (From Ref. [68].)

A single $^{198}Hg^+$ ion, confined in a small Paul trap, has been laser-cooled, in the resolved sideband limit, to near the zero-point energy of motion.[68] The electronic energy levels of Hg^+ are shown in Fig. 4. The procedure was as follows: First, the ion was laser-cooled in the Doppler-cooling limit (discussed in Section V. C. 1.), by radiation scattered from the strongly allowed 194 nm first resonance line. This reduced the temperature to about 1.7 mK and the mean vibrational quantum number $\langle n_a \rangle$ to about 12. The secular frequency ($\omega_a/2\pi$) was about 3 MHz. Laser radiation, tuned to the first lower vibrational sideband of the narrow 282 nm $5d^{10}6s\ ^2S_{1/2}$-to-$5d^9 6s^2\ ^2D_{5/2}$ transition, was then applied to the ions. In order to speed the cooling process, 398 nm laser radiation was used to drive ions from the $^2D_{5/2}$ state to the $5d^{10}6p\ ^2P_{3/2}$ state, which decays rapidly to the ground $^2S_{1/2}$ state. This allowed the absorption of 282 nm photons to proceed faster than the 90 ms natural lifetime of the $^2D_{5/2}$ state would have allowed. After scattering enough photons to remove 12 vibrational quanta from the ion for each degree of freedom, laser radiation of saturating intensity probed the 282 nm sideband spectrum.

The results are shown in Fig. 5. The fact that the intensity of the lower sideband (S_L) is much less intense than the upper sideband (S_U) indicates that $\langle n_a \rangle \ll 1$. From these data, it was deduced that the ion was in the lowest vibrational quantum state, for the y and z degrees of freedom, 95% of the time. This corresponds to a temperature of 47 ± 3 μK. Since the 282 nm probe beam was

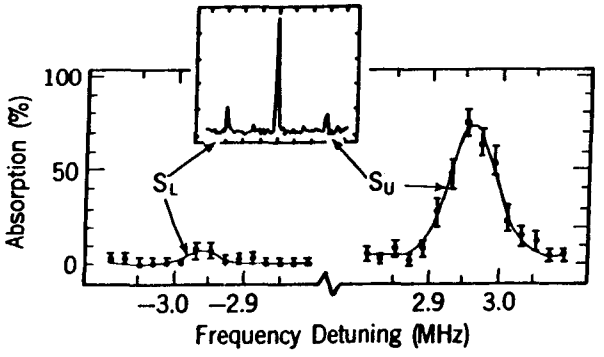

Figure 5: Absorption spectrum of the $^2S_{1/2}$-to-$^2D_{5/2}$ electric quadrupole transition. The vertical scale is the probability that the ion makes a $^2S_{1/2}$-to-$^2D_{5/2}$ transition when the probing 282 nm radiation is applied. The spectrum in the small inset was taken before the sideband cooling was applied. S_L is the first lower sideband (frequency $\omega_b - \omega_a$); S_U is the first upper sideband (frequency $\omega_b + \omega_a$). The asymmetry between S_U and S_L indicates that the average quantum number for the oscillation of the ion is much less than 1. (From Ref. [68].)

nearly orthogonal to the motion in the x direction, the measurement was not very sensitive to the energy in this degree of freedom. For this reason, a precise measurement of the x vibrational energy could not be made in this experiment. Cooling of all degrees of freedom could be done by orienting the 282 nm beam at a different angle with respect to the trap axes.

C. Optical sideband cooling (unresolved sidebands)

In most experiments on optical sideband cooling, a strongly allowed optical transition, which has a natural linewidth γ_b of approximately 20 MHz or more, is used. This is much greater than the motional frequencies of the trapped ion (typically 3 MHz or less). The sideband structure of the optical resonance is not resolved. It is then natural to treat the motion of the ion classically. The scattering of laser light by the ion leads to a force on the ion. The scattering takes place in a time of about γ_b^{-1}, much less than the orbital period, so it can be assumed to give an instantaneous impulse to the ion. In the low intensity limit, the average force on an atom is the product of the average momentum transfer per scattering (the photon momentum $\hbar\vec{k}$) and the scattering rate. The size of the force is maximized when the laser frequency, in the frame moving with the ion, is in resonance with the optical transition. Fluctuations in the force arise from (1) the randomness of the direction of the scattered photon and (2) the random time distribution of the scattering events. As an alternative to this semiclassical approach, the cooling can also be explained with the general concepts of sideband cooling.[55],[64] It is necessary, in this case, to include a large number of sidebands in the analysis.

1. Paul traps

An ion in a Paul trap can be treated, to a good approximation, as a particle in a three-dimensional harmonic potential, undergoing simple harmonic motion at the secular frequencies. The micromotion can be added by numerical simulation,[20],[40],[69] but is hard to treat analytically.

For simplicity, consider an ion moving in a harmonic well in the x direction, interacting with a laser beam propagating in the $+x$ direction. If the ion has a velocity v and the laser detuning is Δ, the average force is

$$\begin{aligned} F_{\rm av} &\approx \frac{I\sigma_0}{\hbar\omega}\frac{\gamma_2^2}{[(\Delta - kv)^2 + \gamma_2^2]}\hbar k \\ &\approx \frac{I\sigma_0\gamma_2^2}{\hbar\omega(\Delta^2+\gamma_2^2)}\left[1 + \frac{2\Delta kv}{\Delta^2+\gamma_2^2}\right]\hbar k \\ &\equiv F_0 - m\Gamma v, \end{aligned} \tag{20}$$

where I is the laser intensity, σ_0 is the resonant scattering cross-section, and m is the mass of the ion. The approximation holds for low intensities $[I\sigma_0/(\hbar\omega) \ll \gamma_2]$ and low velocities $(kv \ll \gamma_2)$.

In the last line of Eq. (20), F_0 is a velocity-independent force, which slightly displaces the equilibrium position of the ion, and $m\Gamma v$ is a damping force (for $\Delta \ll 0$), which leads to cooling. The fluctuations in the force, which are due to the discreteness of the photon scatterings, have properties similar to electronic shot noise.[70] The equation of motion for the ion position is the same as that of a series RCL (resistance-capacitance-inductance) circuit driven by a fluctuating emf. The steady-state energy of the ion can be calculated by methods analogous to those used to calculate noise in electronic circuits.[71]

The steady-state energy is

$$\langle E_a \rangle = \frac{(1+\alpha)\hbar(\gamma_2^2+\Delta^2)}{-4\Delta}. \tag{21}$$

This agrees with Eq. (20) in the limit where $\kappa \to 0$ and $\omega_a/\gamma_2 \to 0$. The lowest energy is obtained when $\Delta = -\gamma_2$:

$$\langle E_a \rangle = k_B T = \frac{(1+\alpha)\hbar\gamma_2}{2}. \tag{22}$$

For $\alpha \approx 1$, this yields the Doppler-cooling limit, $k_B T \approx \hbar\gamma_2 = \hbar\gamma_b/2$, which also holds, under suitable conditions, for free two-level atoms.[55],[64],[72] For typical cases, $T \approx 1$ mK.

Optical sideband cooling in a Paul trap was first demonstrated experimentally by Neuhauser *et al.*[64] They cooled Ba^+ ions, using the 493 nm, $6s\ ^2S_{1/2}$-to-$6p\ ^2P_{1/2}$ transition. One complication in the experiment was that the $6p\ ^2P_{1/2}$ state can decay to the metastable $5d\ ^2D_{3/2}$ state. A 650 nm laser beam drove the ions from the $5d\ ^2D_{3/2}$ state back to the $6p\ ^2P_{1/2}$ state, so that the cooling could continue. The effect of cooling was to increase the storage time of the ions in the trap. Figure 6 shows the effect of laser cooling when the 493 nm laser is tuned below the $6s\ ^2S_{1/2}$-to-$6p\ ^2P_{1/2}$ transition. Later,

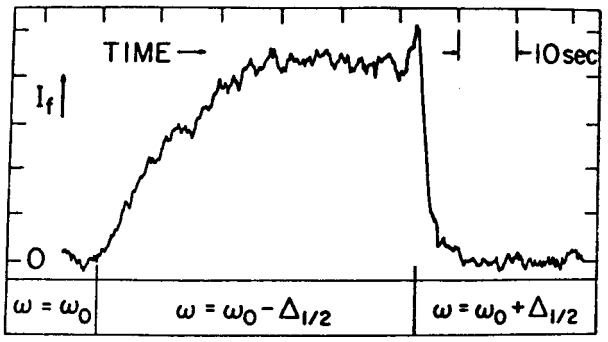

Figure 6: The first optical sideband cooling of ions in a Paul trap. The Ba^+ fluorescence intensity is plotted as a function of time. The trap is initially empty when the laser frequency ω is equal to the atomic resonance ω_0. The fluorescence increases when ω is tuned below the atomic resonance to $\omega_0 - \Delta_{1/2}$. It decreases rapidly when $\omega = \omega_0 + \Delta_{1/2}$. Here, $\Delta_{1/2}$ is the resonance half-width in the discharge tube used as a reference. (From Ref. [64].)

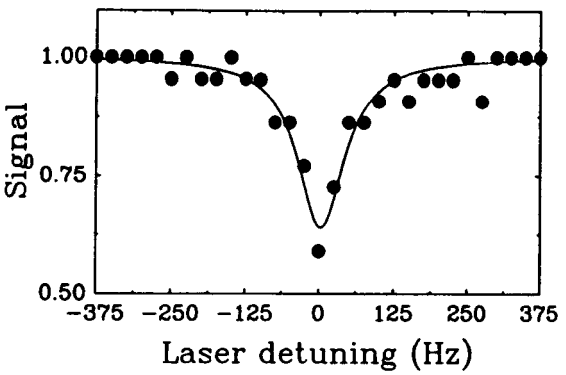

Figure 7: Absorption resonance for a single Zeeman-hyperfine component of the $5d^{10}6s\ ^2S_{1/2}$-to-$5d^96s^2\ ^2D_{5/2}$ transition in a single $^{199}Hg^+$ ion. The probability that the ion did not make a transition to the $^2D_{5/2}$ state is plotted on the vertical axis. The relative frequency of the 563 nm laser, which was frequency doubled to 282 nm, is plotted on the horizontal axis. The curve is a least-squares fit to a Lorentzian. The width of the Lorentzian is about 90 Hz. (From Ref. [74].)

Neuhauser *et al.* were able to estimate the temperature of a single ion to be less than 36 mK, from the size of its photographed image.[73]

Bergquist *et al.*[13] cooled a single Hg^+ ion, confined in a Paul trap, using the 194 nm $5d^{10}6s\ ^2S_{1/2}$ to $5d^{10}6p\ ^2P_{1/2}$ transition. The narrow 282 nm $5d^{10}6s\ ^2S_{1/2}$ to $5d^96s^2\ ^2D_{5/2}$ transition was used to determine the temperature. The relative intensities (transition probabilities) at the frequencies of the carrier (ω_b) and at the first sidebands ($\omega_b \pm \omega_a$) were measured. Equation (44) of Ref. [55] relates the relative intensities of the carrier and sidebands to the temperature. The measured temperature was 1.6 ± 0.5 mK, in good agreement with the Doppler-cooling limit of 1.7 mK. In later experiments with a single $^{199}Hg^+$ ion, the 282 nm carrier was observed with a linewidth of under 200 Hz ($Q \approx 5 \times 10^{12}$).[74] A typical resonance curve is shown in Fig. 7. Similar experiments have been done with a single Ba^+ ion by Nagourney *et al.*[75] They observed the 1.8 μm $6s\ ^2S_{1/2}$-to-$5d\ ^2D_{5/2}$ transition with a linewidth of about 40 kHz.

Optical sideband cooling of ions in Paul traps has been applied to other areas of physics. Radiative lifetimes have been measured in single, laser-cooled ions of Mg^+ (Ref. [76]), Ba^+ (Refs. [77] and [78]), and Hg^+ (Ref. [79]). Rates for quenching of a metastable state by various gases have been measured in single, laser-cooled Ba^+ ions.[78] The fluorescence from a single cooled ion can be easily observed. This has made it possible to observe certain inherently quantum properties of the electromagnetic field, such as photon antibunching[80]–[82] and quantum jumps.[77],[81],[83]

If several ions in a Paul trap are cooled to a low enough temperature, they arrange themselves into spatial configurations which minimize the potential energy. Such ordered patterns have been observed with laser cooled Mg^+ (Ref. [19]), Hg^+ (Ref. [7]), and Ba^+ (Refs. [20] and [21]). Phase transitions of these ions between ordered and disordered states have been studied, especially with regard to classical chaos.[19],[20],[40]

2. *Penning traps*

In some respects, optical sideband cooling of ions in Penning traps and Paul traps are similar. The axial and cyclotron modes are cooled if the laser is detuned below resonance. The differences arise from the $\vec{E} \times \vec{B}$ rotation of the ions around the trap axis. The rotation frequency of a single ion is the magnetron frequency ω_m. The rotation frequency of many ions is increased by electric fields caused by space charge.

Cooling of the rotation requires control of L_z, the z component of the canonical angular momentum of the ions.[84] For a single ion of charge q and mass m, the canonical angular momentum is $\vec{\ell} = \vec{r} \times \vec{p}$, where $\vec{r}$ and $\vec{p}$ are the position and the canonical momentum of the ion. The canonical momentum is defined as $\vec{p} \equiv m\vec{v} + (q/c)\vec{A}(\vec{r})$, where $\vec{v}$ is the velocity of the ion and $\vec{A}(\vec{r})$ is the vector potential at $\vec{r}$. For convenience, we choose the symmetric gauge, where $\vec{A}(\vec{r}) = \frac{1}{2}\vec{B} \times \vec{r}$ and $\vec{B}$ is the magnetic field. The z component of $\vec{\ell}$ is

$$\ell_z \equiv m v_\theta \rho + \frac{qB\rho^2}{2c}, \tag{23}$$

where v_θ and $\rho = (x^2+y^2)^{1/2}$ are the azimuthal component of the velocity and the cylindrical radius of the ion. For N identical ions of charge q and mass m,

$$L_z \equiv \sum_{i=1}^{N} \left(m v_{\theta,i} \rho_i + \frac{qB\rho_i^2}{2c} \right), \tag{24}$$

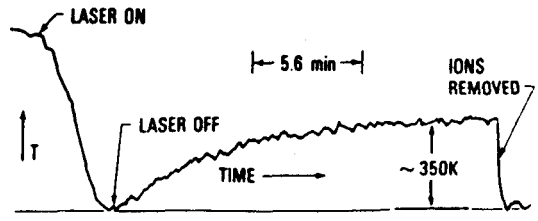

Figure 8: The first optical sideband cooling of ions in a Penning trap. The vertical scale is the axial temperature of the Mg^+ ions. The ions were heated by the laser before the beginning of the data. The laser was turned on with a frequency below the resonance frequency, cooling the ions to below 40 K. After the laser is turned off, the ions thermalize to the ambient temperature of 350 K. (From Ref. [87].)

where v_{θ_i} and ρ_i are the azimuthal component of the velocity and the cylindrical radius of the ith ion. If the trap has perfect axial symmetry and there are no interactions with the outside world, L_z is conserved. For the typical case, where the rotation frequency is much less than the cyclotron frequency $qB/(mc)$, the second term of Eq. (24) dominates. Conservation of L_z is then equivalent to conservation of the mean squared radius of the ion plasma. A more general proof of the fact that conservation of L_z leads to radial confinement has been given by O'Neil.[85]

There are confined thermal equilibria, whose properties are determined by the three conserved quantities: N, L_z, and the energy.[86] One property of these thermal equilibria is uniform rotation. That is, the velocity distribution, with respect to a rotating frame, is Maxwellian. In a continuous fluid model, the zero-temperature equilibria are uniformly rotating spheroids of uniform density.[84]

Asymmetries of the trap fields apply torques to the ions, so that L_z is not perfectly conserved. This leads to radial expansion. The light-scattering force from a laser beam can be used to apply a torque to the ions. This torque can be used to cancel the torques from the trap fields and establish radial confinement. Such a torque could also be applied with a neutral atomic beam.

The first experimental demonstration of optical sideband cooling in a Penning trap was made by Wineland *et al.*[87] Approximately 5×10^4 Mg^+ ions were cooled by a 280 nm laser beam tuned slightly lower in frequency than one Zeeman component of the $3s\ ^2S_{1/2}$-to-$3p\ ^2P_{3/2}$ transition. The temperature of the ions was determined from noise current induced on the electrodes. The ions were cooled below 40 K. This was the measurement limit set by the electronic noise. Figure 8 shows the decrease in temperature when the laser is turned on, with a frequency below the resonance frequency. Collisions with neutral gas molecules cause the ions to rethermalize after the laser is turned off. Optical sideband cooling of Mg^+ ions in Penning traps has also been observed by Plumelle *et al.*[88] and by Thompson *et al.*[89]

Drullinger *et al.*[5] laser-cooled Mg^+ ions to approximately 0.5 K. The temperature was determined from the Doppler width of the transition. They also demonstrated the connection between the plasma radius and the canonical angular momentum. Placing the laser beam on one side or the other of the axis exerted a torque on the ions, which caused radial expansion or contraction of the plasma.

If the cooling laser beam is perpendicular to the rotation axis, the steady state temperature can be calculated from the rotation frequency and the laser frequency and intensity profile.[90] The temperature is calculated from the requirement that the work done on the ions by the laser beam, averaged over the velocity distribution, be zero. The temperature increases with the rotation frequency, the distance between the axis and the laser beam, and the laser intensity. Itano *et al.*[90] obtained good agreement between the calculated and observed temperatures of Be^+ ions, over a wide range of experimental parameters.

In general, lower temperatures are obtained when the cooling laser beam is not perpendicular to the magnetic field. Brewer *et al.*[91] observed temperatures of Be^+ ions as low as 2 mK, when cooling beams were directed simultaneously at 90° and at 55° with respect to the magnetic field. They also confirmed experimentally the relationship between the rotation frequency and the shape of the ion plasma that was predicted in Ref. [84].

The reduction of Doppler shifts and the spatial confinement which results from the cooling has been useful for several experiments. Some examples are in mass spectroscopy[92] and in the observation of single-ion quantum jumps.[93] High-resolution optical spectra of Mg^+ (Ref. [5]) and Be^+ (Ref. [94]), have yielded hyperfine constants, fine structure separations, and isotope shifts. Rf–optical double resonance experiments have yielded ground-state hyperfine constants and g-factors of $^{25}Mg^+$ (Ref. [95]) and $^9Be^+$ (Ref. [92]).

A frequency standard, based on a 303 MHz ground-state hyperfine transition in laser cooled $^9Be^+$ ions, has been demonstrated.[96] The estimated error of this standard, about 1 part in 10^{13}, is comparable to that of primary cesium atomic beam standards. A search was made for any dependence of the frequency on the orientation of the magnetic field, with null results.[97] This is one of the most precise tests of local Lorentz invariance.

Ion plasmas which have high enough densities and low enough temperatures are called strongly coupled. In such plasmas, the positions of neighboring ions are correlated with each other, as in a liquid or a solid. Strongly coupled plasmas have been produced, by optical sideband cooling of ions in Penning traps.[98] Spatial ordering, in the form of concentric shells, has been directly observed by imaging of the resonance fluorescence.[22] Numerical simulations predict the existence of such shells.[99],[100]

VI. Sympathetic laser cooling

Because laser cooling depends on having a favorable energy level structure, it has been demonstrated on only a handful of different ion species (Be^+, Mg^+, Ba^+, Hg^+, and Yb^+). However, the technique of sympathetic laser cooling[6],[101] can be used to cool any ion species to sub-kelvin temperatures. In sympathetic laser cooling, two different ion species are loaded into a trap. One of the ion species is laser cooled. The other species is sympathetically cooled by its Coulomb interaction with the laser-cooled species. In a Penning trap, the sympathetically cooled species must have the same sign of charge as the laser-cooled ion species. Sympathetic laser cooling may enable the formation of cold, strongly coupled plasmas of positrons.[101]

In principle, sympathetic laser cooling can be done in either a Paul or a Penning trap. In a Paul trap, ordered, crystal-like structures in which at least one of the "lattice" sites was occupied by an impurity (perhaps another isotope) that was not directly laser-cooled have been observed.[7],[21],[102] These ions did not fluoresce, but their presence was detected by their effect on the positions of the other ions. This shows that at least a small number of ions can be sympathetically laser-cooled to sub-kelvin temperatures in a Paul trap. (If the nonfluorescing ions had not been cold, the ordered structures would not have formed.) In general, rf heating and the available cooling laser power limits the number of ions that can be sympathetically laser-cooled in a Paul trap. In the Penning trap, which is free of rf heating, experiments have demonstrated sympathetic laser cooling on much larger numbers of ions.[6],[103],[104] Sympathetic laser cooling is a form of collisional cooling between charged particles. However, the low temperatures that can be achieved with laser cooling produce some new features.

For simplicity, consider two ion species with like charges but different masses confined in the same Penning trap. The rotation of the ions about the magnetic field axis of the trap produces a centrifugal separation of the ion species.[105] For typical experimental conditions, where the rotation frequency is much less than the cyclotron frequency, the rotation frequency is nearly independent of ion mass. This is because the rotation is a circular $\vec{E} \times \vec{B}$ drift. The velocity for linear $\vec{E} \times \vec{B}$ drift is independent of mass. However, a more exact calculation shows that the higher mass ions rotate at a higher frequency than the lower mass ions, if they occupy they occupy the same volume in the trap, so that they are subjected to the same electric fields.

The equal and opposite momentum transfers due to the different rotation frequencies will cause the lighter ions to move in (toward the trap axis) and the heavier ions to move out. This separation of the ions continues until both species are rotating at the same frequency, as a result of the change in the space charge induced electric fields. Uniform rotation is one characteristic of thermal equilibrium of a nonneutral plasma.[105] In the low temperature limit, the separation is complete, and a gap forms between the ion species.[6],[101],[105] The higher mass ions form a "doughnut" around the lower mass ions. Although the separation of the ion species limits the thermal coupling between the species, sympathetic laser cooling has been demonstrated experimentally in a number of cases.[5],[6],[103]

The first temperature and density measurements on sympathetically cooled ions were done with Hg^+ ions, sympathetically cooled by laser-cooled Be^+ ions.[6] The ions were confined in a Penning trap. Up to 12 000 Be^+ ions, with temperatures less than 0.2 K, sympathetically cooled a larger number of Hg^+ ions to temperatures less than 1.8 K. As expected, Hg^+ ions were observed at larger distances from the trap axis than the Be^+. However, since the laser beams which probed the spatial distribution of the ions were perpendicular to the trap axis, not along the axis, complete separation could not be experimentally verified. The sympathetic cooling also prevented the radial diffusion of the Hg^+ ions. The Be^+ cooling laser applied a torque to the Be^+ ions, and this torque was transmitted by the Coulomb interaction to the Hg^+ ions. If the Be^+ cooling laser was blocked or tuned off resonance, the Hg^+ ions left the trap in several minutes. With sympathetic cooling, the Hg^+ ions were confined indefinitely.

In another Penning trap experiment, laser-cooled Mg^+ ions were used to sympathetically cool and radially confine Be^+ ions.[103],[104] The sympathetic cooling was used to improve the performance of the $^9Be^+$ frequency standard discussed in Section V. C. 2. In order to avoid ac Stark shifts, the laser used to cool and detect the Be^+ ions was turned off when the hyperfine transition was driven. With this laser off, the Be^+ plasma expanded radially, due to torques originating from trap asymmetries. The radial expansion releases electrostatic potential energy, leading to heating of the ions.[96] The expansion and subsequent heating were eliminated by using approximately 100 000 Mg^+ ions to sympathetically cool about 5000 Be^+ ions to temperatures less than 0.25 K. The ac Stark shift on the Be^+ hyperfine transition, due to the Mg^+ cooling laser, was estimated to be less than 1 part in 10^{15}. This could be neglected relative to other systematic shifts. The Mg^+ ions were observed, with an imaging photomultiplier tube, to form a doughnut around the Be^+ ions. The size of the gap between the two species could not be determined, because the optical system could not be focused at both wavelengths at the same time. In this way a cold, steady-state cloud of Be^+ ions was obtained, independent of the Be^+ cooling laser. The sympathetic cooling enabled long measurement periods (up to 550 s), free from the perturbing effects of the Be^+ cooling laser. The observed resonance linewidth is inversely proportional to the measurement time. For a measurement time of 550 s, 900 μHz linewidth was observed on the 303 MHz hyperfine transition.[103] A typical resonance curve is shown in Fig. 9.

The frequency standard based on this hyperfine transition was used to test the linearity of quantum mechanics.[104] A nonlinearity would have resulted in a dependence of the resonance frequency on the degree of

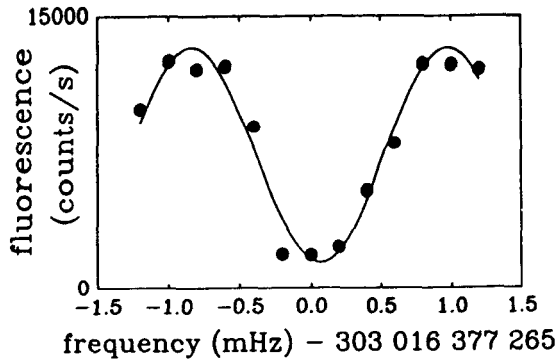

Figure 9: Resonance signal of the 303 MHz hyperfine transition of $^9Be^+$ at a magnetic field of 0.8194 T. A decrease in the fluorescence intensity corresponds to an increase in the transition probability. The width of the resonance is 900 μHz. (From Ref. [103].)

excitation of the transition, analogous to the frequency shift of an anharmonic oscillator with amplitude. An upper limit of 6 μHz was placed on a nonlinear correction to the Hamiltonian of the Be^+ nucleus.

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